PHY 205: Modern Physics (3 units)

Google Classroom Link:

Live Chat/Interactions: 11:00 – 12:00 noon Mondays and Wednesdays.

RECOMMENDED TEXTS:

CONCEPTS OF MODERN PHYSICS – ARTHUR BEISER

COLLEGE PHYSICS – SERWAY

Section 1: Prof J.O. Ojo (Room PY 122, White House)

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2 Major Concepts defining "Modern" Physics (post 1900)

- Quantum Physics –[PHY 205]
- Relativity – [PHY 206]

Module 1:
The Origin of Quantum Theory

- Quanta
- Blackbody
- Thermal equilibrium
- Blackbody radiation spectra
- Exercises

Quanta

- Many quantities in life come as specific packages with discrete values.

  ➔they are quantized.
- Currency notes in Nigeria: N5, N10, N20, N50, N100, N200, N500, N1000
- ➔a valid naira currency can only have one of these 8 possible values.

Quanta

- Sometimes, to get a certain desired result, you need a minimum value of a certain quantity to come as a package. Cumulatives don’t work!
  - Climbing a ladder (the precise amount of energy needed to get to the next rung must come in a single step. And must not be higher either!)’
  - Killing a cancer cell. (a minimum dose required to get result. And in radiation matters, “enough” is always “enough” . i.e. Too much is dangerous!)

EM radiation is quantized

- Electromagnetic Radiation
- One of the major achievements of “modern” Physics is the knowledge that electromagnetic radiation comes and leaves in quanta.
- The quantum theory originated from this knowledge
- Specifically from the spectrum of electromagnetic radiation emitted from a blackbody – blackbody radiation spectra

Blackbody radiation spectra

- A “blackbody” is any object that absorbs all the radiation that falls on it.
- For instance, if the radiation is in the visible range, and all the component colours are absorbed, the object will look black, since it is not reflecting any of the incident radiation.
- A true blackbody absorbs ALL em radiation from all sections of the em spectrum (from radiowaves to gamma rays).

