OBAFEMI AWOLOWO UNIVERSITY, ILE-IFE, NIGERIA
DEPARTMENT OF MATHEMATICS
Rain Mid-Semester Examination, 2021/2022 Academic Session
MTH 202 - Mathematical Methods II

Date: 20th May, 2023. **Type 3** **Time Allowed - One Hour**

Instructions: Attempt all questions. Use HB pencil ONLY. Write and shade your **question type, Names, Registration Number** and **the correct option** in your OMR sheet.

1. Find the matrix associated with the following linear map $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ given by $T(x,y) = (2x - y, x+3y, -x)$; basis $\{(1,0),(0,1)\}$ for $\mathbb{R}^2$ and basis $\{(1,0,0),(0,1,0),(0,0,1)\}$ for $\mathbb{R}^3$.

   (A) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 2 & -1 \\ 1 & 3 \\ -1 & 0 \end{pmatrix}$

   (B) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 3 \end{pmatrix}$

2. Find the divergence of the vector $\bar{A} = r^2\hat{e}_1 + r\sin\theta\hat{e}_2$ in a cylindrical coordinate (r,θ,z) system defined by $x = r\cos\theta, y = r\sin\theta$, $z = z$ where $r \geq 0, 0 \leq \theta < 2\pi, -\infty < z < \infty$.

   (A) $3r + \cos\theta$
   (B) $3r - r\cos\theta$
   (C) $r^2 + r\sin\theta$
   (D) $3r^2 + r\cos\theta$

3. Given the scalar function $\Phi = x^2e^{y^3}\ln(z)$. Evaluate $\nabla \times \nabla\Phi$.

   (A) $2e^{y^3}\ln(z)\underline{i} - 3y^2x^2\ln(z)\underline{j} - \frac{x^2e^{y^3}}{z^2}\underline{k}$
   (B) 0
   (C) $2xe^{y^3}\underline{i} - 3e^{y}\ln(z)\underline{j} + \frac{x^2e^{y^3}}{z}\underline{k}$
   (D) $\vec{0}$

4. Obtain the scale factors for a parabolic cylindrical coordinate (u,v,z) system defined by the transformation equations

$$x = \frac{1}{2}(u^2 - v^2),\ y = uv,\ z = z$$

   where $-\infty < u < \infty, v \geq 0, -\infty < z < \infty$.

   (A) $h_u = u^2 + v^2,\ h_v = u^2 + v^2,\ h_z = 1$
   (B) $h_u = \sqrt{u^2+v^2},\ h_v = \sqrt{u^2-v^2},\ h_z = 1$
   (C) $h_u = \sqrt{u^2+v^2},\ h_v = \sqrt{u^2+v^2},\ h_z = 1$
   (D) $h_u = u^2 + v^2,\ h_v = u^2 - v^2,\ h_z = 1$.

5. Which of the following cannot be a basis for the vector space $\mathbb{R}^2$ over $\mathbb{R}$.

   (A) $\{(1,0),(0,1)\}$
   (B) $\{(1,3),(2,5)\}$
   (C) $\{(10,6),(5,3)\}$
   (D) $\{(5,0),(0,5)\}$

6. Find the Jacobian of the transformation equations $x = \frac{1}{2}(u^2 - v^2), y = uv, z = z$ where $-\infty < u < \infty, v \geq 0, -\infty < z < \infty$.

   (A) uv (C) $u^2 + v^2$
   (B) $2uv$. (D) $u^2 - v^2$

7. Which of the following is not true for $z \in \mathbb{C}$?

   (A) $|z| = |iz|$ (C) $2z > iz$
   (B) $\left|\frac{1}{z}\right| = \frac{1}{|\bar{z}|}$ (D) $z = |z|e^{i\mathrm{Arg}(z)}$

8. One of the values of $\arccos(-2)$ is

   (A) 2π
   (B) $3\pi + i\log_e(2+\sqrt{3})$
   (C) $\pi - i\log_e(2-\sqrt{3})$
   (D) $\pi - i\log_e(2+\sqrt{3})$

9. Calculate the kernel of the linear map $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ given by $T(x,y) = (x+y, x-y)$.

   (A) $\{(1,1)\}$
   (B) $\{(1,0)\}$
   (C) $\{(0,0)\}$
   (D) $\{(0,1)\}$

10. Find the curl of the vector $\bar{V} = r\cos\theta\hat{e_r} - r\sin\theta\hat{e_\theta}$ in a cylindrical coordinate (r,θ,z) system defined by $x = r\cos\theta, y = r\sin\theta, z = z$ where $r \geq 0, 0 \leq \theta < 2\pi, -\infty < z < \infty$.

