OBAFEMI AWOLOWO UNIVERSITY, ILE-IFE
DEPARTMENT OF MATHEMATICS
B.Sc. (Mathematics) Degree Examination
Harmattan Semester, 2021/2022 Session
MTH 101-Elementary Mathematics I

Time Allowed - 2 hours: 10 minutes

Type 4

Instructions: Use HB pencils ONLY. Write and Shade your **Names** and **Registration Number** in the spaces provided on the OMR sheet. **Shade the Question type**. Attempt all questions: Shade the option E if none of the options A-D is correct.
All notations have their usual meanings.

1. Given that $x \neq 0$ and x satisfies $2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$. The values of y satisfying the equation $y = x + \frac{1}{x}$ is

   (A) $y = 1;\ y = 3/2$
   (B) $y = 2;\ y = 5/2$
   (C) $y = 3;\ y = 7/2$
   (D) $y = 4;\ y = 9/2$

2. Given that $x \neq 3$, a solution of the equation

$$\frac{x^3 - 2x^2 - 7x + 12}{x - 3} = 0$$

   is

   (A) $x = 3$
   (B) $x = -2$
   (C) $x = \dfrac{1 + \sqrt{17}}{2}$
   (D) $x = \dfrac{-1 - \sqrt{17}}{2}$

3. If $a, b > 0$ and $a \neq b$, solve for x satisfying $a^x = b^{x-2}$.

   (A) $x = \dfrac{-2\log b}{\log a/b}$ (B) $x = \dfrac{2\log a}{\log a/b}$
   (C) $x = \dfrac{2\log b}{\log a/b}$ (D) $x = \dfrac{-2\log b}{\log ab}$

4. If a and b are positive real numbers, and $a^2 + b^2 = 23ab$, then $\log a + \log b$ is

   (A) $2\log\left(\dfrac{a+b}{5}\right)$ (B) $2\log\left(\dfrac{a+b}{23}\right)$
   (C) $\log\left(\dfrac{a^2+b^2}{23}\right)$ (D) $\log\left(\dfrac{a^2+b^2}{5}\right)$

5. The expression

$$\frac{x^2 - 3x + 5}{(x-1)(x^2-1)}$$

   has the form (A, B and C are constants)

   (A) $1 + \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+1}$
   (B) $1 + \dfrac{A}{x-1} + \dfrac{B}{x-1} + \dfrac{C}{x+1}$
   (C) $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+1}$
   (D) $\dfrac{A}{x-1} + \dfrac{B}{x-1} + \dfrac{C}{x+1}$

6. Which of the following from I. to III. is/are true concerning the function $f : A \subset \mathbb{R} \longrightarrow \mathbb{R}$ defined by

$$f(x) = \frac{x-3}{x^2-9}?$$

   I. Largest possible domain is

$$A = \{x \in \mathbb{R} : x \neq 3\}$$

   II. Range is $f(A) = \{y \in \mathbb{R} : y \neq 0 \text{ and } 1/6\}$

   III. $f(x) = \dfrac{1}{6}$ at $x = 3$

   (A) I. Only (B) II. Only
   (C) I. and III. Only (D) II. and III. Only

7. Let $A = \{a, b, c, d\}$ and $B = \{1, 2, 3\}$. Which of the following from I. to III. is/are true of the function $f : A \longrightarrow B$ whose solution set is

$$\mathcal{F} = \{(a, 1), (b, 2), (c, 2), (d, 3)\}?$$

I. f is surjective
II. f is injective
III. f is not bijective

(A) I. Only (B) II. Only
(C) I. and II. Only (D) I. and III. Only

8. Determine the range $f(A)$ of function $f : A \subset \mathbb{R} \longrightarrow \mathbb{R}$ defined by

$$f(x) = \frac{x^2 - 3x + 2}{x^2 - 1},$$

where $A = \{x \in \mathbb{R} : x \neq 1 \text{ and } x \neq -1\}$.