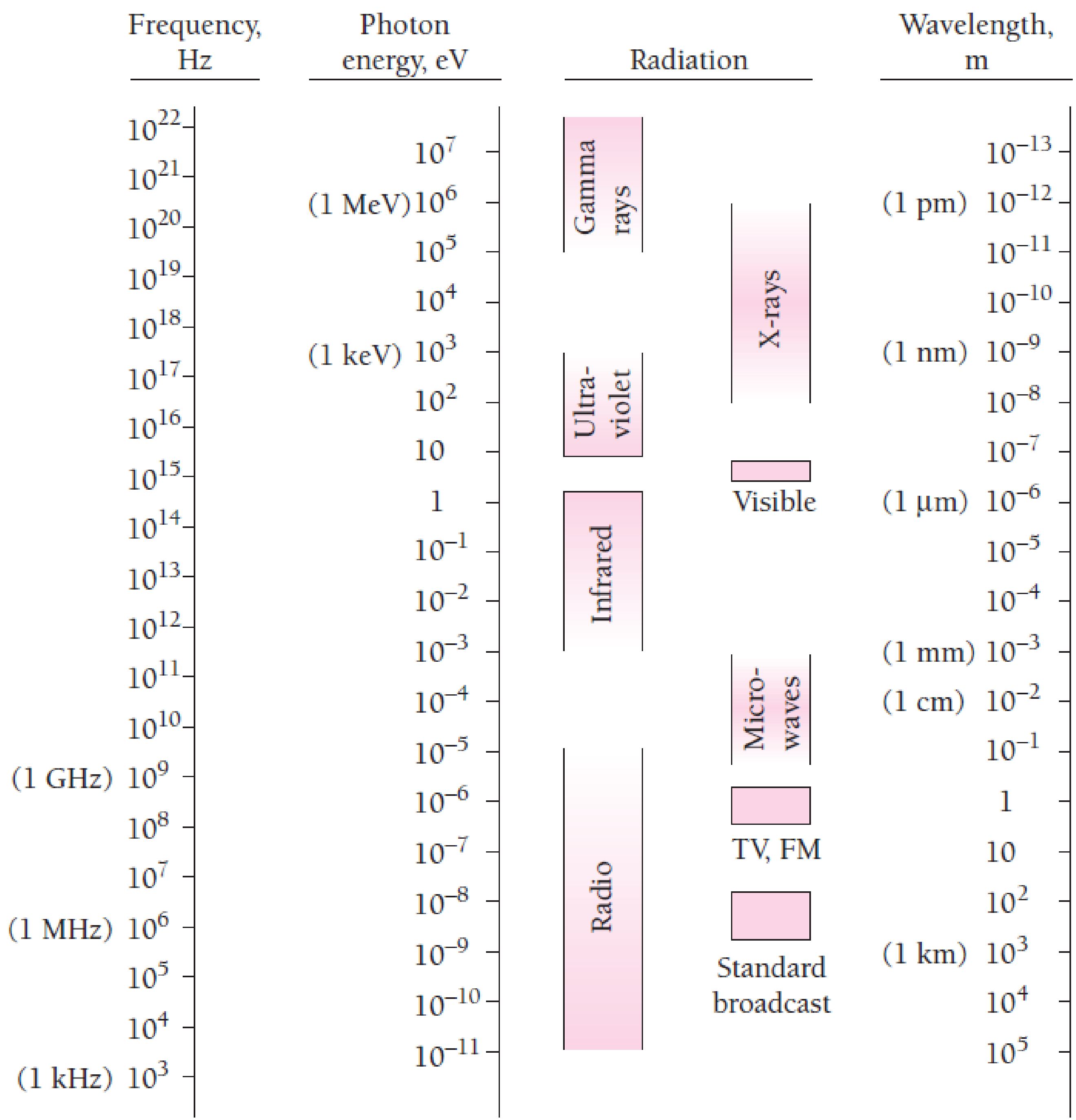


Figure 2.2 The spectrum of electromagnetic radiation.

Blackbody emission spectra

- A blackbody at a specific temperature (in thermal equilibrium with its environment) is also emitting radiation.
- As it is capable of absorbing radiation of ALL frequencies, it is similarly capable of emitting radiation at ALL frequencies.
- Before 1900, NOBODY could explain the spectrum of radiation emitted by a blackbody. All the physics known till then (Classical Physics) not sufficient to explain it. Understanding blackbody radiation marks the birth of Modern Physics.
- What exactly was the problem?

The color and brightness of an object heated until it glows, such as the filament of this light bulb, depends upon its temperature, which here is about 3000 K. An object that glows white is hotter than it is when it glows red, and it gives off more light as well.

- The emission spectrum from a blackbody:
- 1. Intensity depends only on Temperature (for real materials, the type of material will also contribute. BB is idealized)
- 2. Has a peculiar shape (distribution – intensities of the various frequencies) which classical Physics can not explain
- See next slide

The higher the Temperature T, the higher the INTENSITY of total radiation (Stefan's Law)

The higher the Temperature T, the higher the FREQUENCY at which maximum (peak) occurs (Wien's Law)

The intensity after attaining the peak eventually nose-dive to zero (from Planck's equation)

