OBAFEMI AWOLOWO UNIVERSITY, ILE-IFE
DEPARTMENT OF MATHEMATICS
B.Sc. (Mathematics) Degree Mid-Semester Examination
Rain Semester, 2022/2023 Session
MTH 202 - Mathematical Methods II

Time Allowed - 1hr

Type 2

Instructions: Write your **Name** and **Reg. Number** in the spaces provided on the OMR sheet. Fill all other **Required Fields (Course Code, Session, and Combination Code)** on your OMR Sheet. Attempt all questions and shade the correct option for each question. Use **HB pencil only. All notations have their usual meanings as contained in the course materials.**

1. Simplify

$$i^{2000} + i^{1999} + i^{201} + i^{82} + i^{47}.$$

(A) i
(B) -1
(C) $-i$
(D) 1.

C

2. Which of the following is false about scale factors of orthogonal curvilinear systems of coordinates? They are
(A) scalar fields
(B) vector fields
(C) magnitudes of base vectors
(D) at times numbers.

B

3. The component of $\hat{e}_{u_2}$ in the curl $\nabla \times \vec{A}$ of a vector field $\vec{A} = \vec{A}_1\hat{e}_{u_1} + \vec{A}_2\hat{e}_{u_2} + \vec{A}_3\hat{e}_{u_3}$ in a three-dimensional orthogonal curvilinear system of coordinates (u_1, u_2, u_3) with scale factors $h_{u_1}, h_{u_2}, h_{u_3}$ is

(A) $\frac{1}{h_{u_1}h_{u_2}}\left(\frac{h_{u_2}\vec{A}_2}{\partial u_1} - \frac{h_{u_1}\vec{A}_1}{\partial u_2}\right)$

(B) $\frac{1}{h_{u_2}h_{u_3}}\left(\frac{h_{u_3}\vec{A}_3}{\partial u_2} - \frac{h_{u_2}\vec{A}_2}{\partial u_3}\right)$

(C) $\frac{1}{h_{u_2}h_{u_3}}\left(\frac{h_{u_3}\vec{A}_1}{\partial u_1} - \frac{h_{u_1}\vec{A}_2}{\partial u_2}\right)$

(D) $\frac{1}{h_{u_1}h_{u_3}}\left(\frac{h_{u_1}\vec{A}_1}{\partial u_3} - \frac{h_{u_3}\vec{A}_3}{\partial u_1}\right)$.

D

4. Suppose the non-zero vector functions $\vec{F}$ and $\vec{G}$ satisfy the equation

$$\nabla \cdot (\vec{F} \times \vec{G}) + \vec{F} \cdot (\nabla \times \vec{G}) = 0,$$

then $\vec{F}$ is
(A) solenoidal
(B) irrotational
(C) not solenoidal
(D) rotational.

B

5. If the dimension of a linear vector space V is n; $n \in \mathbb{N}$. Which of the following is not necessarily true?
(A) The basis of V has n elements
(B) Any $n+1$-elements of V are linearly dependent
(C) Any subspace of V has dimension n
(D) If U and W are subspaces of V, then $\dim(U+W) = \dim U + \dim W - \dim(U \cap W)$.

C

6. Which of the following set of vectors in $\mathbb{R}^2$ can not form a basis for $\mathbb{R}^2$?
(A) $\{(1,0),(0,1)\}$
(B) $\{(1,3),(2,5)\}$
(C) $\{(0,-2),(-2,0)\}$
(D) $\{(10,6),(5,3)\}$.

D

7. Which of the following vectors is linearly independent on $(-4,14,2) \in \mathbb{R}^3$?
(A) $(-2,7,1)$
(B) $(4,-14,-2)$
(C) $(2,-7,-1)$
(D) $(-1,7,3)$.

D

8. Given that

$$\vec{F} = \nabla\phi, \quad \phi = e^r,$$

where $r = |\vec{r}|$ and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$. Find

$$\nabla \cdot (\nabla \times \vec{F}).$$

(A) $e^r\vec{r}$
(B) $r^{-2}e^r\vec{r}$
(C) $\vec{0}$
(D) 0.