(A) $f(A) = \mathbb{R}$
(B) $f(A) = \mathbb{R} - \{-1/2, 1\}$
(C) $f(A) = \mathbb{R} - \{-1, 1\}$
(D) $f(A) = \mathbb{R} - \{1\}$

9. Determine an expression for the inverse function g^{-1} of the function $g : A \subset \mathbb{R} \longrightarrow \mathbb{R}$ defined by

$$g(x) = 2x^2 - 4x + 5,$$

where $A = \{x \in \mathbb{R} : x \leq 1\}$.

(A) $g^{-1}(x) = 1 - \frac{1}{2}\sqrt{2x - 6}$

(B) $g^{-1}(x) = 1 + \frac{1}{2}\sqrt{2x - 6}$

(C) $g^{-1}(x) = \frac{1}{2x^2 - 4x + 5}$

(D) $g^{-1}(x) = 1 + \frac{1}{2}\sqrt{2x + 6}$

10. Given that $\mathbf{A} = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$; and if $\det(\mathbf{C})$ denotes the determinant of a square matrix $\mathbf{C}$. Which of the following statement is false for all $x \in \mathbb{R}$ and $x \neq \pm\frac{(2n-1)\pi}{2}$, where $n \in \mathbb{N}$?

(A) $\mathbf{AB} = \mathbf{BA}$
(B) $\det(\mathbf{AB}) = 1$
(C) $\det(\mathbf{A}+\mathbf{B}) = \det(\mathbf{A}) + \det(\mathbf{B})$
(D) $\mathbf{ABA} = \mathbf{A}$

11. If $x \in \mathbb{R} - \{-3, 4\}$, solve for u in terms of x in the equation

$$2x^3 + 3x^2 - 29x - 60 = (x + 3)(x - 4)u.$$

(A) $2x + 5$ (B) $2x - 5$
(C) $5x + 2$ (D) $5x - 2$

12. If $a, b \in \mathbb{R}$ such that $a \neq b$, simplify

$$\frac{(a^3 + b^3)(a^2 + ab + b^2)}{a^6 - b^6}$$

(A) $a-b$ (B) $a+b$ (C) $\frac{1}{a-b}$ (D) $\frac{1}{a+b}$

13. If a and b are two real numbers such that $a + b = 1$. Which of the following inequalities is true?
(A) $4ab \geq 1$ and $a^2 + b^2 \leq 1/2$
(B) $4ab \geq 1$ and $a^2 + b^2 \geq 1/2$
(C) $4ab \leq 1$ and $a^2 + b^2 \geq 1/2$
(D) $4ab \leq 1$ and $a^2 + b^2 \leq 1/2$

14. Given that x is a non-zero real constant. Evaluate the coefficient of x^{30} in the expansion of $\left(ax + \frac{b}{x}\right)^{100}$, where a and b are constants.

(A) ${}^{100}C_{65} \times a^{35}b^{65}$
(B) ${}^{65}C_{35} \times a^{35}b^{65}$
(C) ${}^{65}C_{35} \times a^{65}b^{35}$
(D) ${}^{100}C_{65} \times a^{65}b^{35}$

15. Find the possible values of x for which when x^3 and higher powers of x are ignored, the approximation

$$\frac{(1 + 2x)^{3/2} - 4(1 + x)^{1/2}}{1 + x^2} \approx -3 + x + 5x^2$$

is true.