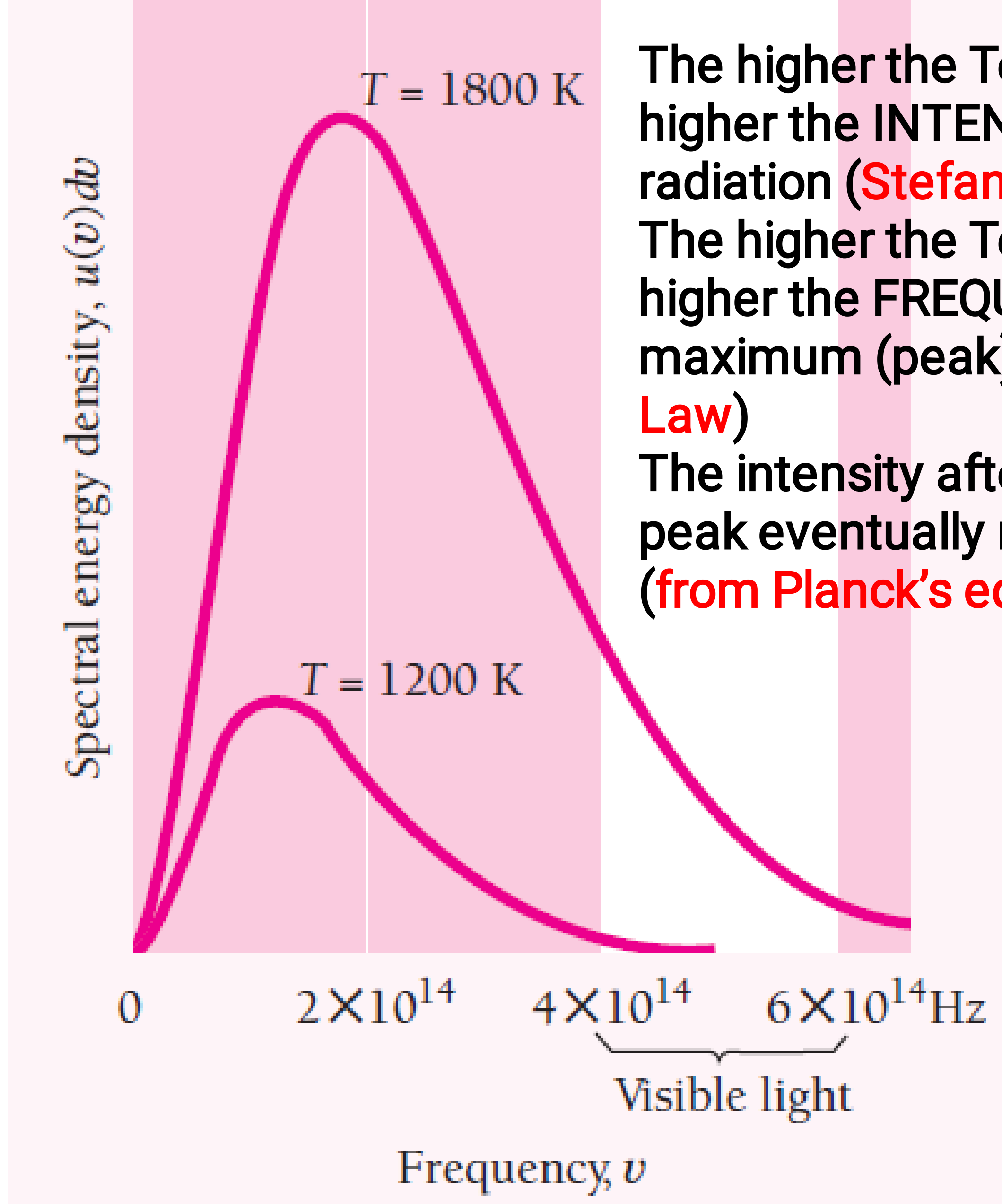


Figure 2.6 Blackbody spectra. The spectral distribution of energy in the radiation depends only on the temperature of the body. The higher the temperature, the greater the amount of radiation and the higher the frequency at which the maximum emission occurs. The dependence of the latter frequency on temperature follows a formula called Wien's displacement law, which is discussed in Sec. 9.6.