C

$< \vec{?}\cdot(\nabla F|$

9. Let $\vec{F} = (3x^2y - z)\vec{i} + (xz^3 + y^4)\vec{j}$ and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, find

$$(\vec{r}\cdot\nabla)\vec{F}.$$

(A) $(9x^2y - z)\vec{i} + 4(xz^3 - y^4)\vec{j}$
(B) $(9x^2y - z)\vec{i} + 4(xz^3 + y^4)\vec{j}$
(C) $(9x^2y + z)\vec{i} + 4(xz^3 - y^4)\vec{j}$
(D) $(9x^2y + z)\vec{i} + 4(xz^3 + y^4)\vec{j}$.

B

10. Let $\vec{A} = \vec{A}_1\hat{e}_{u_1} + \vec{A}_2\hat{e}_{u_2} + \vec{A}_3\hat{e}_{u_3}$ be a vector field in a three-dimensional orthogonal curvilinear system of coordinates (u_1, u_2, u_3). Which of the following is true?
(A) $\nabla^2\vec{A} = (\nabla^2\vec{A}_1)\hat{e}_{u_1} + (\nabla^2\vec{A}_2)\hat{e}_{u_2} + (\nabla^2\vec{A}_3)\hat{e}_{u_3}$
(B) $\nabla^2\vec{A}$ is a scalar field
(C) $\nabla^2\vec{A} = \nabla(\nabla\cdot\vec{A}) + \nabla\times(\nabla\times\vec{A})$
(D) $\nabla^2\hat{e}_{u_j} \neq \vec{0}$, for all $j = 1, 2, 3$.

C

11. Suppose a rotational vector field $\vec{A}$ is such that

$$\nabla(|\vec{A}|^2) = 2(\vec{A}\cdot\nabla)\vec{A},$$

then $\nabla\times\vec{A}$ is
(A) parallel to $\vec{A}$
(B) perpendicular to $\vec{A}$
(C) not parallel to $\vec{A}$
(D) not perpendicular to $\vec{A}$.

B

12. Solve the equation

$$|z| + z = 3 + 4i,$$

where $z = x + iy$ and $x, y \in \mathbb{Z}$.
(A) $z = -\frac{7}{6} + 4i$
(B) $z = \frac{7}{6} + 4i$
(C) $z = 4 + \frac{7}{6}i$
(D) $z = 4 - \frac{7}{6}i$.

A

13. Find the fourth roots of complex number

$$z = -2i.$$

(A) $\sqrt[4]{2}e^{\frac{(3+4k)\pi i}{8}}$, $k \in \{0, 1, 2\}$
(B) $\sqrt[4]{2}e^{\frac{(3+4k)\pi i}{8}}$, $k \in \{0, 1, 2, 3\}$
(C) $\sqrt[4]{2}e^{\frac{(3+4k)\pi i}{2}}$, $k \in \{0, 1, 2, 3\}$
(D) $\sqrt[4]{2}e^{\frac{(3+4k)\pi i}{4}}$, $k \in \{0, 1, 2, 3\}$.

D

14. Let $\mathbb{Q}$ be set of rational numbers, which of the following vectors does not belong to the linear vector space $\mathbb{Q}^3$ over $\mathbb{R}$.
(A) $(0, 0, 0)$ →
(B) $(0, 1, 0)$
(C) $(\sqrt{2}, \sqrt{3}, \sqrt{5})$
(D) $(-1, 0, -2)$.

C

15. Let the scalar field ϕ be defined by

$$\phi = \nabla^2 \log r, \quad r = |\vec{r}| \quad and \quad \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}.$$

Find $\nabla\times(\nabla\phi)$.
(A) 0
(B) $r^{-3}\vec{r}$
(C) $\vec{0}$
(D) $r^{-2}\vec{r}$.

