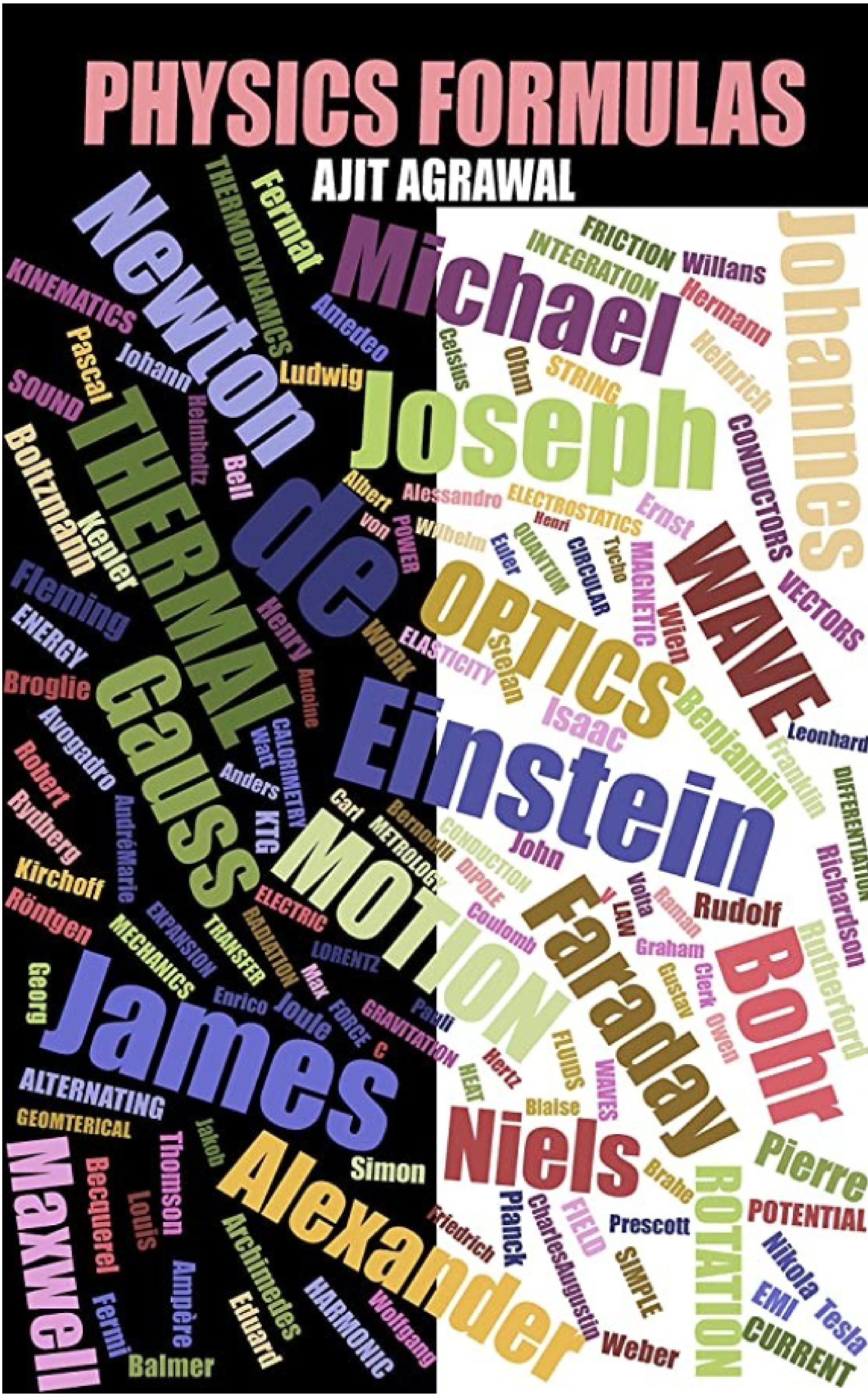
PHYSICS FORMULAS
AJIT AGRAWAL

DEDICATION

I felt very inspired to write a book because of my Dearmost teacher

Prof. P. V. Krishnan (retired professor, IIT Delhi),

and his wife, *mother* ***Krishangi Devi***.

Because of them my life has got meaning.

Whatever good anyone sees in me, I owe to them.

PREFACE

This book is specially meant for those who are preparing for IIT ADVANCED. In physics there are numerous concepts and formulas which are very difficult to remember and recall. It is quite possible that students may not even have any idea about particular concepts and formula. Every formula that is needed for IIT ADVANCED is here in this book.

Advantages of this book.

a) All formulas and concept at one place.

b) A student can easily identify what they do not know.

c) While practicing problem solving in a particular chapter, a student can easily know the concepts/formulas which they did not know/understand, because of which the they could not solve the problem.

d) Wherever needed, the concepts are tabulated, so as to see all the concepts at one place.

e) Neat and coloured diagrams/sketches to further enhance the understanding.

I hope everyone find this book helpful and wish everyone success in their life.

AJIT AGRAWAL

ACKNOWLEDGEMENT

I have always desired to write a book, but never wanted to get it printed. So I decided to launch it in amazon kindle. The advantage of Kindle store is that one may not waste papers; a little step in saving the environment.

Firstly I would like to thank my teacher **Dr. P. V. Krishnan** and his wife, mother **Krishangi Devi**. It is because of them that there is meaning to my life.

I would also like to thank my friend Mr. Manish Singhal who was a constant inspiration especially at the time of needs. I would also like to thank Mr. Santosh Mishra for being a continuous encouragement in this service.

A very special thanks to my wife, Mrs. Amita Gupta, for supporting me while I was writing the book and hence could not give her the time. Entire book designing ideas were given by her.

AJIT AGRAWAL

ABOUT AUTHOR

AJIT AGRAWAL

I have done B.Tech in "Industrial and Production Engineering" from IIT Delhi (2002). Thereafter, I was associated with training students for IIT (advanced) and have achieved ranks within Top 10 several times. I have experience of more than 14 years in this profession. Presently I am working in ALLEN Career Institute, KOTA.

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1. If $\vec{a} = \overline{const}$, the three kinematics' equations are given as:

$\vec{v} = \vec{u} + \vec{a}\,t$ — $\vec{r}_o$: initial position vector

$\vec{r} = \vec{r}_o + \vec{u}\,t + \frac{1}{2}\vec{a}\,t^2$ — $\vec{r}$: final position vector

$|\vec{v}|^2 - |\vec{u}|^2 = 2\vec{a}.(\vec{r} - \vec{r}_o)$ — $\vec{r} - \vec{r}_o$: displacement vector

2. $s_n - s_{n-1} = u + \frac{a}{2}(2n-1)$: Displacement in n^{th} second.

3. $\langle \vec{p} \rangle = \frac{\int \vec{p}\,dt}{\int dt}$: Average definition.

4. Position Vector: $\vec{r} = x\hat{i} + y\,\hat{j} + z\,\hat{k}$

5. Distance: $s = \int v\,dt = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$

6. Displacement: $\vec{s} = \vec{r}_f - \vec{r}_i$: Final position vector – initial position vector

7. Average speed $\langle |\vec{v}| \rangle = v = \frac{\text{total distance}}{\textit{total time}}$ or *mean value of velocity modulus*

8. Average velocity: $\langle \vec{v} \rangle = \frac{\text{total displacement}}{\textit{total time}}$

9. $|\langle \vec{v} \rangle|$: *magnitude of average velocity* or *Modulus of mean velocity vector*

10. Instantaneous velocity $(\vec{v}) = \frac{d\,\vec{r}}{dt}$

11. Instantaneous speed $|\vec{v}| = \left|\frac{d\,\vec{r}}{dt}\right|$

12. Instantaneous acceleration $(\vec{a}) = \frac{d\vec{v}}{dt}$

13. $\left|\frac{d\vec{v}}{dt}\right|$: Magnitude of acceleration

14. $\frac{d\,|\vec{v}|}{dt} = a_\tau$: *tangential acceleration* or *acceleration in the direction of velocity* or projection of $\vec{a}$ on $\vec{v}$

15. Average acceleration = $\frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$

16. Area under curve
   a. Area under velocity-time graph gives displacement
   b. Area under speed time graph gives distance.
   c. Area under acceleration-time graph gives change in *velocity*.
   d. Area under acceleration-displacement graph gives $\frac{v^2 - u^2}{2}$

17. While calculating distance, find the time where v = 0. [As the particle might have changed its direction at those times].

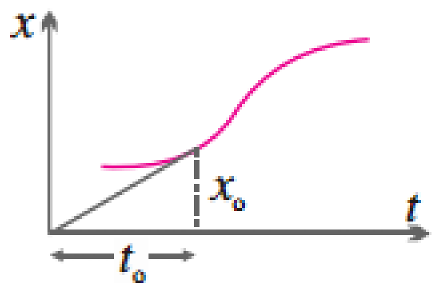
18. v +ive a+ive $\Rightarrow$ accelerating

    v +ive a-ive $\Rightarrow$ decelerating

19. If a question asked is to find a quantity (say, displacement) in some other reference frame; most of the time it's useful to change the reference frame and solve the question.

20. For motion under gravity, it is always easy to solve if we start from top most point as the speed is zero there.

21. Condition under which $\langle \vec{v} \rangle = \vec{v}_{inst}$:



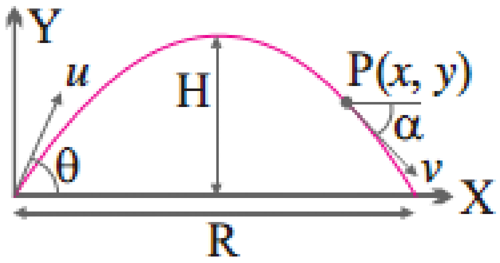
From origin draw a tangent to graph.

$$slope = \tan\theta = \frac{x_o}{t} = v_{inst} \qquad \langle v \rangle = \frac{displacement}{time} = \frac{x_o}{t} \qquad \therefore \langle \vec{v} \rangle = \vec{v}_{inst}$$

PROJECTILE MOTION

Projectile Motion

1. *Ground to Ground*

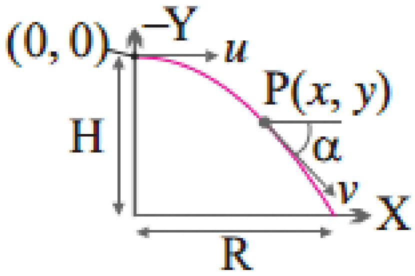


$$y = x\tan\theta - \frac{g\,x^2}{2u^2\cos^2\theta} = x\tan\theta\left(1-\frac{x}{R}\right)$$

$$v\cos\alpha = u\cos\theta$$

$$T = \frac{2u\sin\theta}{g} \qquad R = \frac{u^2\sin 2\theta}{g} = \frac{2u_x u_y}{g} \qquad H = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^2}{2g}$$

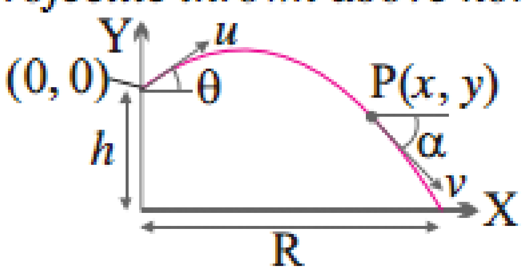
2. *Projectile thrown horizontally from tower*



$$T = \sqrt{\frac{2h}{g}} \qquad R = u\sqrt{\frac{2h}{g}}$$

Equation of Trajectory: $y = \frac{1}{2}g\left(\frac{x}{u}\right)^2$

3. *Projectile thrown above horizontal*



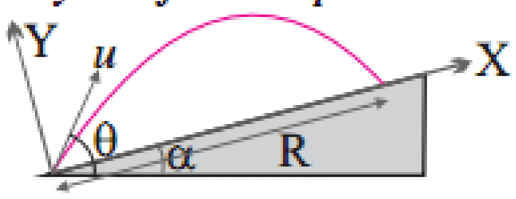
$$T = \frac{u\sin\theta + \sqrt{u^2\sin^2\theta + 2g\,y}}{g}$$

$$R = (u\cos\theta)T$$

$$R_{max} = \frac{u\sqrt{u^2+2g\,y}}{g} \quad \text{at } \theta = \frac{1}{2}\cos^{-1}\left(\frac{g\,y}{u^2+g\,y}\right)$$

Equation of Trajectory: $y = -x\tan\theta + \frac{g\,x^2}{2u^2\cos^2\theta}$

4. *Body Projected up an inclined plane*



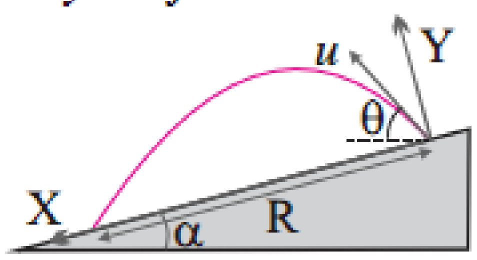
$$R = \frac{u^2\left[\sin(2\theta-\alpha)-\sin\alpha\right]}{g\,\cos^2\alpha}$$

$$T = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$$

$$R_{max} = \frac{u^2}{g\,(1+\sin\alpha)} \quad \text{at } \theta = \frac{\pi}{4}+\frac{\alpha}{2}$$

For same range: $\theta' = \frac{\pi}{2} - (\theta-\alpha)$

5. *Body Projected down an inclined plane*



$$R = \frac{u^2\left[\sin(2\theta+\alpha)+\sin\alpha\right]}{g\,\cos^2\alpha}$$

$$T = \frac{2u\sin(\theta+\alpha)}{g\cos\alpha}$$

$$R_{max} = \frac{u^2}{g\,(1-\sin\alpha)} \quad \text{at } \theta = \frac{\pi}{4}-\frac{\alpha}{2}$$

For same range: $\theta' = \frac{\pi}{2} - (\theta+\alpha)$

RELATIVE VELOCITY

Reference frame consist of co-ordinate axis and time. Some physical quantities those are reference frame independent are:

- Mass
- Time
- Force
- Heat transfer.
- Impulse
- Constraint equation.
- Quantity of a substance (moles)
- Electrical Current and Heat current.
- WD by friction on a system.
- Relative velocity between two bodies.

- **Collision**: *Two bodies will crash into each other only if they reach the same point simultaneously*. Therefore, velocity of one body w.r.t. other should be directed towards other.
- If the points (or destinations) are on the ground, then the direction of velocity is w.r.t. ground.
- A man swims in water, implies its relative velocity is w.r.t water.
- The air speed of plane, or the speed of plane in air means that the velocity of plane w.r.t air is given.
- Swimmer keeps himself at an angle of 30° with the river flow means the velocity of swimmer w.r.t to river flow direction is given.
- A swimmer aims to or heads to (velocity is **not** w.r.t ground)
- Swimmer takes shortest time if its velocity w.r.t. river is normal to the river flow
- u: river speed, v: Boat speed w.r.t river

  For shortest route when $\sin\theta = \dfrac{u}{v}$ $\quad (u < v) \quad$ $drift = 0$

  For shortest route when $\sin\theta = \dfrac{v}{u}$ $\quad (v < u) \quad$ $drift = \dfrac{\sqrt{u^2 - v^2}}{v} l$

NEWTONS LAW

1. Atwood machine: $a = \dfrac{m_1 - m_2}{m_1 + m_2} g$, $\quad T = \dfrac{2m_1 m_2}{m_1 + m_2} g$
2. Constraint Equation can be written in any reference frame, just do not forget to show tension at every location it is acting.
3. *While writing constraint equation* for a system inside lift, you should not forget to apply tension at the ceiling of the lift.
4. For finding velocity of point on thread, just replace the point with a mass and find velocity of the mass.
5. If a body is on ground, take reference as ground.
6. If a body A is on body B, take reference of body A to be body B.
7. But when both bodies A and B are having horizontal surface (i.e. g is perpendicular to the contact surface of both bodies, one may or may not change the reference frame).
8. When you apply pseudo force, reference frame changes.
9. When you want to change the reference frame, apply pseudo force.
10. Pseudo force acts at the COM of the system = ma (of reference point).
11. When you change the reference frame to some mass M, then mass M is at rest in its frame.
12. The reading of weighing machine = $\dfrac{N}{g}$
13. Weightlessness is the effect when normal reaction becomes zero.
14. Newton's I law is definition of inertial reference frame.
15. For solving spring cutting problem, draw FBD just before the spring is cut and draw FBD just after it is cut.

FRICTION

16. $f_K = \mu N$, $\qquad f_S \le \mu N$
17. Direction of static friction depends on the direction of *acceleration* while the direction of kinetic friction is in direction opposite to the direction of *relative velocity*.
18. One can afford to do mistake in direction of f_s but not f_k.
19. T < 0 means tension is zero. N < 0 means normal is zero.
20. $f_s \le \mu N$: **Condition for no slipping or skidding.**
21. $\tan\theta > \mu_s$: Condition for a block to slide down the incline.

CIRCULAR MOTION

1. θ: scalar   ω: vector   α: vector

2. ***For variable α*:** $\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$; $\omega = \frac{d\theta}{dt}$.

3. *Angular displacement is different for different observers in the same reference frame.*

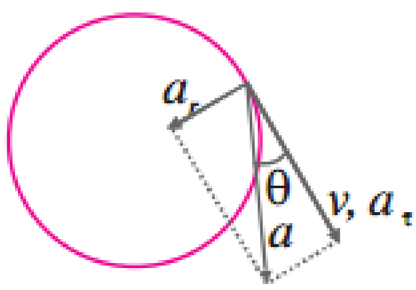
4. $\vec{v} = \vec{\omega}\times\vec{R}$ $\vec{a} = \vec{\alpha}\times\vec{R}$

5. $a_R = \sqrt{a_t^2 + a_r^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$; $a_\tau = \frac{dv}{dt} = a\cos\theta = \vec{a}\cdot\hat{v} = \alpha R$

   r: radius of curvature and not actually radius of wheel.

6. Radius of curvature in curvilinear path: $R_c = \frac{\left[1+\left(dy/dx\right)^2\right]^{3/2}}{d^2y/dx^2}$

7. $\tan\theta = \frac{a_r}{a_\tau}$ $a_\tau = a\cos\theta$ $a_r = a\sin\theta$



8. For solving dynamics question, always look for center of circle. Since acceleration is towards center of circle and hence net force should be written towards the center of circle.

9. Ofcourse one should not forget the tangential acceleration $a_\tau = \frac{dv}{dt}$.

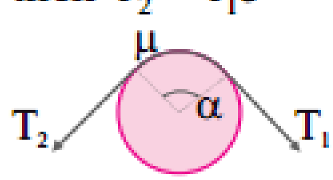
10. Conical Pendulum: $\omega = \sqrt{\frac{g}{l\cos\theta}}$ $\omega \ge \sqrt{\frac{g}{l}}$

11. Car on Plane circular road: $v \le \sqrt{\mu g r}$

12. Car on Banked Road: $v = \sqrt{g r\tan\theta}$ for no friction to act

    $$\sqrt{g r\left(\frac{\tan\theta - \mu_s}{1+\mu_s\tan\theta}\right)} \le v \le \sqrt{g r\left(\frac{\tan\theta + \mu_s}{1-\mu_s\tan\theta}\right)}$$

13. If a string is wrapped on a hinged massless pulley such that friction exists between string and the pulley, then $T_2 = T_1 e^{\mu\alpha}$



1. Work energy theorem can be applied in both inertial and non-inertial reference frame.
2. WD by internal forces on a rigid body is always zero.
3. Internal force **can** do work on a non-rigid body.
4. WD by statics friction on a system is always zero.
5. WD by kinetic friction on a system is always negative.
6. WD by forces on a particle: $WD = \int \vec{F}_{net} \bullet d\,\vec{r} = \Delta K$
7. WD against external forces on a particle: $WD = -\int \vec{F}_{net} \bullet d\,\vec{r}$
8. Conservative force definition:
   a. *If a body moves under the action of a force that does no total work during any round trip, then the force is conservative; otherwise non-conservative.*
   b. *If the work done by a force in moving a body from an initial position to a final position is independent of path taken between the two positions, then the force is conservative; otherwise non-conservative.*
   c. *It is state dependent* (depends on initial & final position only).
   d. *It forms exact differential.*
   e. *Potential energy is defined only for conservative forces.*
   f. *The field of conservative force never forms a closed loop.*
9. WD by following forces is path independent (conservative).
   - Gravity: $U = m\,g\,h$
   - Spring: $U = \frac{1}{2} k\,x^2$
   - Buoyant force
   - Electrostatic force
10. WD by following forces is path dependent (non-conservative).
    - Friction
    - Pressure force inside piston
    - Induced electric field.
11. Mechanical energy definition: $T = K + U$
12. *Internal energy*: Microscopically, it is KE of atoms; macroscopically it is temperature.
13. Equilibrium:
    a) Stable Equilibrium: $\frac{d^2U}{dx^2} > 0 \; or \quad \frac{dF}{dx} < 0$
    b) Unstable Equilibrium: $\frac{d^2U}{dx^2} < 0 or \quad \frac{dF}{dx} > 0$
    c) Neutral equilibrium: U = constant, F = always zero
    d) Saddle point:

14. *Most forgotten formula*: $W_s = W_c = -\Delta U$: This is called WD by system.

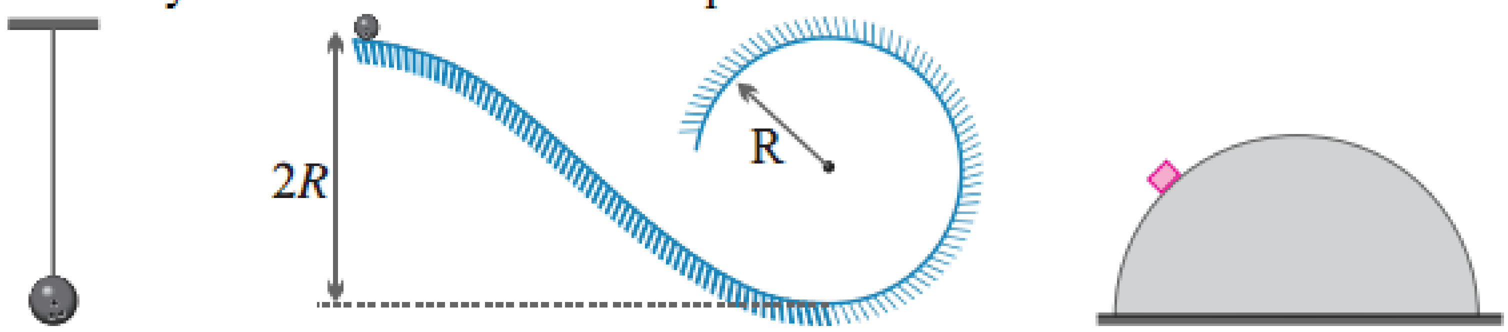
15. $P = \dfrac{dW}{dt} = \vec{F}.\vec{v} \qquad P = \int \tau \, d\theta$

16. *Average power* $= \dfrac{\int P dt}{\int dt} = \dfrac{WD}{t}$

Motion in vertical circle

Body moving in circular path can follow two trends:

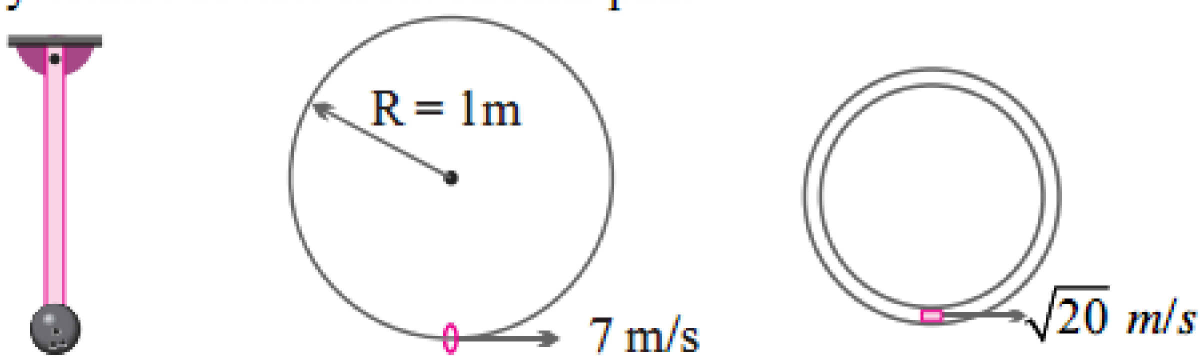
a) The body can deviate from circular path



Summary:

$u \geq \sqrt{5gl}$	The bob will complete the circle.
$\sqrt{2gl} < u < \sqrt{5gl}$	Bob will not complete circle and move in parabolic path
$u < \sqrt{2gl}$	The bob would oscillate to-&-fro.

b) The body cannot deviate from circular path



$u \geq \sqrt{4gl}$	Bob will complete the circle. For $\theta < \cos^{-1}(2/3)$ with upward vertical, tension in rod will become negative.
$\sqrt{2gl} < u < \sqrt{4gl}$	Bob will move in circular path but will not complete full circle.
$u \leq \sqrt{2gl}$	The bob would oscillate to-&-fro in circular path.

COM of uniform bodies

1. Semicircular ring of radius R. Ans: $2R/\pi$
2. Semicircular disc of radius R. Ans: $4R/3\pi$
3. Hemispherical shell of radius R. Ans: R/2
4. Solid hemisphere of radius R. Ans: 3R/8
5. Triangular plate of height *h* and base *2b*. Ans: h/3
6. hollow cone of base radius R and height *h*. Ans: h/3
7. solid cone of base radius R and height *h*. Ans: h/4

8. $\vec{r}_{COM} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i \quad \vec{v}_{COM} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{v}_i \qquad \vec{a}_{COM} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{a}_i$

9. $\vec{J} = \int \vec{F}(t)\,dt = \vec{F}_{avg}\,\Delta t = \vec{p}_f - \vec{p}_i$

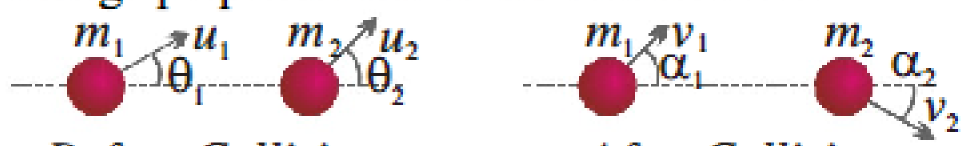
10. collision in one dimension:

Before Collision *After Collision*

$v_{1G} = (1+e)v_{CG} - e\,u_{1G}$

$v_{2G} = (1+e)v_{CG} - e\,u_{2G}$

11. Coefficient of restitution is written only along the common normal. Since the forces will be acting only along the common normal, the velocity will not change perpendicular to the common normal.



Before Collision *After Collision*

$$\frac{v_1\cos\alpha_1 - v_2\cos\alpha_2}{u_1\cos\theta_1 - u_2\cos\theta_2} = -e$$

12. Momentum of system in C- frame is zero.

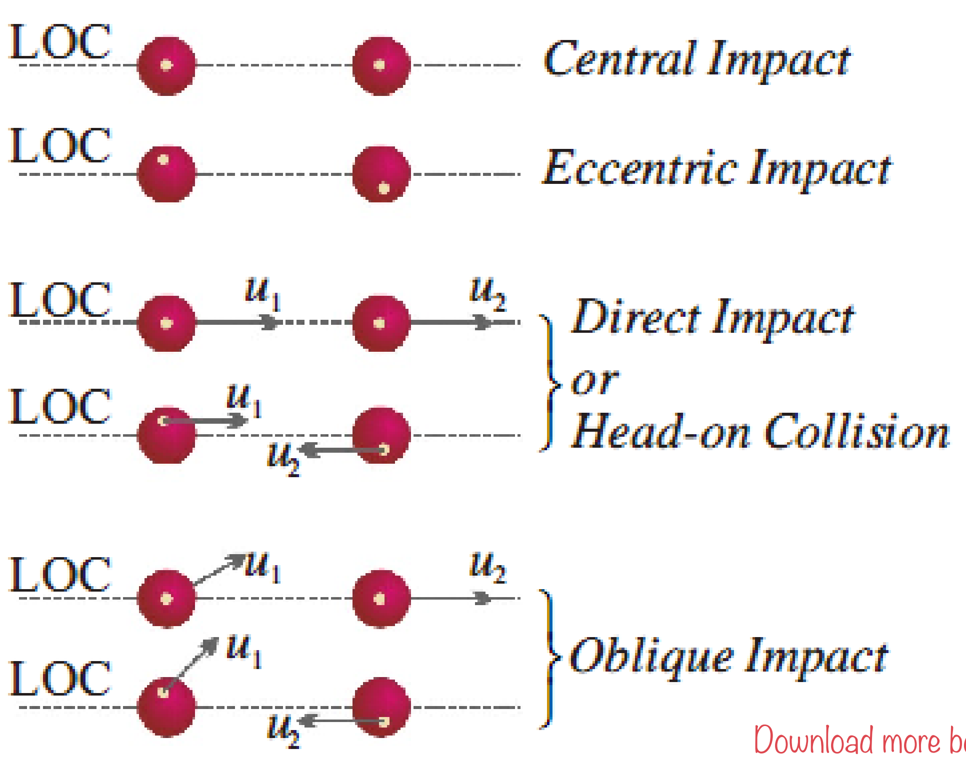
13. KE of system is minimum in C-frame: $KE_{min} = \frac{1}{2}\frac{m_1m_2}{m_1+m_2}(\vec{v}_2 - \vec{v}_1)^2$

14. KE in ground reference frame in terms of velocity of COM

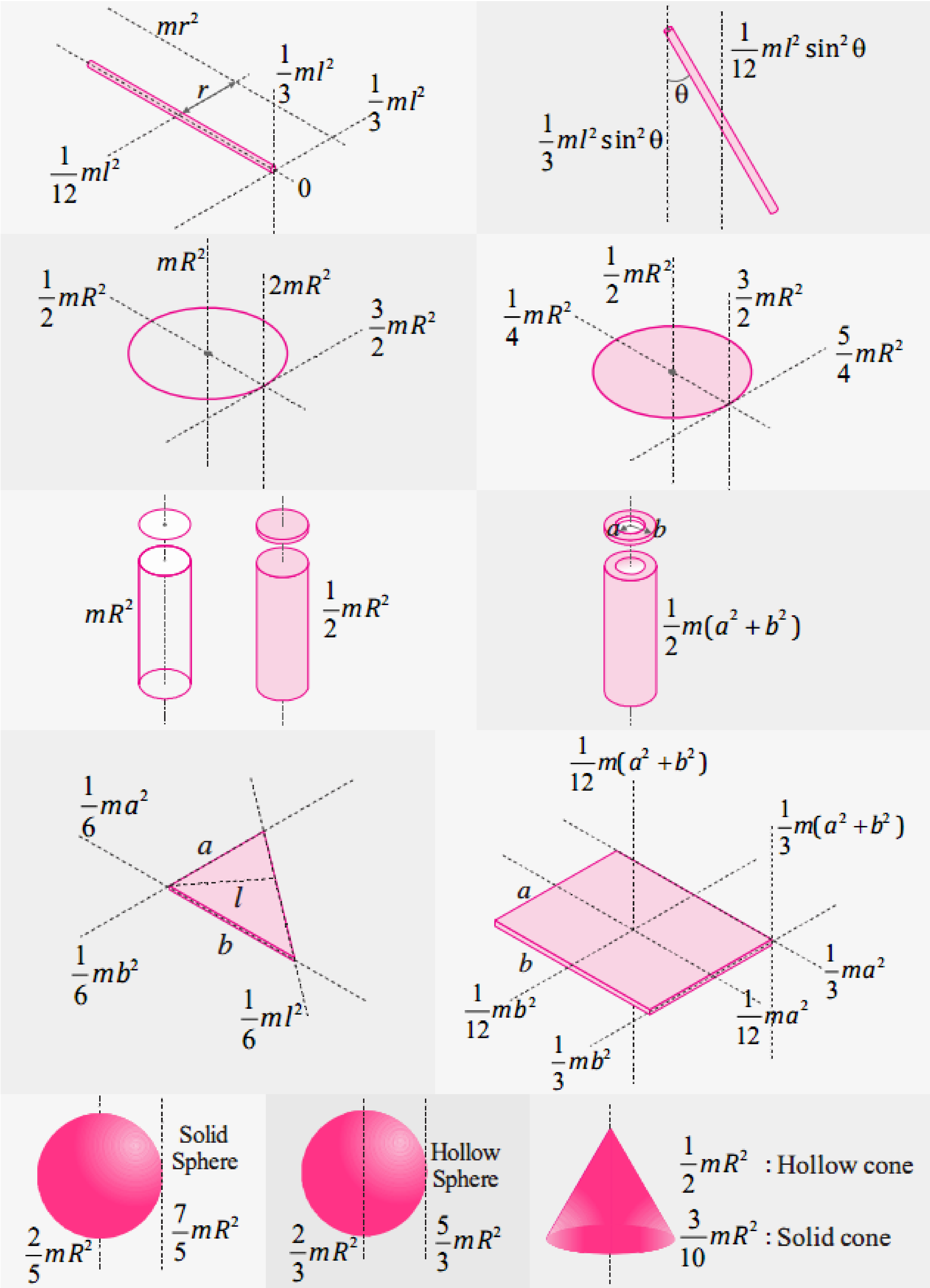
$$KE_G = \frac{1}{2}m_1v_{1G}^2 + \frac{1}{2}m_2v_{2G}^2 = \frac{1}{2}\mu\vec{v}_r^{\,2} + \frac{1}{2}M\,\vec{v}_{CG}^{\,2}$$

15. Deformation energy is maximum when the colliding body has same velocity along the *common normal*.

16.