“Expected” versus “Observed” – The Ultraviolet Catastrophe

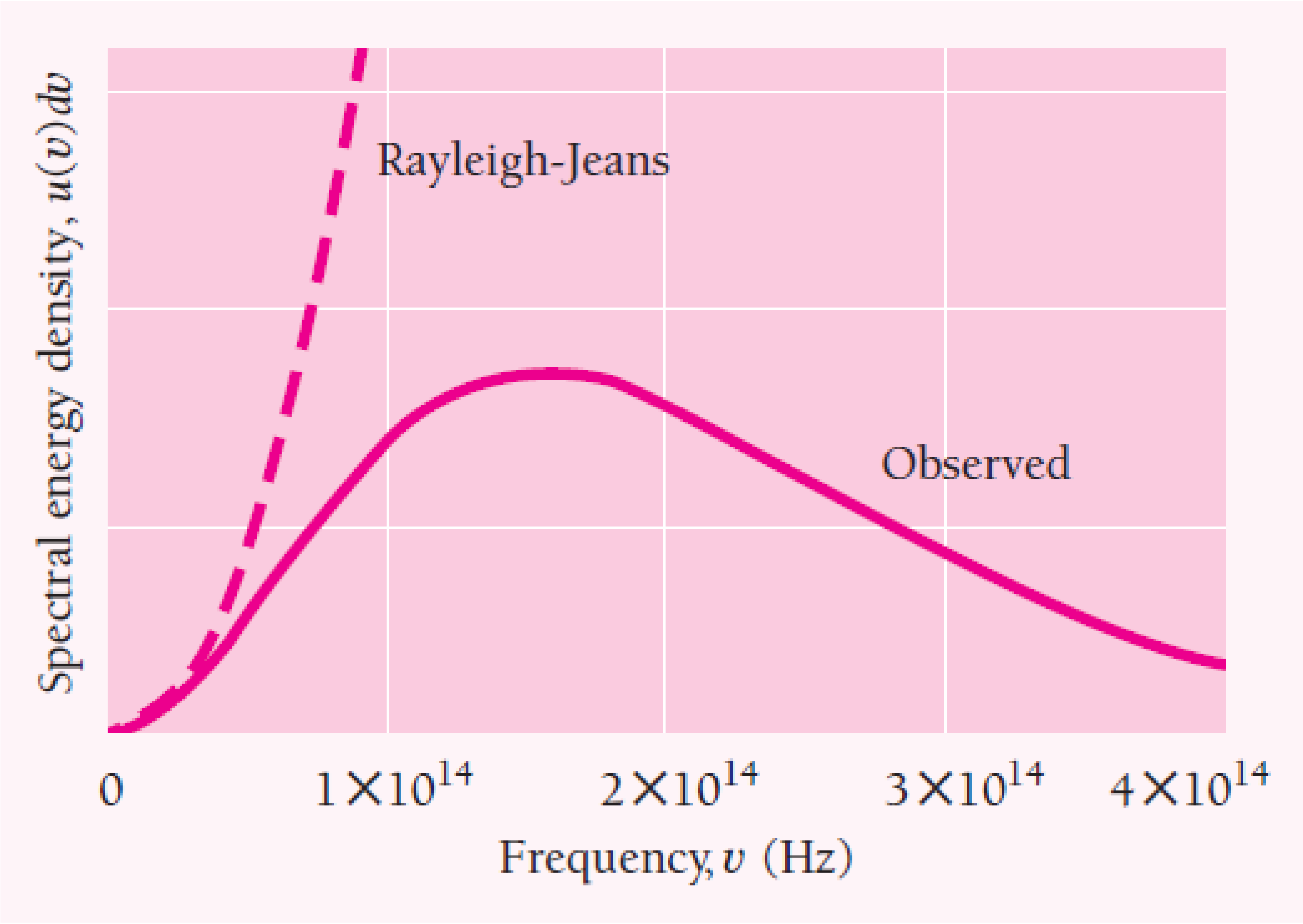


Figure 2.8 Comparison of the Rayleigh-Jeans formula for the spectrum of the radiation from a blackbody at 1500 K with the observed spectrum. The discrepancy is known as the ultraviolet catastrophe because it increases with increasing frequency. This failure of classical physics led Planck to the discovery that radiation is emitted in quanta whose energy is $h\nu$.

The equation obtained from Classical Physics considerations for Blackbody radiation (Obtained by the pair of Lord Rayleigh and James Jeans)

$$u(\nu)\, d\nu = \bar{\epsilon} G(\nu)\, d\nu = \frac{8\pi kT}{c^3} \nu^2\, d\nu \qquad (2.3)$$

The energy density u as a function of frequency ν [u(ν)] determines the radiation rate from the blackbody. This Rayleigh-Jeans formula which gives radiation rate increasing proportionately as square of ν implies that by the time we reach ultraviolet frequency range, we already have radiation rates tending to infinity! This phenomenon is called the "ultraviolet catastrophe". It of course doesn't occur in real life. So the formula is wrong.

Max Planck, in 1900, first obtained an equation that fits the shape very accurately.

$$u(\nu)\, d\nu = \frac{8\pi h}{c^3} \frac{\nu^3\, d\nu}{e^{h\nu/kT} - 1} \qquad (2.4)$$

$$h = 6.626 \times 10^{-34}\ \text{J} \cdot \text{s}$$

- Note:
- we can obtain Planck’s formula by making the substitution

$$kT \rightarrow \frac{h}{e^{\frac{hV}{kT}}-1}$$

- At low frequencies ν, R-J equation agrees with Planck’s

- At high frequencies, $h\nu \gg kT$ and $e^{h\nu/kT} \rightarrow \infty$,

  which means that $u(\nu)\ d\nu \rightarrow 0$

  which means, the ultraviolet catastrophe has been resolved.

- But Physics is much more than mere “curve fitting.” Why do we have this equation (which Planck attributed to “lucky guesswork”)?

Understanding the Physics of Planck's equation

- Blackbody is an idealized contraption. Can absorb ALL radiation frequencies perfectly
- In thermal equilibrium with environment, must also emit ALL frequencies perfectly
- BB can be approximated by a cylinder with a tiny hole in one side. All energies incident can enter, and they are largely all trapped within the cavity
- Radiation from the "Cavity Radiator" (BB) results from interaction between radiation within the cavity and radiation emitted by oscillating atoms on its walls.
- We know that electromagnetic radiation can, IN PRINCIPLE, have a continuous distribution of frequencies.
- Hence, in principle, these atomic oscillators can have any value for their frequencies (within certain range, of course, depending on the temperature)