A

16. Let $\{u, v\}$ be a basis for $\mathbb{R}^2$ over $\mathbb{R}$. Which of the following is true?
(A) $\alpha u + \beta v = 0$, for any $\alpha, \beta \in \mathbb{R}$
(B) For any $w \in \mathbb{R}^2$, there exists nonzero scalars α, β such that $w = \alpha u + \beta v$.
(C) $u = \alpha v$ for some scalars β
(D) $\alpha u + \beta v = 0$ for nonzero scalars α, β.

B

17. Suppose the set $\mathbb{R}^2 = \mathbb{R}\times\mathbb{R} = \{(x, y)|x, y \in \mathbb{R}\}$, find the real numbers x and y for which

$$\frac{x-3}{3+i} + \frac{y-3}{3-i} = i.$$

(A) $(-2, 8)$
(B) $(8, -2)$
(C) $(2, 8)$
(D) $(-8, 2)$.

A

18. Suppose z is an arbitrary complex number, express

$$z = (1-i)^{10}(\sqrt{3}+i)^5$$

in Euler form $re^{i\theta}$.
(A) $2^5 e^{\frac{55\pi}{3}i}$
(B) $2^{10} e^{\frac{5\pi}{3}i}$
(C) $2^5 e^{\frac{55\pi}{33}i}$
(D) $2^{10} e^{\frac{55\pi}{3}i}$.

D

19. Which of the following is false about base vectors of orthogonal curvilinear systems of coordinates? They are
(A) mutually orthogonal
(B) at times scalar fields
(C) constant vectors
(D) normal vectors to coordinate surfaces.

C

20. Which of the following is true of the line element ds of the spherical polar system of coordinates (ρ, θ, ϕ)?
(A) $(ds)^2 = (d\rho)^2 + \rho^2(d\theta)^2 + \rho^2(d\phi)^2$
(B) $(ds)^2 = \rho^2(d\rho)^2 + (d\theta)^2 + \rho^2\sin^2\theta(d\phi)^2$
(C) $(ds)^2 = (d\rho)^2 + \rho^2(d\theta)^2 + \rho^2\sin^2\theta(d\phi)^2$
(D) $(ds)^2 = \rho^2(d\rho)^2 + \rho^2\sin^2\theta(d\theta)^2 + (d\phi)^2$.

C

MTH202 TEST SOLUTIONS

⇒ Boardmaths

TYPE 2

1) Simplify $i^{2000} + i^{1999} + i^{201} + i^{82} + i^{47}$

$\Rightarrow (i^2)^{1000} + (i^2)^{999} i + (i^2)^{100} i + (i^2)^{41} + (i^2)^{23} i$

But $[i^2 = -1]$

$\Rightarrow (-1)^{1000} + (-1)^{999} i + (-1)^{100} i + (-1)^{41} + (-1)^{23} i$

$\Rightarrow 1 - i + i - 1 - i$

$= -i$ (C)

2) Scale factors in orthogonal curvilinear coordinates are scale quantities, not ~~scalar~~ vector fields (B)

3) For given vector field $\vec{A} = \vec{A}_1 \hat{e}_{u_1} + \vec{A}_2 \hat{e}_{u_2} + \vec{A}_3 \hat{e}_{u_3}$

The curl of a vector field A in orthogonal curvilinear coordinates is given by

$$\nabla \times A = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{e}_{u_1} & h_2 \hat{e}_{u_2} & h_3 \hat{e}_{u_3} \\ \partial/\partial u_1 & \partial/\partial u_2 & \partial/\partial u_3 \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix}$$

The component of $\nabla \times A$ along $\hat{e}_{u_2}$ is

$(\nabla \times A)_{u_2} =$

$$\cancel{(-)\frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(h_3 A_3) - \frac{\partial}{\partial u_3}(h_1 A_1)\right]}$$

by ~~Multiplying through~~ by the (-)

$$\Rightarrow \cancel{\frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_3}(h_1 A_1) - \frac{\partial}{\partial u_1}(h_3 A_3)\right]}$$

~~A... Focus on just column~~.