1. $I=\sum_{i=1}^{n} m_i r_i^2=\int r^2\,dm=MK^2$, K : radius of gyration



$I_z=I_x+I_y$

Applicable for plane laminas only

$I_P=I_{COM}+M\,h^2$

MI is minimum through COM

2. **Equilibrium:** The two requirements for equilibrium are: $\vec{F}_{net} = 0 \qquad \vec{\tau}_{net} = 0$
If forces are concurrent, then net torque is definitely zero and therefore, the body will be in rotational equilibrium.

3. $\vec{\tau} = \vec{r} \times \vec{F} = I\,\vec{\alpha} \qquad \dfrac{d\vec{L}}{dt} = \vec{\tau} \quad \therefore \int \vec{\tau}\,dt = \vec{L}_f - \vec{L}_i$: *Angular impulse*
Torque (*moment of force*) equation can be written about any axis. But MI should be written about the same axis about which torque is written.

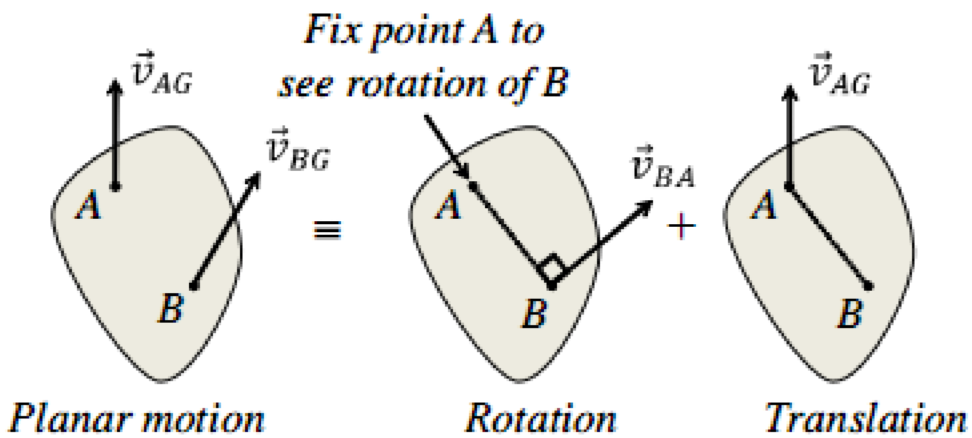
4. For a body at rest, translationally and rotationally, $\sum \vec{\tau} = 0$ about any point.

5. For an accelerating body torque can be written about any axis but we should not forget to apply **pseudo force** at the **COM**.
pseudo force = mass * acceleration of reference point.
Note: This is valid only for torque equation.

6. For no toppling of a vehicle on circular tracks: $v < \sqrt{\dfrac{g\,r\,a}{h}}$
To make sure that car skids rather than topples: $\mu_s \le \dfrac{a}{h}$

7. *Chasel's Theorem*: 'ω' about all parallel axis is the same.

8. *Velocity & acceleration* of a particle in a rigid body in planar motion:

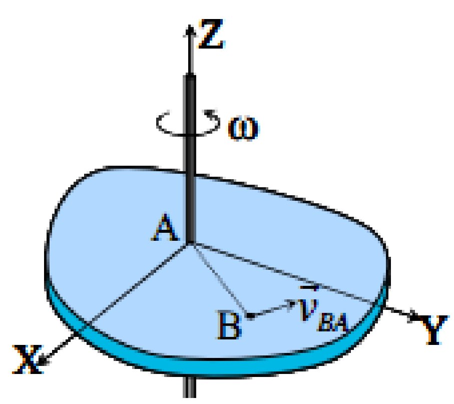


$\vec{v}_{BG} = \vec{v}_{BA} + \vec{v}_{translation}$
But, $\quad \vec{v}_{BG} = \vec{v}_{BA} + \vec{v}_{AG}$
$\therefore \vec{v}_{B\_translation} = \vec{v}_{AG}$

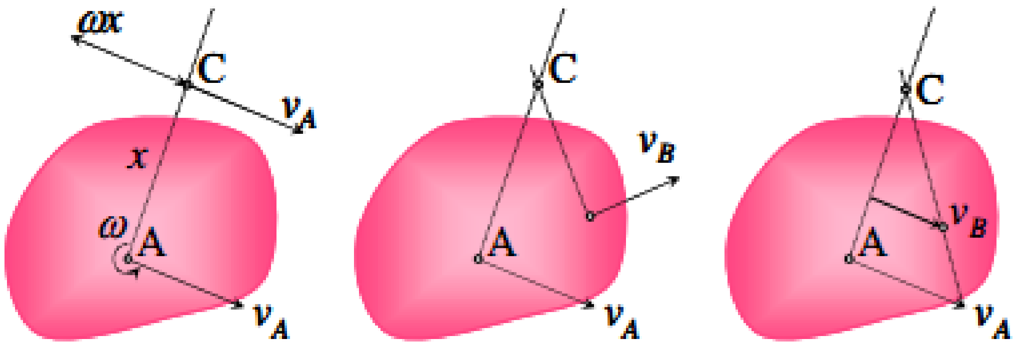
If we analyze w.r.t. COM, we can say that: $\vec{v}_{BG} = \vec{v}_{CG} + \vec{\omega} \times \vec{r}_{BC}$

9. $\vec{a}_{BA} = \vec{a}_{BG} - \vec{a}_{AG} = \alpha\, r(\hat{\tau}) + \omega^2 r(-\hat{r})$

$$\therefore \vec{a}_{BG} = \underbrace{\vec{a}_{AG}}_{\text{Translation Component}} + \underbrace{\overbrace{\alpha\, r(\hat{\tau})}^{\text{tangential}} + \overbrace{\omega^2 r(-\hat{r})}^{\text{Radial}}}_{\text{Rotational Component}}$$



10. ICR:
Case (a): $v_C = v_A - \omega x = 0$: As for ICR translation velocity should be 0.
Case (b): By dropping perpendicular on velocity vector $\vec{v}_A$ & $\vec{v}_B$, one can find ICR as shown below.
Case (c): The intersection of dotted lines gives ICR.
Because point C is ICR, $\therefore \vec{v}_A \perp CA$ & $\vec{v}_B \perp CB$.



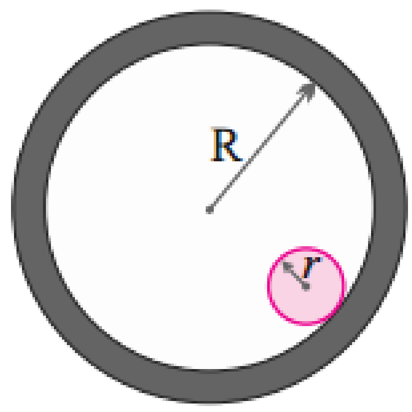
(a) (b) (c)

11. **Constraint equation**: Written at contact points where no slipping (sliding) takes place and that can also be thought as pure rolling.

$$\begin{bmatrix} \text{velocity of a contact} \\ \text{point on } 1^{st} \text{ rigid body} \end{bmatrix} = \begin{bmatrix} \text{velocity of same contact} \\ \text{point on } 2^{nd} \text{ rigid body} \end{bmatrix}$$

12. Pure rolling is a condition under which the point of contact in one body has the same velocity as the point of contact in other body.

13. **Wheel rolling inside a circular region:**
For the situation shown, find the relationship between rotational angular velocity & angular velocity due to circular motion.



$v_c = \omega r \qquad ---(1)$

$v_c = \dfrac{d\theta}{dt}(R-r) \qquad ---(2) \quad \therefore \omega r = \dfrac{d\theta}{dt}(R-r)$

For a rigid body moving in circular motion, we should see the radius of the circle in which the COM of the body is moving.

$\dfrac{d\theta}{dt} = \dfrac{v}{R \pm r}$: Angular velocity of circular motion

$\omega = \dfrac{v}{r}$: Angular velocity of rotation motion

r: radius of body *R*: radius of outer (or inner) circle

14. *Friction in rolling*: Newton's law should not be violated. The direction of friction (f_s) is determined by $\vec{a}$ & $\vec{\alpha}$ and not $\vec{v}$ & $\vec{\omega}$

15. A free wheel moving with a constant velocity in pure rolling does not experience any friction force, although the surface has friction.

16. If *friction coefficient* is infinite, then pure rolling can take place, but if ($f_s \le \mu_s N$) criteria is not satisfied pure rolling will not take place.
μ_s should be sufficiently large for pure rolling to take place.

17. Due to the friction between string and pulley, the tension in string in contact with pulley is not constant. And since static friction acts, it does not do any work. If there is no friction between the string and the pulley, even if the pulley has mass, it will slip and hence no rotation will take place.

18.

	Hinged	Not hinged
KE	$\frac{1}{2} I_{axis} \omega^2$	$\frac{1}{2} M v_c^2 + \frac{1}{2} I_{COM} \omega^2$
WD	$\int \vec{\tau}_{net} \cdot d\vec{\theta}$	$\int \vec{F}_{net} \cdot d\vec{r}_c + \int \vec{\tau}_{net} \cdot d\vec{\theta}$
Power	$\vec{\tau}_{net} \cdot \vec{\omega}$	$\vec{F}_{net} \cdot \vec{v}_c + \vec{\tau}_{net} \cdot \vec{\omega}$
Angular momentum	$I_{axis} \vec{\omega}$	$m(\vec{R} \times \vec{v}_c) + I_{COM} \vec{\omega}$
Angular momentum of Point mass		$m(\vec{r} \times \vec{v})$

The equation above suggests that *Moment of inertia* is written about COM only (for not hinged case).

19. **KE in Rolling:**

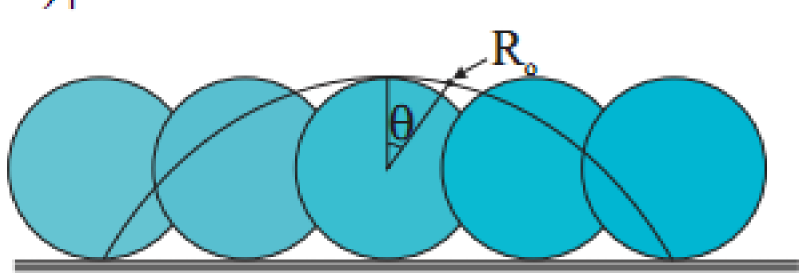
Body	$KE = \frac{1}{2} M v_c^2 + \frac{1}{2} I_{COM} \omega^2 = \frac{1}{2} I_{ICR} \omega^2$
Ring	$m v^2$
Disc	$3 m v^2 / 4$
Hollow sphere	$5 m v^2 / 6$
Solid sphere	$7 m v^2 / 10$

20. **Angular momentum** is conserved if no net torque acts about an axis, i.e. Angular Momentum is to be conserved only about that axis.

Caution: $\vec{\tau}$ & $\vec{L}$ must be measured relative to the same axis (both initially and finally). Usually we conserve $\vec{L}$ about the axis of rotation. The **moment of inertia** is also written about the very **same axis**.

21. If you are not able to identify a axis about which angular momentum is conserved, then go for angular impulse to solve the question.

22. Radius of curvature of a point on periphery for a rolling wheel:
$R_C = 4R\,|\cos(\theta/2)|$



1. A particle is said to be in SHM if $\frac{d^2x}{dt^2} \alpha - x$ *or* $\frac{d^2\theta}{dt^2} \alpha - \theta$.

2. SHM can be described as the projection of uniform circular motion along the diameter of the circle. Or, Circular motion can be regarded as the combination of two SHM's at right angles, with equal amplitudes and frequencies but differing in phase by 90°.

3.

$$X = x - x_o = A\sin(\underbrace{\omega t + \varphi}_{\text{Phase}})$$

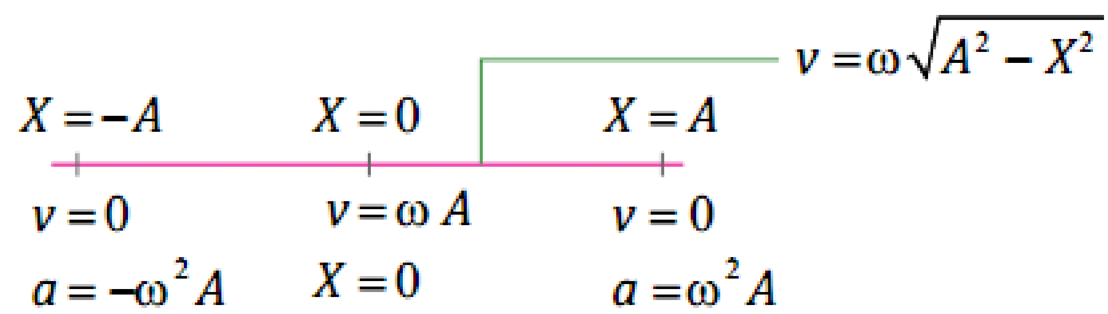
Initial Phase/Epoch
Angular frequency
Amplitude
Mean position
Position of particle
Displacement from mean position

Time Period (T): Time necessary to complete one cycle (oscillation).

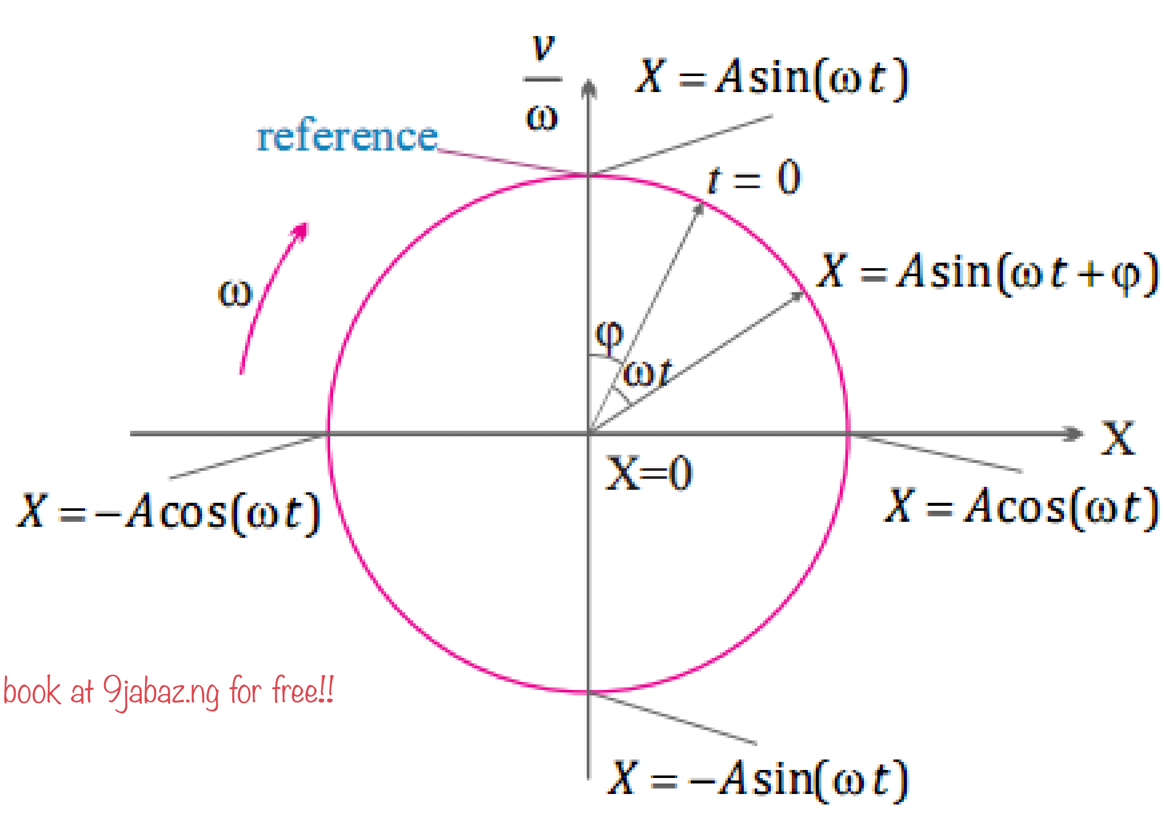
4. *Frequency* (*f*): The number of cycles per unit time (how frequently a particular event is occurring).

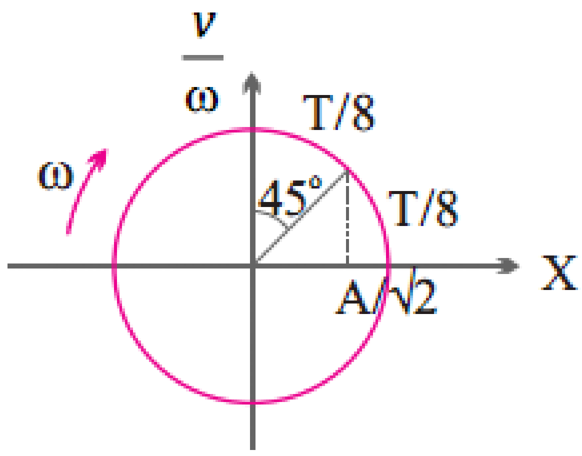
5. In SHM, the period of oscillations is independent of amplitude. Such a motion is called *isochronous motion*.

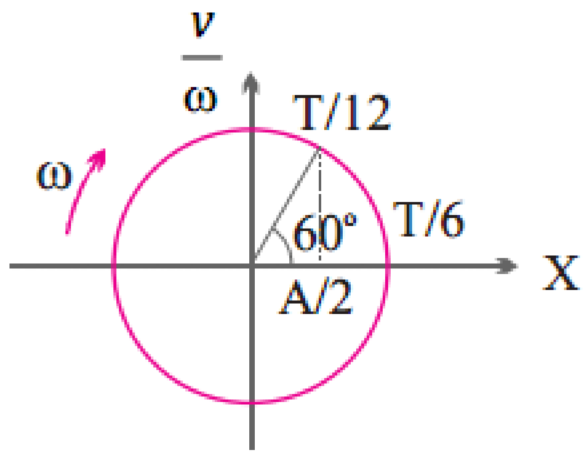
6.



7. **Phasor:**







8. TE in SHM + Extra Energy = Total Energy of system

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2\omega t = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$PE = \frac{1}{2}Kx^2 = \frac{1}{2}m\omega^2x^2 \qquad \therefore TE = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}KA^2$$

PE at equilibrium (Extra Energy): $\frac{1}{2}kx_o^2$

GPE = 0 at equilibrium (standard).

9. **Two blocks connected by spring (blocks given velocity shown)**

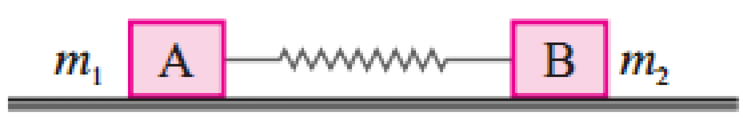
$$\omega = \sqrt{\frac{K}{\mu_r}}$$

$$A_1 = \left(\frac{m_2}{m_1 + m_2}\right)\frac{v_{rel}}{\omega} \qquad A_2 = \left(\frac{m_1}{m_1 + m_2}\right)\frac{v_{rel}}{\omega} \qquad v_{rel} = v_1 + v_2$$

$$x_{1C} = -A_1\sin\omega t \qquad \Rightarrow x_1 - x_C = -A_1\sin\omega t \therefore x_1 = v_ct - A_1\sin\omega t$$

$$x_{2C} = A_2\sin\omega t \Rightarrow x_2 - x_C = A_2\sin\omega t \qquad \therefore x_2 = v_ct + A_1\sin\omega t$$

10. **Two blocks connected by spring (spring initially compressed/ elongated and at rest)**



$$A_1 = \frac{x_1}{2} = \left(\frac{m_2}{m_1 + m_2}\right)x_o \qquad A_2 = \frac{x_2}{2} = \left(\frac{m_1}{m_1 + m_2}\right)x_o$$

$$x_{1G} = +A_1\cos\omega t \qquad \therefore x_1 = A_1\cos\omega t$$

$$x_{2G} = -A_2\cos\omega t \qquad \therefore x_2 = -A_2\cos\omega t$$

11. Amplitude is distance from *equilibrium position* to extreme *position*.

12. Composition of two SHM's in same direction:

$$A_{rel} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta\phi)}$$

13. Compostion of SHM in mutualy perpendicular direction:

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}=\sin^2\phi+\frac{2xy\cos\phi}{A_1A_2}$$

This equation is a trajectory of a particle representing ellipse.

(i) $\phi = 0, \pi$ – equation of straight lines

(ii) $\phi = \pi/2$ or $A_1 = A_2$ – equation of circle.

14. Always make sure that when you write force equation or torque equation, the value of x or θ is always increasing.
It is easier to solve a question if x is calculated from natural length of spring.

15. If amplitude is asked, you must write initial condition correctly, if you want to get the write answer.

16. Time period of simple pendulum: $T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

17. Time period of compound Pendulum: $T = 2\pi\sqrt{\frac{I}{Mgh}}$

18. For more than one body system, kept on each other [in horizontal plane], SHM will take place if no sliding is there between them.

19. For more than one body system, kept on each other [in vertical plane], SHM will take place if N > 0 all the time.

20. In vertical oscillation; one should be careful while finding amplitude as the mean position might change during the process.

21. Torsional Pendulum

$$\tau = -\kappa\theta \quad \Rightarrow I\frac{d^2\theta}{dt^2} = -\kappa\theta \quad \text{where, } \kappa = \frac{1}{2}\frac{\pi\eta R^4}{l} \quad \therefore T = 2\pi\sqrt{\frac{I}{\kappa}}$$

22. Angular frequency is directly proportional to angular velocity although they are different quantities (we understand from phasors).

So while conserving energy $\frac{1}{2}I\omega_o^2$, one can use $\frac{1}{2}I\omega^2$.

ω_o : Angular velocity. ω : Angular frequency.

WAVE TRAVELING ON STRING

Traveling Wave Equation: In wave motion, momentum and energy are transferred from one location to other without mass transfer.
Mechanical properties of the wave depend on elasticity & inertia of the medium.

General equation of wave: $$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

General solution: $y(x,t) = \frac{1}{2}\left[f^*(x-ct) + f^*(x+ct)\right]$.

Note carefully, that the above result is the *superposition* of *wave*.

Three specific solutions of our interests are:

$y = A\sin(kx - \omega t + \phi)$	:	Plane progressive simple harmonic wave. I = constant. (source is at large distance).
$y = \frac{A}{\sqrt{r}}\sin(kx - \omega t + \phi)$	:	Cylindrical progressive wave. $I = P/2\,\pi rl$ (line source).
$y = \frac{A}{r}\sin(kx - \omega t + \phi)$	:	Spherical progressive wave. $I = P/4\pi r^2$ (point source).

'*A*' is a constant; *r*: separation of the point in space from the source.

Traveling wave in a string

$$K = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$v = \frac{\omega}{K} = \frac{\lambda}{T} = f\lambda$$

$$y = A\sin(\overbrace{Kx - \omega t + \varphi}^{\text{Phase}})$$

Initial Phase/Epoch

Angular frequency

Angular Wave Number or Propagation constant

Amplitude

Displacement from mean position

NOTE:

- The frequency of wave is set by the external source.
- The speed of a wave depends only on medium.
- The wavelength is then fixed by $\lambda = v/f$
- Here, λ is reference frame independent; v is reference frame dependent, and thus f is also reference frame dependent.

Apparent variation in f due to the relative motion between the source and the observer is known as *Doppler effect*.

Linear wave equation: $$|v_p| = \frac{\partial y}{\partial t} = v\left|\frac{\partial y}{\partial x}\right|$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{(\mu/A)\,A}} = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{\text{stress}}{\rho}} = \sqrt{\frac{Y \times \text{strain}}{\rho}}$$: wave speed

$y = A\sin(Kx - \omega t + \phi)$: Displacement of particle in a wave

$v_P = -A\omega\cos(Kx - \omega t + \phi)$: velocity of particle in wave

$a_P = -A\omega^2 \sin(Kx - \omega t + \phi)$: acceleration of particle in wave

- $\begin{bmatrix} Kx - \omega t \\ -Kx + \omega t \end{bmatrix} \Rightarrow \left\langle \begin{matrix} \textit{wave is moving} \\ \textit{in } +X \textit{ direction} \end{matrix} \right\rangle$ & $\begin{bmatrix} Kx + \omega t \\ -Kx - \omega t \end{bmatrix} \Rightarrow \left\langle \begin{matrix} \textit{wave is moving} \\ \textit{in } -X \textit{ direction} \end{matrix} \right\rangle$

	Transverse	longitudinal
speed	$v = \sqrt{\tau/\mu}$	$c = \frac{\partial P}{\partial \rho} = \sqrt{B/\rho}$; $c_{gas} = \sqrt{\frac{\gamma P}{\rho}}$
∂U	$\frac{T}{2}\left(\frac{\partial y}{\partial x}\right)^2 \partial x$	$\frac{BA}{2}\left(\frac{\partial \varepsilon}{\partial x}\right)^2 \partial x$
	$\partial K = \partial U$	$\partial K = \partial U$
$\partial K + \partial U$	$\mu\omega^2 A^2 \cos^2(Kx - \omega t)\partial x$	$\rho A\, \varepsilon_m^2 \omega^2 \sin^2(Kx - \omega t)\partial x$
P	$\mu v \omega^2 A^2 \cos^2(Kx - \omega t)$	$\rho A c\, \varepsilon_m^2 \omega^2 \sin^2(Kx - \omega t)$
$\langle P \rangle$	$\frac{1}{2}\mu v \omega^2 A^2$	$\frac{1}{2}\rho A c\, \varepsilon_m^2 \omega^2 = \frac{A(\Delta p_m)^2}{2\rho c}$
I	$\rho v \omega^2 A^2 \cos^2(Kx - \omega t)$	$\rho c\, \varepsilon_m^2 \omega^2 \sin^2(Kx - \omega t)$
$\langle I \rangle$	$\frac{1}{2}\rho v \omega^2 A^2$	$\frac{1}{2}\rho c\, \varepsilon_m^2 \omega^2 = \frac{(\Delta p_m)^2}{2\rho v}$
$TE_{\lambda/2}$	$\frac{1}{4}\mu \lambda A^2 \omega^2$	$\frac{1}{4}\mu \lambda\, \varepsilon_m^2 \omega^2$
	$T_y = T\frac{\partial y}{\partial x}$ $= \mu v \omega A \cos(Kx - \omega t)$	$\Delta p(x,t) = -B\frac{\partial \varepsilon}{\partial x}$; $\Delta p > 0$ $(comp)$ $\Delta p_m = \rho c \omega \varepsilon_m = B\varepsilon_m K$

Reflection from boundary:

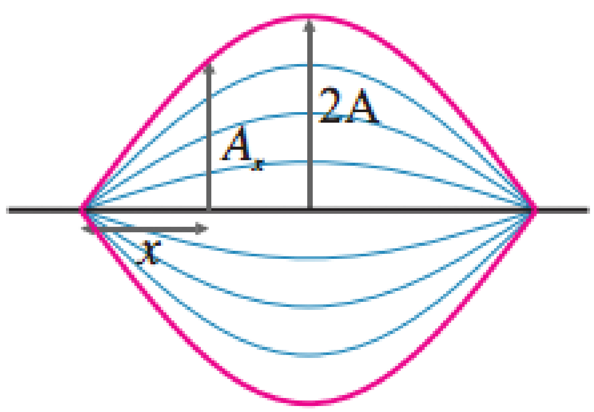
When a wave goes from a rarer medium to a denser medium, the reflected wave undergoes a phase change of π [& node is formed]. These phase changes are mathematical aspect only. [Antinode is formed in case of no phase change].

Rarer to denser:	$A_r = \frac{v_1 - v_2}{v_1 + v_2} A_i$ & $A_t = \frac{2v_2}{v_1 + v_2} A_i$	$v_1 > v_2$ $\mu_1 < \mu_2$
Denser to rarer:	$A_r = \frac{v_1 - v_2}{v_1 + v_2} A_i$ & $A_t = \frac{2v_1}{v_1 + v_2} A_i$	$v_1 < v_2$ $\mu_1 > \mu_2$

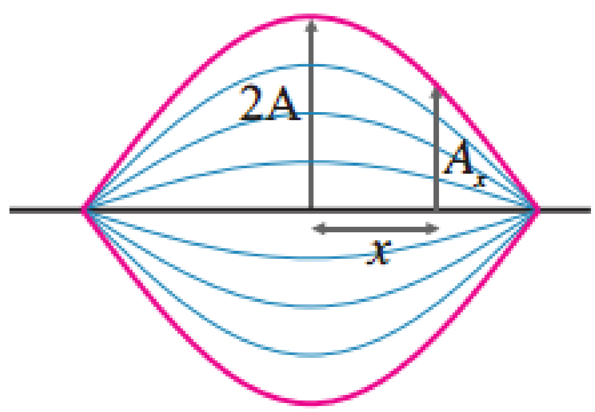
$$R = \frac{P_r}{P_i} = \left(\frac{A_r}{A_i}\right)^2 = \left(\frac{v_1 - v_2}{v_1 + v_2}\right)^2 = \left(\frac{\sqrt{\mu_2} - \sqrt{\mu_1}}{\sqrt{\mu_2} + \sqrt{\mu_1}}\right)^2 \qquad T = \frac{4\sqrt{\mu_1 \mu_2}}{\left(\sqrt{\mu_2} + \sqrt{\mu_1}\right)^2}$$

Superposition of waves: Conditions for superposition are:
(*i*) The superposing wave must be of same nature.
(*ii*) All superposing waves must reach the same location simultaneously.
Let E_1, E_2, E_3 … be total energies of *n* traveling waves before the superposition & E_1', E_2', E_3'... be the corresponding total energies of after the superposition. Then $E_1' = E_1$, $E_2' = E_2$, $E_3' = E_3$ … .
During the superposition, total energy of the resulting wave is: $E_1 + E_2 + E_3 \ldots + E_n$
Superposition of a *transverse & longitudinal mechanical* wave results in ripples.

Standing Wave: formed only for ***discreet*** frequencies.
Condition to produce standing wave: Two progressive waves of same nature having the same frequency and traveling in opposite direction should superpose each other.
When amplitude of both these waves is equal, a perfect standing wave is formed.
The general equation of a standing wave is given as:
$y = 2A\sin(Kx + \theta_1)\cos(\omega t + \theta_2)$.



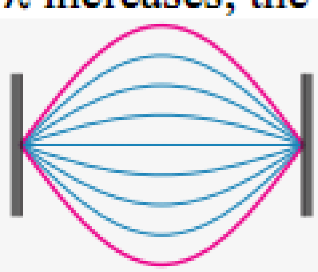
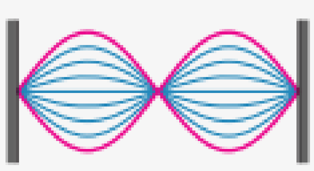
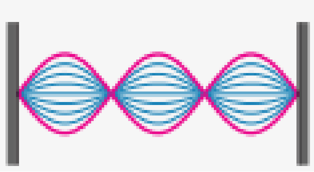
$A_x = 2A\sin Kx.$



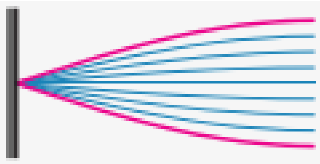
$A_x = 2A\cos Kx.$

All the particles in standing wave is performing SHM and come to its extreme position simultaneously. Moreover, they come to their mean position simultaneously.

From the equation $L = p\dfrac{\lambda}{2}$, we can understand that for a particular length of string if λ increases; the number of *modes* decreases.