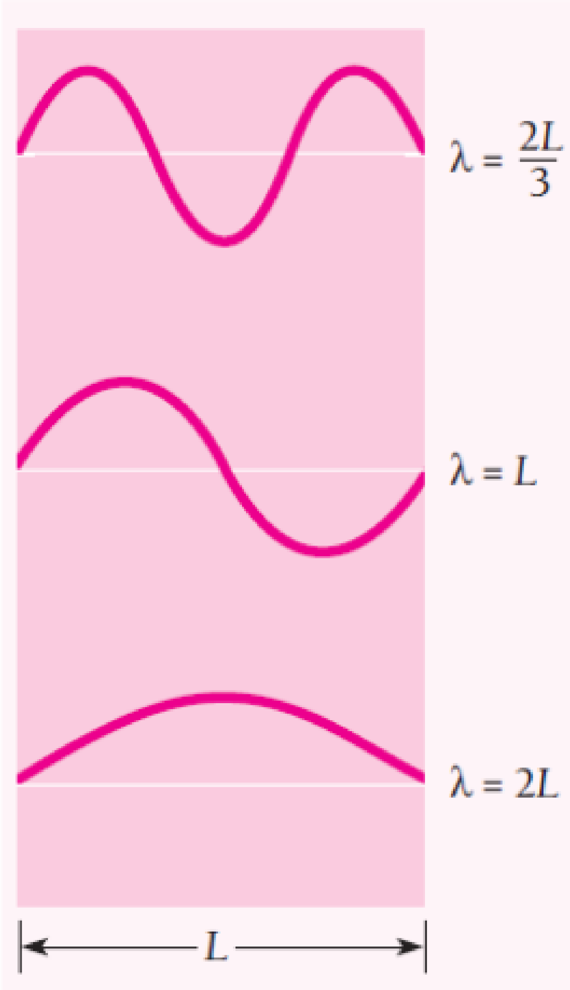


Figure 2.7 Em radiation in a cavity whose walls are perfect reflectors consists of standing waves that have nodes at the walls, which restricts their possible wavelengths. Shown are three possible wavelengths when the distance between opposite walls is L.

- Within a certain frequency range ($\nu + d\nu$), there are a specific number of states that radiation of a particular frequency can exist in.
-
- Take for example, on a given length of rope fixed at both ends, there are certain number of waves (of different wavelengths) that can be set on.
- These are called modes. See diagram
-
- The higher the value of frequency, the more the number of standing waves (modes of waves) that can be accommodated
- (e.g. For the same container size, you can put in more number of pebbles (smaller wavelength →higher frequency) than big stone)

- IF the atomic oscillators actually oscillate (and therefore radiate energy) at every frequency permitted IN PRINCIPLE, then the higher the frequency, the higher the "energy density" of the emission spectrum.
- This is precisely the ultraviolet catastrophe predicted by Classical Physics.
- Planck came up with his postulate (after several weeks of "the most strenuous work of my life") in 1900 stating that an oscillator cannot oscillate at every frequency permitted IN PRINCIPLE, but only at certain frequencies

Oscillator energies $$\epsilon_n = nh\nu \qquad n = 0, 1, 2, \ldots \qquad (2.5)$$

$$\epsilon_n = nh\nu \qquad n = 0, 1, 2, \ldots$$

- (Imagine crossing a river on a ladder used as an *emergency* bridge. You can not take every step allowed you in principle, but must be constrained to those steps dictated by the gaps between the rungs of the ladder)
- This POSTULATE (A postulate doesn't give any proof!) is the famous Planck's Postulate; and it solves the Blackbody radiation problem.
- Question how real is this? It turned out to be applicable in every area of life, once we are considering them at atomic level!
- (See for example this article written by me in 2006 applying quantization to a very real social problem: https://nigeriaworld.com/articles/2006/feb/072.html)
- Next Module, we see some other Physics applications.

Some Exercises

1. From Planck's equation (equation 2.4 of Beiser), obtain the total energy density of blackbody radiation as a function of Temperature.
[Hint: Integrate $E(\nu)d\nu$ from $\nu=0$ to $\nu \rightarrow \infty$]
You are free to check textbooks or the internet.

The final answer is the Stefan-Boltzman Law.
It is usually written as:

$$F = e\sigma T^4$$

Where
F is the Radiation Emittance (defined as Energy E emitted by the object per unit Area and per unit time)
σ is the Stefan-Boltzman constant = $5.67 \times 10^{-8} Wm^{-2}K^{-4}$
and
e is the emissivity of the object.
Values for e range from 0 to 1. Only a blackbody has an emissivity of 1.

- **Example:** A body of emissivity e = 0.75, with surface area of 300 cm^2 and temperature 227 ºC is kept in a room at temperature 27 ºC. Using the Stephens Boltzmann law, calculate the initial value of net power emitted by the body. (Answer: 69.4 Watts.)

2. Express the Planck's radiation equation (eqn 2.4 in Arthur Beiser) in terms of wavelength λ.

$$u(\nu)\, d\nu = \frac{8\pi h}{c^3} \frac{\nu^3\, d\nu}{e^{h\nu/kT} - 1} \qquad (2.4)$$

[Hint: from the relationship $c = \nu\lambda$, find expression for $d\nu$, and substitute]

3. From the expression you obtained in Exercise 2, it is possible (perhaps a little difficult calculus involved) to obtain the wavelength at which the energy density is a maximum (λ_{max}). This result, called the Wien's displacement Law, can be shown to be given by:

$$\lambda_{max} = \frac{b}{T}$$

- Where b is the Wien's displacement constant = 2.8977×10^{3} m.K and T is the Temperature in K.