Focus on just $\hat{e}_{u_2}$ using determinant

$$\Rightarrow (-)\frac{1}{h_1 h_2 h_3} \times h_2 \hat{e}_{u_2} \begin{vmatrix} \partial/\partial u_1 & \partial/\partial u_3 \\ h_1 A_1 & h_3 A_3 \end{vmatrix}$$

$$\Rightarrow (-)\frac{\hat{e}_{u_2}}{h_1 h_3}\left(\frac{\partial}{\partial u_1}(h_3 A_3) - \frac{\partial}{\partial u_3}(h_1 A_1)\right)$$

$$\Rightarrow -\frac{\hat{e}_{u_2}}{h_1 h_3}\left(\frac{\partial}{\partial u_1}(h_3 A_3) - \frac{\partial}{\partial u_3}(h_1 A_1)\right)$$

$$\Rightarrow \frac{\vec{e}_{u_2}}{h_1 h_3}\left(\frac{\partial}{\partial u_3}(h_1 A_1) - \frac{\partial}{\partial u_1}(h_3 A_3)\right)$$

The component is

$$\cancel{\frac{1}{h_{u_1} h_{u_3}}\left(\frac{\partial u_1 A}{\partial u_3}\right)}$$

$$\frac{1}{h_{u_1} h_{u_3}}\left(\frac{h_{u_1} \vec{A}_1}{\partial u_3} - \frac{h_{u_3} \vec{A}_3}{\partial u_1}\right)$$ (D)

4) $\nabla\cdot(\vec{F}\times\vec{G})+\vec{F}\cdot(\nabla\times\vec{G})=0$

From Vector Calculus Identities

$\nabla\cdot(\vec{F}\times\vec{G}) = G\cdot(\nabla\times\vec{F}) - F\cdot(\nabla\times\vec{G})$

However

$\nabla\cdot(\vec{F}\times\vec{G})+\vec{F}\cdot(\nabla\times\vec{G})=0$

$\Rightarrow$

$G\cdot(\nabla\times\vec{F})-\vec{F}\cdot(\nabla\times\vec{G})+\vec{F}\cdot(\nabla\times\vec{G})$
$=0$

$\Rightarrow G\cdot(\nabla\times\vec{F})=0$

Since the dot product of $\vec{G}$ with $\nabla\times\vec{F}$ is zero, then G is orthogonal to $\nabla\times\vec{F}$. For this to hold, for any non-zero G, the only possibility is that $\nabla\times\vec{F}=0$

This means that F is irrotational (i.e, F has no curl) (B)

5) Let's analyze each statement

A) The basis of V has n elements

This is true, By definition, the basis of a vector space of dimension n has exactly n linearly Independent vectors

B) Any n+1 element of V are linearly dependent.

This is true. In a vector space of dimension n, any set of more than n vectors must be linearly dependent because there cannot be more than n linearly Independent vectors

c) Any subspace of V has dimension n. This is false - A subspace of V can have any dimension from 0 to n.

D) ~~If~~ U and W are subspaces of V, then

$\dim(U+W)=\dim U+\dim W-\dim(U\cap W)$

This is true. (C)

6) To determine which set of vectors cannot form a basis of $\mathbb{R}^2$, we need to check whether the vectors in each set are linearly Independent and span $\mathbb{R}^2$

For a pair of vectors (a,b) and (c,d) to be linearly Independent, ~~the deter~~ the determinant of the matrix formed by ~~these~~ vectors as column must be non-zero

$\det\begin{pmatrix} a & c \\ b & d\end{pmatrix} = ad-bc \neq 0$

A) $\{(1,0),(0,1)\} \rightarrow \begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix} \Rightarrow 1\neq 0$