(A) $-1 < x < 1$
(B) $-2 < x < 2$
(C) $0 < x < 1$
(D) $-1/2 < x < 1/2$

16. If $-1 < x < 1$ such that powers of x higher than x^4 are ignored. Obtain the expansion of $\sqrt{9 + x^2}$.

(A) $3 + \frac{x^2}{6} - \frac{x^4}{81}$

(B) $3 + \frac{x^2}{6} + \frac{x^4}{27}$

(C) $3 + \frac{x^2}{6} - \frac{x^4}{216}$

(D) $3 + \frac{x^2}{6} - \frac{x^4}{243}$

17. Suppose $r, N \in \mathbb{N}$ and $N > r$, then

$^{N+1}C_r - {}^{N}C_r$

is

(A) $^{N}C_{r-1}$

(B) $^{N+1}C_n$

(C) $^{N}C_r$

(D) $^{N+1}C_{r-1}$

18. Evaluate the real part of $\sqrt{40 + 42i}$, where $i^2 = -1$.

(A) ± 3 (B) ± 5 (C) ± 7 (D) ± 9

19. If $x, y \in \mathbb{R}$, find y satisfying the equation (where $i^2 = -1$)

$$x(3 + 4i) - y(1 + 2i) + 5 = 0$$

(A) -5 (B) -8 (C) -10 (D) -11

20. If $-1 < x < 1$, the series $\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{x}{2}\right)^n$ is equal to

(A) $-(x+1)^{-1}$ (B) $-(x+2)^{-1}$
(C) $(x+2)^{-1}$ (D) $(x+1)^{-1}$

21. If $m \in \mathbb{R}$, then the roots of the equation

$$x^2 - \frac{5x}{2} - \frac{11}{16} = mx$$

are

(A) distinct and non-real roots
(B) distinct and real roots
(C) equal roots
(D) nature of roots cannot be determined

22. Let the roots of the equation $3x^2 - 7x - 1 = 0$ be α and β, where $\alpha > \beta$. The value of $\alpha - \beta$ is

(A) $\frac{\sqrt{61}}{9}$, (B) $\frac{\sqrt{61}}{3}$, (C) $\sqrt{\frac{61}{3}}$, (D) $\frac{61}{9}$

23. x in the inequality $\frac{1}{x-1} > 2$ satisfies which of the following inequalities?

(A) $x - 1 < 0$
(B) $(x-1)(2x-3) < 0$
(C) $(x-1)(2x-3) > 0$
(D) $2x - 3 > 0$

24. Suppose a is a non-zero real number and satisfies $a \leq 1/4$, which of the following is a real value of x satisfying the equation

$$(x^2 + x)^2 = 2ax^2 + 2ax - a^2?$$

(A) $x = -1 + \frac{\sqrt{1-4a}}{2}$

(B) $x = -1 - \frac{\sqrt{1+4a}}{2}$

(C) $x = 1 - \frac{\sqrt{1-4a}}{2}$

(D) $x = 1 + \frac{\sqrt{1+4a}}{2}$

25. Given a non-zero real constant a, a relationship between x and y satisfying the system of equations

$$x^2 - 6xy + 11y^2 = 3a^2$$

and $$x^2 - 2xy - 3y^2 = 5a^2$$

is

(A) $x = y$ (B) $x = 2y$ (C) $x = 3y$ (D) $x = 4y$

26. If A and B are non-empty subsets of the universal set $\mathcal{U}$, the statement $A \subseteq B$ will generally imply the following except

I. $A \cup B^c = \mathcal{U}$. II. $A^c \cup B = \mathcal{U}^c$.
III. $A - B = \{\}$ IV. $B - A = \{\}$
V. $A \cap B = A$ VI. $A \cup B = B$

(A) I. and IV. Only (B) II. and III. Only
(C) V. Only (D) VI. Only

27. If the universal set $\mathcal{U} = \mathbb{R}$, and $A := \{x \in \mathbb{R} : x^2 + 1 > 0\}$. Which of the following statements(s) is/are true?

I. A is a finite set
II. $A \neq \{-\sqrt{-1}, \sqrt{-1}\}$
III. $A = \{\}$
IV. $A \subseteq \mathbb{R}$
V. $A^c \neq \mathbb{R}$

(A) I., II. and V. Only
(B) I., III. and IV. Only
(C) II., IV. and V. Only
(D) II., III. and V. Only

28. The following from I. to V. define the symmetric difference $A \Delta B$ of set A and B except

I. $(A - B) \cup (B - A)$
II. $[(A \cap B^c) \cup B] \cap [(A \cap B^c) \cup A]$
III. $(A \cap B) - (A \cup B)$
IV. $(A \cup B) - (A \cap B)$
V. $(A \cup B) \cap (A^c \cup B^c)$

(A) I. Only (B) II. and IV. Only
(C) III. Only (D) V. Only

29. From 3552 students taking the MTH 101 and PHY 101 examinations at the Obafemi Awolowo University for the 2019/2020 session, each student did at least one of the MTH 101 and PHY 101. At most 2052 passed MTH 101 and at most 2490 passed PHY 101. The following from I. to III. is/are the possible number of students that would have passed both examinations

I. 701 II. 851 III. 991

(A) III. Only (B) I. and II. Only
(C) II. and III. Only (D) I., II. and III. Only

30. Suppose the union of a set with eight elements and a set with nine elements is formed. Which of the following values from I. to IV. would be possible cardinalities of the union?