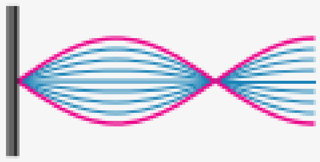
$L = \dfrac{\lambda_1}{2}$		First Harmonic or Fundamental Frequency	$L = p\dfrac{\lambda}{2}$
$L = \dfrac{2\lambda_2}{2}$		Second Harmonic First Overtone	$f = \dfrac{p}{2L}\sqrt{\dfrac{T}{\mu}} = \dfrac{p}{2L}\sqrt{\dfrac{Stress}{\rho}}$ *p*: Harmonic Number or loops or mode
$L = \dfrac{3\lambda_3}{2}$		Third Harmonic 2^{nd} Overtone	Octave means twice

$L = \frac{\lambda_1}{4}$



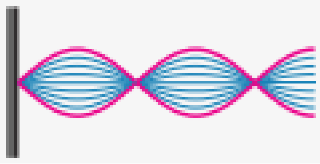
First Harmonic or Fundamental Frequency

$L = \frac{3\lambda_2}{4}$



Third Harmonic
First Overtone

$L = \frac{5\lambda_3}{4}$



Fifth Harmonic
2^{nd} Overtone

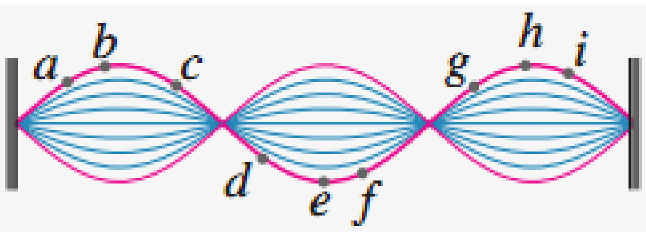
$$L = (2p-1)\frac{\lambda}{4}$$

$$f = \frac{2p-1}{4L}\sqrt{\frac{T}{\mu}}$$

$2p - 1$: Harmonic Number
$p - 2$: Overtone

If the frequency of the external source doesn't match with any natural frequency of the string, then resulting oscillation of the string will be small.

Phase in standing wave: $y = [2A\sin Kx]\cos\omega t = A'\cos\omega t$



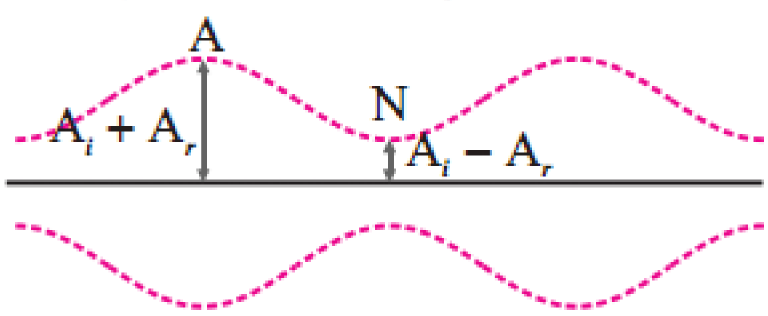
All particles contained within a loop are in same phase. Thus, in this case *a, b, c, g, h, i* are in same phase, while *d, e, f* are in opposite phase.

Energy in standing waves: $y = 2A\sin Kx\cos\omega t$

$$K_{\lambda/2} = \frac{1}{2}\mu\lambda\omega^2A^2\sin^2\omega t; \quad U_{\lambda/2} = \frac{1}{2}\mu\lambda\omega^2A^2\cos^2\omega t; \quad TE_{\lambda/2} = \frac{1}{2}\mu\lambda\omega^2A^2$$

When the string is in its mean position, the total energy for a loop is entirely kinetic, and when the string is in its extreme position, the energy is entirely potential.

Partial Standing wave: It is formed when the amplitudes of the superposing waves are not equal. An imperfect standing wave can be resolved into a standing wave and a traveling wave given as: $y = 2A_s\sin Kx\cos\omega t + A_t\sin(Kx - \omega t)$



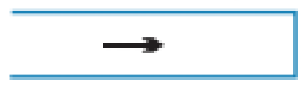
$$\text{Standing wave ratio (SWR)} = \frac{a_{max}}{a_{min}} = \frac{A_i + A_r}{A_i - A_r}$$

$$\text{Reflection Coefficient, } R = \left(\frac{A_r}{A_i}\right)^2 = \left(\frac{a_{max} - a_{min}}{a_{max} + a_{min}}\right)^2$$

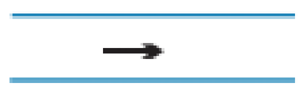
$$\text{Transmission Coefficient, } T = 1 - R = \frac{4a_{max}a_{min}}{(a_{max} + a_{min})^2}$$

- $\delta P > 0 \Rightarrow$ compression
- $\delta P << P \Rightarrow$ pressure change is much less than original P_o.
- $P + \delta P$: maximum atmospheric pressure
- $P - \delta P$: minimum atmospheric pressure
- $2\delta P$: maximum variation in pressure.
- $\beta = 10\log\frac{I}{I_o}$ $\quad I_o = 10^{-12}$ W/m^2 (Threshold intensity).

Standing waves in organ pipe: *Assumption*: Perfect reflection.



Displacement Node (Phase change of π)
Pressure Antinode (No Phase Change)



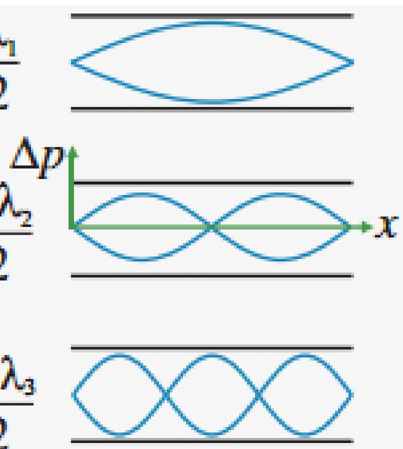
Displacement Antinode (No Phase Change)
Pressure Node (Phase change of π)

At a section where phase difference between incident wave and reflected wave is π node is formed. And the section where the incident wave and reflected wave are in phase, antinode is formed.

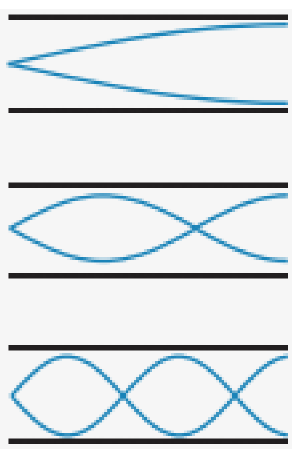
$\Delta p = 2\Delta p_m \sin Kx \cos \omega t$

$\Delta p_x = 2\Delta p_m \sin Kx$ $\quad$ 'x' is calculated from node.

$\Delta p_x = 2\Delta p_m \sin(Kx + \phi)$ $\quad$ 'x' is calculated from general point.



$L = \frac{\lambda_1}{2}$	First Harmonic or Fundamental Frequency	$L = p\frac{\lambda}{2}$ $\quad f = \frac{p}{2L}c$
$L = \frac{2\lambda_2}{2}$	Second Harmonic First Overtone	p: Harmonic Number
		End Correction: $L' = L + 0.3D*2$
$L = \frac{3\lambda_3}{2}$	Third Harmonic 2^{nd} Overtone	$f = \frac{p}{2L'}c$



$L = \frac{\lambda_1}{4}$	First Harmonic or Fundamental Frequency	$L = (2p-1)\frac{\lambda}{4}$
$L = \frac{3\lambda_2}{4}$	Third Harmonic First Overtone	$f = \frac{2p-1}{4L}c$
$L = \frac{5\lambda_3}{4}$	Fifth Harmonic 2^{nd} Overtone	$2p - 1$: Harmonic Number $p - 2$: Overtone

End Correction: $L' = L + 0.3D*2$ $\quad\quad f = \frac{2p-1}{4L'}c$

Beats: $\quad f_{beat} = f_1 - f_2$

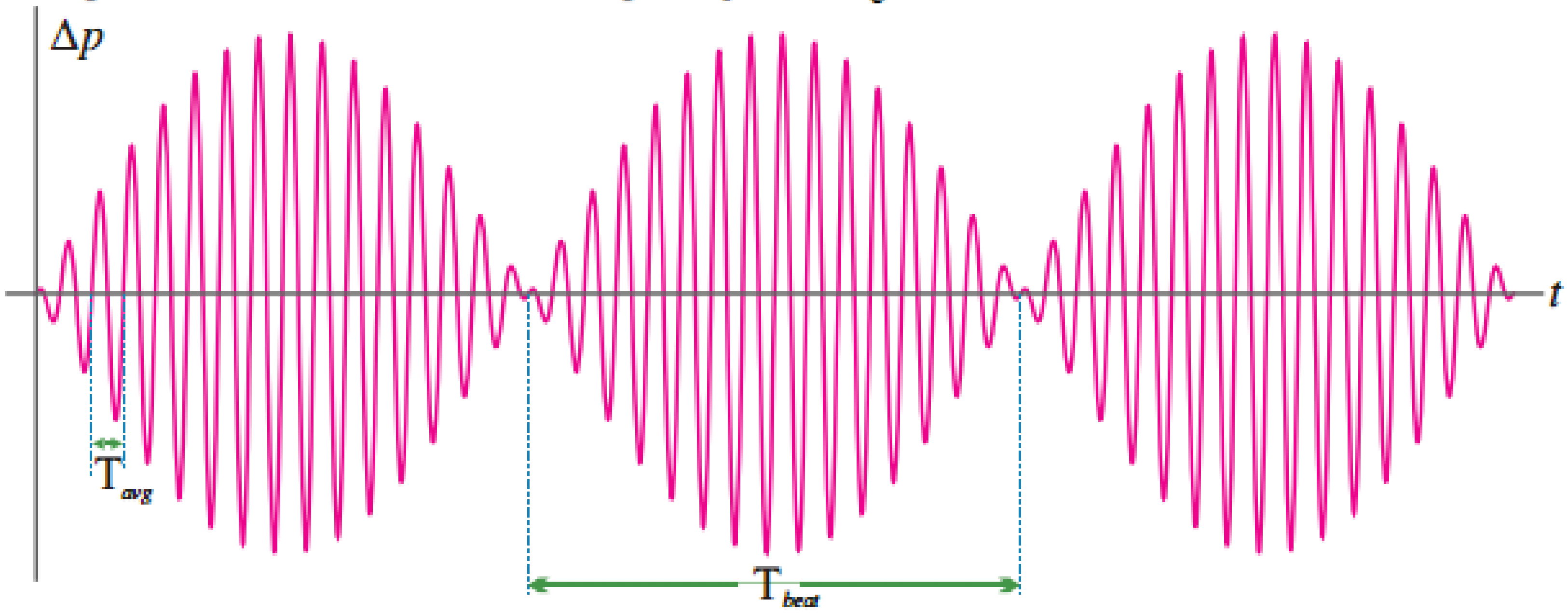
Conditions for beats to occur:

(i) Two or more waves of slightly different frequencies must be superposed.

(ii) The waves may move in any direction in space.

Frequency at which maximum amplitude is heard $= f_1 - f_2$

The particles oscillate with a frequency of ω_{avg}.



The time taken to reach from one minimum to another minimum is called beat time (T_{beat}). And the frequency is called *beat frequency*.
Apart from that we can also see that the intensity drops to zero many times during one beat cycle. The rate at which it does so is (on an average only, not exactly) is the average frequency. Or the time taken for that is $T_{avg} = 1/f_{avg}$

$f_{beat} > 10$: the beats are not audible to human ears.

If the prongs of the tuning fork are filed, its frequency increases and if the stem of the tuning fork is filed, its frequency decreases.
The prongs of the tuning fork are vibrated in its own plane in the same phase.

Doppler Effect: Wavelength changes due to motion of source **only**

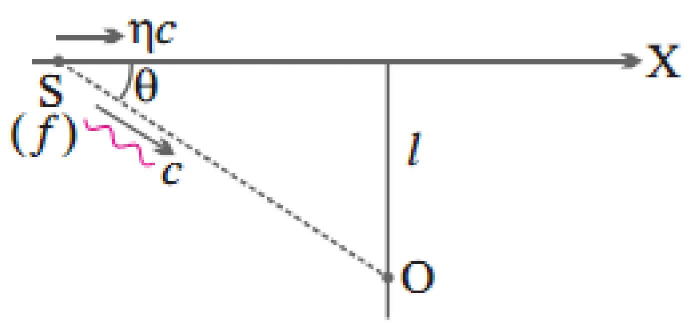
$\lambda' = cT - vT$ $\quad f' = \dfrac{c-u}{c-v} f$

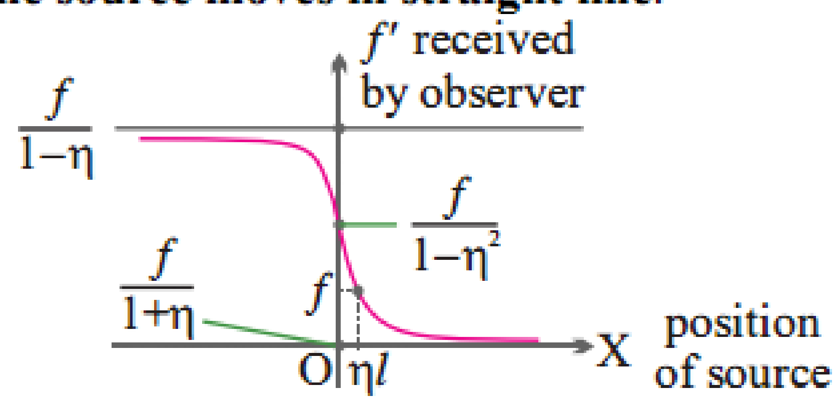
c: speed of sound $\quad$ v: speed of source

All the velocities above are written in same reference frame.

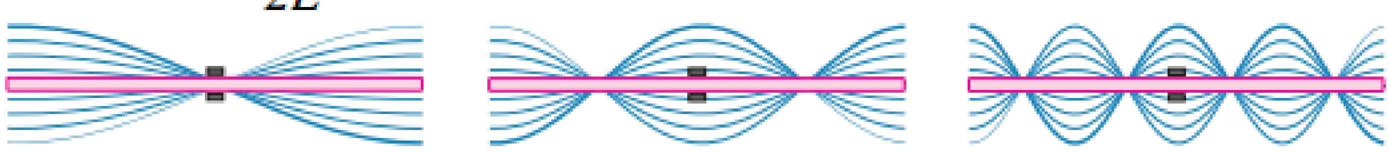
When *wind blows*, the speed of sound changes, but the frequency does not change, as frequency depends on source.

Frequency heard by observer when the source moves in straight line.





Rods: $f = \dfrac{2p-1}{2L} v_{rod}$ (*see below*) $\qquad$ **Rod clamped at one end:** $f = \dfrac{2p-1}{4L} v_{rod}$



$v_{rod} = \sqrt{\dfrac{Y}{\rho}}$

CALORIMETRY

Zeroth Law of Thermodynamics: It states that if bodies A & B are in thermal equilibrium with a body C, then they are in thermal equilibrium with each other.

$Q = \int_{T_i}^{T_f} C' dT$ ---(1) C': Heat capacity

$Q = m\int_{T_i}^{T_f} s\, dT$ ---(2) $s(c)$: Specific heat capacity

$Q = n\int_{T_i}^{T_f} C\, dT$ ---(3) C: Molar heat capacity

C', s, C : all are temperature dependent

$W = JH$; J = 4.2 'H is in calorie'

'C' of elemental solids, with few exceptions, have values close to 25 J/mol.K.

Water Equivalent:
When heat capacity of a particular body is equal to the mass of water in CGS system, then it is called water equivalent $C' = ms = m_w s_w = m_w$

Expansion of Solid – Liquid – Gases

For isotropic materials: $L = L_o e^{\alpha \Delta T} \approx L_o(1 + \alpha\,\Delta T)$

$\Delta L = L\alpha\Delta T$	:	linear expansion
$\Delta A = A.2\alpha.\Delta T$	:	Superficial expansion
$\Delta V = V.3\alpha.\Delta T$	:	Volume expansion
$\rho = \dfrac{\rho_o}{1+\gamma\Delta T}$	:	Density

α: Coefficient of linear expansion (it varies with temperature)

Metallic Scale: L: actual length of rod

$$\text{Reading} = \frac{L(1+\alpha_r\,\Delta T)}{1+\alpha_m\,\Delta T} = L[1 + (\alpha_r - \alpha_m)\Delta T]$$

α_r: expansion coefficient of rod; α_m: expansion coefficient of metallic scale

Simple Pendulum: $P = 2\pi\sqrt{\dfrac{l}{g}}$

If g = constant; $dP = 0.5\,P\,\alpha\,dT$

dP: error in P; P: time for which error is P is to be found (example: 1 year)

Thermal stress:

$$2\text{ m} \xrightarrow{\text{heating}} 2.5\text{ m} \xrightarrow[\text{stress}]{\text{compressive}} 2.2\text{ m}$$

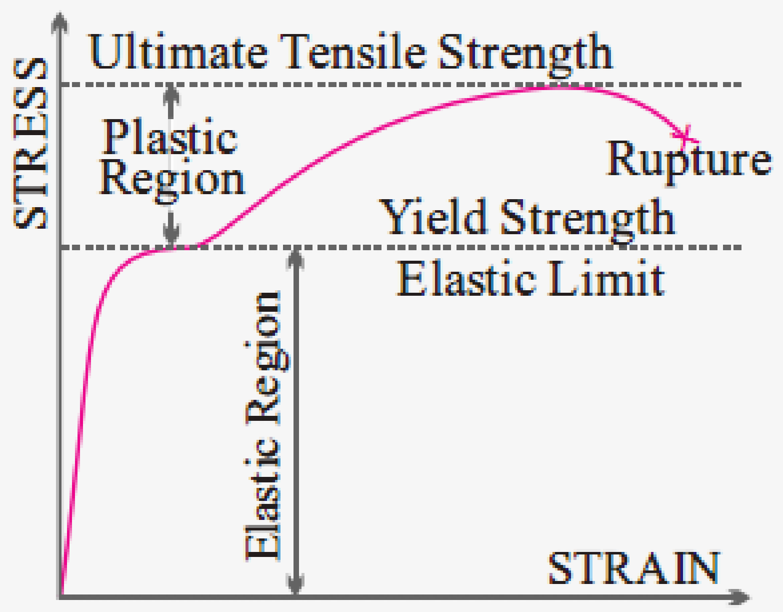
strain = 0   strain $= \dfrac{0.3}{2.5}$

stress = 0   stress ≠ 0

$$\left(\frac{\Delta L}{L_o}\right)_{thermal} + \left(\frac{\Delta L}{L_o}\right)_{tension} = \alpha\,\Delta T + \frac{F}{AY} = 0$$

$$\therefore \frac{F}{A} = -Y\,\alpha\,\Delta T$$

ELASTICITY



For linear portion,

stress = constant × strain.

$$\Rightarrow \frac{F}{A} = Y\frac{\Delta L}{L}$$

$F = \frac{YA}{L}\Delta L = K\Delta L$: Hooke's Law

PE stored in wire (U) = $= \frac{1}{2}\frac{YA}{L}(\Delta L)^2$

Energy density: $u = \frac{Y}{2}(\text{strain})^2 = \frac{1}{2}(\text{stress})\ \text{x}\ (\text{strain}) = \frac{1}{2Y}(\text{stress})^2$

***Bulk Modulus* & Compressibility**	:	$Volume\ Strain = \frac{\Delta V}{V}$; Normal stress $= \frac{F}{A}$ $K(B) = -\frac{\Delta P}{\Delta V/V}$ $\quad C = -\frac{1}{V}\frac{\Delta V}{\Delta P}$
Shearing stress	:	$Shear\ Stress = \frac{F}{A}$ $\quad Shear\ Strain = \frac{\Delta x}{y} = \tan\theta \approx \theta$ $G(\eta) = \frac{F/A}{\Delta x/y}$ (Shear/Rigidity Modulus)
Torsion	:	$\tau = k\theta$ where, $k = \frac{\pi G R^4}{2L}$: Torsional Rigidity
Poisson's Ratio	:	$\frac{dr}{r}\alpha\frac{dl}{l} \Rightarrow \frac{dr}{r} = \mu\frac{dl}{l}$

KTG

Equipartition theory: $k = R/N_A$

Energy of gas molecules per degree of freedom per mole is: $\frac{1}{2}RT$

Energy of gas molecule per degree of freedom per molecule is: $\frac{1}{2}kT$

Conclusion:

- $KE = E_{\text{int}} = \frac{3}{2}nRT = \frac{3}{2}NkT$ connects macroscopic property temperature with the microscopic property; the kinetic energy of the molecules.
- The average translational KE in C-frame of a gas contributes to its temperature or temperature is defined in C-frame.
- The KE associated with the motion of COM has no bearing on the gas temperature.
- Although the derivation is done assuming that the molecules doesn't collide

with each other, but the conclusion remains the same even if they did so.

- Real gases behave as an ideal gas at high temperature and low pressure. High temperature ensures randomness and large container ensures low pressure.

Internal Energy of gas (A Macroscopic Property): $\Delta E = nC_v\Delta T = \frac{1}{2} f\, nRT$

- It is intrinsic property of system.
- It doesn't depend on kind of process that system goes through.
- It depends only on initial & final state (thus a state function), $\therefore$ conservative
- $\Delta E = 0$ in a cyclic process.
- It cannot be defined for single gas molecule.
- Internal energy is $\langle KE \rangle_{translational}$ of gas in C-frame, which is equivalent to representing temperature of gas molecules.

	Translation	*Rotation*	*DOF* $f = 2C_v/R$	*Internal energy* (*E*) per molecule
Monatomic	3	0	3	$(3/2)kT$
Diatomic	3	2	5	$(5/2)kT$
Polyatomic	3	3	6	$(6/2)kT$

CO_2; although polyatomic, has 5 degrees of freedom only as its shape is linear.

$v_{avg} = \dfrac{\sum_{i=1}^{N} \lvert \vec{v}_i \rvert}{N}$	$v_{rms} = \sqrt{\dfrac{\sum_{i}^{N_1} v_i^2}{N_1}}$	$v_{avg} = \sqrt{\dfrac{8RT}{\pi M}}$	$v_{probable} = \sqrt{\dfrac{2RT}{M}}$

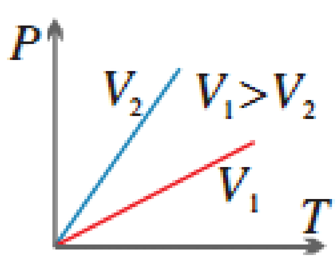
$$\bar{\lambda} = \frac{1}{\sqrt{2}\pi \times d^2 \frac{N}{V}} = \frac{m}{\sqrt{2}\pi d^2 \rho} = \frac{KT}{\sqrt{2}\pi d^2 P}$$: Mean Free Path

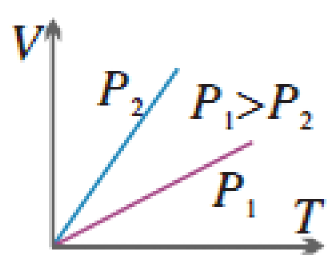
Variation of pressure in the atmosphere:

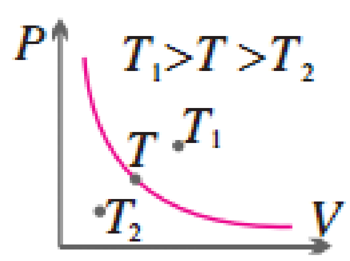
$$\left.\begin{aligned} dP &= -\rho\, g\, dy & ---(1) \\ dP &= -\frac{PM}{RT} g\, dy & ---(2) \end{aligned}\right\}$$ Equation (1) & (2) are always true.

$P = P_o e^{-\frac{Mgy}{RT}}$: True if T, M and g do not vary with height.

THERMODYNAMICS





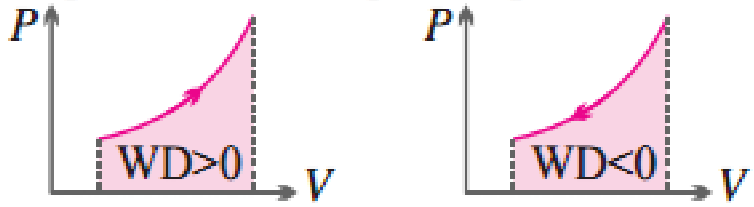


Sign Convention: $\partial Q_g = \partial W_s + \Delta E_{int}$: ∂Q_g & ∂W_s are *non-conservative*.

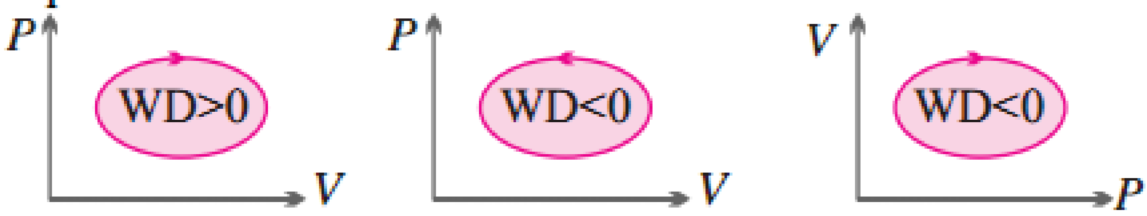
Heat gained, Increase in internal energy & WD by/of system is positive.

WD by gas (definition): $= \int P\, dV$

- WD by gas is always against an external force. If there is no external force acting on piston (other than force applied by gas), then *WD* by gas = 0.
- WD is area under P-V plot (since it is path dependent, ∴ non conservative).



- $W = 0$ for a process in which $V = constant$ at all the time.

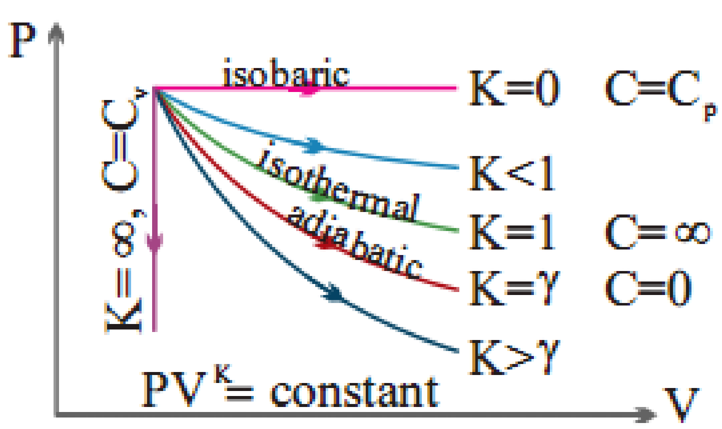


- $\frac{1}{\gamma P}$: Adibatic compressibility γP: Adibatic Bulk Modulous
  P: Isothermal Bulk Modulous $\frac{1}{P}$: Isothermal compressibility
- Work is not an intrinsic property of the system. We cannot say that a particular system possesses 10 J of work.

Diathermic: one which allows heat to flow.
Athermic (Adiabatic): *one which doesn't allow heat to flow.*

Thermodynamic Processes	WD by system	Increase in system's E_{int}	Heat gained by system
Isobaric V/T = c	$P(V_2 - V_1)$	$nC_v\Delta T$	$nC_p\Delta T$
Isothermal PV = c	$P_f V_f \ln\frac{V_f}{V_i} = nRT\ln\frac{P_i}{P_f}$	0	$P_f V_f \ln\frac{V_f}{V_i}$
Isochoric P/T = c	0	$nC_v\Delta T$	$nC_v\Delta T$
Adiabatic: $\Delta Q = 0$ $PV^\gamma = c$	$-nC_v\Delta T$	$nC_v\Delta T$	**0**
Polytropic $PV^K = c$	$\partial Q_g = nC\Delta T$	$nC_v\Delta T$	$\frac{nR\Delta T}{1-K}$

Polytropic Processes: $C = C_v + \frac{P}{n}\frac{dV}{dT} = \frac{R}{\gamma - 1} + \frac{R}{1-K}$; $K = \frac{C - C_p}{C - C_v}$



K	C	ΔE	ΔW	ΔQ
K=0	C=C_p	>0	>0	>0
K<1		>0	>0	>0
K=1	C=∞	=0	>0	>0
K=γ	C=0	<0	>0	=0
K>γ		<0	>0	<0

How to understand a process:

- Any sudden process is adiabatic.
- If the system is isolated, no heat leaves or enters the system (no matter the process is sudden or slow). So it's adiabatic.
- If the gas exchange heat from the surrounding, then for slow processes, the temperature is constant. So, it's isothermal.
- If the volume of container is not changed at all times (due to constraints), the process is isochoric. But, if $V_i = V_f$, $WD_g \neq 0$. Example cyclic process.
- To find if the process is isobaric, we must draw the free body diagram of the piston. And if we get "P = constant" inside the cylinder, then the process is isobaric.

Free Expansion (non-equilibrium process):

$W = 0, Q = 0$ $\therefore E_{int} = 0$ [free expansion]

Since the internal energy doesn't change, the temperature remains constant.
The free expansion is a good example of non-equilibrium process.

Efficiency

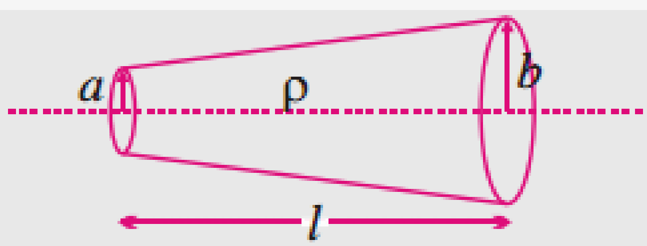
$$\eta = \frac{\text{What we get}}{\text{What we pay for}} = \frac{\text{work done by gas}}{\text{Heat supplied}} = \frac{|W|}{|H|} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

Efficiency of Carnot Cycle: $\eta = 1 - \dfrac{T_L}{T_H}$

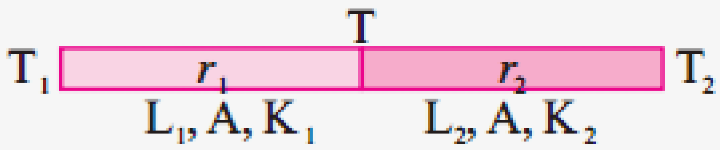
HEAT TRANSFER

$$H(i) = \frac{dQ}{dt} = K\,A\frac{d(-\theta)}{dx};\quad H(\text{or, } i) = K\,A\frac{T_H - T_C}{L};\quad \frac{d\theta}{dx}: \text{temperature gradient}$$

K: *thermal conductivity*

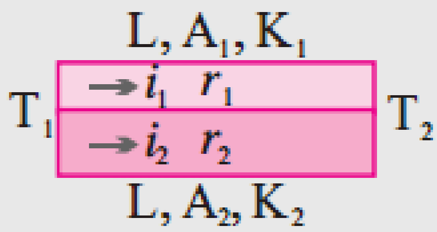
Body	Thermal Resistance
uniform slab	$R = \dfrac{L}{K\,A}$
Spherical shell:	$R = \dfrac{1}{4\pi K}\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$
Cylindrical shell: between inner & outer curved surface	$R = \dfrac{1}{2\pi\,K\,L}\ln\left(\dfrac{b}{a}\right)$
Cylindrical shell: (end faces)	$R = \dfrac{L}{\pi K\,(b^2 - a^2)}$
	$R = \dfrac{\ell}{\pi\,K\,a\,b}$

Thermal Resistance in Series and Parallel



$$T = \frac{T_2 r_1 + T_1 r_2}{r_1 + r_2}$$

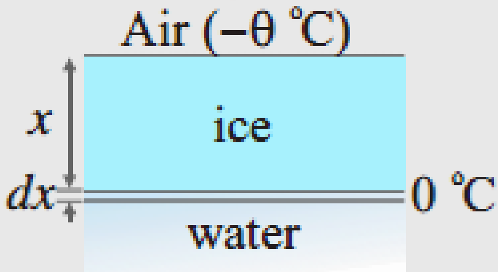
$$r_{eq} = r_1 + r_2 \qquad K_{eq} = \frac{L_1 + L_2}{r A}$$



$$i_1 = \frac{R_2}{R_1 + R_2} i; \qquad i_2 = \frac{R_1}{R_1 + R_2} i$$

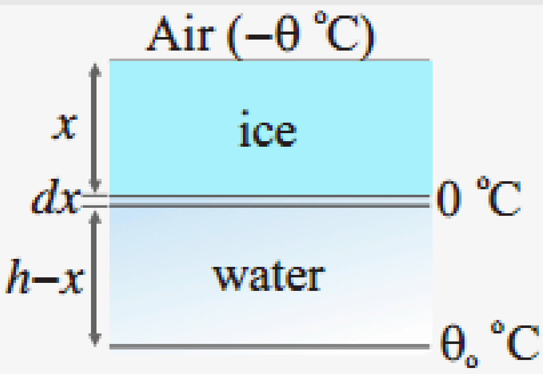
$$\therefore K_{eq} = \frac{L}{r(A_1 + A_2)} = \frac{(K_1 A_1 + K_2 A_2)}{A_1 + A_2}$$

Growth of ice on pond:



$$\rho A L_f \frac{dx}{dt} = \frac{K A \theta}{x}$$

$$\Rightarrow \frac{x_2^2 - x_1^2}{2} = \frac{K \theta t}{\rho L_f}$$



$$\frac{K_w A(\theta_o - 0)}{h - x} + \frac{(\rho A dx)}{dt} L_f = \frac{K_i A \theta}{h}$$

L: Latent heat

ρ: Density of ice

THERMAL RADIATION

The radiation emitted by a hot body depends on following factors:

- *Temperature, Material of body, wavelength distribution*
- *Shape and size of the body* – greater size implies more intense.
- *Nature of its surface*: radiation increases if the surface is roughened.