B) $\{(1,3),(2,5)\} \Rightarrow \begin{vmatrix}1 & 2\\ 3 & 5\end{vmatrix} \Rightarrow 5-6=-1\neq 0$

c) $\{(0,-2),(-2,0)\} \Rightarrow \begin{vmatrix}0 & -2\\ -2 & 0\end{vmatrix} \Rightarrow -4\neq 0$

D) $\{(10,6),(5,3)\} \Rightarrow \begin{vmatrix}10 & 5\\ 6 & 3\end{vmatrix} \Rightarrow 0$

(D)

7) To determine which of the given vector is linearly independent in the subspace $(-4, 14, 2) \in \mathbb{R}^3$, we need to check if the vector can be expressed as a scalar multiple of the given vector

A) $(-2, 7, 1)$

$\Rightarrow (-4, 14, 2) = K(-2, 7, 1)$

$K = 1/2$

B) $(4, -14, 2)$

$\Rightarrow (-4, 14, 2) = K(4, -14, 2)$

$K = -1$

c) $(2, -7, -1)$

$\Rightarrow (-4, 14, 2) = K(2, -7, -1)$

$K = -1/2$

D) $(-1, 7, 3)$

$\Rightarrow (-4, 14, 2) = K(-1, 7, 3)$

cannot be expressed as a scalar multiple

8) $\vec{F} = \nabla\phi$, $\phi = e^r$

$r = |\vec{r}|$, $\vec{r} = \vec{i}x + \vec{j}y + \vec{k}z$.

To find $\nabla\cdot(\nabla\times\vec{F})$

Since $\vec{F} = \nabla e^r$

$\Rightarrow \nabla\cdot(\nabla\times\vec{F}) = \nabla\cdot(\nabla\times\nabla e^r)$

The divergence of a curl is a zero vector $\vec{0}$ (C)

9) $\vec{F} = (3x^2y - z)\vec{i} + (xz^3 + y^4)\vec{j}$ and $\vec{r} = \vec{i}x + \vec{j}y + \vec{k}z$

$(\vec{r}\cdot\nabla)\vec{F} =$

$$\left[(x\vec{i} + y\vec{j} + k\vec{z})\cdot\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\right]\cdot\vec{F}$$

$$= \left[x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}\right]\cdot\vec{F}$$

$$\Rightarrow x\frac{\partial\vec{F}}{\partial x} + y\frac{\partial F}{\partial y} + z\frac{\partial F}{\partial z}$$

$$x\left[6xy\vec{i} + z^3\vec{j}\right] + y\left[3x^2\vec{i} + 4y^3\vec{j}\right] + \vec{z}\left[-\vec{i} + 3xz^2\vec{j}\right]$$

$$\Rightarrow 6x^2y\vec{i} + xz^3\vec{j} + 3x^2y\vec{i} + 4y^4\vec{j} - z\vec{i} + 3xz^3\vec{j}$$

$$\Rightarrow (9x^2y - z)\vec{i} + (4y^4 + 4xz^3)$$

$$\Rightarrow (9x^2y - z)\vec{i} + 4(xz^3 + y^4)\vec{j}$$

B

10) $\nabla^2 \vec{A} = \nabla(\nabla \cdot \vec{A}) + \nabla \times (\nabla \times \vec{A})$

(C)

11) $\nabla(|\vec{A}|^2) = 2(\vec{A} \cdot \nabla)\vec{A}$ (B)

Bc. $\nabla \times \vec{A}$ is perpendicular to $\vec{A}$

12) $|z| + z = 3 + 4i$

$z = x + iy$

$|x+iy| + (x+iy) = 3+4i$

$|x+iy| = \sqrt{x^2+y^2}$

$\Rightarrow \sqrt{x^2+y^2} + (x+iy) = 3+4i$

Real part :

$\sqrt{x^2+y^2} + x = 3$

Imaginary part:

