OBAFEMI AWOLOWO UNIVERSITY, ILE-IFE
DEPARTMENT OF MATHEMATICS
B.Sc. (Mathematics) Degree Mid-Semester Examination
Harmattan Semester, 2022/2023 Session
MTH 201 - Mathematical Methods I

Time Allowed - 1hr

Type 1

Instructions: Write your **Name** and **Reg. Number** in the spaces provided on the OMR sheet. Fill all other **Required Fields (Course Code, Session, and Combination Code)** on your OMR Sheet. Attempt all questions and shade the correct option for each question. Use **HB pencil** only. **All notations have their usual meanings as contained in the course materials.**

1. The derivative of the function $f(x) = x|x|$ with respect to $x \in \mathbb{R}$ is
   (A) $2|x|$
   (B) $2x$
   (C) $-2x$
   (D) Does not exist.

2. Suppose f is a real-valued function which is *continuous in* $[a, b]$ and differentiable in (a, b), *there exists* $x_o \in (a, b)$ such that
   (A) $f(x_0) = 0$
   (B) $f'(x_0) = 0$
   (C) $f(x_0) = \dfrac{f(a) - f(b)}{a - b}$
   (D) $f'(x_0) = \dfrac{f(b) - f(a)}{b - a}$.

3. The nth derivative of $(x^2 + 1)e^{2x}$ with respect to x is
   (A) $2^{n-2}e^{2x}(4x^2 + 4nx + n^2 - n + 4)$
   (B) $2^{n-2}e^{2x}(4x^2 - 4nx + n^2 - n + 4)$
   (C) $2^{n-2}e^{2x}(4x^2 + 4nx - n^2 - n + 4)$
   (D) $2^{n-2}e^{2x}(4x^2 + 4nx + n^2 + n + 4)$.

4. Which of the following is not true about sequences of real numbers?
   (A) If a sequence $\{x_n\}$ converges to a limit l, then the limit is unique
   (B) If a sequence $\{x_n\}$ converges to a limit l, then the limit of sequence $\{x_{n+p}\}$ is l for a fixed $p \in \mathbb{R}$.
   (C) If a sequence $\{x_n\}$ converges to a limit l, then the limit of sequence $\{x_n + p\}$ is l for a fixed $p \in \mathbb{R}$
   (D) If a sequence $\{x_n\}$ converges to a limit l and $x_n \geq 0$ for all $n \in \mathbb{N}$, then $l \geq 0$.

5. Which of the following statements is not true about boundedness of sequences of real numbers?
   (A) Every bounded sequence is convergent
   (B) Every bounded sequence is bounded above
   (C) Every bounded sequence is bounded below
   (D) Every Cauchy sequence is bounded below.

6. Which of the following statements is not true about the sequence $\{x_n\}$, defined as $x_1 = \sqrt{3}$ and $x_{n+1} = \sqrt{3x_n}$, $n \geq 1$?
   (A) $\{x_n\}$ converges
   (B) $\{x_n\}$ is a bounded sequence
   (C) $\{x_n\}$ is a monotone decreasing sequence
   (D) $\{x_n\}$ is bounded below.

7. If Newton's method is used to locate a root of the equation

$$f(x) = \cos\left(\frac{\pi(x+1)}{8}\right) + 0.148x - 0.9062$$

and the initial approximation is $x_0 = -0.5$, then the first approximation x_1 is
   (A) -0.508219
   (B) -0.508192
   (C) -0.508129
   (D) -0.508199

8. In finding the maximum and minimum values of function $f(x, y) = x^2y$ which is subject to the constraint $x^2 + y^2 = 1$. Which of the following relations must hold (I) $y = -\lambda$, (II) $x^2 = -2\lambda y$, (III) $2x + y = 2\lambda y$?
   (A) I only
   (B) II only
   (C) I, II, and III
   (D) I and II only.

9. Suppose that $f(0) = 1, f(0.5) = 2.5, f(1) = 2$, and $f(0.25) = f(0.75) = \beta$. Find β if the Trapezoidal rule with $n = 4$ gives the value of $\int_0^1 f(x)dx = 1.75$
(A) 2.0
(B) 2.5
(C) 1.5
(D) 5.5

10. Find the approximate value of $\int_3^7 x^2 \ln(x)dx$ using the Simpson's $\frac{1}{3}$ rule with $h = 1$, correct to six decimal places.
(A) 177.483772
(B) 177.481735
(C) 177.481724
(D) 170.481723

11. Which of the following is true about
$$\lim_{(x,y)\to(0,0)} \frac{5y^4 \cos^2 x}{x^4 + y^4}?$$
(A) limit exists
(B) $f(0, y) = -5$
(C) limit does not exist
(D) $f(x, 0) = x^4$.

12. If function $f(x, y, z) = \sqrt{\sin^2 x + \sin^2 y + \sin^2 z}$ then the value of $f_z(x, y, z)$ at the point $(0, 0, \frac{\pi}{4})$ is
(A) $\frac{\sqrt{5}}{5}$ (B) $\sqrt{2}$ (C) $\frac{\sqrt{3}}{3}$ (D) $\frac{\sqrt{2}}{2}$.

13. If $u^2 - v = 3x + y$ and $u - 2v^2 = x - 2y$, then find $\frac{\partial u}{\partial y}$.
(A) $\frac{-2(1+2v)}{1-8uv}$ (B) $\frac{-(4u+1)}{1-8uv}$ (C) $\frac{2u-3}{1-8uv}$ (D) $\frac{-2(1+2u)}{1-8uv}$,
where $1 - 8uv \neq 0$.

14. At the critical points of the curve $f(x, y) = x^3 + y^3 - 3x - 12y + 20$, which of the following equations must hold
(I) $x^2 - 1 = 0$; (II) $y^2 + 4 = 0$; (III) $y^2 - 4 = 0$?
(A) I, II, and III
(B) I and II only
(C) I and III only
(D) I only.

15. Find the open interval of convergence of the series
$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}.$$
(A) $(2, 4)$
(B) $(2, -4)$
(C) $(-2, 4)$
(D) $(-2, -4)$.

16. Which of the following is true about Integral test for convergence of function $f(x)$?
(A) $\int_0^\infty f(x)\,dx$ exists
(B) $\int_0^\infty f(x)\,dx = \infty$
(C) $\int_0^\infty f(x)\,dx = -\infty$
(D) None of the above.

17. Find the limit of $\sum_{n=1}^{\infty} e^{-n} n!$
(A) $-\infty$
(B) e^1
(C) ∞
(D) e^{-1}.

18. Evaluate
$$\lim_{x\to 0} \left(\frac{\sqrt{1-x^3} - \sqrt{1-x}}{\sqrt{1+x^2} - \sqrt{1+x}} \right).$$
(A) -1
(B) 1
(C) 2
(D) -2.

19. Let
$$f(x) = \begin{cases} A^2x^2, & x < 2 \\ (1-A)x, & x \geq 2 \end{cases}$$
For what values of A is $f(x)$ continuous at $x = 2$?
(A) $-1/2,\ 1$
(B) $1,\ -1$
(C) $1/2,\ -1$
(D) $2,\ -1$.

20. Evaluate
$$\lim_{x\to 1} \left(\frac{1-x^2}{|x-1|} \right).$$
(A) -2
(B) 1
(C) -1
(D) Does not exist.