Wien’s Displacement Law

Wien’s Law states that the wavelength at which the emitted radiation has peak intensity is inversely proportional to Temperature. When a piece of metal is heated, it first becomes ‘red hot’. This is the longest visible wavelength. On further heating, it moves from red to orange and then yellow. At its hottest, the metal will be seen to be glowing white. This is the shorter wavelengths dominating the radiation. This relationship can thus be used in thermometry:

- Some examples of the use of Wien’s displacement law:

1. If the peak radiation from a wood fire is found at 2000 nm. What is the temperature of the fire? (Note: the human eye is not very sensitive to radiation at this wavelength. So we may not be able to use the wood fire for light, but it could be an excellent source of heat – if we mind the pollution)

2. The temperature of the sun’s surface is 5700 K. Using the Wien displacement law; calculate the wavelength at which the radiation from the sun has maximum intensity. (Check the Literature, How visible is this peak radation to the human eyes?)

Assignments – check assignment tab in google classroom and submit there.

1.a. How much money can be found in the pockets/purses of the 250 students in PHY205 class if no student may carry 2 identical banknotes with them? (Give the range: minimum – maximum. Ignore coins)

1.b. Repeat question 1 if each student may carry up to 20 identical banknotes with them.

1.c. Repeat question 1 to determine how much money can be found in the bank account of the 250 students (noting that various bank charges of different values are being added or subtracted in real time)

2. Assuming that the sun is a spherical blackbody with a radius of 7 x 10^{8}m. Calculate the sun's temperature and the radiation energy density within it. The intensity of the Sun's radiation at the surface of the earth (which is 1.5 x 10^{11}m distant from the sun) is 1.4 x 10^{3}Wm^{-2}

3. A hot black body emits the energy at the rate of 16 J m^{-2} s^{-1} and its most intense radiation corresponds to 20,000 Å. When the temperature of this body is further increased and its most intense radiation corresponds to 10,000 Å, then find the value of energy radiated in Jm^{-2} s^{-1}.

Module 2: **Electrons and quanta**

Cathode rays
The specific charge of electrons
The charge and mass of electrons

- Items to cover:

- Cathode rays. Thermionic emission, Properties
- Applications: CRO, TV,
- Cathode rays are electrons!
- Charge/mass ratio
- Mass spectrometers, accelerators, cyclotrons
- Various forms of electrons:,
- Bound electrons (inner shell, outer shell,) Conduction electrons. Free electrons (Fast electrons, slow electrons); auger electrons, beta minus particles
- Refs. Halliday Ch.28; Serway ch. 15:7. Internet sources

Introduction: Electric Charge is quantized

- Two of the basic properties of matter are mass and electric charge.
- Electric charge is a good example of a quantized quantity, with the smallest charge possible* being the charge carried by an electron.
- Other quantities of charge come in multiples of e, and they are usually obtained either by adding electrons to a system (to get a negatively-charged system) or removing electrons from them (to get positive charges)
- i.e. the charge Q on any body can be written as
- $Q = ne$
- where e is the electronic charge ($- 1.602 \times 10^{-19}$ coulombs),
- and n = 1, 2, 3……
- *some particles called hadrons can be modeled as comprising of substructures (called quarks) which have fractional electronic charges).

The Electron is probably the most important particle in the practice of Modern Physics.

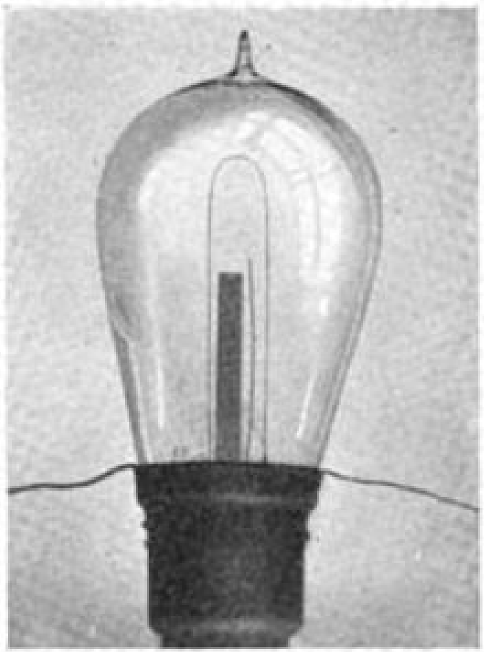
SIR JOSEPH JOHN THOMSON,
English Physicist (1856–1940)