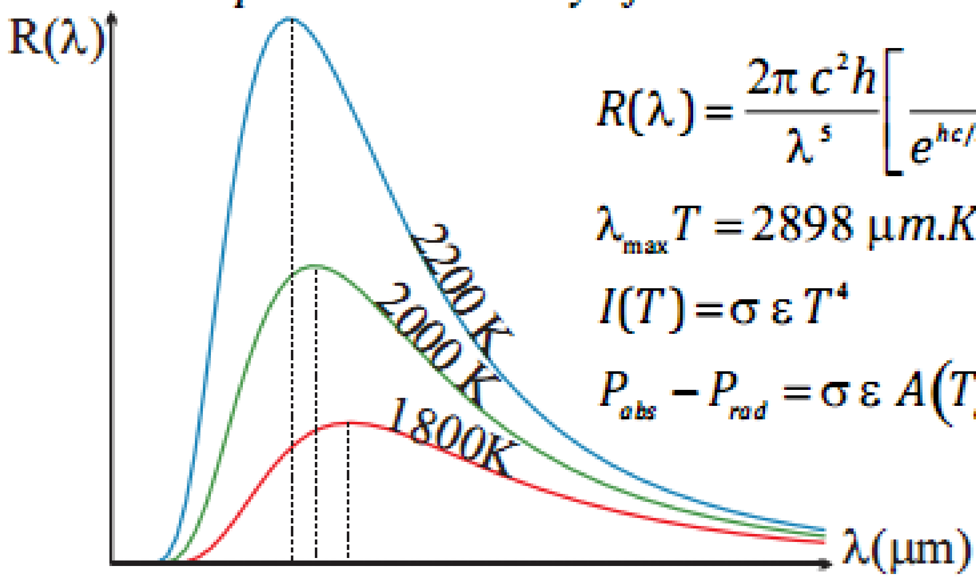
Kirchoff's Law: *A good absorber is a good emitter* ($a = e$).

Properties of Black body:

- Perfect emitter in each direction
- Perfect emitter at every wavelength & all incident angle.
- Radiation is isotropic for a black body.

R(λ) or **dEλ/dλ**: *Spectral Radiancy* (or ***spectral emissive power***)

I: *Emissive power* or *intensity of emitted radiation*



$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5}\left[\frac{1}{e^{hc/\lambda KT} - 1}\right] \qquad ---(1)$$

$$\lambda_{max} T = 2898\ \mu m.K \qquad ---(2)$$

$$I(T) = \sigma\,\varepsilon\,T^4 \qquad ---(3)$$

$$P_{abs} - P_{rad} = \sigma\,\varepsilon\,A\left(T_{env}^4 - T^4\right) \qquad ---(4)$$

Radiation from Sun: Energy received by earth is: $S.\pi R_e^2$

Solar constant is the amount of energy received by earth from sun.

$$S = \frac{P_{rad}}{A_{spherical\ surface}} = \frac{\sigma(4\pi R_s^2)T_s^4}{4\pi d^2} = \sigma\,T_s^4\left(\frac{R_s}{d}\right)^2$$

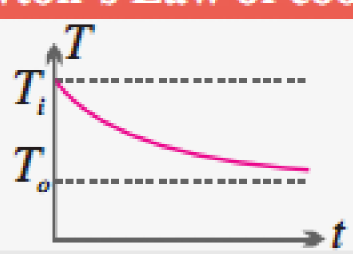
d : distance between sun & earth

R_e : Radius of Earth

R_s : Radius of Sun

T_s : Temperature of Sun

Newton's Law of cooling

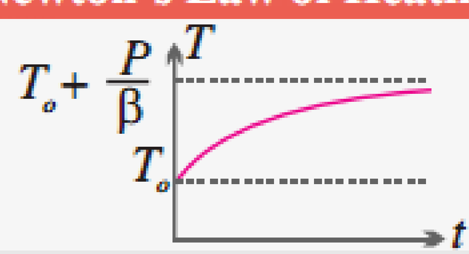


$$\frac{dQ}{dt} = \sigma A\left[T^4 - T_o^4\right]$$

$$-m\,s\frac{dT}{dt} = \beta(T - T_o)$$

$$T - T_o = (T_i - T_o)e^{-\frac{\beta}{ms}t}$$

Newton's Law of Heating



Rate of gain of heat $= P - \sigma A\left[T^4 - T_o^4\right]$

$$\frac{dQ}{dt} \approx P - \beta(T - T_o)$$

$$\therefore T - T_o = \frac{P}{\beta}\left(1 - e^{-\frac{\beta t}{ms}}\right)$$

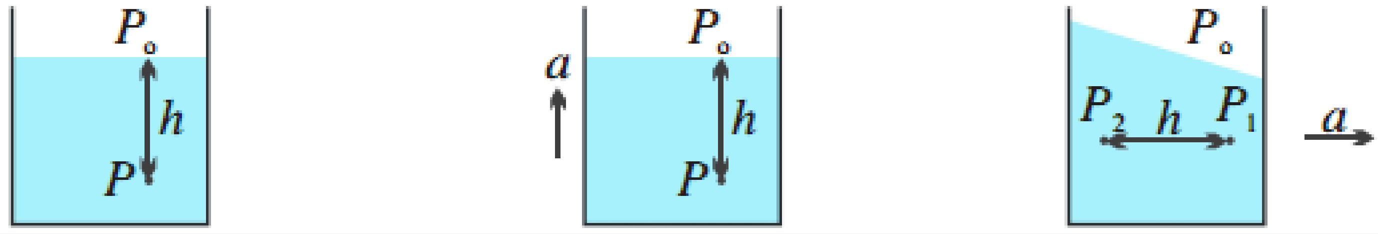
Average temperature method

$$\frac{T_1 - T_2}{t_o} = \frac{\beta}{m\,s}\left[\frac{T_1 + T_2}{2} - T_o\right]$$

Average temperature method

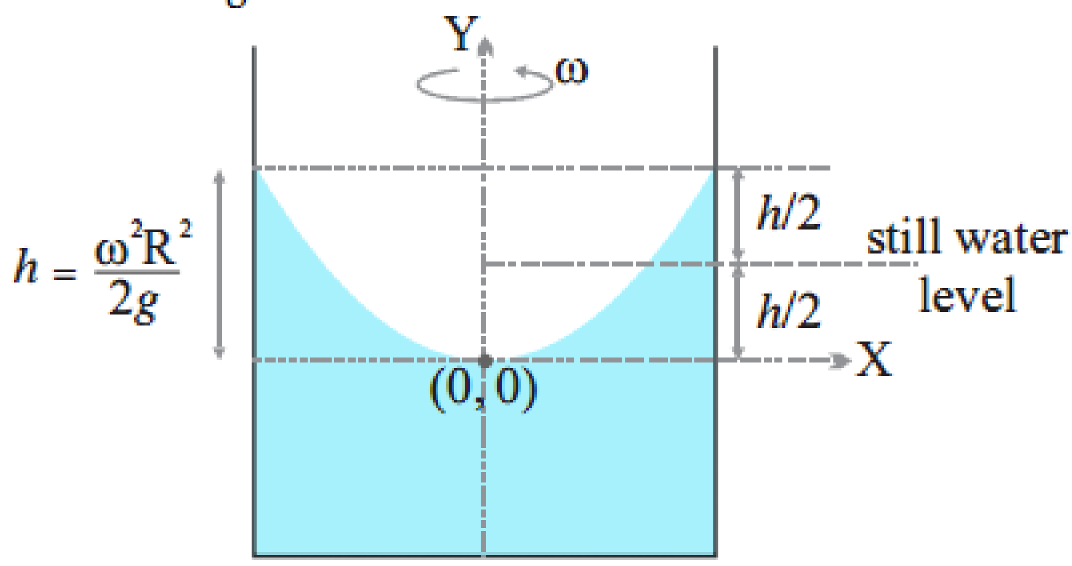
$$m\,s\frac{T_1 - T_2}{t_o} = P - \beta\left[\frac{T_1 + T_2}{2} - T_o\right]$$

Fluid is a substance that cannot sustain shear stress when at rest.



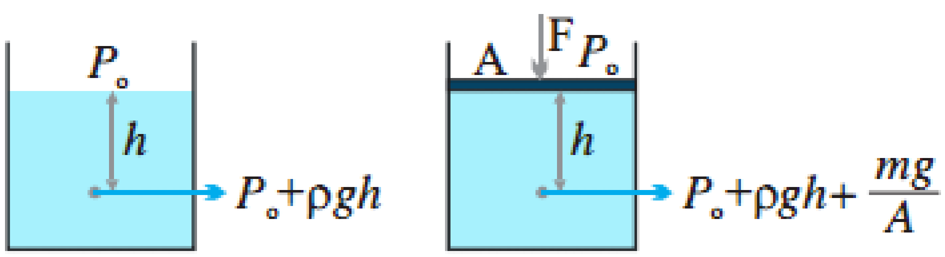
$P = P_o + \rho g h$	$P = P_o + \rho(g+a)h$	$P_2 - P_1 = \rho a h$

Viscous fluid in a rotating vessel:



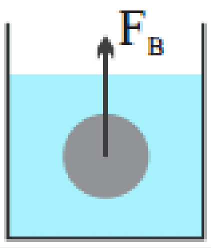
***Force applied by fluid on walls of a container*:**

Horizontal direction	$F_x = (P_{CM}$ at chosen section$) \times (A_{proj}$ of that section$)$
Vertical direction	Draw FBD of fluid/solid (after replacing it with fluid).

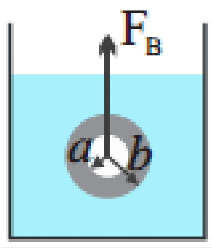


Pascal's Principle: A change in pressure applied to an *incompressible* fluid is transmitted undiminished to every portion of the fluid & to the walls of its container

Archimedes' Principle:



$$F_B = \rho V g = \frac{mg}{\gamma}$$
$$\gamma = \rho_{body}/\rho_{fluid}$$



$$F_B = \rho V g$$
$$V = \frac{4}{3}\pi\left(b^3 - a^3\right)$$

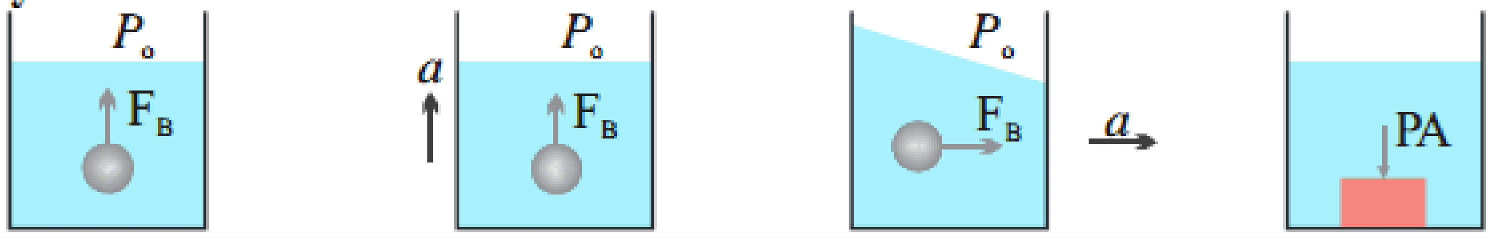
For bodies with no cavities: $B = \frac{m'g}{\gamma}$	$B = m_f g = \rho_f V_f g$

Condition for floatation: $m = m_f$

m: mass of body, m_f: mass of displaced fluid, m': mass of body inside fluid

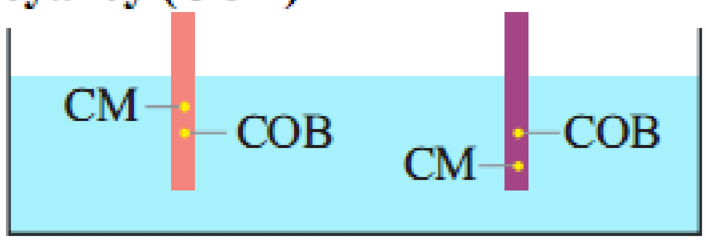
- Buoyant force is pressure difference force. When we draw FBD, either we show the pressure difference, or the buoyant force.
- Buoyant force should act at the COM of displaced water. And that location is called *Center of Buoyancy*.
- Buoyant force is conservative in nature.
- If an object experiences buoyant force, fluid experiences the same force in opposite direction. [Every action has equal and opposite reaction]

Buoyant force in various situations:



$F_B = \rho_f V_f\, g$ $\quad$ $F_B = \rho_f V_f (g + a)$ $\quad$ $F_B = \rho_f V_f\, a$ $\quad$ $F_B = 0$

COM vs Center of buoyancy (COB)



unstable equilibrium $\quad$ stable equilibrium

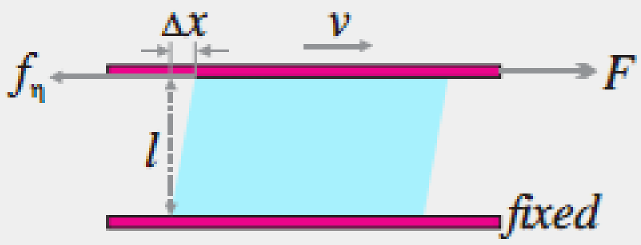
Definitions:

1. *Streamline*: The locus of point where the tangent at each point indicates direction of fluid velocity at that point is a streamline. Since streamlines are tangent to the velocity vector at every point in the flow field, there cannot be any flow across the *streamline*.
2. *Non-uniform flow*: It is one where velocity is dependent on one, two, or three-dimension depending on the number of space coordinate required to specify the velocity field
3. *Steady flow*: $d\eta/dt = 0$, where η is fluid property (viz; velocity, pressure, density). Thus, in steady flow, any property may vary from point to point in the field, but all properties remain constant with time at any point 'P'.
   - *In steady flow, pattern of streamline does not change with time.*
   - *A non-uniform flow can be steady in nature.*
   - *Whether or not the flow is steady; depends on reference frame.*
4. *Velocity flux*: $\phi_v = \int \vec{v} \cdot d\vec{A}$
5. *Irrotational flow*: Suppose a grain is kept on a moving fluid. Then the grain does not rotate about its axis. For irrotational flow: $\oint \vec{v} \bullet d\vec{l} = 0$ across any closed loop.
6. *Incompressible flow: $\rho = const.$*

Equation of continuity	***Bernoulli's Equation***
It is *Mass conservation principle* • The flow is steady. • The fluid is incompressible. • No flow across bounding streamline. • $Q_V = Av =$ *volume flow rate* • $Q_m = \rho AV =$ *mass flow rate* • **Momentum flux:** $\phi_P = \dfrac{\Delta m}{\Delta t} v = Q_m\, v = \rho Q v.$	It is *work energy theorem* • The fluid is non-viscous. • The flow is steady. • The fluid is incompressible. • The flow is irrotational. [if the flow is rotational, the it is applied along one streamline only] • Flow along a streamline. • $P_1 + \frac{1}{2}\rho v^2 + \rho g\, y = \text{constant}$

VISCOSITY & SURFACE TENSION

Viscous force between parallel plates:



$$f_\eta = \eta A\frac{v}{l} = \eta A\frac{dv}{dl}$$

$$\frac{v}{l} = \frac{dv}{dx}: \text{ velocity gradient}$$

η: *Dynamic viscosity*	*Kinematic Viscosity* (μ): $\mu = \eta / \rho$
SI Unit : Pa.s, Ns/m^2, Poiseuille CGS Unit : Dyne.s.cm^2, Poise	SI Unit : m^2/s CGS Unit : cm^2/s or stoke
1Pa.s = 10 Poise	1 m^2/s = 10^4 stoke
η↑ Temperature ↓ for gas	η↑ Temperature ↑ for liquid

Stokes Law: *Viscous force on ball moving in a medium*: $F = 6\pi\eta r v$

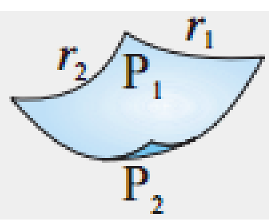
Ball Falling down	***Air bubble rising up***
$v = \frac{2(\rho-\sigma)g r^2}{9\eta}\left(1 - e^{-\frac{9\eta}{2r^2\rho}t}\right)$	$v = \frac{2\sigma g r^2}{9\eta}\left(1 - e^{-\frac{9\eta}{2r^2\rho}t}\right)$
ρ: density of ball/bubble σ: density of medium	*r*: radius of ball/bubble η: coefficient of dynamic viscosity

SURFACE TENSION

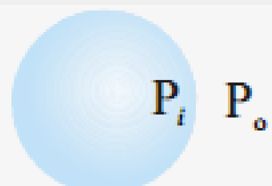
Surface Tension: It is a surface phenomenon.

$W = T\Delta A \leftarrow$ surface energy [W: internal free energy]

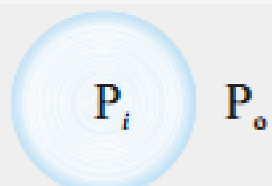
Excess pressure (general) : $P_1 - P_2 = T\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$



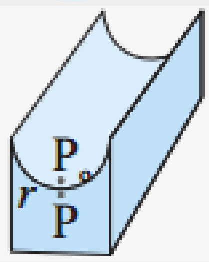
Drop and air bubble : $P_i - P_o = \frac{2T}{R}$



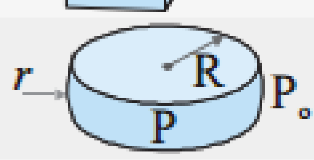
Soap bubble : $P_i - P_o = \frac{4T}{R}$



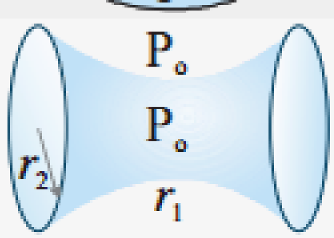
Straight Cylinder : $p = \frac{T}{r}$



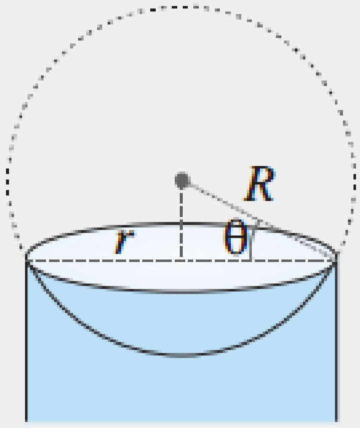
Tablet : $p = T\left(\frac{1}{r} + \frac{1}{R}\right)$



Catenoid : $\frac{1}{r_1} - \frac{1}{r_2} = 0$



Capillary rise (sufficient length tube)

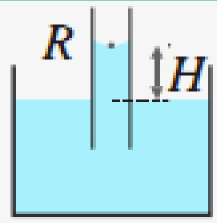


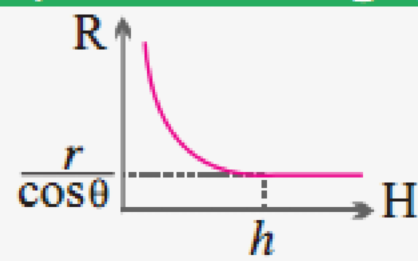
$$R\cos\theta = r$$

$$h = \frac{2T\cos\theta}{\rho g r}$$

$$Q_L = \frac{2\pi T^2\cos^2\theta}{\rho g}$$

Capillary rise (insufficient length)





$$R = \frac{2T}{\rho g H}$$

Effect of temperature on surface tension

Surface Tension always decreases with increase in temperature.

$$\alpha = \mu B\left(1-\frac{T}{T_c}\right)\left[1+b\left(1-\frac{T}{T_c}\right)\right]$$

$\mu = 1.256,\ B = 235.8\ mN/m,\ b = -0.625,$

T : Temperature, T_c: crictical temperature

T_c: temperature at which surface tension becomes zero.

This happens so as the liquid doesn't have any surface at that temperature.

The critical temperature of a substance is the temperature at and above which vapor of the substance cannot be liquefied, no matter how much pressure is applied. Critical temperature of water: 374°C

Effect of pressure on surface tension: As pressure increases, surface tension reduces. The critical pressure of a substance is the pressure required to liquefy a gas at its critical temperature.

Effect of temperature on Contact Angle: Contact angle may either increase or decrease with temperature, depending on the relative magnitude of surface entropies of the two phases. But the change in contact angle with change in temperature is very small of the order of 0.05 degrees/°C, at ordinary temperatures. However, at temperature near boiling point, the liquid surface tension reduces rapidly with temperature, and the contact angle will rapidly approach zero. Thus, for water the contact angle is zero at 100°C.

Surfactant compounds reduce the surface tension considerably.

Basics

1. Electric charge is quantized. Electric charge is independent of frame of reference; it's an absolute property. There is no physical process in which the electrical charge of a system is not conserved.

2. Like charges repel, unlike charges attract each other.

3. A charged body can attract a neutral conducting body.

4. A charged body can attract a neutral non-conducting body.

5. The force between two charged particles is given by:

   $\vec{F} = K\dfrac{q_1 q_2}{r^3}\vec{r} \qquad K = 9\times10^9\, N\,m^2/C^2$

   The direction of $\vec{r}$ is towards the body on which force is found.

   It's a central force, so conservative in nature. $\oint \vec{E}\cdot d\vec{l} = 0$

   Note: The law is valid only for static point charges. If the charges are moving with velocities comparable with the speed of light, then coulombs law is not applicable. In such a case time lag must be taken into account.

6. $\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \ldots + \vec{F}_{1n}$: Superposition Principle.

7. Electrostatic field lines (EFL):
   - The tangent to electric field lines gives the direction of electric field.
   - Denser the field electric field line more is the electric field.
   - Denser the potential lines, more is the electric field.
   - Neither do they form any closed loop, nor do they intersect.
   - Number of EFL is proportional to the magnitude of charge present.
   - A uniform electric field line can never drop to zero suddenly.
   - Field lines come from a positive charge and goes in the negative charge.

8. Electric field is dependent upon the charge distribution and not where the charge is distributed.

9. For stable equilibrium along X-axis: $q\dfrac{dE}{dx} < 0$

   Example: A positive charge is in stable equilibrium between two positive charges along X-axis

Electric field and potential for various charge configurations.

$\vec{E} = -\dfrac{KQ}{r^3}\vec{r}$. *The direction of $\vec{r}$ is towards the point where we wish to find the field. Thus, the direction of $\vec{E}$ is always towards the charge that causes the field.*

Point charge: $V = \dfrac{KQ}{r}$. It's a scalar quantity.

Body	Electric Field	Electric potential
Ring	$\dfrac{Kqx}{\left(R^2+x^2\right)^{3/2}}$; $E_{\max}\big\|_{x=R/\sqrt{2}}$	$\dfrac{Kq}{\sqrt{x^2+R^2}}$
Disc	$\dfrac{\sigma}{2\varepsilon_o}\left(1-\dfrac{x}{\sqrt{x^2+R^2}}\right)$	$\dfrac{\sigma}{2\varepsilon_o}\left[\sqrt{x^2+R^2}-x\right]$
	$\dfrac{K\lambda}{r}\begin{bmatrix}\left(\sin\theta_1+\sin\theta_2\right)\hat{i}\\ -\left(\cos\theta_1-\cos\theta_2\right)\hat{j}\end{bmatrix}$	For finite rod: Can find ΔV for ∞ rod.
Apex of cone	∞	$\sigma R/2\varepsilon_o$
infinite plate	$E=\sigma/2\varepsilon_o$ *surface density* σ (*charge on one side only*)	$\Delta V=\dfrac{\sigma}{2\varepsilon_o}d$
infinite plate	$E=\begin{cases}\dfrac{\sigma}{\varepsilon_o}\text{:outside the conductor}\\ 0\ :\ \text{inside the conductor}\end{cases}$ *surface density* σ (*charge on both sides*)	$\Delta V=\dfrac{\sigma}{\varepsilon_o}d$
infinite wire	$E=\lambda/2\pi\varepsilon_o r$	$\Delta V=\dfrac{\lambda}{2\pi\varepsilon_o}\ln\dfrac{r_1}{r_2}$
Hollow Sphere	$\vec{E}=\begin{cases}0 & 0\le r<R\\ \dfrac{Kq}{r^2}\hat{r} & r\ge R\end{cases}$	$V=\begin{cases}\dfrac{Kq}{R} & 0\le r<R\\ \dfrac{Kq}{r} & r\ge R\end{cases}$
Solid sphere	$\vec{E}=\begin{cases}\dfrac{Kq\vec{r}}{R^3}=\dfrac{\rho\vec{r}}{3\varepsilon_o} & 0\le r<R\\ \dfrac{Kq}{r^2}\hat{r} & r\ge R\end{cases}$	$V=\begin{cases}\dfrac{Kq}{2R}\left(3-\dfrac{r^2}{R^2}\right) & 0\le r<R\\ \dfrac{Kq}{r} & r\ge R\end{cases}$
Hollow Cylinder	$\vec{E}=\begin{cases}0 & r<R\\ \dfrac{\sigma R}{\varepsilon_o r}\hat{r} & r\ge R\end{cases}$	$\Delta V=\begin{cases}0 & r<R\\ \dfrac{\sigma R}{\varepsilon_o}\ln\dfrac{r_2}{r_1} & r\ge R\end{cases}$
Solid Cylinder	$\vec{E}=\begin{cases}\dfrac{\rho r}{2\varepsilon_o}\hat{r} & r<R\\ \dfrac{\rho R^2}{2\varepsilon_o r}\hat{r} & r\ge R\end{cases}$	$\Delta V=\begin{cases}\dfrac{\rho(R^2-r^2)}{4\varepsilon_o} & r<R\\ \dfrac{\rho R^2}{2\varepsilon_o}\ln\dfrac{r_2}{r_1} & r\ge R\end{cases}$

10. $V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{r}$ & $W_s = -\Delta U = -(U_f - U_i) = q\left(V_i - V_f\right)$

$q\left(V_i - V_f\right)$ indicates the work done *by the electric field* to move charge q from potential V_i to potential V_f. *Don't forget sign of charge*.

11. Electric field direction will be towards maximum potential gradient.

$$\therefore \vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) \quad \text{: In Cartesian Co-ordinate system}$$

$$\vec{E} = -\left[\frac{\delta V}{\delta r}\hat{e}_r + \frac{1}{r}\frac{\delta V}{\delta \theta}\hat{e}_\theta + \frac{1}{\sin\theta}\frac{\delta V}{\delta \phi}\hat{e}_\phi\right] \quad \text{: In Polar Co-ordinate system.}$$

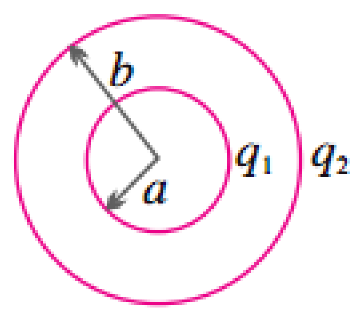
12. $$\frac{\rho}{\varepsilon_o} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial z} + \frac{\partial E_z}{\partial z} = -\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right) \text{: Cartesian Coordinate}$$

$$\rho(r) = -\varepsilon_o\left[\frac{\partial^2 V}{\partial r^2} + \frac{2}{r}\frac{\partial V}{\partial r}\right] \text{: Polar Coordinate}$$

13. $V = \sum_{i=1}^{n} V_i = K\sum_{i=1}^{n}\frac{q_i}{r_i}$, *Potential at a point due to point charges.*

14. *Consider two concentric spherical shells of radius a & b respectively.*

$$V = \begin{cases} \left(\dfrac{K\,q_1}{a} + \dfrac{K\,q_2}{b}\right) & 0 \le r \le a \\ \left(\dfrac{K\,q_1}{r} + \dfrac{K\,q_2}{b}\right) & a \le r \le b \\ \left(\dfrac{K\,q_1}{r} + \dfrac{K\,q_2}{r}\right) & b \le r < \infty \end{cases}$$



$$U = \frac{1}{8\pi\varepsilon_o}\frac{q_1^2}{a} + \frac{1}{8\pi\varepsilon_o}\frac{q_2^2}{b} + \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{b}. = (self + interaction)\ energy$$

Electrostatic energy doesn't follow superposition principle.

15. *Electrical potential energy (interaction energy):* $U = qV$

16. *Potential at a particular point cannot be defined.*

17. To change reference of potential from ∞ to point $r = r_o$; subtract the potential at $r = r_o$ at every location.

18. *Self energy of a solid* sphere: $U = \dfrac{Q^2}{40\pi\varepsilon_o R} + \dfrac{Q^2}{8\pi\varepsilon_o R} = \dfrac{3Q^2}{20\pi\varepsilon_o R}$

19. *Energy density:* $u = \dfrac{1}{2}K\varepsilon_o E^2$

20. *Electrostatic pressure,* $P = \dfrac{1}{2}\varepsilon_o E^2 = \dfrac{\sigma^2}{2\varepsilon_o}$

Gauss law:

Area of Gaussian surface

Electric flux $\longrightarrow \phi_E = \oint \vec{E}.d\vec{A} = \dfrac{q_{enc}}{\varepsilon_o} \longleftarrow$ *Charge inside Gaussian surface*

Electric field on Gaussian surface

Note: Although we consider the charge enclosed, the *electric field* (*on the Gaussian surface*) is due to the charges both *inside* and *outside* the Gaussian surface.

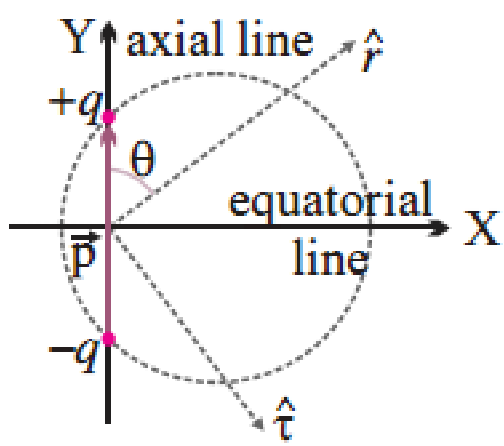
Problem solving Approach: Our goal will always to choose a Gaussian surface so as to reduce $\oint \vec{E}.d\vec{A} = E\,A$.

Dipole:

- Dipole moment is characteristic of ref. point & not charge. For a pure dipole it is directed from – ive to +ive charge.
- Dipole moment of charge q (kept at $\vec{r}_f$) w.r.t reference A (at $\vec{r}_i$) is given as:

  $\vec{p} = q(\vec{r}_f - \vec{r}_i)$
- Dipole moment of charge configuration is independent of reference point if its total charge is zero.
- If a charge distribution has a nonzero charge, the location about which the dipole moment is zero is: $\vec{r}_o = \sum_{i=1}^{n} q_i \vec{r}_i \Big/ \sum_{i=1}^{n} q_i$

Potential due to electric dipole:



$$V = \frac{1}{4\pi\varepsilon_o}\frac{\vec{p}\cdot\vec{r}}{r^3}$$

$$\vec{E} = \frac{1}{4\pi\,\varepsilon_o r^3}[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}]$$

$$\vec{E}_{axial} = \frac{2\vec{p}}{4\pi\,\varepsilon_o r^3} \qquad \vec{E}_{equitorial} = \frac{-\vec{p}}{4\pi\,\varepsilon_o r^3}$$

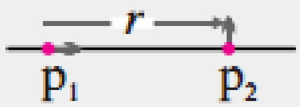
$$\therefore E_{axial} = 2E_{equitorial}$$

Torque on a pure dipole in a uniform Electric field: $\vec{\tau} = \vec{p}\times\vec{E}$

Potential Energy of an Electric Dipole in external Electric field: $U = -\vec{p}\cdot\vec{E}$

PE (U) and force between the two dipoles shown.

$$U = -\frac{1}{4\pi\,\varepsilon_o r^3}\left[3(\vec{p}_1\cdot\hat{r})(\vec{p}_2\cdot\hat{r}) - (\vec{p}_1\cdot\vec{p}_2)\right] \qquad \vec{F} = -\frac{dU}{dr}\hat{r}$$

Dipole configuration	Interaction Potential energy	Interaction force
	$U = -\dfrac{2p_1p_2}{4\pi\varepsilon_o r^3}$	$\vec{F} = -\dfrac{6p_1p_2}{4\pi\varepsilon_o r^4}\hat{r}$ attractive
	$U = \dfrac{p_1p_2}{4\pi\varepsilon_o r^3}$	$\vec{F} = \dfrac{3p_1p_2}{4\pi\varepsilon_o r^4}\hat{r}$
	$U = 0$	$\vec{F}_{net} = \dfrac{3p_1p_2}{4\pi\varepsilon_o r^4}\hat{r}$

The last one is tough to derive so better memorize it.

monopole: $V \,\alpha\, \dfrac{1}{r}$, $E \,\alpha\, \dfrac{1}{r^2}$

dipole: $V \,\alpha\, \dfrac{1}{r^2}$, $E \,\alpha\, \dfrac{1}{r^3}$, $U \,\alpha\, \dfrac{1}{r^3}$, $F \,\alpha\, \dfrac{1}{r^4}$

Force on a dipole in a non- uniform $\vec{E}$

$$F_x = p_x\frac{\delta E_x}{\delta x} + p_y\frac{\delta E_x}{\delta y} + p_z\frac{\delta E_x}{\delta z}$$

$$F_y = p_x\frac{\delta E_y}{\delta x} + p_y\frac{\delta E_y}{\delta y} + p_z\frac{\delta E_y}{\delta z}$$

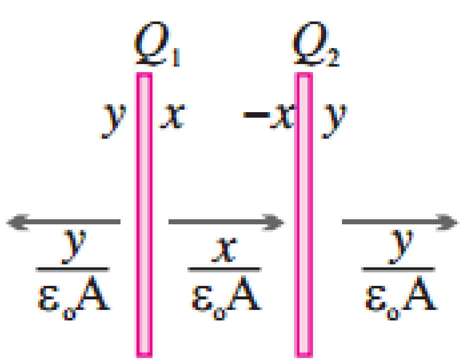
$$F_z = p_x\frac{\delta E_z}{\delta x} + p_y\frac{\delta E_z}{\delta y} + p_z\frac{\delta E_z}{\delta z}$$

Charged-isolated conductor: At electrostatic equilibrium

i. Electric field at all points inside a conductor is zero.
ii. No excess charge can reside inside a conductor.
iii. A conductor is an equipotential surface.
iv. Electric field is always normal to the surface of a conductor and has a magnitude of σ/ε_o near the surface of conductor.
v. A conductor behaves as a Faraday Cage.
vi. Charge is distributed on the surface of a conductor according to equation:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Parallel plates



On Earthing a conductor, the potential of conductor becomes zero. Charge may or may not become zero.

