OBAFEMI AWOLOWO UNIVERSITY, ILE-IFE
DEPARTMENT OF MATHEMATICS
B.Sc. (Mathematics) Degree Mid-Semester Examination
Harmattan Semester, 2021/2022 Session
MTH 101-Elementary Mathematics I

Time Allowed - 1 Hour

Type 1

Instructions: Use HB pencils ONLY. Write and Shade your **Names** and **Registration Number** in the spaces provided on the OMR sheet. **Shade the Question type**. Attempt all questions: Shade the option **E** if none of the options **A-D** is correct.
All notations have their usual meanings.

1. If the universal set $\mathcal{U} = \mathbb{R}$, and $A = \{x \in \mathbb{R} : x^2 + 1 = 0\}$. Which of the following statement(s) is/are not true?
I. A is a finite set
II. $A \neq \{-\sqrt{-1}, \sqrt{-1}\}$
III. $A = \{\ \}$
IV. $A \subseteq \mathbb{R}$
V. $A^c \neq \mathbb{R}$.

(A) I. and III. (B) II. (C) IV. (D) V.

Solution

Answer: D
$x^2 + 1 = 0$ has no real solution. Thus, A is an empty set. But $\mathbb{R} = \mathcal{U}$, so that $A^c = \mathbb{R}$. Hence, $A^c \neq \mathbb{R}$ is not true.

2. The following define the symmetric difference $A\Delta B$ of sets A and B except
(A) $(A - B) \cup (B - A)$
(B) $[(A \cap B^c) \cup B] \cap [(A \cap B^c) \cup A^c]$
(C) $(A \cap B) - (A \cup B)$
(D) $(A \cup B) \cap (A^c \cup B^c)$

Solution

Answer: C

$A\Delta B \neq (A \cap B) - (A \cup B)$

3. The cardinality of an arbitrary finite set X is denoted by $n(X)$. Which of the following from I. to III. is/are true?
I. $n(A) = n(A - B) + n(A \cap B)$
II. $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$
III. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

(A) I. only (B) III. only
(C) I. and III. only (D) I., II. and III.

Solution

Answer: D

4. Let $A = \{1, 4, 9\}$ and $B = \{-3, -2, -1\}$. If the propositional function $p(x, y)$ on $A \times B$ means $x = y^2$. Which of the following from I. to IV. about the relation $f = (A, B, p(x, y))$ is/are true?
I. f is a function from A to B
II. f is not a function from A to B
III. Solution set of f is
$\{(-3, 9), (-2, 4), (-1, 1)\}$.

(A) I. only (B) II. only
(C) I. and III. only (D) II. and III. only

Solution

Answer: A
f is a function from A to B.

5. If A and B are respectively the largest possible domains of the real-valued functions f and g of real variable x defined by

$$f(x) = \frac{x-3}{x^2-9} \quad \text{and} \quad g(x) = \frac{1}{x+3}.$$

Which of the the following reasons from I. to III. justify why f is not the same as g?
I. $A \neq B$ II. $A \subset B$ III. $B \subset A$

(A) I. only (B) I. and II. only
(C) I. and III. only (D) II. only.

Solution

Answer: B
Domain of f is $A = \{x \in \mathbb{R} : x \neq -3 \text{ and } 3\}$
Domain of g is $B = \{x \in \mathbb{R} : x \neq -3\}$
Clearly, $A \neq B$ and $A \subset B$.

6. Evaluate N, given that $\displaystyle\sum_{k=0}^{N}(5 \times 2^k) = 315$.
(A) 8 (B) 7 (C) 6 (D) 5

Solution

Answer: D
Clearly, this is a geometric series with first term $a = 5$, common ratio $r = 2$. The sum is given by

$$S = a\frac{(r^{N+1} - 1)}{r - 1} = 315$$

$$\implies 315 = 5(2^{N+1} - 1) \implies N = 5.$$

7. If $\displaystyle\sum_{k=1}^{N} U_k = \frac{3N^2 + N}{2}$. Evaluate U_{15}.
(A) 345 (B) 144 (C) 45 (D) 44

Solution

Answer: D
Clearly,
$S_{15} = \dfrac{3 \times (15)^2 + 15}{5} = 345$
and $S_{14} = \dfrac{3 \times (14)^2 + 14}{2} = 301$.
$U_{15} = S_{15} - S_{14} = 345 - 301 = 44$.