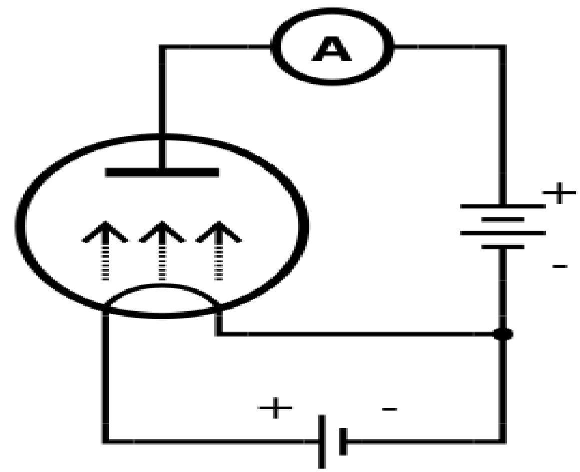
Thomson, usually considered the discoverer of the electron, opened up the field of subatomic particle physics with his extensive work on the deflection of cathode rays (electrons) in an electric field. He received the 1906 Nobel prize for his discovery of the electron.

- Others e.g. Johann W. Hittorf and William Crookes have come across electrons before, in cathode ray tubes, but didn't appreciate the true nature and significance.
- Cathode-ray studies began in 1854 when Heinrich Geissler, a glassblower and technical assistant to German physicist Julius Plücker, improved the vacuum tube.
- Eddison is credited with discovering thermionic emission which is a principal way of obtaining electrons for studies and applications.

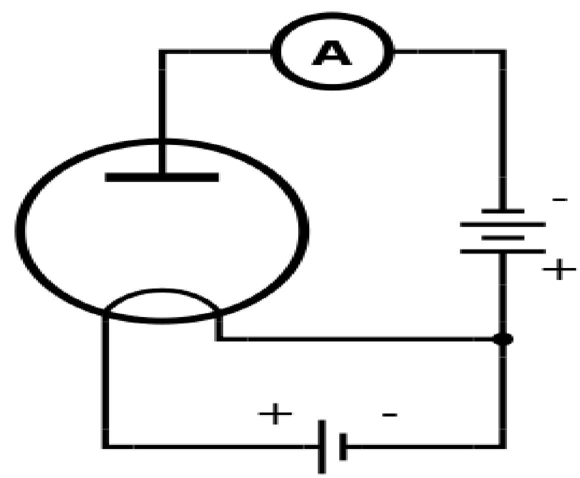
Thermionic Emission:

liberation of electrons from an electrode by virtue of its temperature (releasing of energy supplied)





Electron flow



No current

One of the bulbs with which Edison discovered thermionic emission. It consists of an evacuated glass light bulb containing a carbon filament *(hairpin shape)*, with an additional metal plate attached to wires emerging from the base. Electrons released by the filament were attracted to the plate when it had a positive voltage

[NOTE THE CONVENTIONS AND SYMBOLS IN

How do you get electrons?

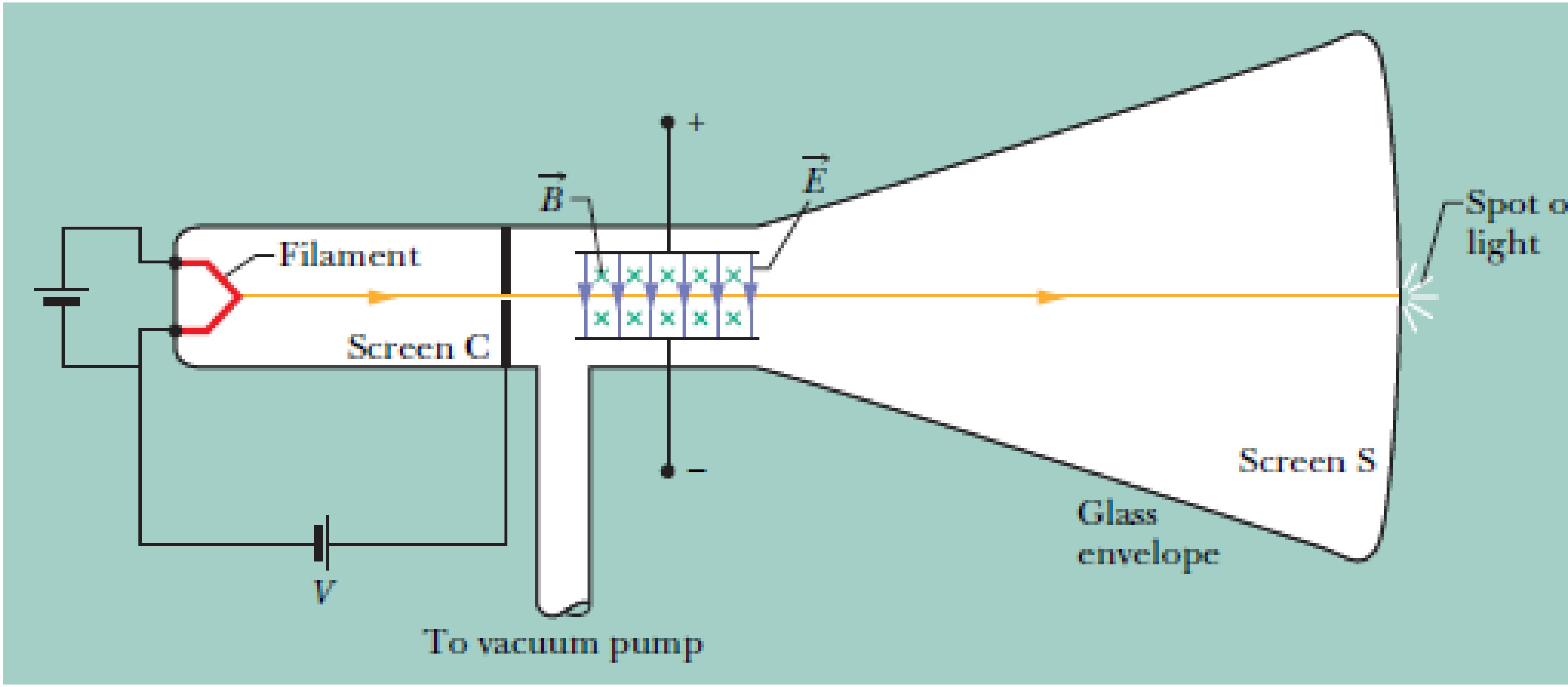
Historically, electrons called cathode rays, because they are obtained from electrodes (usually metals) serving as the cathode in some circuits containing 2 or more electrodes. Another electrode which absorbs the electrons/cathode rays is called the anode, and it completes the circuit.