1. The force of attraction forms Newton's third law action-reaction pair & is given by:

$$\vec{F} = -G\frac{m_1 m_2}{r^3}\vec{r} \quad G = 6.67\times10^{-11} m^3 / Kg.s^2$$

The direction of $\vec{r}$ is towards the body on which force is found.

2. **Gravitation field and potential for various configurations.**

$\vec{E} = -\frac{GM}{r^3}\vec{r}$. *The direction of $\vec{r}$ is towards the point where we wish to find field.*

Thus, the direction of $\vec{E}$ is always towards the mass that causes the field.

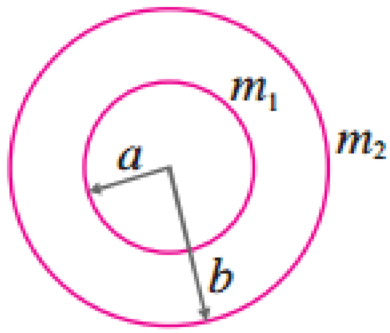
Body	Gravitational Field	Gravitational potential
Point mass	$-\frac{Gm}{r^2}$	$-\frac{Gm}{r}$
Ring	$\frac{Gmx}{\left(R^2+x^2\right)^{3/2}}$; $E_{max}\big\vert_{x=R/\sqrt{2}}$	$-\frac{Gm}{\sqrt{x^2+R^2}}$
Disc	$2\pi G\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$	$-2\pi G\sigma\left[\sqrt{x^2+R^2}-x\right]$
infinite wire	$E = 2G\lambda / r$	$\Delta V = 2G\lambda \ln\frac{r_1}{r_2}$
Along wire of length L at a distance x from one end	$\frac{G\lambda L}{x(x+L)}$	$-G\lambda\ln\left(\frac{x+L}{x}\right)$
Apex of cone	∞	$-2\pi G\sigma R$
infinite plate	$E = 2\pi G\sigma$	$\Delta V = 2\pi G\sigma d$
Hollow Sphere	$\vec{E} = \begin{cases} 0 & 0 \le r < R \\ -\frac{GM}{r^2}\hat{r} & r \ge R \end{cases}$	$V = \begin{cases} -\frac{GM}{R} & 0 \le r < R \\ -\frac{GM}{r} & r \ge R \end{cases}$
Solid sphere	$\vec{E} = \begin{cases} -\frac{GM\vec{r}}{R^3} = -\frac{4\pi G\rho}{3}\vec{r} & 0 \le r < R \\ -\frac{Kq}{r^2}\hat{r} & r \ge R \end{cases}$	$V = \begin{cases} -\frac{GM}{2R}\left(3-\frac{r^2}{R^2}\right) & 0 \le r < R \\ -\frac{GM}{r} & r \ge R \end{cases}$

3. Gravitational field is dependent upon the mass distribution and not where the mass is distributed.

4. $V_f - V_i = -\int_i^f \vec{E}.d\vec{r}$ & $\Delta U = -W_s = U_f - U_i = m\left(V_f - V_i\right)$

5. $\vec{E} = -\frac{\delta V}{\delta x}(\hat{i}) - \frac{\delta V}{\delta y}(\hat{j}) - \frac{\delta V}{\delta z}(\hat{k})$

6. *Consider two concentric spherical shells of radius a & b respectively.*



$$V = \begin{cases} -\left(\frac{G m_1}{a} + \frac{G m_2}{b}\right) & 0 \le r \le a \\ -\left(\frac{G m_1}{r} + \frac{G m_2}{b}\right) & a \le r \le b \\ -\left(\frac{G m_1}{r} + \frac{G m_2}{r}\right) & b \le r < \infty \end{cases}$$

$$U = m_1 V_2 = -\frac{G m_1 m_2}{b}$$

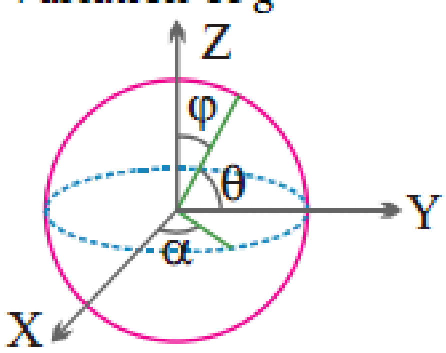
7. *Gravitational interaction energy:* $U = -\frac{G M m}{r}$

8. *Self-energy of solid sphere:* $U = -\frac{G M^2}{2R} - \frac{G M^2}{10R} = -\frac{3}{5}\frac{G M^2}{R}$

9. *Energy density:* $u = -\frac{E^2}{8\pi G}$

10. *Orbital speed:* $v = \sqrt{\frac{G M}{r}}$; *Escape speed:* $v = \sqrt{\frac{2G M}{R}} = \sqrt{2 g R}$

11. Variation of g



θ: *latitude*
φ: *co-latitude*
α: *longitude*

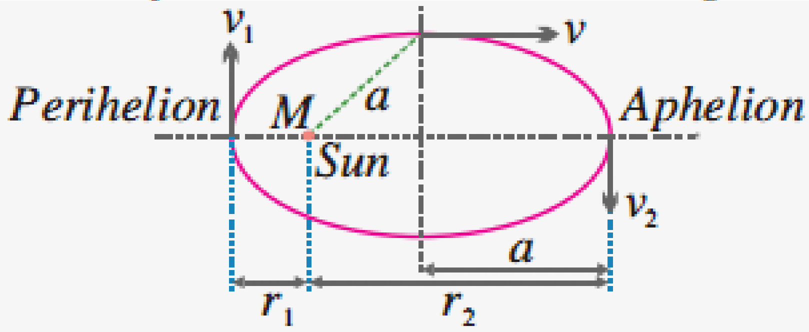
outside earth: $g' = g\left(1 + \frac{h}{R}\right)^{-2} \approx g\left(1 - \frac{2h}{R}\right)$ *for* $h \ll R$

inside earth: $g' = g\left(1 - \frac{h}{R}\right)$

with latitude: $g' \approx g\left(1 - \frac{\omega^2 R}{g}\cos^2\theta\right)$

Kepler's Law: $\vec{L} = const$

12. *For a satellite in an elliptical orbit; semi-major 'a' revolving around Sun. Semi-major axis is also called average radius of elliptical orbit.*



$$E = -\frac{GMm}{2a}; \quad T^2 = \left(\frac{4\pi^2}{GM}\right)a^3$$

$$v_1 = \sqrt{\frac{GM}{a}\left(\frac{r_2}{r_1}\right)}$$

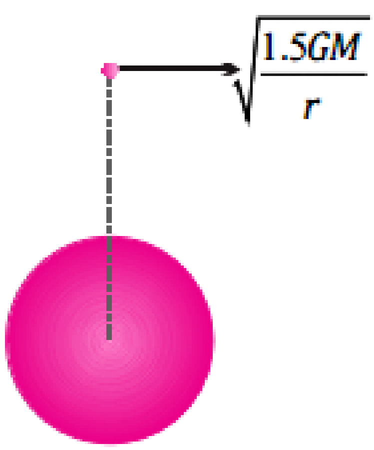
$$v_2 = \sqrt{\frac{GM}{a}\left(\frac{r_1}{r_2}\right)}$$

$$v = \sqrt{\frac{GM}{a}}$$

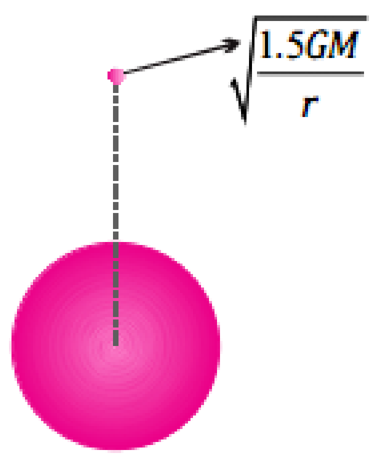
13. *For double star system of total mass M separated by distance r:*

$$\omega^2 = \frac{GM}{r^3} \qquad i.e.\ T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

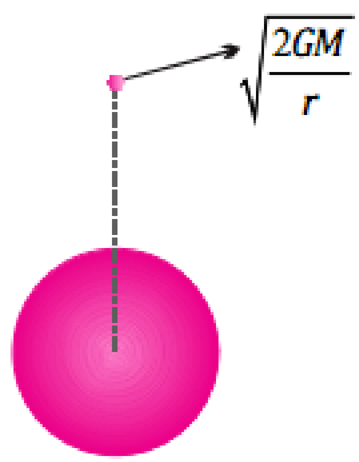
14. **Trajectory of satellite for different speeds:**



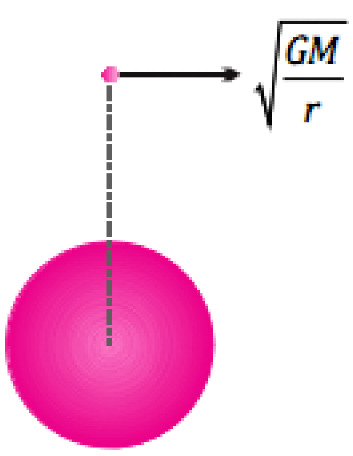
Elliptical
(Perihelion)



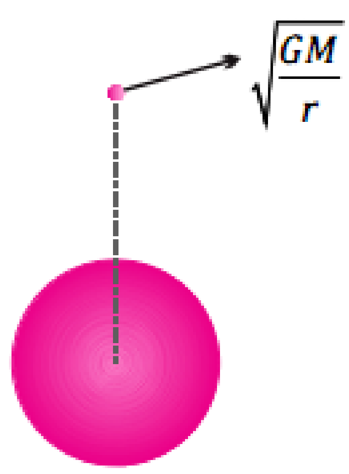
Elliptical



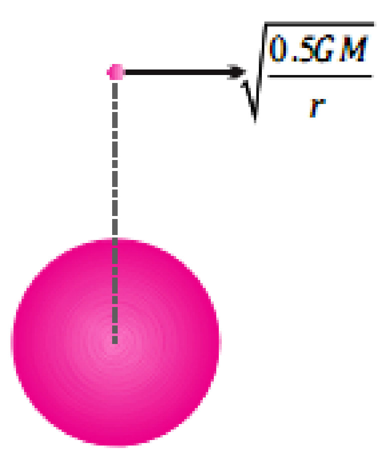
Parabolic



Circular



Elliptical
(Semi-Minor Axis)



Elliptical
(Aphelion)

$TE_{particle}$	Destination	Trajectory
$E > 0$ (unbound)	Particle reaches infinity ($K_\infty > 0$).	*hyperbola*
$E = 0$ (unbound)	Particle just reaches infinity.	*parabola*
$E < 0$ (bound)	Particle does not reach infinity.	*ellipse*

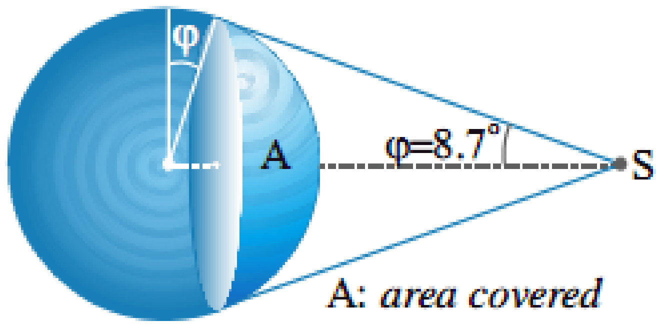
- $v_e > \sqrt{\frac{2GM}{r}}$: projectile will escape in hyperbolic path.

15. Inverse law of gravitation holds true for central forces iff $1 < n < 3$.

16. *Geostationary Satellite*: A satellite which appears to be stationary when seen from earth is called a geostationary satellite. Communication satellites are *geostationary satellite*. It must revolve from west to east in circular orbit in Earth's equatorial plane with a time period of 24 hrs. Its distance from Earth's center must be 42200 km approximately.

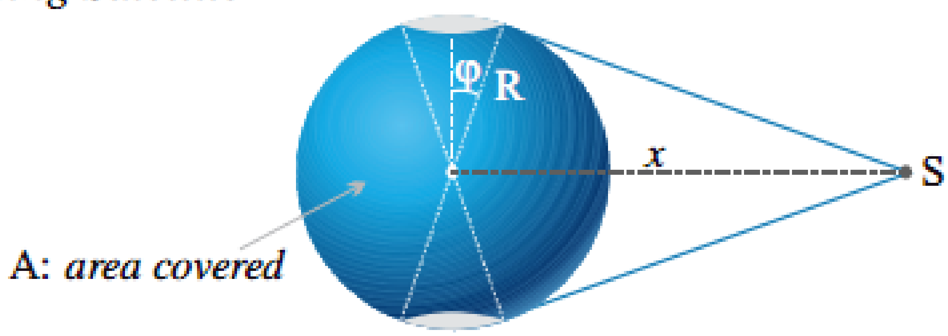
For geostationary satellite, the orbital speed and radial acceleration are:

$$v = \sqrt{\frac{GM}{r}} = 3.07 \text{ km/s}; \qquad a = \omega^2 r = 0.233 \text{ m/s}^2$$



Area covered by Geostationary Satellite: $A = 2\pi R^2 (1 - \sin\varphi)$

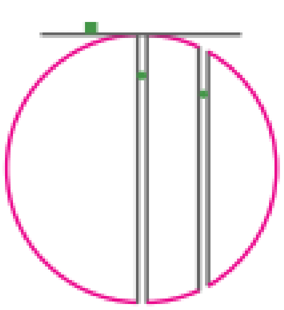
17. *Remote Sensing Satellite*



Area covered $= 4\pi R^2 \cos\varphi$ $\qquad \therefore A = 4\pi R^2 \dfrac{\sqrt{x^2 - R^2}}{x}$

18. *Simple pendulum when length of string is comparable to earth's radius*

$$T = 2\pi \sqrt{\frac{1}{g\left(\frac{1}{R} + \frac{1}{l}\right)}} \qquad \text{If } l \to \infty \text{, then } T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{R^3}{GM}} = 84.6 \text{ min}$$

19. 

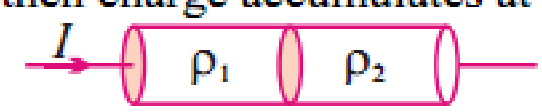
$$T = 2\pi\sqrt{\frac{R^3}{GM}}$$

Time taken by Earth satellite to move around Earth close to Earth surface = 84.6 min

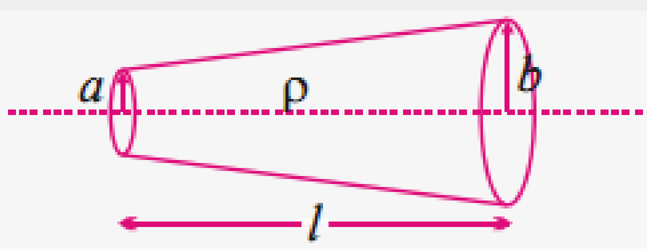
Basics

1. Because of electric field inside the wire, the current flows. The conductor is not in electrostatic equilibrium in this chapter.
2. Battery is not a charge supplier. When there is excess charge on the wire, it comes on its surface.
3. Current is reference frame independent
4. At steady state, current through every cross section is same.
5. $i = \dfrac{dq}{dt}$, $\quad i = neAv_d$
6. $\vec{J} = ne\vec{A}$ (A : cross-section Area) : $\vec{J}$ is characteristic of a point on a conductor, rather than conductor as a whole.
7. $\rho = \dfrac{E}{J}$ (resistivity) $\qquad$ Conductivity, $\sigma = \dfrac{1}{\rho}$
8. $R = \dfrac{\rho l}{A}$; $\qquad$ Conductance $= \dfrac{1}{R}$
9. $\alpha = \dfrac{1}{\rho}\dfrac{d\rho}{dT}$: Temperature coefficient of resistivity.
10. $\rho = \rho_o[1 + \bar{\alpha}\,\Delta T]$
11. The relationship $R = \dfrac{V}{i}$ & $\rho = \dfrac{E}{J}$ remains the general definition of resistance and resistivity. It is not ohm's law. We call it ohm's law when R and ρ are constant at a particular temperature.
12. If two resistances are in series, then charge accumulates at their junction.

$$q = \varepsilon_o I(\rho_2 - \rho_1)$$



13. Resistance of rigid bodies:

Body	Resistance
Spherical shell:	$R = \dfrac{\rho}{4\pi}\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$
Cylindrical shell: between inner & outer curved surface	$R = \dfrac{\rho}{2\pi L}\ln\left(\dfrac{b}{a}\right)$
Cylindrical shell: (end faces)	$R = \dfrac{\rho L}{\pi(b^2 - a^2)}$
	$R = \dfrac{\rho \ell}{\pi a b}$

14. WD by the battery implies work done by chemicals in battery (i.e. non conservative electric field inside the battery). $WD_{battery} = \Delta q V$ $\qquad$ $P_{battery} = iV$

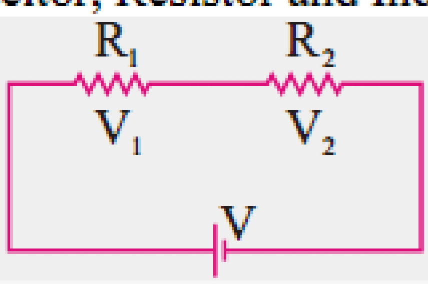
15. $\xi = \int_{-}^{+} \vec{E}_{NC} \cdot d\vec{r} = -\int_{-}^{+} \vec{E}_C \cdot d\vec{r}$: definition of emf of battery

16.

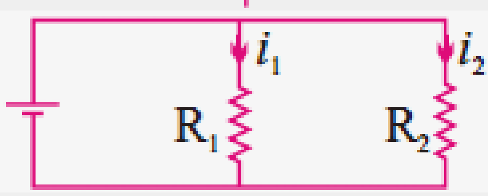
Discharging battery 	**Charging battery** 
Battery is doing positive work against the electric field inside the battery. $\Delta V = \varepsilon - ir$: terminal voltage	Battery is doing negative work against the electric field inside the battery. $\Delta V = \varepsilon + ir$: terminal voltage

17. *Note*: $V_a - V_b = 0 = iR \equiv \begin{cases} R = 0 & i \text{ may or maynot be zero} \\ R \neq 0 & i \text{ must be zero} \end{cases}$

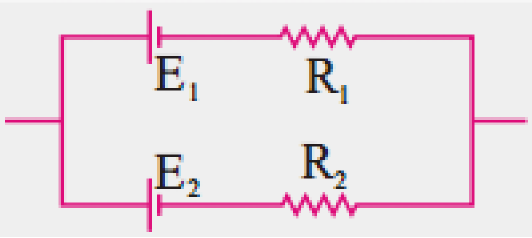
18. Capacitor, Resistor and Inductor in series and parallel.



$$V_1 = \frac{R_1}{R_1 + R_2} V; \qquad V_2 = \frac{R_2}{R_1 + R_2} V$$



$$i_1 = \frac{R_2}{R_1 + R_2} i; \qquad i_2 = \frac{R_1}{R_1 + R_2} i$$



$$E_o = \frac{E_1/R_1 + E_2/R_2}{1/R_1 + 1/R_2}; \; R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

19. $\oint \vec{J} \cdot d\vec{A} = 0$: Junction rule – *charge conservation* principle

20. $Q = \int i^2 R\, dt \qquad P = i^2 R = \dfrac{V^2}{R}$

21. When two resistances are connected in series, one having larger resistance dissipates more power.

22. When two resistances are connected in parallel, one having smaller resistance dissipates more power.

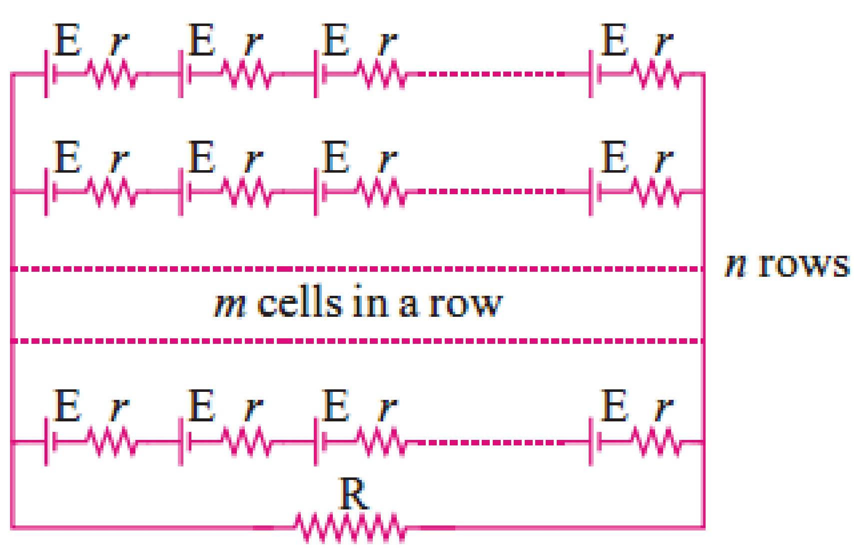
23. Brightness of bulb α power dissipated.

24.

$$i = \frac{NE}{mr + nR}$$

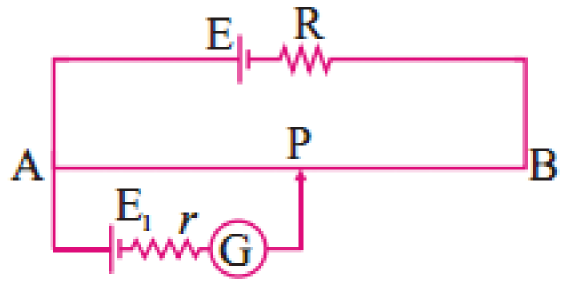
For $I_{\max}$: $\dfrac{m}{n} = \dfrac{R}{r}$

If m or n is not an integer then take nearest value to find I_{max}



Potentiometer

If R = 0, then in balanced condition:



$$E_1 = \frac{E}{l}x = \text{potential gradient} * \text{balancing length}$$

$$r = \frac{R(x-x')}{x'}$$

After the jockey is moved to the right extreme B & the current doesn't change direction, then we have done mistakes.

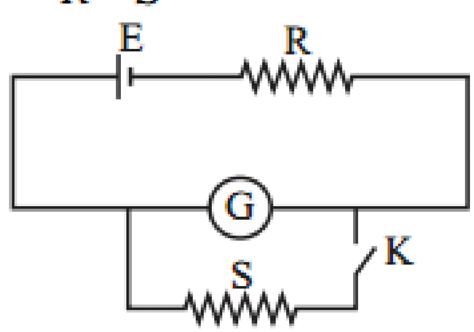
- We made a wrong connection (the terminal of E was connected oppositely).
- *Emf* 'E_1' is greater than Potential difference across AB.
- And if $E_1 > V_{AB}$ then galvanometer will show deflection only in one direction.

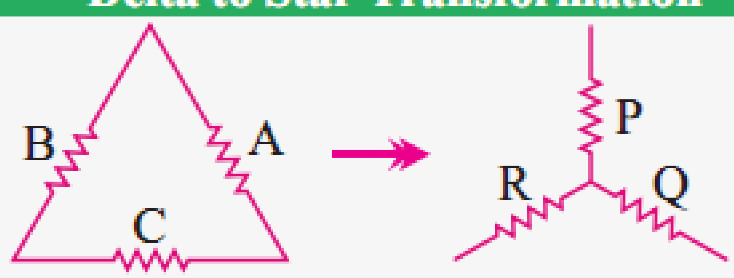
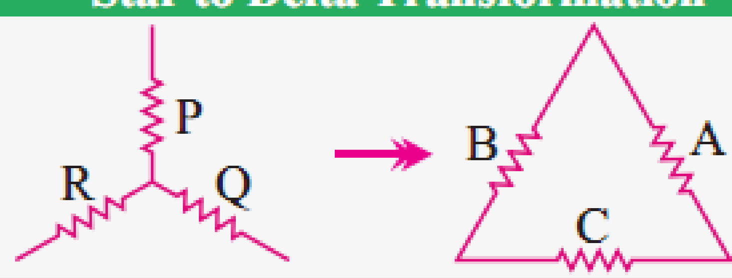
Sensitivity of an instrument

It tells us how quickly a measuring instrument will respond to electrical signals.

- *Current Sensitivity*: Pointer's deflection per unit current $CS = \frac{\theta}{\mathrm{I}}$.
- *Voltage Sensitivity*: Pointer's deflection per unit voltage $VS = \frac{\theta}{V}$.
- *Sensitivity of Potentiometer*: Sensitivity $\alpha \frac{L}{V_{AB}}$

Half-Deflection Method: $G = \frac{RS}{R-S}$



Delta to Star Transformation	Star to Delta Transformation
	
$P = \frac{AB}{A+B+C}$	$A = \frac{PQ+QR+RP}{R}$

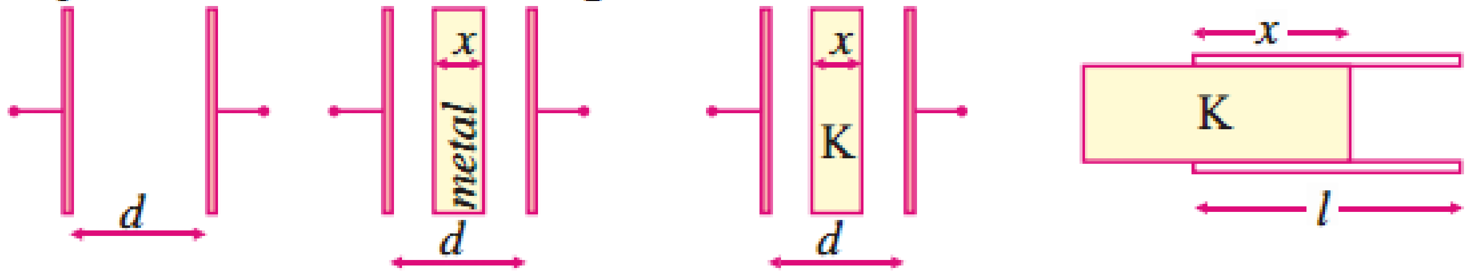
1. 'C' depends on geometrical property and medium only.

$$V = \frac{q}{C}$$

Charge on inner plate only
Capacitance of the system
Potential difference across the plates

- Plates being conductors are equipotential in nature.
- Charges $+q$ & $-q$ appear on the inner surface of the capacitor. However, we still refer to the charge on the capacitor as q.

2. Capacitance for various configurations:



$$C = \frac{\varepsilon_o A}{d} \qquad C = \frac{\varepsilon_o A}{d - l} \qquad C = \frac{\varepsilon_o A}{d - x(1 - 1/\mathrm{K})} \qquad C = C_o\left[1 + \frac{x}{l}(K-1)\right]$$

3. Spherical capacitor: $C = \dfrac{4\pi\varepsilon_o ab}{b-a}$

4. Solid conducting sphere: $C = 4\pi\varepsilon_o R$

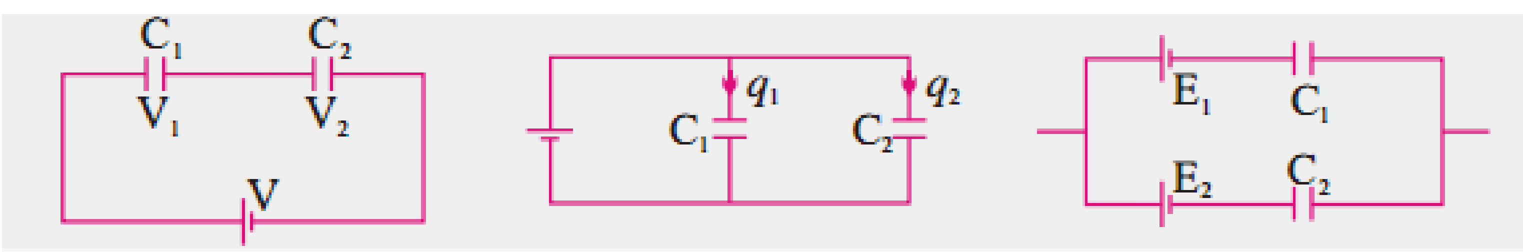
5. Spherical capacitor with dielectric (C between plates): $C = \dfrac{4\pi\varepsilon_o Kab}{b-a}$

6. Spherical capacitor with dielectric (C of system): $C = \dfrac{4\pi\varepsilon_o}{\frac{1}{K}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{1}{b}}$

7. Cylindrical capacitor (C between plates): $C = \dfrac{2\pi\varepsilon_o L}{\ln(b/a)}$

8. We know that
- 'V' is fixed (if battery remains connected)
- 'q' is fixed (if battery is disconnected)
- since C is geometrical property, $\therefore$ using $q = CV$, we can easily find one of 'q' or 'V'.
- And then calculate $E = -\partial V/\partial x$.

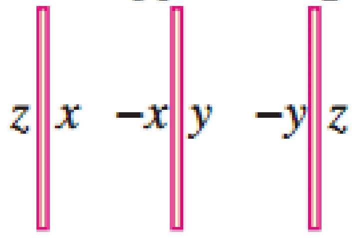
9.



$$V_1 = \frac{C_2}{C_1 + C_2}V \qquad q_1 = \frac{C_1}{C_1 + C_2}q \qquad V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$V_2 = \frac{C_1}{C_1 + C_2}V \qquad q_2 = \frac{C_2}{C_1 + C_2}q \qquad C_{eq} = C_1 + C_2$$

10. For parallel plate, the charge on outer plate is always the same while the charge on facing plates always has opposite sign.



11. Force between plates of capacitor: $F_s = -\dfrac{q^2}{2\varepsilon_o A}$

This attractive force between the plates doesn't change even if the dielectric is present.

12. *Charging a capacitor*:

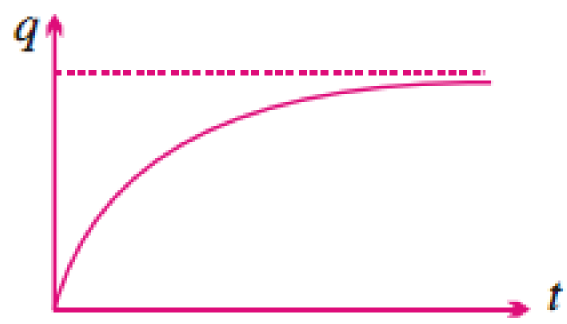
$q = CV_o(1 - e^{-t/CR})$ $\quad\quad i = \dfrac{V}{R}e^{-t/\tau}$ $\quad\quad \tau$: time constant

$V_R = Ve^{-t/\tau}$ $\quad\quad V_C = V\left(1 - e^{-t/\tau}\right)$

Uncharged capacitor acts as a short-circuit and a fully charged capacitor acts as an open circuit.

13.

time	q/q_o
τ	63%
2τ	86%
3τ	95%
4τ	98%
5τ	99%



14. *Discharging a capacitor*: by touching the two plates with a conducting wire, a capacitor is discharged.

$q = q_o e^{-t/CR}$ $\quad\quad I = \dfrac{q_o}{RC}e^{-t/\tau}$ $\quad\quad V_R = V_C = \dfrac{q_o}{C}e^{-t/\tau}$

Time constant is same for charging and discharging condition

15.