$y = 4$

$\sqrt{x^2+4^2} + x = 3$

$\sqrt{x^2+16} = 3 - x$

Square both sides

$\left(\sqrt{x^2+16}\right)^2 = (3-x)^2$

$x^2+16 = 9 - 6x + x^2$

$x^2+16-9+6x-x^2 = 0$

$7 + 6x = 0$

$x = -7/6$

but $z = x + iy$

$z = -\frac{7}{6} + 4i$ (A)

13) Given $z = -2i$

$k \in \{0,1,2,3\}$

$z = \sqrt[n]{r}\left[\cos\left(\frac{\theta+2k\pi}{n}\right) + i\sin\left(\frac{\theta+2k\pi}{n}\right)\right]$

$z = -2i$

$r = \sqrt{4} = 2$, then $n = 4$

z is purley Imaginary and Negative

$\Rightarrow \sqrt[4]{2}\, e^{\frac{(3+4k)}{4}\pi i}$, $k \in \{0,1,2,3\}$

(D)

14) A) $(0,0,0)$ all Components are rational number, so it belongs to $\mathbb{Q}^3$

B) $(0,1,0) \Rightarrow$ all components are rational numbers (0 and 1 are rational), so it belongs to $\mathbb{Q}^3$

C) $(\sqrt{2}, \sqrt{3}, \sqrt{5}) \Rightarrow$ all Components are Irrational, so this does not belong to $\mathbb{Q}^3$

D) $(-1, 0, 2)$ all components are rational, it belongs to $\mathbb{Q}^3$

(C)

15) $\phi = \nabla^2 \log r, \ r = |\vec{r}|$

$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$\nabla \times (\nabla \phi) = 0$

The curl of a grad is a zero scalar

16) For any $w \in \mathbb{R}^2$, there exist non-zero scalars a, b such ~~that~~

$W = \alpha u + \beta v$

17) $\dfrac{x-3}{3+i} + \dfrac{y-3}{3-i} = i$

$\dfrac{(3-i)(x-3) + (3+i)(y-3)}{9-3i+3i+1} = i$

$\Rightarrow 3x - 9 - ix + 3i + 3y - 9 + yi - 3i = 10i$

The real part $\Rightarrow$

$3x - 9 + 3y - 9 = 0$

$3x + 3y = 18$

$x + y = 6$ — (1)

Imaginary part

$-x + 3 + y - 3 = 10$

$y - x = 10$ — (11)

Solve simultaneously, $y = 8$

$x = -2$

$(-2, 8)$ (A)

2.

18) $z = (1-i)^{10}(\sqrt{5}+i)^5$

In the form $z = re^{i\theta}$

For $(1-i)^{10}$ $r = \sqrt{2}$, $\theta = 315^\circ$

Using De'Moivre's theorem

$z' = (\sqrt{2})^{10}\left[\cos 3150 + \sin 3150 i\right]$

$= 2^5 [c$

In the form $re^{i\theta}$

$\Rightarrow 2^5 e^{3150i}$ $\quad 2^5 e^{3150i}$

For $(\sqrt{3}+i)^5$ r

$r = \sqrt{(\sqrt{3})^2 + 1} = 2$

$\theta = 30^\circ$.

$z = 2^5\left[\cos 150^\circ + i\sin 150^\circ\right]$

$= 2^5 e^{150i}$

Then $z = (1-i)^{10}(\sqrt{5}+i)^5$

$\Rightarrow 2^5 e^{3150i} \times 2^5 e^{150i}$

$\Rightarrow 2^{10} e^{3300i}$ $\quad \Rightarrow 2^{10} e^{\frac{55\pi}{3} i}$

Since $\pi = 180^\circ$

(D)

2

19) Base vectors are not constant vectors (C)

20) For spherical polar coordinates

$h_\rho = 1, \ h_\theta = \rho, \ h_\phi = \rho \sin\phi$

$(ds)^2 = (d\rho)^2 + \rho^2 (d\theta)^2 + \rho^2 \sin^2\theta (d\phi)^2$

(C)