8. Evaluate N, if $\displaystyle\sum_{n=3}^{N}(18 - 5n) = -116$.
(A) 10 (B) 9 (C) 8 (D) 7

Solution

Answer: A
Clearly, series is an arithmetic series with $(N - 2)$ terms. The sum is given by

$$S_{N-2} = -116 = \frac{(N-2)}{2}(2\times3+(N-3)\times(-5))$$

$$\implies -232 = (N - 2)(21 - 5N) \implies N = 10.$$

9. Suppose α and β are the roots of the equation $x^2 - 2sx + 3s^2 + r^2 = 0$, given that s and r are real numbers. Then
(A) $\alpha, \beta \in \mathbb{R}$ and $\alpha \neq \beta$
(B) $\alpha = \beta$
(C) $\alpha, \beta \in \mathbb{C} - \mathbb{R}$
(D) α and β are undefinable

Solution

Answer: C
The discriminant $D \equiv b^2 - 4ac$ is

$$D \equiv 4s^2 - 4(3s^2 + r^2) = -(8s^2 + 4r^2) < 0.$$

Hence, $\alpha, \beta \in \mathbb{C} - \mathbb{R}$.

10. Supose α and β are the roots of the equation $5x^2 - 3x - 1 = 0$. Which of the following is the value of $\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha}$?
(A) $-\dfrac{72}{25}$, (B) $-\dfrac{18}{5}$ (C) $-\dfrac{9}{5}$ (D) $-\dfrac{3}{5}$.

Solution

Answer: A

Recall that $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
$\dfrac{\alpha^3 + \beta^3}{\alpha\beta} = -\dfrac{72}{25}$,
where $\alpha + \beta = 3/5$ and $\alpha\beta = -1/5$.

11. x in the inequality $\dfrac{1}{x - 1} < 2$ satisfies two of the following inequalities:
I. $x - 1 < 0$
II. $(x - 1)(2x - 3) < 0$
III. $(x - 1)(2x - 3) > 0$
IV. $2x - 3 < 0$

(A) I. and II. only (B) III. and IV. only
(C) I. and III. only (D) II. and IV.

Solution

Answer: C

The inequality $\frac{1}{x-1} < 2$ is equivalent to

$$\frac{(x-1)}{(x-1)^2} < 2 \implies (x-1) < 2(x-1)^2$$

$$\implies (x-1)(2x-3) > 0.$$

We thus have

$$x < 1 \text{ and } x > \frac{3}{2}$$

$$\implies x-1 < 0 \text{ and } (x-1)(2x-3) > 0.$$

12. Evaluate the values of the constants a and b for which

$$\frac{6x^3 - 8x + 5}{ax+b} = 3x^2 + 6x + 8 + \frac{37}{ax+b}.$$

(A) $a = 2,\ b = -4$ (B) $a = 2,\ b = 2$
(C) $a = 2,\ b = 4$ (D) $a = 2,\ b = -2$

Solution

Answer: A

The equation implies

$$6x^3 - 8x + 5 =$$

$$3ax^3 + 6ax^2 + 8ax + 3bx^2 + 6bx + 8b + 37.$$

By comparison, we have

$$a = 2 \text{ and } b = -4.$$

13. A solution of the equation

$$\frac{x^3 - 2x^2 - 7x + 12}{x-3} = 0$$

is

(A) $x = 3$
(B) $x = -2$
(C) $x = \frac{-1+\sqrt{17}}{2}$
(D) $x = \frac{1-\sqrt{17}}{2}$