Polar dielectric	**Non-Polar dielectric**
If $\vec{E}_{ext} = \vec{0}$, then $\vec{p}_{ind} \neq \vec{0}$, $\vec{p}_{net} = \vec{0}$	If $\vec{E}_{ext} = \vec{0}$, then $\vec{p}_{ind} = \vec{0}$, $\vec{p}_{net} = \vec{0}$
If $\vec{E}_{ext} \neq \vec{0}$, then $\vec{p}_{ind} \neq \vec{0}$, $\vec{p}_{net} \neq \vec{0}$	If $\vec{E}_{ext} \neq \vec{0}$, then $\vec{p}_{ind} \neq \vec{0}$, $\vec{p}_{net} \neq \vec{0}$
If $\vec{E}_{ext} \neq \vec{0}$, then $\vec{p}_{net\_POLAR} \gg \vec{p}_{net\_NON\,POLAR}$	
If temperature increases, then $\vec{p}_{ind} = \overrightarrow{const}$, $\vec{p}_{net}$ decreases	If temperature increases, then $\vec{p}_{ind} = \overrightarrow{const}$, $\vec{p}_{net} = \overrightarrow{const}$

16. $\oint K\,\vec{E}\cdot d\vec{A} = \dfrac{q_{\text{free charge}}}{\varepsilon_o}$.

17. Revised Energy Density : $u = \dfrac{1}{2}K\,\varepsilon_o E^2$

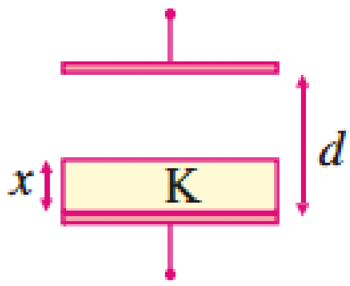
18. ε_r: relative permittivity, ε : permittivity, $\varepsilon = \varepsilon_r \varepsilon_o$

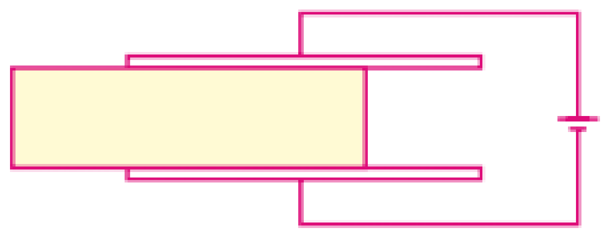
19. The charge induce in the dielectric is: $q' = q\left(1 - \frac{1}{K}\right)$ — *q': bound charge*

20. ***Force on dielectric***

$$F_s = \begin{cases} -\dfrac{Q^2}{2}\dfrac{d(1/C)}{dx} & \text{closed system (battery disconnected)} \\ +\dfrac{V^2}{2}\dfrac{dC}{dx} & \text{open system (battery connected)} \end{cases}$$

21.





$$F_s = \frac{Q^2}{2\varepsilon_o A}\left(1 - \frac{1}{K}\right) \qquad F_s = +\frac{V^2}{2}\frac{\varepsilon_o A}{d\,l}(K-1)$$

22. **Torque on dielectric:**

$$\tau_s = \begin{cases} -\dfrac{Q^2}{2}\dfrac{d(1/C)}{d\theta} & : \text{closed system} \\ +\dfrac{V^2}{2}\dfrac{d(C)}{d\theta} & : \text{open system} \end{cases}$$

MAGNETIC FIELD I

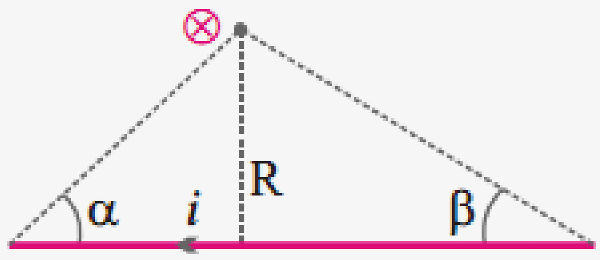
1. $d\vec{B} = \dfrac{\mu_o}{4\pi} i \dfrac{d\vec{s} \times \vec{r}}{r^3}$ $\mu_o = 4\pi \times 10^{-7}$ T.m/A (permeability constant)

   $d\vec{s}$: Along the direction of current

   $\vec{r}$: directed from current element to location where field is evaluated

2. $d\vec{B} = \dfrac{\mu_o q}{4\pi} \dfrac{\vec{v} \times \vec{r}}{r^3}$: Magnetic field due to the moving charge

3. Magnetic field due to various configurations:

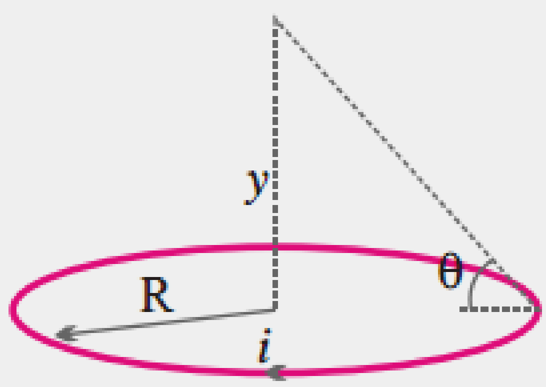


$$B = \frac{\mu_o i}{4\pi R}(\cos\alpha + \cos\beta)$$

∞ straight wire: $B = \dfrac{\mu_0 i}{2\pi R}$

semi-∞ straight wire: $B = \dfrac{\mu_0 i}{4\pi R}$

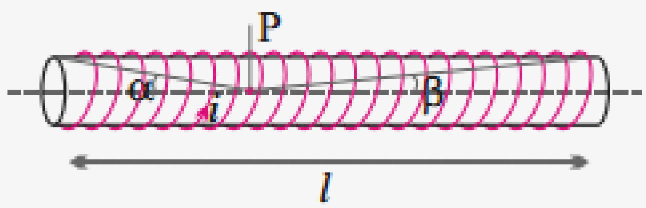
α = β: $B = \dfrac{\mu_o i}{2\pi R}\cos\alpha$



$$B_y = \frac{\mu_o i R^2}{2(R^2 + y^2)^{3/2}}$$

$B = \dfrac{\mu_o i}{2R}$ at loop's center

$B = \dfrac{\mu_o i}{2R}\left(\dfrac{\theta}{2\pi}\right)$ at loop's center subtending an angle θ at its center



n: number of turns/unit length

$B = \dfrac{1}{2}\mu_o n i(\cos\theta_1 + \cos\theta_2)$

$B = \mu_o n i$: *∞ solenoid*

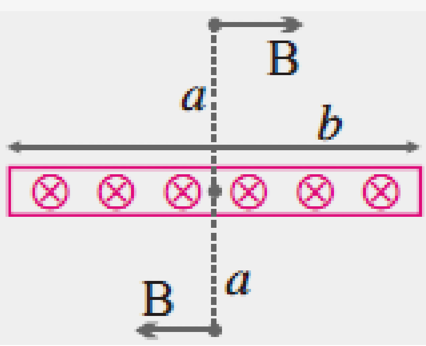
$B = \dfrac{1}{2}\mu_o n i$: at the end of solenoid

∞ straight hollow wire

$$B = \begin{cases} 0 & r < R \\ \dfrac{\mu_o i}{2\pi r} & r \geq R \end{cases}$$

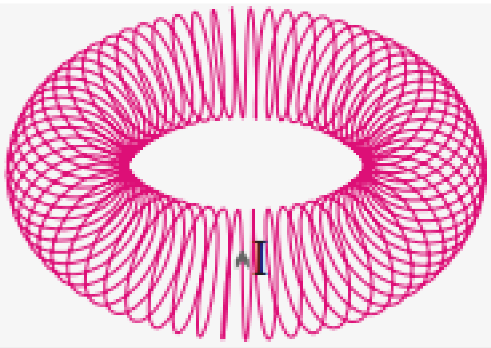
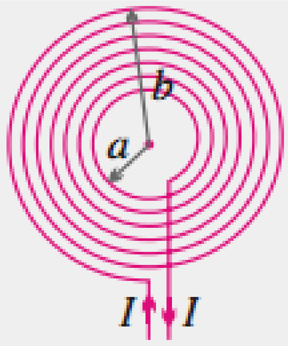
∞ straight solid wire

$$B = \begin{cases} \dfrac{\mu_o i}{2\pi}\dfrac{r}{R^2} = \dfrac{\mu_o \vec{j} \times \vec{r}}{2} & r < R \\ \dfrac{\mu_o i}{2\pi r} & r \geq R \end{cases}$$



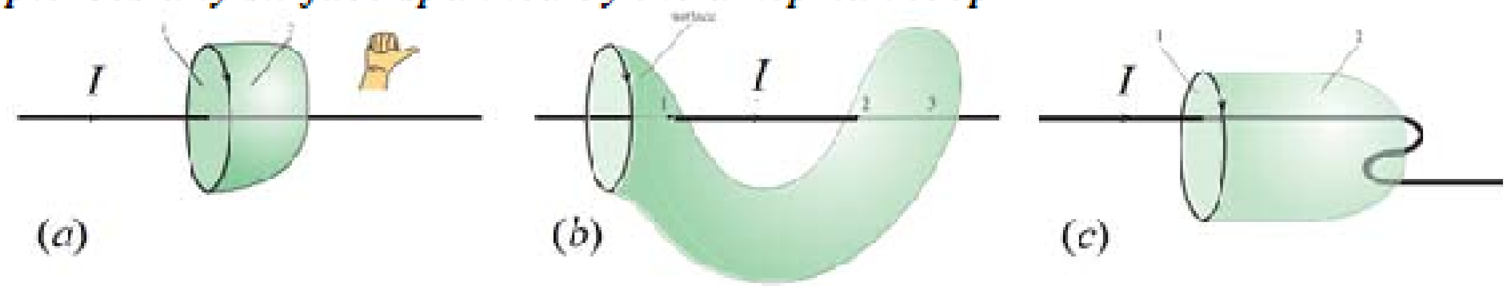
$B = \dfrac{\mu_o K}{\pi}\tan^{-1}\left(\dfrac{b}{2a}\right)$ $K = \dfrac{I}{b}$

$b \to \infty$ $\therefore B = \dfrac{\mu_o K}{2}$: ∞ sheet

	Magnetic field inside toroid at distance r from the center of toroid: $B = \dfrac{\mu_o i}{2\pi r} N$ N: number of turns
	$B = \dfrac{\mu_o I N}{2(b-a)} \ln\left(\dfrac{b}{a}\right)$
Rotating charged ring	$B = \dfrac{\mu_o q\,\omega}{4\pi R}$
Rotating charged disc	$B = \dfrac{\mu_o \sigma\,\omega R}{2}$
Rotating charged sphere	$B = \dfrac{2\mu_o\,\omega \sigma R}{3}$
Rotating cylinder	$B = \mu_o \sigma\,\omega R$

4. **Amperes Law**:

$\oint \vec{B} \cdot d\vec{l} = \mu_o i_{enc}$: i_{enc} is *current through any ameprian loop* or *current that pierces any surface spanned by the ameprian loop*.



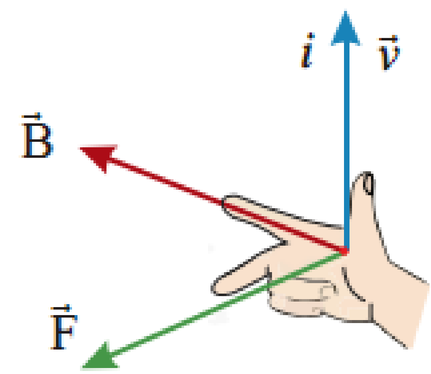
$\oint \leftarrow$ also indicates loop around which element of length $d\vec{l}$ is to be integrated.

This law is valid when:

- ✓ Electric field inside the amperian loop remains constant.
- ✓ When a steady current flows through the wire.

- Although we are calculating magnetic field for the current enclosed, the magnetic field so evaluated is due to the current both outside and inside the loop
- The magnetic field calculated is on the amperian loop.
- $\oint \vec{B}.d\vec{l} \neq 0$: $\therefore$ The magnetic field is non-conservative in nature.

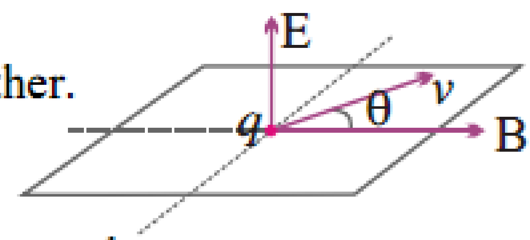
1. *Magnetic field does no work.*
2. $\vec{F}_B = q(\vec{v} \times \vec{B})$: Force acting on a charged particle.
3. $\vec{F}_B = i(\vec{L} \times \vec{B}) = V(\vec{j} \times \vec{B})$: Force on a current carrying wire.

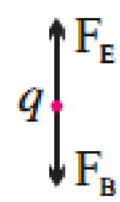


4. If $\vec{B}(x,t) = \overrightarrow{\text{constant}}$:

Straight line path	Circular path	Helical Path
$\vec{v} \measuredangle \vec{B} = 0^o$ or 180^o	$\vec{v} \measuredangle \vec{B} = 90^o$	$\vec{v} \measuredangle \vec{B} \neq 0^o/180^o/90^o$
	$r = \frac{mv}{qB}, T = \frac{2\pi m}{qB}, \omega = \frac{qB}{m}$	$r = \frac{mv_\perp}{qB}, T = \frac{2\pi m}{qB}, p = v_\parallel T$

5. In case both $\vec{E}$ & $\vec{B}$ are present, then for straight line path:
From the figure it is very clear that
- E & B must be perpendicular to each other.
- E must be perpendicular to v
- v may not be perpendicular to B.

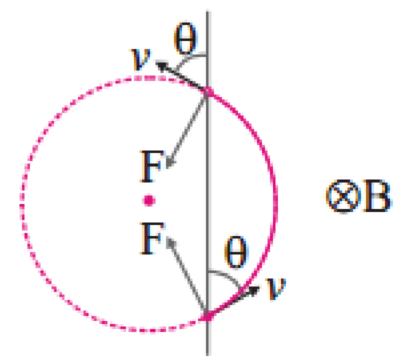




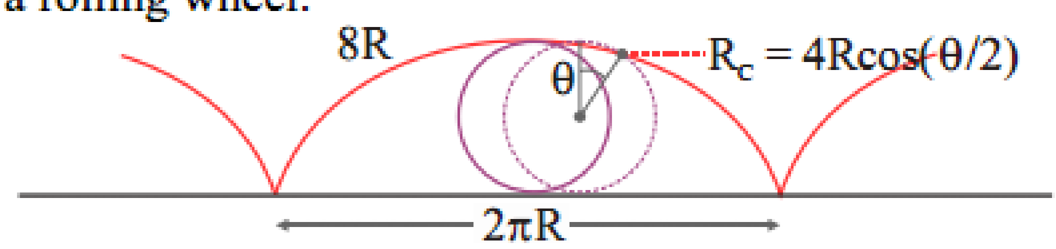
$\therefore$ For the particle to move in the straight line path:

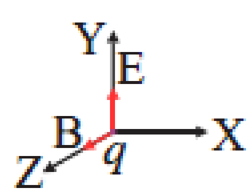
$$q v B \sin\theta = q E \quad \therefore v = \frac{E}{B \sin\theta}$$

6. If a charged particle enters a magnetic field as shown, then the angle at which it enters, at that angle it will come out. The center of circle is always towards the direction in which force is acting on the charged particle. We first find the center of circle before solving the question.



7. **Cycloidal path:** When electric and magnetic field is uniform and perpendicular, then the charged particle which has started from rest; moves in a cycloidal path. Such a cycloidal path is same as that of the motion of a point on a rolling wheel.

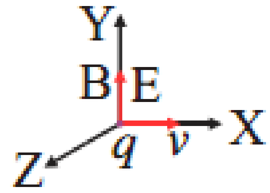




$$\therefore v = \omega R = \frac{E}{B} \qquad \omega = \frac{qB}{m}$$

$\vec{r} = R(\omega t - \sin\omega t)\hat{i} + R(1 - \cos\omega t)\hat{j}$: Equation of path

8. Helical Path with increasing pitch:



9. **Hall's Effect:** $n = \dfrac{i\,d\,B}{eAV}$, $v_d = \dfrac{V}{B\,d}$, $V = E\,d$

10. If $\vec{B}(\mathrm{x},\mathrm{t}) = \overrightarrow{\text{constant}}$, then force on a closed current carrying loop is zero.

11. $\vec{B}(\mathrm{x},\mathrm{t}) = \overrightarrow{\text{constant}}$, then force on a current carrying wire is B*il* if B, *i*, and *l* are perpendicular to each other.

12. If current in two wires are in same direction, then they attract, else repel.

13. **Magnetic dipole moment:** $\vec{\mu} = N\,i\,\vec{A} = \dfrac{q}{2m}\vec{l}$

    Note**:** Mass and MI is of charge distribution and not of the body on which it is distributed.

14. Torque and potential energy

    If $\vec{B}(\mathrm{x},\mathrm{t}) = \overrightarrow{\text{constant}}$, then $\vec{\tau} = \vec{\mu} \times \vec{B}$ $\quad \theta = 0^o$: stable equilibrium

    $U = -\vec{\mu}\cdot\vec{B}$ always. $\quad \theta = 180^o$:unstable equilibrium

15. Magnetostatic *energy density* and Magnetostatic *pressure* is given by: $P = u_B = \dfrac{B^2}{2\mu_o}$: This pressure that one applies on itself due to its own current.

16. **D'Arsonval galvanometer:**

    $I = \dfrac{k}{N\,A\,B}\theta = c\,\theta$: $c \equiv$ Galvanometer Constant

    $\dfrac{\theta}{I} = \dfrac{N\,A\,B}{k}$: $\dfrac{\theta}{I} \equiv$ Current sensitivity

    $\dfrac{\theta}{V} = \dfrac{N\,A\,B}{k}\dfrac{1}{R}$: $\dfrac{\theta}{V} \equiv$ Voltage sensitivity

17. Two protons move parallel to each other with an equal velocity *v*:

    $$\frac{F_B}{F_E} = \mu_o \varepsilon_o v^2 = \left(\frac{v}{c}\right)^2$$

1. **Faraday's law of induction**:

- Emf is induced in loop when magnetic lines of forces (**flux**) change through it.
- Emf induced in the loop depends on relative motion of the coil and the magnet.
- Emf induced is independent of chosen reference frame.
- $\phi_B = \int \vec{B} \cdot d\vec{A}$ [By definition] *SI* unit: Weber
- $\xi = -N \dfrac{d\phi_B}{dt}$ *for* N turns coil

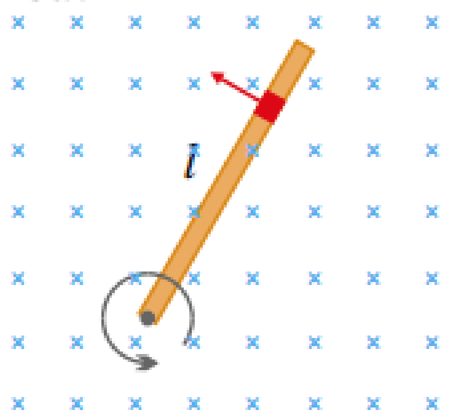
The direction of induced emf is worked out using *Lenz's Law*. It states that an emf is induced in such a way that it opposes the cause that induces emf.

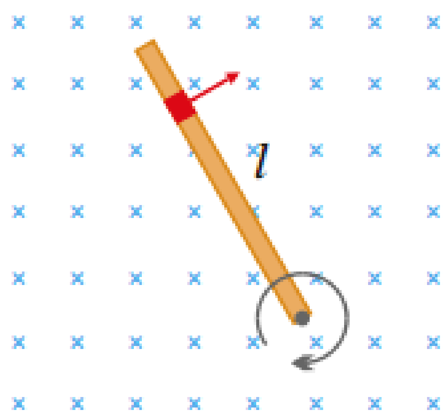
2. **Average emf:** $\langle \xi \rangle = \dfrac{\phi_i - \phi_f}{\tau}$. **Charge Flown:** $q = \dfrac{\varphi_i - \varphi_f}{R}$

3. **Motional Emf:** It is calculated using $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$.

Constant Speed: $\vec{F}_{net} = \vec{0}$	***Constant Acceleration***
$\vec{E} = \vec{B} \times \vec{v}$ If $B \perp v \perp l$, then $\xi_{ind} = \Delta V = Bvl$	$m_e \vec{a} = e(\vec{E} + \vec{v} \times \vec{B})$ $\vec{E} = \dfrac{m_e \vec{a}}{e} - \vec{v} \times \vec{B} \quad \therefore \Delta V = \int \vec{E} \cdot d\vec{l}$
Even if magnetic field is not present, still potential difference will develop.	

Rotating rod:





$$\xi_{ind} = \frac{m_e \omega^2 l^2}{2e} + \frac{1}{2} B\omega l^2 \approx \frac{1}{2} B\omega l^2 \qquad \xi_{ind} = -\frac{m_e \omega^2 l^2}{2e} + \frac{1}{2} B\omega l^2 \approx \frac{1}{2} B\omega l^2$$

4. *Changing magnetic field induces electric field.*: $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\phi}{dt}$

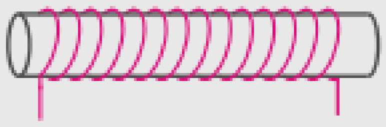
$$\xi = \begin{cases} \pi r^2 \dfrac{d\vec{B}}{dt} & 0 < r \le R \\ \pi R^2 \dfrac{d\vec{B}}{dt} & R < r \le \infty \end{cases} \qquad \vec{E}_{ind} = \begin{cases} \dfrac{r}{2} \dfrac{d\vec{B}}{dt} & 0 < r \le R \\ \dfrac{R^2}{2r} \dfrac{d\vec{B}}{dt} & R < r \le \infty \end{cases}$$

- Direction of $\vec{E}_{ind}$ is found using Lenz's law.
- $\vec{E}_{ind}$ doesn't have any radial component.
- $\vec{E}_{ind}$ is not associated with charges. It has no origin & end.
- $\vec{E}_{ind}$ forms a closed loop, therefore is non-conservative.
- Electric potential has no meaning for $\vec{E}_{ind}$.

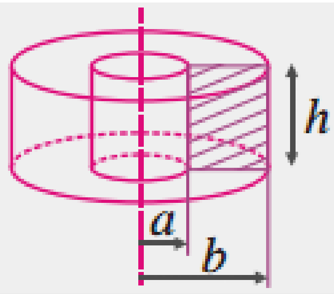
Inductor doesn't allow sudden change in flux. $\varphi_B = Li$.

If 'L' doesn't change suddenly, then current will also not change.
But, if 'L' changes suddenly, then flux still remains the same and current changes.

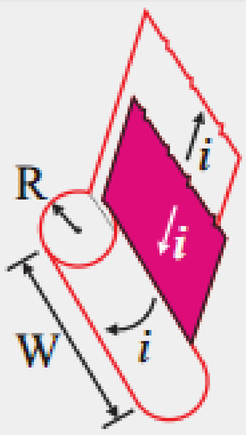
If the inductance of a solenoid coil is reduced suddenly from 'L' to 'nL'; then flux through it remains the same just before and just after the change in the inductance. If i_o is the current flowing through the inductor before the change, then
$Li_o = nL\,i$ $\quad\quad \therefore\ i = i_o/n.$ $\quad\quad$ i.e. the current increases.



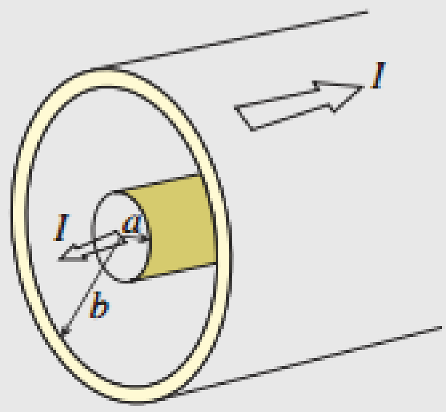
$$L = \mu_o n^2 l\,A = \frac{\mu_o N^2 A}{l}$$



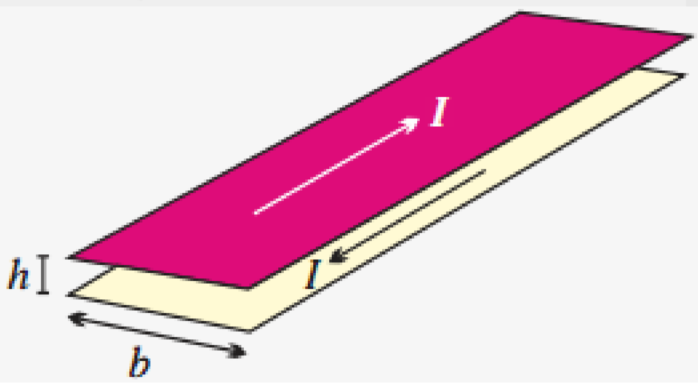
$$L = \frac{\mu_o N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)$$



$$L = \frac{\mu_o \pi R^2}{w}$$



$$\frac{L}{l} = \frac{\mu_o}{2\pi}\ln\left(\frac{b}{a}\right)$$



$$\frac{L}{l} = \frac{\mu_o h}{b}$$

RL Circuit: In the direction of current, we write $L\dfrac{di}{dt}$

$i = \dfrac{V}{R}\left(1 - e^{-R_o t/L}\right)$ $\quad\quad$ $i = \dfrac{V}{R}e^{-Rt/L}$ $\quad\quad$ $\tau = \dfrac{L}{R}$ (one time constant)

Demagnetized inductor acts as an open circuit.
An inductor acts as a short circuit after a long time.

Mutual Inductance: $M = \dfrac{\varphi_{2\_net}}{I_1} = \dfrac{\varphi_{1\_net}}{I_2}$, $\xi_2 = M\dfrac{di_1}{dt}$, $\xi_1 = M\dfrac{di_2}{dt}$

Inductors in **series**:

i. Current in both the coils is in the same sense: $L_{eq} = L_1 + L_2 + 2M$

ii. Current both the coils are in the opposite sense: $L_{eq} = L_1 + L_2 - 2M$

And if flux is not lost, then $M_{\max} = \sqrt{L_1 L_2}$

Inductors in **Parallel**:

iii. Current in both the coils is in the same sense: $L = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$

iv. Current both the coils are in the opposite sense: $L = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$

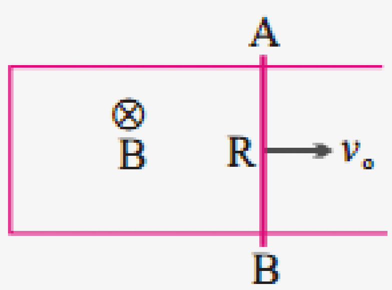
Magnetic field does not follow superposition principle:

$$u_B = \frac{1}{2\mu_o}\left(\vec{B}_1 + \vec{B}_2\right)^2 = \underbrace{\frac{\vec{B}_1^2}{2\mu_o} + \frac{\vec{B}_2^2}{2\mu_o}}_{\text{self inductance}} + \underbrace{\frac{\vec{B}_1 \cdot \vec{B}_2}{\mu_o}}_{\text{mutual inductance}}$$

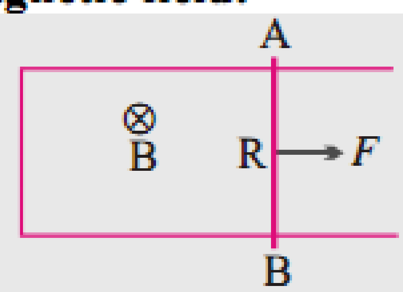
$$U_B = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 \pm M\, I_1 I_2$$

Energy stored in the magnetic field: $U_B = \dfrac{1}{2}Li^2$

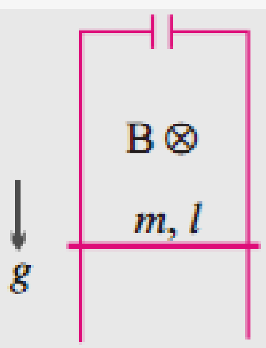
Behavior of various electrical elements in magnetic field:



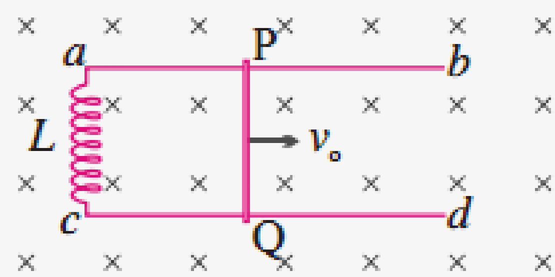
$$v = v_o e^{-\frac{B^2 l^2}{m R}t}$$



$$v = v_o\left(1 - e^{-\frac{B^2 l^2}{m R}t}\right); \quad v_o = \frac{F R}{B^2 l^2}$$



$$a = \frac{m g}{B^2 l^2 C + m}$$



$$x = \frac{v_o}{\omega}\sin \omega t$$

For a superconductor, $\varphi_B = const$

LC Oscillator: Potential difference across C and L is always same in LC circuit.

$q = Q_m \cos \omega t$: When initially capacitor is fully charged.

Thus, time period of complete oscillation: $T = \dfrac{2\pi}{\omega}$; $\omega = \dfrac{1}{\sqrt{LC}}$

- 10 V AC source means RMS value (or *effective value*)
- V = 10 sin (120*t*) means peak value.
- A *hot wire* ammeter measures current in *RMS* value. A *hot wire* voltmeter measures voltage in *RMS* value. Both can measure DC current and voltages also.
- A moving coil galvanometer measures DC but gives zero reading for AC.
- Power is sent to cities at high voltage so as to reduce power loss

$$\frac{\xi_2}{\xi_1}=\frac{N_2}{N_1} \qquad P = I_1\xi_1 = I_2\xi_2$$

$N_2 > N_1$: *step-up* transformer.
$N_2 < N_1$: *step-down* transformer.