I. 9 II. 10 III. 17 IV. 20

(A) I., II. and III. Only
(B) I., II. and IV. Only
(C) I., III. and IV. Only
(D) II., III. and IV. Only

31. Let $A = \{1, 4, 9\}$ and $B = \{-3, -2, -1\}$. If the propositional function $p(x, y)$ on $A \times B$ means $x = y^2$. Then the following statements from I. to IV. about the relation $f = (A, B, p(x, y))$ is/are true

I. f is a function from A to B
II. f is surjective
III. f is injective
IV. Solution set of f is $\{(9, -3), (4, -2), (1, -1)\}$.

(A) I., II. and III. Only
(B) I., III. and IV. Only
(C) I. II. and IV. Only
(D) I., II., III. and IV. Only

32. If $\mathbf{A} = \mathbf{B} + \mathbf{C}$, where $\mathbf{B} = \begin{pmatrix} y & 2 \\ 7 & y - 9 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} x & -1 \\ 1 & 2 + x \end{pmatrix}$. Which of the following is a relationship between x and y for which $\mathbf{A}$ is singular?

(A) $x^2 + y^2 + 2xy - 7x + 7y = 8$
(B) $x^2 + y^2 + 2xy + 7x + 7y = 8$
(C) $x^2 + y^2 + 2xy - 7x + 7y = 8$
(D) $x^2 + y^2 + 2xy + 7x - 7y = 8$

33. Find the real values of x for which

$$\begin{vmatrix} x - 4 & -x - 1 \\ x + 1 & x - 5 \end{vmatrix} > 16.$$

(A) $1 < x < 5/2$ (B) $x < 1$; $x > 5/2$
(C) $x = 1$; $x = 5/2$ (D) No real values of x

34. Find the real values of k for which the system of equations

$$x + 7y + 9 = 0, \quad x + (k + 1)y = -1$$

$$(k + 2)x + 5y = 10$$

has a unique solution

(A) 2 and $-16/3$ (B) -2 and $16/3$
(C) $2/3$ and -16 (D) $-2/3$ and 16

35. If $\mathbf{A} = \begin{pmatrix} 2 & -5 & 11 \\ 1 & 1 & -6 \\ 4 & -3 & 8 \end{pmatrix}$. Which of the following statement is false?

(A) $\mathbf{A}$ has an inverse.
(B) $\mathbf{A}$ is non-singular
(C) $\mathbf{A}$ is singular
(D) determinant of $\mathbf{A}$ is a positive integer

36. Evaluate $\sum_{r=1}^{N} (2r - 1)$

(A) $\frac{N(N-1)}{2}$

(B) $\frac{N(N+1)}{2}$

(C) N^2

(D) $(N-1)^2$

37. Find the sum of the infinite series

$$(\sqrt{2}+1) + 1 + (\sqrt{2}-1) + (3 - 2\sqrt{2}) + \cdots$$

(A) $(3\sqrt{2}+4)/2$
(B) $(3\sqrt{2}-4)/2$
(C) $2\sqrt{2}+1$
(D) $2\sqrt{2}-1$

38. If $\sum_{j=2}^{N} 2^{2j-5} = 174.762.5$, evaluate N.

(A) 13 (B) 12 (C) 11 (D) 10

39 If $\sum_{k=1}^{N} U_k = \frac{3N^2+N}{2}$ Which of the following is a correct expression for U_N?

(A) $3N - 1$

(B) $3N + 1$

(C) $1 - 3N$

(D) $\frac{N(3N+1)}{2}$

40. Suppose α and β are the roots of the equation $x^2 - 2sx - 3s^2 - r^2 = 0$, given that s and r are non-zero real numbers. Then

(A) $\alpha, \beta, \in \mathbb{R}$ (B) $\alpha = \beta$,
(C) $\alpha, \beta, \in \mathbb{C} - \mathbb{R}$, (D) $\alpha > \beta$