Solution

Answer: BONUS

Clearly, we have

$$\frac{x^3 - 2x^2 - 7x + 12}{x-3} = x^2 + x - 4 = 0$$

$$\implies x = \frac{-1 \pm \sqrt{17}}{2}.$$

14. If a and b are non-negative real numbers and $a^2 + b^2 = 23ab$, then $\log a + \log b$ is

(A) $2\log\left(\frac{a+b}{5}\right)$ (B) $2\log\left(\frac{a+b}{23}\right)$
(C) $\log\left(\frac{23}{a^2+b^2}\right)$ (D) $\log\left(\frac{a^2+b^2}{5}\right)$

Solution

Answer: A

Note that

$$(a+b)^2 = a^2 + b^2 + 2ab = 23ab + 2ab = 25ab$$

$$\implies ab = \frac{(a+b)^2}{25}$$

$$\implies \log a + \log b = 2\log\left(\frac{a+b}{5}\right).$$

15. The expression

$$\frac{x^3 - x^2 - 3x + 5}{(x-1)(x^2-1)}$$

has the form (A, B and C are constants)

(A) $1 + \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$
(B) $1 + \frac{A}{x-1} + \frac{B}{x-1} + \frac{C}{x+1}$
(C) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$
(D) $\frac{A}{x-1} + \frac{B}{x-1} + \frac{C}{x+1}$

Solution

Answer: A

16. If $a, b > 0$ and $a \neq b$, solve for x satisfying $a^x = b^{x-y}$ and $2 - y = 0$.

(A) $x = \frac{-2\log b}{\log b/a}$ (B) $x = \frac{2\log b}{\log a/b}$
(C) $x = \frac{2\log b}{\log b/a}$ (D) $x = \frac{-2\log b}{\log ab}$

Solution

Answer: C

$a^x = b^{x-2} \implies x \log a = (x-2)\log b$
$x = \dfrac{2\log b}{\log b/a}$.

17. Evaluate the imaginary part of $\sqrt{40+42i}$, where $i^2 = -1$.

(A) ± 3 (B) ± 5 (C) ± 7 (D) ± 9

Solution

Answer: A

$\sqrt{40+42i} = x + iy$, where $x, y \in \mathbb{R}$.

$$\implies (x+iy)^2 = 40 + 42i$$

$$\implies x^2 - y^2 = 40,\ xy = 21$$

$$\implies x = \pm 7,\ y = \pm 3.$$

Hence, imaginary part is $y = \pm 3$.

18. If $x, y \in \mathbb{R}$, find x satisfying the equation

$$x(3+4i) - y(1+2i) + 5 = 0.$$

(A) -2 (B) -3 (C) -4 (D) -5

Solution

Answer: D

The equation $x(3+4i) - y(1+2i) + 5 = 0$ implies solving the system of equation

$$3x - y = -5, \quad 4x - 2y = 0.$$

The value of $x = -5$.

19. If a and b are two real numbers such that $a + b = 1$. Which of the following inequalities are true?
(A) $4ac \leq 1$ and $a^2 + b^2 \geq 1/2$
(B) $a^2 + b^2 \leq 1/2$ and $4ac \geq 1$
(D) $4ac \geq 1$ and $a^2 + b^2 \geq 1/2$
(D) $4ac \leq 1$ and $a^2 + b^2 \leq 1/2$

Solution

Answer: BONUS

Recall that $a^2 + b^2 \geq 2ab$ and it is given that $a + b = 1$. Thus, we have

$$a^2 + b^2 + 2ab = 1 \implies a^2 + b^2 = 1 - 2ab \geq 2ab$$

$$\implies 4ab \leq 1.$$

Also, $a^2 + b^2 = 1 - 2ab \geq 1 - 1/2 = 1/2$.

20. Suppose "$\cdot$" is the multiplication sign on any pair of real numbers (eg. $2 \cdot 5 = 10$). Which of the following is true concerning the solution x of the equation

$$0 \cdot x = 0?$$

(A) x is undefinable
(B) Equation has infinite number of solutions
(C) Equation has only one non-zero solution
(D) The only solution is $x = 0$

Solution

Answer: B