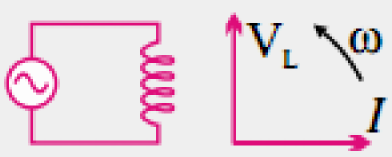
$$<P> = \frac{\int P dt}{\int dt} \qquad P_{RMS} = \sqrt{<P^2>} = \sqrt{\frac{\int P^2 dt}{\int dt}}$$

$<\sin\theta>_{0-2\pi} = 0$	$<\sin\theta>_{0-\pi} = \frac{2}{\pi}$	$\sqrt{<\sin^2\theta>_{0-\pi}} = \frac{1}{\sqrt{2}}$

If a physical quantity 'P' is constant then $<P> = P$, $\sqrt{<P^2>} = P$

For straight line with varying from I_o to zero: $<I> = I/2$, $I_{RMS} = I/\sqrt{3}$

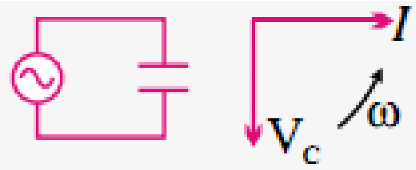
Behavior of L, C & R in AC: Applied voltage be $\xi = \xi_m \cos(\omega t)$



$$i = \frac{\xi_m}{X_L}\cos\left(\omega t - \frac{\pi}{2}\right)$$

$$X_L = \omega L, \quad \tilde{X}_L = jX_L$$

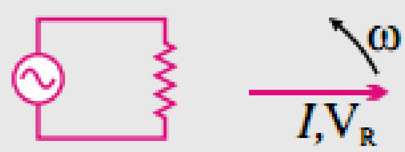
I_L lags V_L by $\pi/2$



$$i = \frac{\xi_m}{X_C}\cos\left(\omega t + \frac{\pi}{2}\right)$$

$$X_C = \frac{1}{\omega C}, \quad \tilde{X}_C = -jX_C$$

I_C leads V_C by $\pi/2$



$$i = \frac{\xi}{R}\cos(\omega t)$$

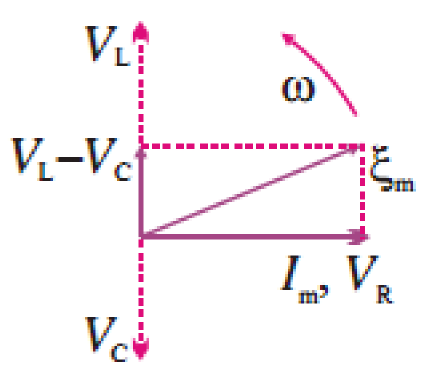
$$R$$

I_R & V_R are in phase

LCR series circuit: *LIC: Read clockwise; but we take anticlockwise positive*
Note: In diagram below, V_L, V_C & V_R are all peak values.

$$I_m = \frac{\xi_m}{Z}, \quad V_R = I_m R, \quad V_C = I_m X_C, \quad V_L = I_m X_L$$

$$\xi = \xi_m \cos\omega t, \; i = \frac{\xi_m}{Z}\cos(\omega t - \phi)$$



$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\xi_m = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\xi = v_R + v_C + v_L$$

$$v_R = \frac{\xi_m}{Z} R\cos(\omega t - \phi)$$

$$v_C = \frac{\xi_m}{Z} X_C\cos(\omega t - \phi - \pi/2)$$

$$v_L = \frac{\xi_m}{Z} X_L\cos(\omega t - \phi + \pi/2)$$

Power delivered by AC is power lost by resistor:

$P = i\xi$: instantaneous value $\qquad <P> = I_{rms}\,\xi_{rms}\cos\phi \qquad \cos\phi$: power factor

$P = i^2 R; \qquad \langle P\rangle = \xi_{rms} I_{rms}\cos\phi = \xi_{rms}^2 R/Z^2$

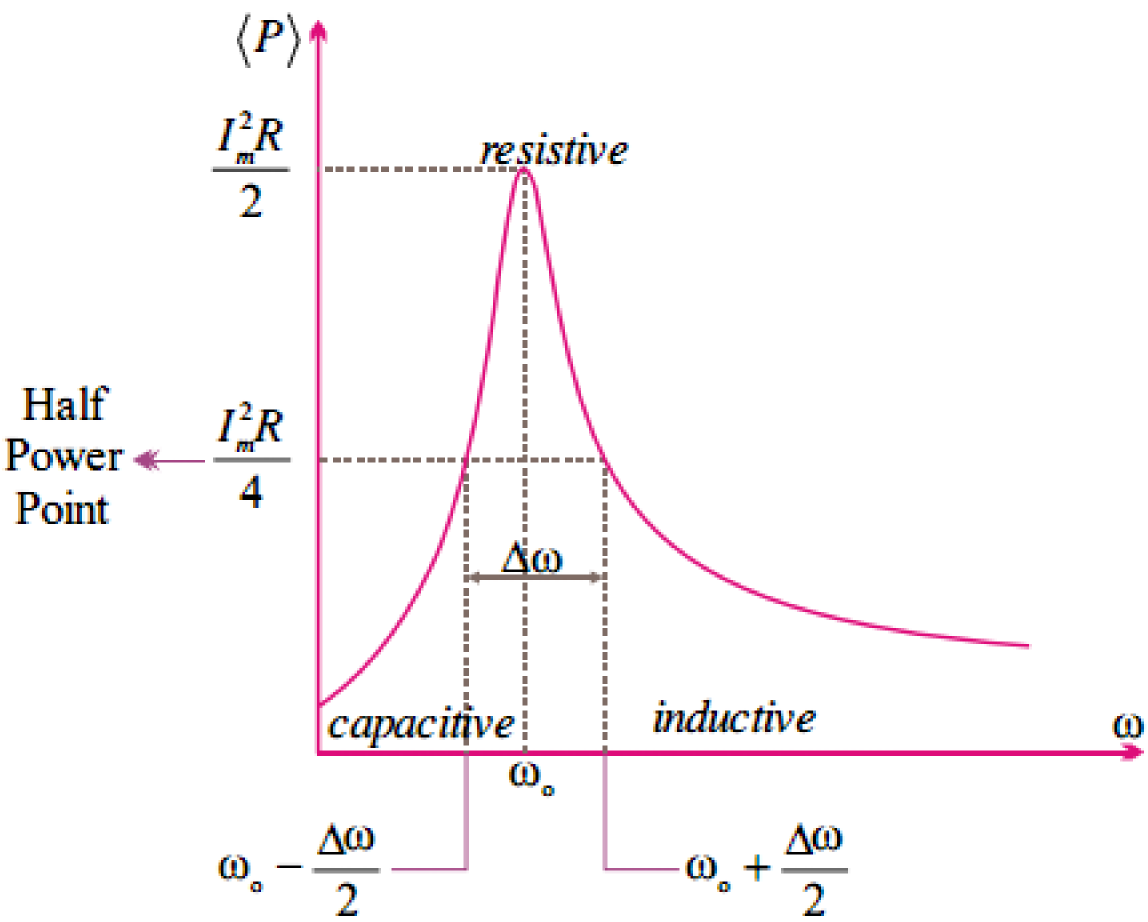
Power in inductor, capacitor & Resistor:

$P_L = i v_L \quad \therefore < P_L > = 0 \qquad P_C = i v_C \quad \therefore < P_C > = 0$: ***Wattless Component.***

Heat lost in resistor in one cycle: $Q = \int i^2 R\, dt = \dfrac{I^2 R}{2} T$

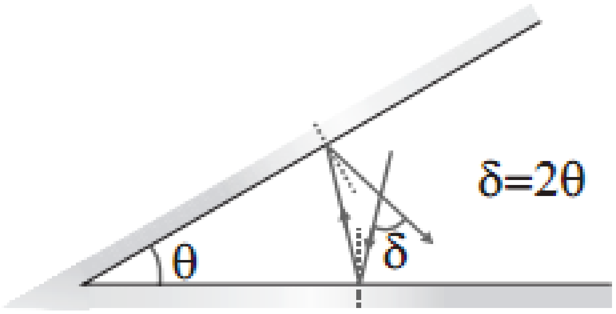
LCR and Radio: *Capacitance* is changed to tune radio.

Inductive circuit	$V_L > V_C$	$X_L > X_C$	$\phi > 0$	$\omega > \omega_o$	$\omega_o = \dfrac{1}{\sqrt{LC}}$
Capacitive circuit	$V_C > V_L$	$X_C > X_L$	$\phi < 0$	$\omega < \omega_o$	
Resistive circuit	$V_C = V_L$	$X_C = X_L$	$\phi = 0$	$\omega = \omega_o$	
RESONANCE	$\phi = 0$		$I_{max} = \dfrac{\xi_{max}}{R}$, $P = \dfrac{\xi_{rms}^2}{R}$		

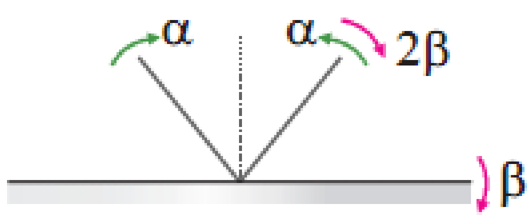


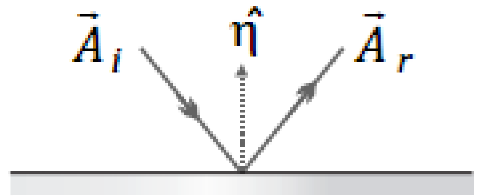
$$\Delta\omega = \frac{R}{L}$$

$$Q = \frac{\omega_o}{\Delta\omega} = \frac{1}{R}\sqrt{\frac{L}{C}}$$



Deviation is angle between incident ray and reflected/refracted ray.





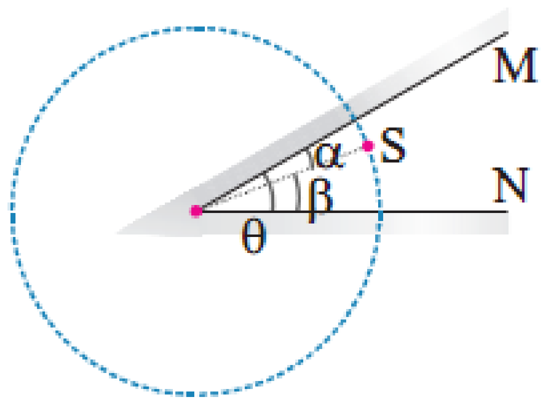
$$\hat{A}_r = \hat{A}_i - 2\left[\hat{A}_i \cdot \hat{\eta}\right]\hat{\eta}$$

Real object : If the incident light rays are diverging, then object is real.
Virtual Object: If incident light rays are converging, then the object is virtual.
Real image : If outgoing light rays are converging, then the image is real.
Virtual image : If outgoing light rays are diverging, then the image is virtual.

The total number of images if P-series is given by:

$$N_P = \begin{cases} \dfrac{180^o - \alpha}{\theta} & \text{if } \dfrac{180^o - \alpha}{\theta} \in I \\ \left[\dfrac{180^o - \alpha}{\theta}\right] + 1 & \text{if } \left[\dfrac{180^o - \alpha}{\theta}\right] \notin I \end{cases}$$

$$\text{if } \left[\frac{180^o}{\theta}\right] \in I \Rightarrow \text{final P \& Q image will coincide}$$

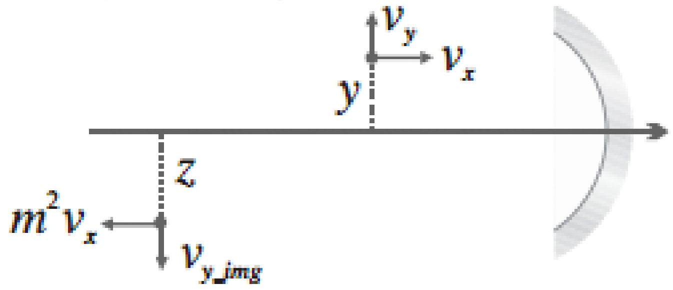


$$\therefore \text{Number of images: } N = \begin{cases} N_P + N_Q & \left[\dfrac{180^o}{\theta}\right] \notin I \\ N_P + N_Q - 1 & \left[\dfrac{180^o}{\theta}\right] \in I \end{cases}$$

Mirror	Lens
$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}; M = -\frac{v}{u}; \lvert M \rvert = \frac{\text{image size}}{\text{object size}}$	$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = \left(\frac{\mu}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
$M \equiv \begin{cases} > 0 & I \;\&\; O: \text{same orientation} \\ < 0 & I \;\&\; O: \text{opposite orientation} \end{cases}$	Lateral Magnification: $\frac{dv}{du} = m^2$
$f < 0$ for concave mirror (diverging) $f > 0$ for convex mirror (converging)	$f < 0$ for diverging lens $f > 0$ for converging lens
$x_1 x_2 = f^2;\ M = -\frac{f}{x_1}$	$x_1 x_2 = -f^2;\ M = \frac{f}{x_1}$
$v_{image} = -m^2 v_o$	$v_{image} = m^2 v_o$

- virtual image is not formed for virtual object if the rays are converging.
- real image is not formed for real object if rays are diverging.

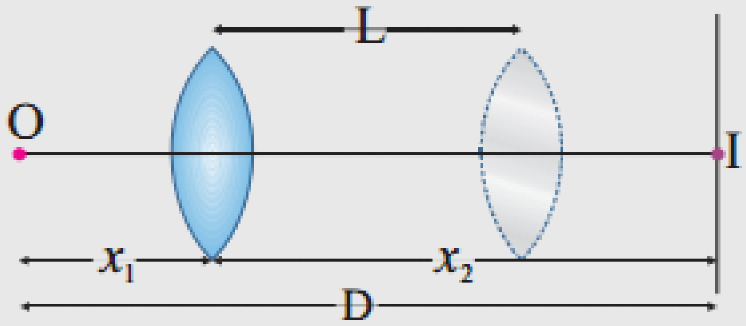
Velocity of image in Y- direction:



$$\frac{dx}{dt} = -m^2 v_x$$

$$\frac{dz}{dt} = m v_y + \frac{y}{f} m^2 v_x$$

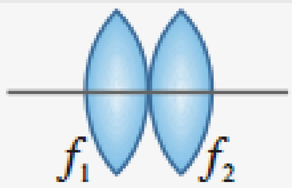
Displacement Method



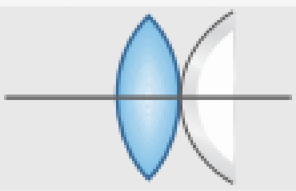
$$m_1 = \frac{D - x_1}{x_1} = \frac{D+L}{D-L}$$

$$m_2 = \frac{D - x_2}{x_2} = \frac{D-L}{D+L}$$

$$\therefore m_1 m_2 = 1 = \frac{I_1}{O} = \frac{I_2}{O} \qquad \therefore O = \sqrt{I_1 I_2}$$



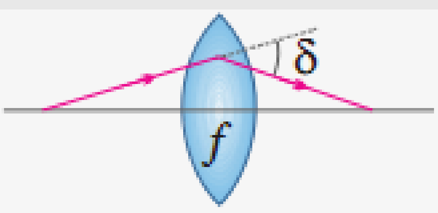
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$



$$\frac{1}{F} = -\frac{2}{f} + \frac{2}{R_m}$$

Put f as positive for converging lens and negative for diverging lens.
Put R as positive for convex mirror and negative for concave mirror.
If $F > 0$, it's convex mirror; If $F < 0$, it's concave mirror.

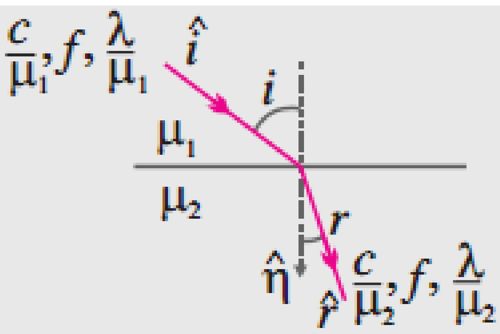
Optical power for mirror: $P = -\dfrac{1}{f}$. Optical power of lens: $P = \dfrac{1}{f}$.

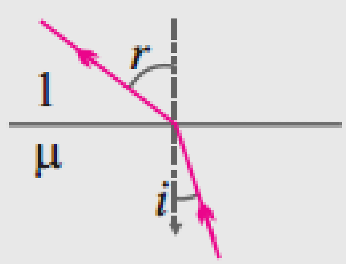


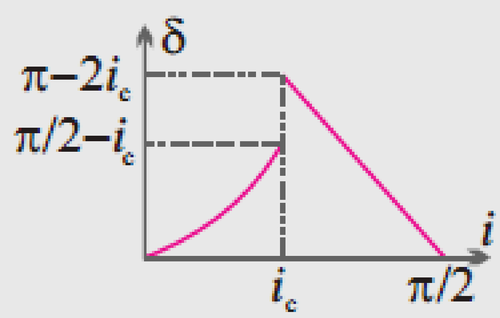
Deviation of light through a thin lens:

$$\delta = \frac{h}{f}$$

Refraction: $\mu_1 \left(\hat{i} \times \hat{n} \right) = \mu_2 \left(\hat{r} \times \hat{n} \right)$; $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

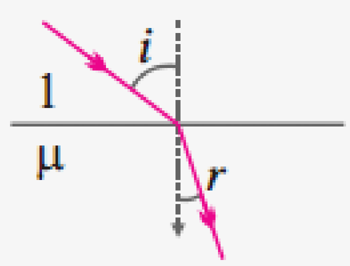


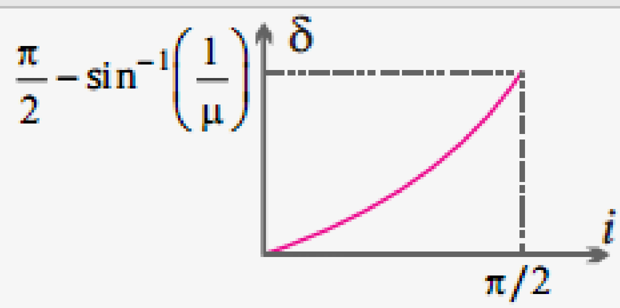




Cauchy's Law:

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \ldots$$

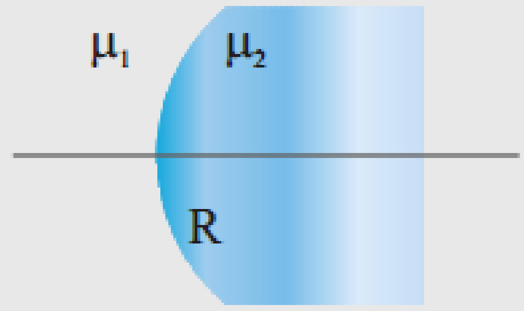




Refraction through plane & curved surface:

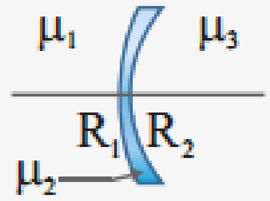
Plane surface: $\frac{\mu_2}{v}-\frac{\mu_1}{u}=0,\ M=1$

Curved Surface: $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R},\ M=\frac{\mu_1 v}{\mu_2 u}$

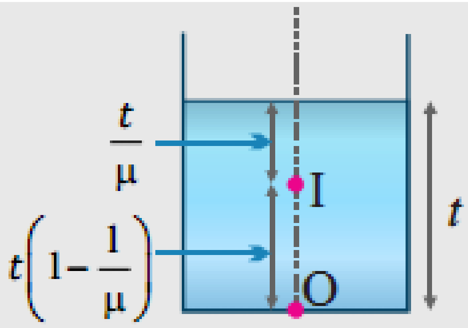


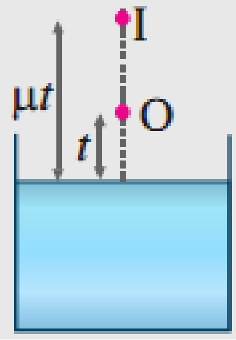
Refraction from thin lens:

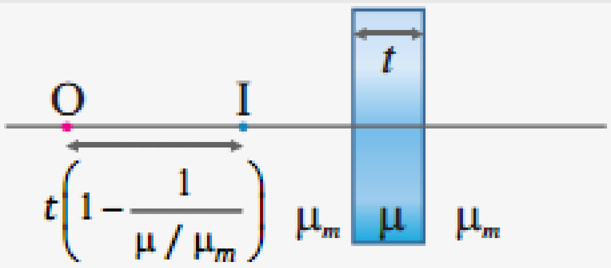
$$\frac{\mu_3}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R_1}+\frac{(\mu_3-\mu_2)}{R_2}$$



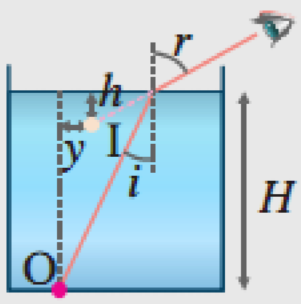
Apparent shift:





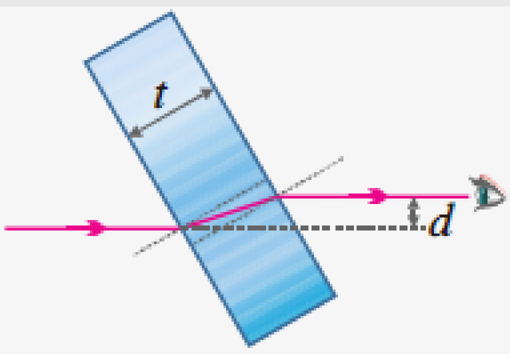


Apparent shift $=t\left(1-\frac{1}{\mu_r}\right)$ is always in the direction of incident light



$$h=\frac{\mu^2 H\cos^3 r}{\left(\mu^2-\sin^2 r\right)^{3/2}}$$

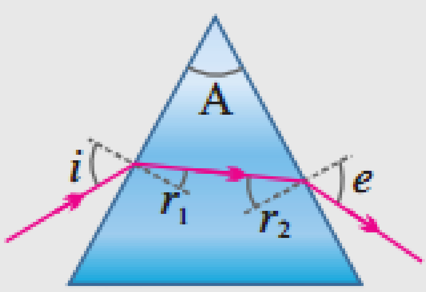
$$y=\frac{H\sin^3 r\left(\mu^2-1\right)}{\left(\mu^2-\sin^2 r\right)^{3/2}}$$

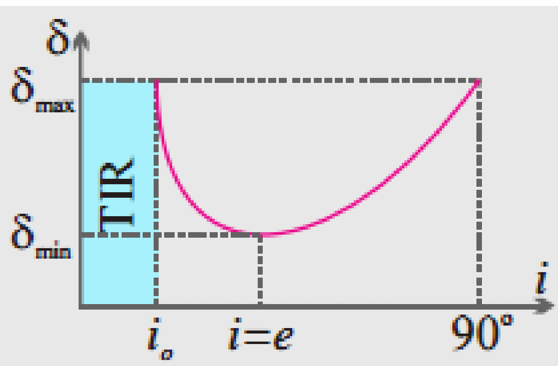


$$d=\frac{t}{\cos r}\sin(i-r)$$

$$d=t\sin i\left(1-\frac{\cos i}{\sqrt{\mu^2-\sin^2 i}}\right)$$

Refraction through Prism:





$$\delta=i+e-A$$

$$r_1+r_2=A$$

$$\mu=\frac{\sin\left(\frac{A+\delta_{\min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

A: refracting angle or apex angle or angle of prism.

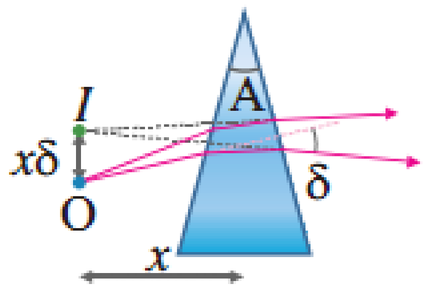
Minimum Deviation: It occurs when: $r_1=r_2=A/2$ & $i=e$

Maximum deviation: It occurs for two angle of incidence: i_o & 90°.

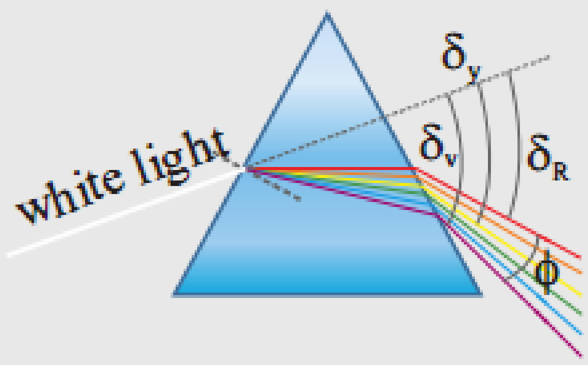
$$\sin i_o=\sin A\sqrt{\mu^2-1}-\cos A$$

$$\mu \equiv \begin{cases} \le \operatorname{cosec} A & \text{NO TIR takes place} \\ \operatorname{cosec} A < \mu \le \operatorname{cosec}\left(\dfrac{A}{2}\right) & \begin{array}{l} 0 \le i < i_o \quad \text{TIR takes place} \\ \hline i_o \le i < 90^o \ \text{TIR doesn't takes place} \end{array} \\ > \operatorname{cosec}\left(\dfrac{A}{2}\right) & \text{TIR definitely takes place} \end{cases}$$

Small angled prism: $\partial = (\mu - 1)A$



Dispersion through prism: It is phenomenon of splitting of light in 7 colors.

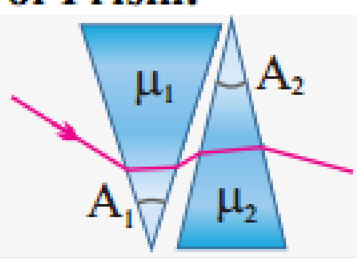


Dispersion: $\phi = \delta_v - \delta_r = (\mu_v - \mu_r)A$

$$\delta_m = (\mu_y - 1)A = \left(\frac{\mu_v + \mu_r}{2} - 1\right)A$$

$$\text{Dispersive Power } (\omega) = \frac{\phi}{\delta_m} = \frac{\mu_v - \mu_r}{\mu_y - 1}$$

Combination of Prism:



$$\partial_v = (\mu_{v1} - 1)A_{v1} - (\mu_{v2} - 1)A_{v2}$$

$$\partial_R = (\mu_{r1} - 1)A_{r1} - (\mu_{r2} - 1)A_{r2}$$

$$\partial_y = (\mu_{y1} - 1)A_{y1} - (\mu_{y2} - 1)A_{y2}$$

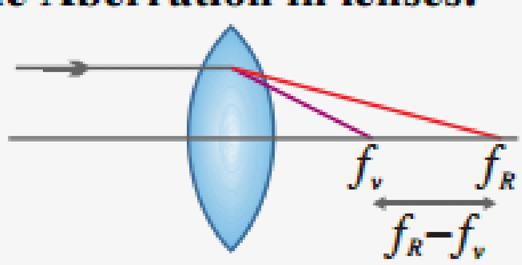
$$= \partial_{y1} - \partial_{y2}$$

$$\phi = \partial_v - \partial_R = \phi_1 - \phi_2$$

Achromatic combination: $\phi = \phi_1 - \phi_2 = 0$

Direct vision combination: $\partial_y = \partial_{y1} - \partial_{y2} = 0$

Chromatic Aberration in lenses:



$f_r - f_v \equiv$ Longitudinal chromatic aberration

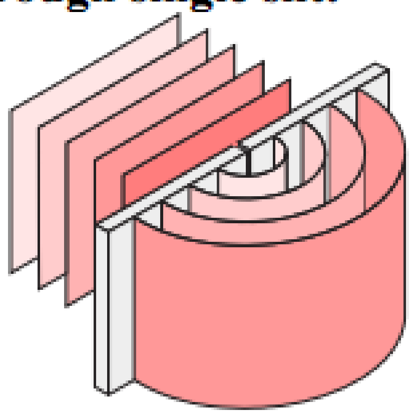
$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{f_r - f_v}{f_{yellow}}$$

Condition for achromatism in lenses: $\dfrac{\omega_1}{f_1} + \dfrac{\omega_2}{f_2} = 0$

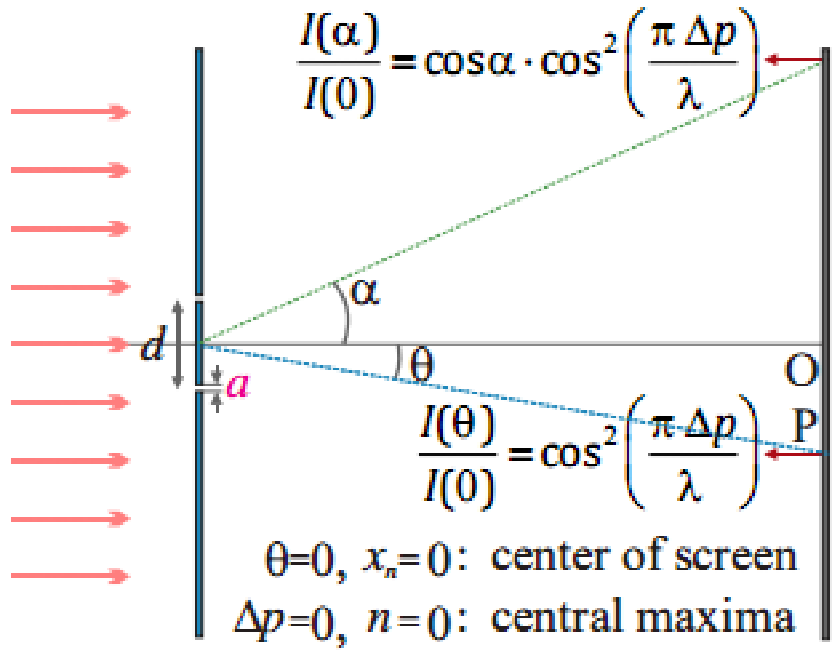
Wavefront: The surfaces joining all points of equal phase are known as *wavefronts*. [*Phase is defined as*: $(Kx-\omega t+\phi)$.]

Perpendicular to the wavefront will give the direction of ray.

Planar wavefront passing through single slit:



The widths of the slits must be of the same order as the wavelength of the light.



$\beta=\frac{\lambda D}{d};\ \beta_\theta=\frac{\lambda}{d}$

$x_n=\frac{D}{\sqrt{(d/n\lambda)^2-1}}$

$x_n=\frac{n\lambda D}{d}$

θ : small angle $\rightarrow \Delta p=d\sin\theta\approx d\tan\theta$

α : large angle $\rightarrow \Delta p=d\sin\alpha$

Δp : Optical Path Difference

n: order of fringe

Coherence: $\Delta\phi$ = constant

$I_R=I_1+I_2+2\sqrt{I_1I_2}\,\cos\Delta\phi$ $\qquad \Delta\phi=\frac{2\pi}{\lambda}\Delta p$ $\qquad \lambda$: Wavelength in **vacuum**

For interference pattern to be distinct $a<\frac{\lambda D}{2d}$.

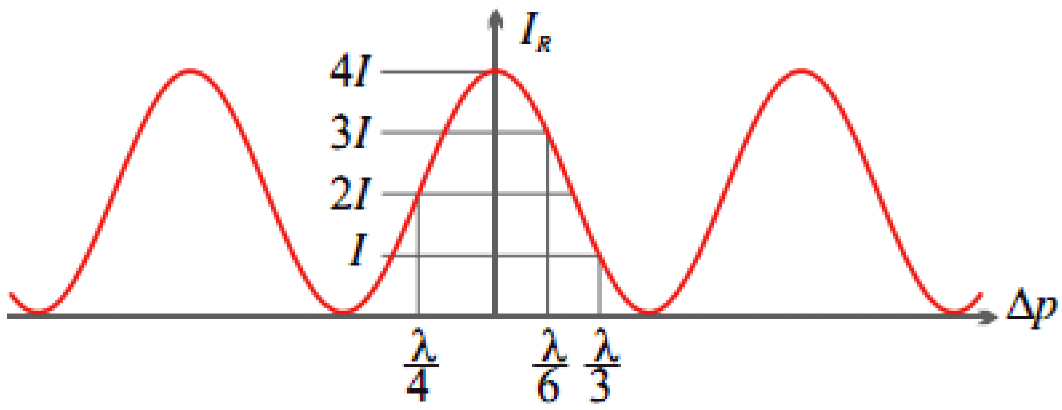
Incoherence: $\Delta\phi\neq$ constant $\quad\Rightarrow I_R=I_1+I_2$ throughout the screen.

Constructive interference	***Destructive interference***
$\Delta\phi=2n\pi$ *or* $\Delta p=n\lambda$	$\Delta\phi=(2n+1)\pi$ *or* $\Delta p=\left(n+\frac{1}{2}\right)\lambda$
$\therefore I_{\max}=\left(\sqrt{I_1}+\sqrt{I_2}\right)^2$	$\therefore I_{\min}=\left(\sqrt{I_1}-\sqrt{I_2}\right)^2$
$N=\begin{cases}2\frac{d}{\lambda}+1 & \frac{d}{\lambda}\in I\ \text{[inculding } \infty]\\ 2\left[\frac{d}{\lambda}\right]+1 & \frac{d}{\lambda}\notin I\ \text{[max. not at } \infty]\end{cases}$	$N=\begin{cases}2\left(\frac{d}{\lambda}+\frac{1}{2}\right) & \left(\frac{d}{\lambda}+\frac{1}{2}\right)\in I\ \text{[inculding } \infty]\\ 2\left[\frac{d}{\lambda}+\frac{1}{2}\right] & \left(\frac{d}{\lambda}+\frac{1}{2}\right)\notin I\ \text{[min. not at } \infty]\end{cases}$

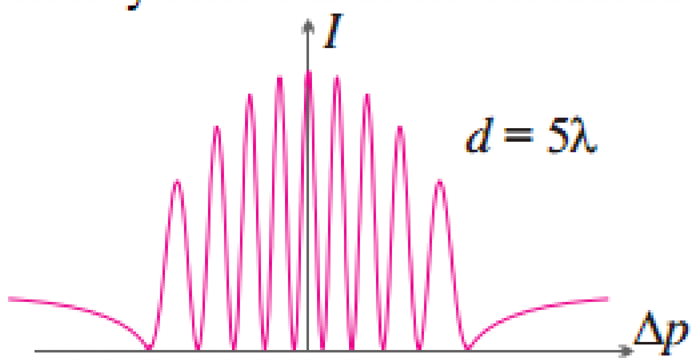
N: Number of maxima/minima formed on the screen.

μL: optical path length = $OPL=\int\mu\,ds$.