    (A) $-\sin\theta\hat{e_\theta}$
    (B) $-r\sin\theta\hat{e}_z$
    (C) $-r\sin\theta\hat{e}_r$
    (D) $-\sin\theta\hat{e}_z$

11. Under what conditions on $a, b.c \in \mathbb{R}$ is the the vector (a,b,c) in the vector space $\mathbb{R}^3$ over $\mathbb{R}$ a linear combination of the vectors $(1,2,3)$ and $(2,3,1)$?

    (A) $5a + 2b - 3c = 0$
    (B) $a + 5b - c = 0$
    (C) $7a - 5b + c = 0$
    (D) $2a + 5b - c = 0$

12. Let the solution set of $z^3 = 1$ be $\{1, \alpha, \beta\}$, where $z \in \mathbb{C}$. Then

    (A) $\alpha = -\beta$
    (B) $\alpha = \beta^3,\ \beta = \alpha^3$
    (C) $\alpha = \beta^2,\ \beta = \alpha^2$
    (D) $\alpha = \beta$

$z =$

$(z-3)^3 = 0$

13. Which of the following is the solution set for the equation $z^3 - 27 = 0$?

(A) $\left\{\frac{3}{2}(-1+i\sqrt{3}), \frac{3}{2}(-1-i\sqrt{3}), 3\right\}$

(B) $\left\{\frac{3}{\sqrt{2}}(1-i), \frac{3}{\sqrt{2}}(1+i), 3\right\}$

(C) $\left\{\frac{3}{2}(1-i\sqrt{3}), \frac{3}{2}(1+i\sqrt{3}), 3\right\}$

(D) $\{+3, -3\}$

14. Find the value of β for which the vector $\vec{A} = (yz^2 - x)\underline{i} + (z^2x^3 + \beta y)\underline{j} + (ye^{3x} - 4z)\underline{k}$ is solenoidal.

(A) 2

(B) 1

(C) 4

(D) 5

$\nabla \cdot \vec{A} = -1 + \beta - 4 = 0$

$= -5 + \beta = 0$

$\beta = 5$

15. Which of the following vectors does not belong to the vector space $\mathbb{R}^3$ over $\mathbb{R}$.

(A) $(0,0,0)$

(B) None of the above.

(C) $(\sqrt{2}, \sqrt{3}, \sqrt{5})$

(D) $(-1,0,2)$

16. Calculate $\nabla^2\Omega$ given that $\Omega = e^{4x-3y+2z}$.

(A) $20e^{4x-3y+2z}$

(B) $3e^{4x-3y+2z}$

(C) $29e^{4x-3y+2z}$

(D) $13e^{4x-3y+2z}$

17. Determine the gradient of the scalar $\Phi = r\cos\theta + \cos 2\phi$ in a spherical coordinate (r, θ, ϕ) system defined by $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ where $r \geq 0$, $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$ and the scale factors are $h_r = 1$, $h_\theta = r$, $h_\phi = r\sin\theta$.

(A) $\sin\theta\hat{e_r} - \sin\theta\hat{e_\theta} - \frac{2\sin 2\phi}{\sin\theta}\hat{e_\phi}$

(B) $\cos\theta\hat{e_r} + \sin\theta\hat{e_\theta} - \frac{\sin 2\phi}{r\sin\theta}\hat{e_\phi}$

(C) $\cos\theta\hat{e_r} - \sin\theta\hat{e_\theta} - \frac{2\sin\phi}{r\sin\theta}\hat{e_\phi}$

(D) $\cos\theta\hat{e_r} - \sin\theta\hat{e_\theta} - \frac{2\sin 2\phi}{r\sin\theta}\hat{e_\phi}$

18. Given the scalar function $\Psi = \frac{1}{r^m}$, where $r = |\vec{r}|$, $\vec{r} = x\underline{i} + y\underline{j} + z\underline{j}$ and $0 \neq m \in \mathbb{R}$. Evaluate $\nabla\Psi$.

(A) $-mr^{-m-1}\vec{r}$

(B) $mr^{-m-1}\vec{r}$

(C) $-mr^{-m-2}\vec{r}$

(D) $mr^{-m-2}\vec{r}$

r^{-m}

$-m r^{-m-2}\vec{r}$

19. The principal argument of the complex number $z = -1 - i$ is

(A) $\frac{3\pi}{4}$

(B) $-\frac{\pi}{4}$

(C) $-\frac{3\pi}{4}$

(D) $\frac{\pi}{4}$

20. Given a vector field $\vec{B} = (-y + \sin x)\underline{i} + (y^3 - 1)\underline{j} - 14e^z\underline{k}$, evaluate $\nabla \times \vec{B}$.

(A) $\underline{i}$

(B) $\underline{k}$

(C) $\underline{j}$

(D) $\underline{i} + \underline{j}$

$180 + 45$

$225 \times \frac{\pi}{180}$

$\frac{5\pi}{4}$