Intensity of fringes: If $I_1 = I_2 = I$, then $I_R = 4I\cos^2\left(\dfrac{\Delta\phi}{2}\right) = I_o\cos^2\left(\dfrac{\pi\,\Delta p}{\beta}\right)$

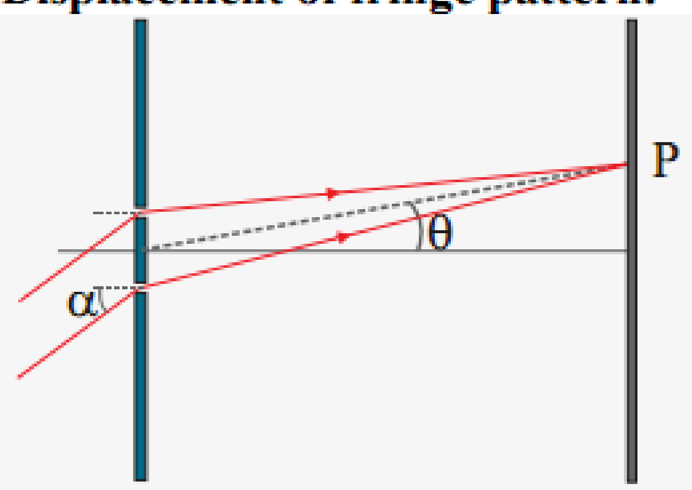


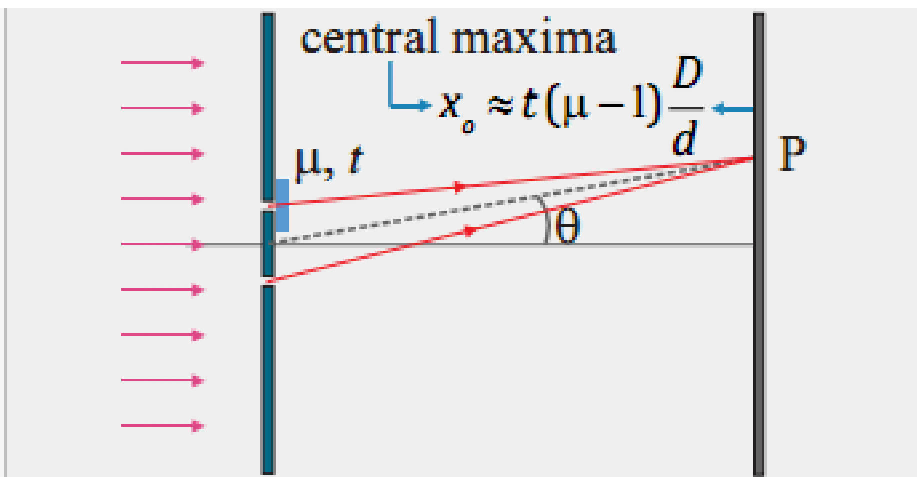
When $d \sim \lambda$, then the number of fringes formed in finite distance are 9. And we can see that intensity of nearby maximas do not remain constant.



Amplitudes of electric field can be changed by using different slit sizes or by using slabs in front of slits which reduces the intensity of transmitted light.

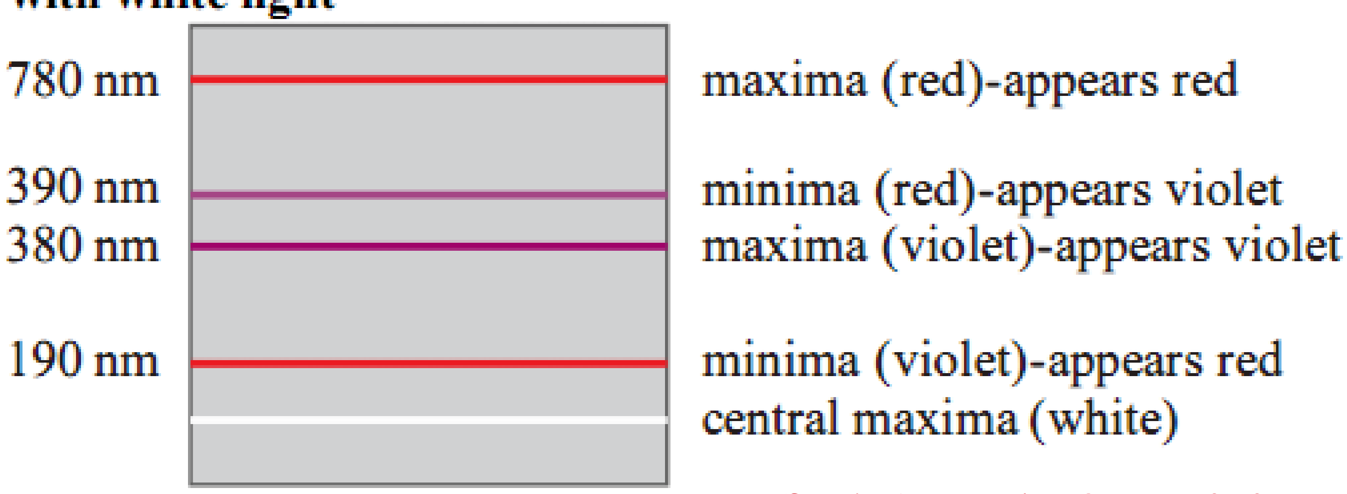
Displacement of fringe pattern:



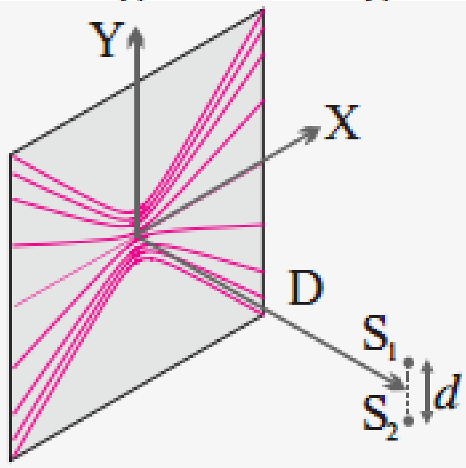


$\Delta p = d\sin\theta - d\sin\alpha$	$\Delta p = d\sin\theta - (\mu-1)t$
Location of central maxima: $\theta = \alpha$	Location of central max: $0 = d\sin\theta - (\mu-1)t$
At center of screen: $\Delta p = -d\sin\alpha$	At center of screen: $\Delta p = -(\mu-1)t$
For small θ: $\Delta p \approx d\tan\theta - d\sin\alpha$	For small θ: $\Delta p = d\tan\theta - (\mu-1)t$

YDSE with white light



Shape of fringes in Young's Pin Hole Experiment

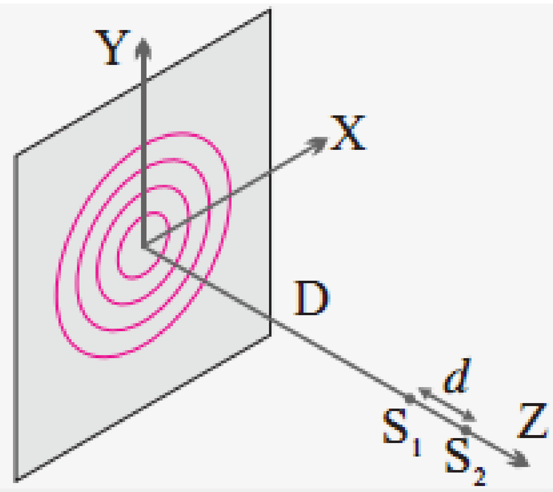


Bright fringes:

$$\frac{y^2}{n^2\lambda^2} - \frac{x^2}{d^2 - n^2\lambda^2} \approx \frac{D^2}{(d^2 - n^2\lambda^2)}$$

Location of apex of hyperbola:

$$y = \frac{D}{\sqrt{(d/n\lambda)^2 - 1}}$$



Bright fringes:

$$\left(\frac{dD}{n\lambda}\right)^2 - D^2 \approx x^2 + y^2$$

Radius of circle: $x \simeq D\sqrt{\left(\frac{d}{n\lambda}\right)^2 - 1}$

For interference pattern to be obtained in the screen $d > n\lambda$

The number of maxima's and minima's in this case is same as in regular YDSE.

Phase change in reflection:

When a ray of light goes from *rarer to denser* medium, *reflected* ray undergoes a phase shift of π.

Note: Light ray reflected from a mirror undergoes a phase change of π.

YDSE with 3 slits:

$$I_R = I_o[1 + 2\cos\Delta\phi]^2$$

$\Delta\phi$: phase difference between light coming from any two consecutive slits.

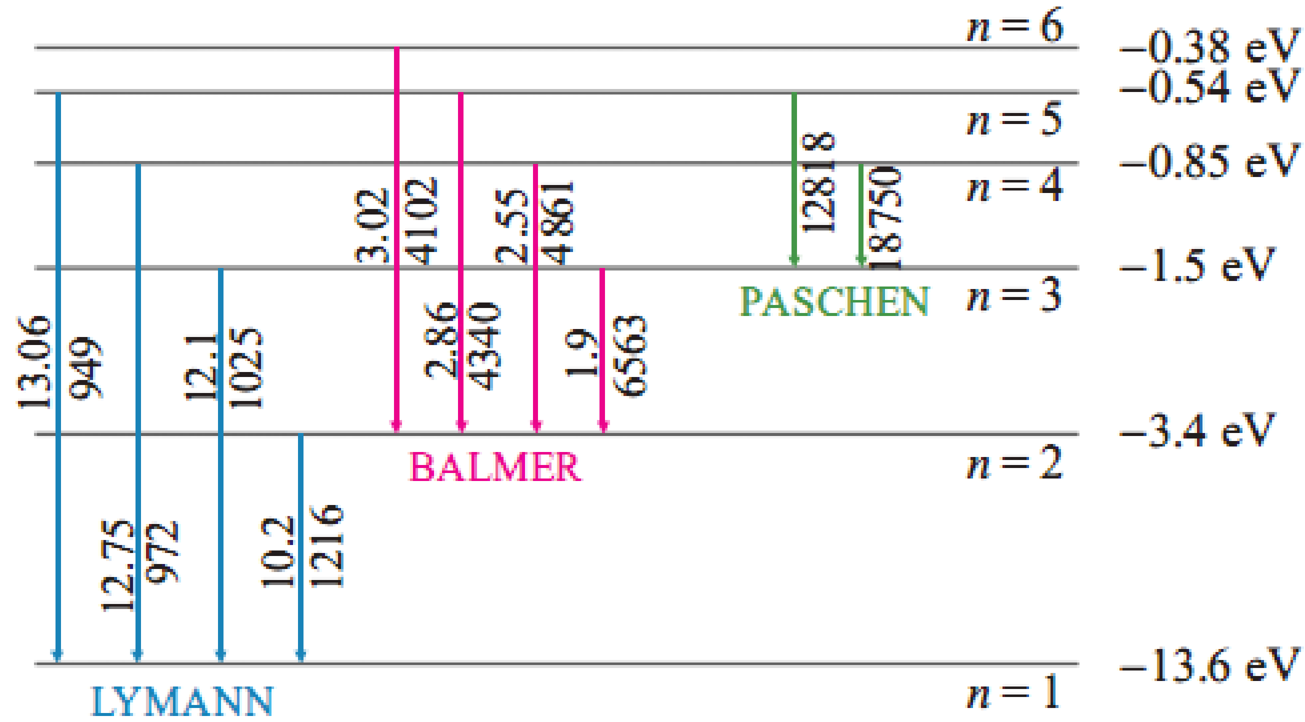
N-slit interference: $D >> Nd$

For small angles: $I_{net}(\theta) = I_{net}(0)\left(\frac{\sin(N\alpha/2)}{N\sin(\alpha/2)}\right)^2$: where, $\alpha = k\,d\sin\theta$

Interference in thin films: $\Delta p = 2\mu d\cos\alpha$ where α is refracted angle

Bohr's Theory: We assume Hydrogen atom at rest in our study.

1. Energy of electron in n^{th} energy level: $E_n = -\dfrac{hcR}{n^2}$



2. Photon (emitted/absorbed) in transition: $h\upsilon_{nm} = E_n - E_m = hcR\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

3.

Case I: H-atom at rest	*Case* II: Electron & nucleus revolving about their common COM
$r_n = \dfrac{h^2\varepsilon_o}{\pi m_e e^2}\dfrac{n^2}{Z} = 0.53\dfrac{n^2}{Z}$ Å $v_n = \dfrac{e^2}{2\varepsilon_o h}\dfrac{Z}{n} = 2.165\times10^6\dfrac{Z}{n}$ m/s $E_n = -\dfrac{m_e e^4}{8\varepsilon_o^2 h^2}\dfrac{Z^2}{n^2} = -13.6\dfrac{Z^2}{n^2}$ eV/atom $= -2.178\times10^{-18}\dfrac{Z^2}{n^2}$ J/atom $= -1312\dfrac{Z^2}{n^2}$ KJ/mole $KE = 13.6\dfrac{Z^2}{n^2}$; $PE = -2\times13.6\dfrac{Z^2}{n^2}$ $\nu_n = \dfrac{m_e e^4}{4\varepsilon_o^2 h^3}\dfrac{Z^2}{n^3} = 6.53\times10^{15}\dfrac{Z^2}{n^3}$ Hz; $T = 1.53\times10^{-16}\dfrac{n^3}{Z^2}$	$r_n = \dfrac{h^2\varepsilon_o}{\pi\mu e^2}\dfrac{n^2}{Z}$ $v_n = \omega(r_1 + r_2) = \dfrac{e^2}{2\varepsilon_o h}\dfrac{Z}{n}$ $E_n = \dfrac{1}{2}\mu v_n^2 - \dfrac{1}{4\pi\varepsilon_o}\dfrac{(Ze)e}{r} = \dfrac{\mu e^4}{8\varepsilon_o^2 h^2}\dfrac{Z^2}{n^2}$ $\nu_n = \dfrac{\mu e^4}{4\varepsilon_o^2 h^3}\dfrac{Z^2}{n^3}$ $R' = \dfrac{\mu e^4}{8\varepsilon_o h^3 c}Z^2$ $\dfrac{R - R'}{R} = \dfrac{m}{M} = \dfrac{1}{1837}$

4.

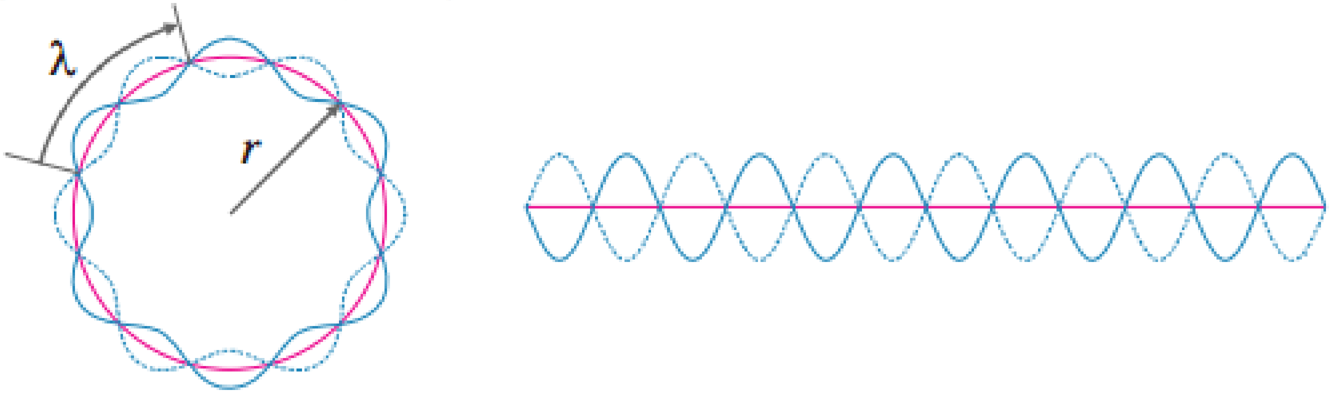
	r	v (speed)	E	$\nu(f)$	R
Case I	$1/m_e$	independent	m_e	m_e	m_e
Case II	$1/\mu$	independent	μ	μ	μ
	$\dfrac{n^2}{Z}$	$\dfrac{Z}{n}$	$\dfrac{Z^2}{n^2}$	$\dfrac{Z^2}{n^3}$	Z^2
	h^2	$1/h$	$1/h^2$	$1/h^3$	$1/h^3$

4. *Excitation energy*: defined from ground state: $\Delta E = 13.6\left(1-\frac{1}{n^2}\right)Z^2 \ eV$

	When transition is from
n = 1: ground state	$n_1 = 1$ to $n_2 = 2$ – 1st excitation energy
n = 2: 1st excited state	$n_1 = 1$ to $n_2 = 3$ – 2nd excitation energy
n = 3: 2nd excited state	$n_1 = 1$ to $n_2 = 4$ – 3rd excitation energy

- The spectral lines are defined for H-atom only and not for hydrogen like ions.
- *Series limit*: n = ∞ → home. It is the shortest wavelength in the series.
- In general, the number of spectral lines emitted when an electron falls from state n_2 to state $n_1 \rightarrow \frac{(n_2-n_1)(n_2-n_1+1)}{2}$

Debroglie Wave and Hydrogen atom.



$n\lambda = 2\pi r \qquad also \qquad \lambda = \frac{h}{mv} \qquad \therefore mv_n r_n = \frac{nh}{2\pi}$

Collision of atoms:

$$KE_G = \underbrace{\frac{1}{2}\mu\vec{v}_r^2}_{\text{Convertible KE}} + \underbrace{\frac{1}{2}M\,\vec{v}_{CG}^2}_{\text{Non-Convertible KE}} \qquad KE_C = \frac{1}{2}\mu\vec{v}_r^2$$

The only mechanism by which the collision can be inelastic is: $\frac{1}{2}\mu\vec{v}_r^2 \geq \Delta E$.

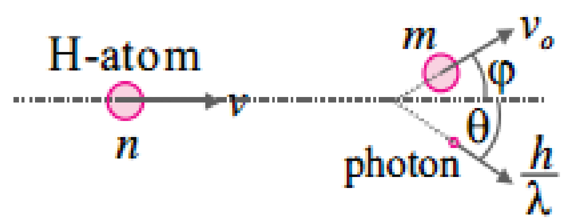
Bohr's theory with atoms in motion:

λ_o, υ_o: wavelength & frequency of emitted photon when hydrogen atom is at rest

$$\upsilon \approx \upsilon_o\left(1+\frac{v}{c}\cos\theta\right)$$

$$\lambda \approx \lambda_o\left[1+\frac{\Delta E}{2mc^2}\right]\left(1-\frac{v}{c}\cos\theta\right)$$

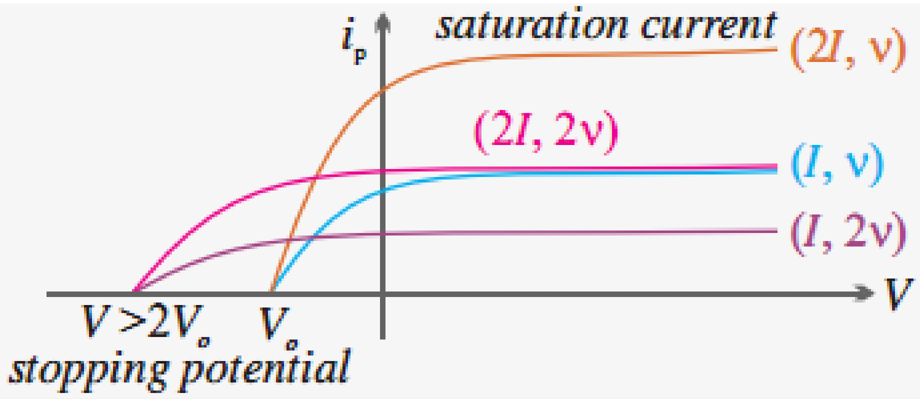
$$v_o \approx v - \frac{\Delta E}{mc}\cos\theta$$



One Dimensional Electron Trap: $E_n = \frac{p_n^2}{2m} = \frac{n^2h^2}{8mL^2}; \qquad p_n = \frac{h}{\lambda_n} = \frac{nh}{2L}$

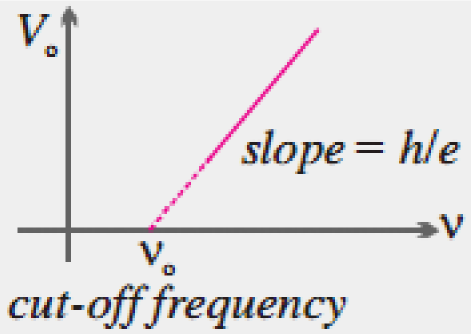
PHOTOELECTRIC EFFECT

Photocurrent: $i = \frac{dq}{dt} = e\frac{dn}{dt}$. $\quad$ n: number of photoelectron emitted.



Principle: $I = \frac{nh\nu}{At}$

$\frac{n}{At}$: *Photon flux*



$\phi = h\nu_o = \frac{hc}{\lambda_o}$

$K_{max} = eV_o$

$\frac{hc}{\lambda} = \phi + K_{max}$

$\lambda_o \equiv$ threshold wavelength

- *velocity* of photoelectrons is w.r.t to the target material.
- *KE* of photoelectrons is a random distribution ranging from 0 to K_{max}.
- An electron loses its *KE* in the following ways:
  - Collision of free electrons with the lattice of metals.
  - Loss of *KE*, as the electrons cross the surface of the metal and comes out. This loss accounts for the work function (ϕ).

Point source:	$P_R = IA\cos\theta = \frac{P}{4\pi r^2}A\cos\theta$	$\left(\frac{n}{t}\right)_{Emitted} = \frac{P}{4\pi r^2}.\frac{1}{h\nu}.\eta$
Line source:	$P_R = IA\cos\theta = \frac{P}{2\pi rl}A\cos\theta$	$\left(\frac{n}{t}\right)_{Emitted} = \frac{P_R}{h\nu}.\eta$
Uniform intensity:	$P_R = IA\cos\theta$	$\left(\frac{n}{t}\right)_{Emitted} = \frac{P_R}{h\nu}.\eta$

III)

	A photon	A particle (e.g. electron)
Momentum	$p = \frac{h}{\lambda} = \frac{E}{c}$	$p = mv$
Wavelength	$\lambda = c/\nu$	$\lambda = h/mv = h/p$
Energy	$E = h\nu$	$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

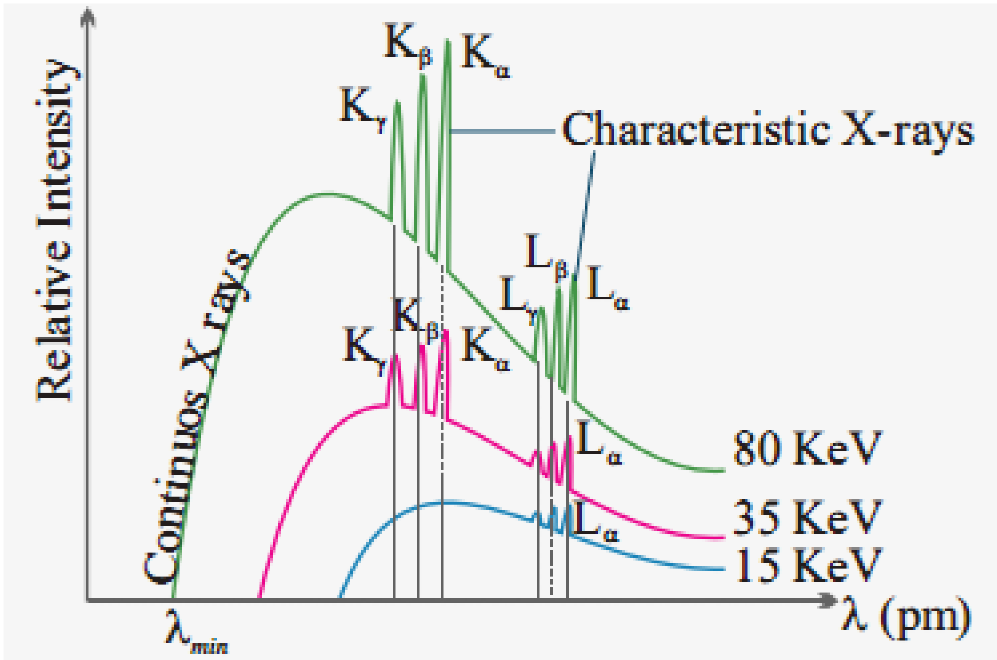
Intensity of electromagnetic radiation: I = ń $h\nu$,
ń: number of photoelectrons per unit area of the wavefront per unit time.

Electromagnetic momentum/time: $\frac{p}{\Delta t} = \frac{I.\Delta A.\cos\theta}{c}$

Electromagnetic pressure = I/c (perfect absorption)

Force due to electromagnetic pressure (P): $F = PA_\perp$ $\quad$ $A_\perp$: Projection area

X-RAYS



$$\begin{bmatrix} K_\alpha : & 2 \to 1 \\ K_\beta : & 3 \to 1 \\ K_\gamma : & 4 \to 1 \end{bmatrix} \quad \begin{bmatrix} L_\alpha : & 3 \to 2 \\ L_\beta : & 4 \to 2 \\ L_\gamma : & 5 \to 2 \end{bmatrix}$$

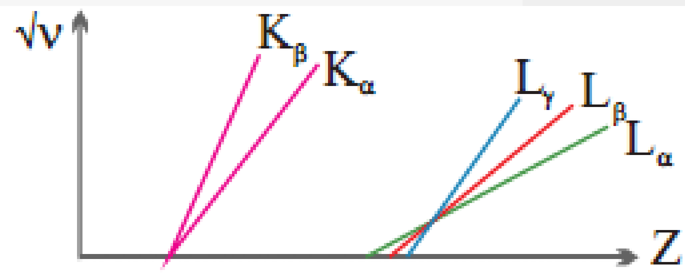
$$\sqrt{\upsilon} = a(Z - b),$$

$$a = 4.95 \times 10^7 \ Hz^{\frac{1}{2}} \quad for \quad K_\alpha$$

For K_α b = 1. For L_α = 7.4

$$\lambda_{\min} = \frac{hc}{eV}$$

V: *accelerating voltage*



Intensity (I) of emitted electrons on changing filament current

- When we only increase filament voltage (V_o); more number of thermal electrons will be emitted. Thus, $\frac{n}{t}$ increases. Therefore, I increases.
- On increasing accelerating voltage (V); λ_{min} decreases, Therefore, I increases.
- If υ increases, then it is called Hard X-rays.

NUCLEAR PHYSICS

Forces	*Relative Strength*	*Range*	*Mediating Particle*
Nuclear	1	Short (1 fm)	Gluon (π)
Electromagnetic	1/137	Long	Photon (just as concept of field)
Weak Forces	10^{-6}	(10^{-3} fm)	bosons
Gravitational	10^{-38}	Long	Gravitation

There are 5 kinds of forces that operate within the nucleus.

i) Attractive forces
ii) Hard core repulsive forces
iii) Coulomb forces of repulsion
iv) Weak forces: *Weak forces are responsible for beta decay.*
v) Tensor forces: Range of tensor forces is 3 *fermi*.

Nuclear Stability:

A typical heavy nucleus is stable when $1.2 < \frac{N}{Z} < 1.4$.

No elements with Z > 92 occur naturally.

Lead (Z = 82) is a stable nucleus. Nuclei of Z > 82 are radioactive.

Symbols in nucleus: $_{Z}X^{A}$ (called a *nuclide*)

A: mass no. = no. of protons + no. of neutrons = no. of nucleons

Z: atomic number = no. of protons

Atomic Mass Unit (unified mass unit):

one atom of the isotope ^{12}C is exactly 12 u, where $1\text{ u} = 1.660559\times10^{-27}$ Kg.

$$m_n > m_p > m_e \qquad \frac{m_e}{m_p} = \frac{1}{1837}$$

Binding energy: $\Delta m = Z\,m_p + N\,m_n - M$ $\qquad M$: Mass of nucleus

$BE = 931.5$ MeV/amu

$BE \equiv E_B = \left(Zm_p + Nm_n - M\right)\times 931.5$ MeV/atom

$$E_B/\text{nucleon} = \frac{\Delta m\times 931.5}{A}\ \text{MeV/atom} = \frac{\Delta m\times 931.5}{A}\times 1.6\times10^{-13}\ \text{J/atom}$$

Binding energy (E_B): Energy supplied to separate the constituent *of a nucleus* to a large distance.	**Reaction energy (Q-value)** It is the energy released in reaction

Note: In both the cases, we will take "reactant – product".

$E_B = BE(\text{reactant}) - BE(\text{product})$

$E_B = BE(^2_1H) - BE(^1_1H)$

$= \left[m\left(^1_1H\right) + m_n - m\left(^2_1H\right)\right]c^2$

$= \Delta m\,c^2$

Δm : mass defect

$$^2_1H \rightarrow {}^1_1H + {}^1_0n$$

$= \textit{mass of reactant} - \textit{mass of product}$

$\Delta m = m\left(^2_1H\right) - m\left(^1_1H\right) - m_n$

(Δm): *Mass difference*

$$\therefore E_B = -Q$$

$Q > 0$ *or* $E_B < 0$: *Energy is released* (*spontaneous/exothermic react^n*)

$Q < 0$ *or* $E_B > 0$: *Energy is supplied* (*induced /endothermic reaction*)

Reactant mass > Product mass (*Spontaneous reaction*).

Product mass > Reactant mass (*Induced Reaction*).

If $E_B > 0$,

$BE(\text{reactant}) > BE(\text{product})$ $\qquad\Rightarrow$ reactant more stable

Nuclear Reaction: A nuclear reaction is represented by $X(a,b)Y$.

$Q = (m_X + m_a)c^2 - (m_Y + m_b)c^2$ $\qquad$ ---(1)

$Q = (K_Y + K_b)c^2 - (K_X + K_a)c^2$ $\qquad$ ---(2)

Equations 1 and 2 are valid only when Y and b are in their ground states.

Scattering: If a and b are identical particles, which requires that X and Y also be identical, we describe the reaction as *scattering*.

a) *Elastic scattering*: If $K_i = K_f$, $Q = 0$ and all nuclides remain the same and in their ground states.

b) *Inelastic scattering*: $K_i \neq K_f$, $Q \neq 0$, in this case Y ***or*** b may be left in an excited state. *We are talking about nuclear energy level.* When they deexcites back, gamma radiation is emitted.

Alpha Decay: $K_\alpha = \frac{m_Y}{m_X}Q = \frac{m_X - m_\alpha}{m_Y + m_\alpha}Q$; $\quad K_Y = \frac{m_\alpha}{m_X}Q$

βeta Decay:

β^- *decay*: ${}_on^1 \rightarrow {}_1H^1 + {}_{-1}e^o + \bar{\upsilon}$ (*antineutrino*)

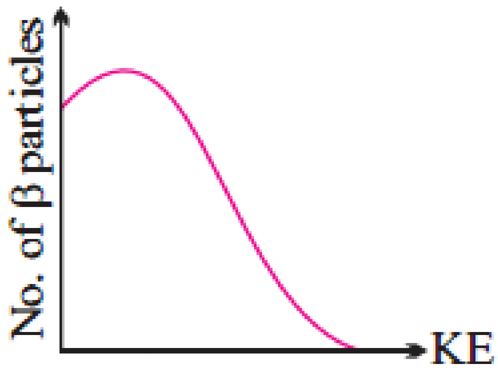
${}_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\upsilon}$

$\therefore \Delta m = {}_ZX^A - {}_{Z+1}Y^A$

β^+ *decay*: ${}_1H^1 \rightarrow {}_on^1 + {}_{+1}e^o + \upsilon$ (*neutrino*)

${}_ZX^A \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + \upsilon$

$\Delta m = {}_ZX^A - {}_{Z+1}Y^A - 2m_e$



- In beta decay, the KE of the emitted electrons have a continuous spectrum of energies, from zero up to a maximum K_{max}.
- Direction of emitted electrons & recoiling nuclei are never exactly opposite.

Electron Capture (or K-capture, or inverse beta decay): ${}_1^1H + {}_{-1}^0e \rightarrow {}_0^1n + \nu$

${}_ZX^A + {}_{-1}e^0 \rightarrow {}_{Z-1}Y^A + \upsilon \qquad \Delta m = \left[m\left({}_ZX^A\right) - m\left({}_{Z-1}Y^A\right)\right]$

Thus, the charge in the atom remains the same still.

Electron capture leads to X-ray with equation: $\sqrt{\upsilon} = a(Z - 1 - b)$

Positrons have a very short life-time. ${}_{+1}e^o + {}_{-1}e^o \rightarrow 2\gamma$

RADIOACTIVITY

A_t unit: 1 Curie = 3.7 x 10^{10} *dps* (or *Bequerel* or *counts/sec*)

Specific Activity: $\frac{A}{m}$, where m is the mass of sample .

$N_t = N_oe^{-\lambda t} = N_o(0.5)^{t/t_{1/2}} \quad t_{1/2} = \ln 2/\lambda. \quad \langle t \rangle = \frac{1}{\lambda}$

Successive disintegration

Case 1: Production rate is constant: $\frac{dN_t}{dt} = R - \lambda N_t \quad \therefore N_t = \frac{R}{\lambda}\left(1 - e^{-\lambda dt}\right)$

Case 2: Production rate is not constant

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \qquad ----(1)$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \qquad ----(2)$$

$$\frac{dN_3}{dt} = \lambda_2 N_2 \qquad ----(3)$$

$$N_1 = N_oe^{-\lambda_1 t}$$

$$N_2 = \frac{\lambda_1 N_o}{\lambda_2 - \lambda_1}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$$

$$N_3 = N_o\frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}\left[\frac{1 - e^{-\lambda_1 t}}{\lambda_1} - \frac{1 - e^{-\lambda_2 t}}{\lambda_2}\right]$$

$$N_1 + N_2 + N_3 = const. = N_o$$

$P_{disintegration} = 1 - e^{-\lambda T} \; P_{surv} = e^{-\lambda t} = (0.5)^{t/t_{1/2}} = \frac{N_t}{N_o} \qquad \lambda_{eff} = \lambda_1 + \lambda_2$