There are many ways, a metal (cathode) can be made to give off electrons

- Heating : Thermionic emission (See https://en.wikipedia.org/wiki/Thermionic_emission)
- Light: Photoelectric emission (more on this later!)
- Kinetic energy from other electrons: Secondary Emission (Scattering, Auger, etc)
- Electrical Voltage: Field Emission
- Chemical potential difference (e.g. in a battery/ electric cell):

Crossed Fields and the Discovery of the electron

- The video earlier posted documents some interesting historical facts, as J.J. Thomson tried to solve some riddles associated with cathode ray tubes. https://www.youtube.com/watch?v=n3c77C-69wg
- By placing both a magnetic field and an electric field at right angles (perpendicularly) across the path of the “cathode rays”, Thomson identified the rays as some “corpuscles” of fundamental importance in the structure of matter. Today the corpuscles are called electrons. And Thomson is credited with the discovery.



A modern version of J. **J.Thomson's apparatus** for measuring the ratio of mass to charge for the electron (Fig. 28.7 Halliday and Resnick)

An electric field **E** is established by connecting a battery across the deflecting-plate terminals.

The magnetic field **B** is set up by means of a current in a system of coils (not shown).

The magnetic field shown is into the plane of the figure, as represented by the array of **X**s (which resemble the feathered ends of arrows)

Cathode rays, released by thermionic emission from the filament stream in from the left and eventually hits the Screen S on the far right. The Glass envelope is evacuated to allow free motion of the electron via scattering and ionization. Motion is now completely controlled by the applied E and B fields.

Steps taken by JJ Thomson in unmasking the identity of the electron

1. Set $E = 0$ and $B = 0$ and note the position of the spot on screen S due to the undeflected beam.

2. Turn on Electric field **E** and measure the resulting beam deflection.

3. Maintaining **E**, now turn on magnetic field **B** and adjust its value until the beam returns to the undeflected position.(With the forces in opposition, they can be made to cancel.)

- From simple kinematics (e.g $v^2 = u^2 + 2as$), it can be shown that the deflection y of a charged particle from the centre, as it passes through 2 plates of a capacitor is given by:

$$y = \frac{|q|EL^2}{2mv^2},$$

where *v* is the particle's speed, *m* its mass, *q* its charge, and *L* is the length of the plates providing the electric field E.

- When the two fields in Fig. 28-7 are adjusted so that the two deflecting forces cancel (step 3),
- Electric Force q**E** equals Magnetic Force = q**v**x**B**
- i.e. $$|q|E = |q|vB\sin(90°) = |q|vB$$
- which gives

$$v = \frac{E}{B}.$$

- or

$$\frac{m}{|q|} = \frac{B^2L^2}{2yE},$$

Thus, the crossed fields allow us to measure the ratio {*m*/|*q*|} of the particles moving through Thomson's apparatus.
Thomson claimed that these **particles are found in all matter**. He also claimed that they are lighter than the lightest known atom (hydrogen) by a factor of more than 1000. (The exact ratio proved later to be 1836.15.)
His *m*/|*q*| measurement, coupled with the boldness of his two claims, is considered to be the "discovery of the electron."

Recall: The right hand rule stating direction of the vector **F=qvxB**

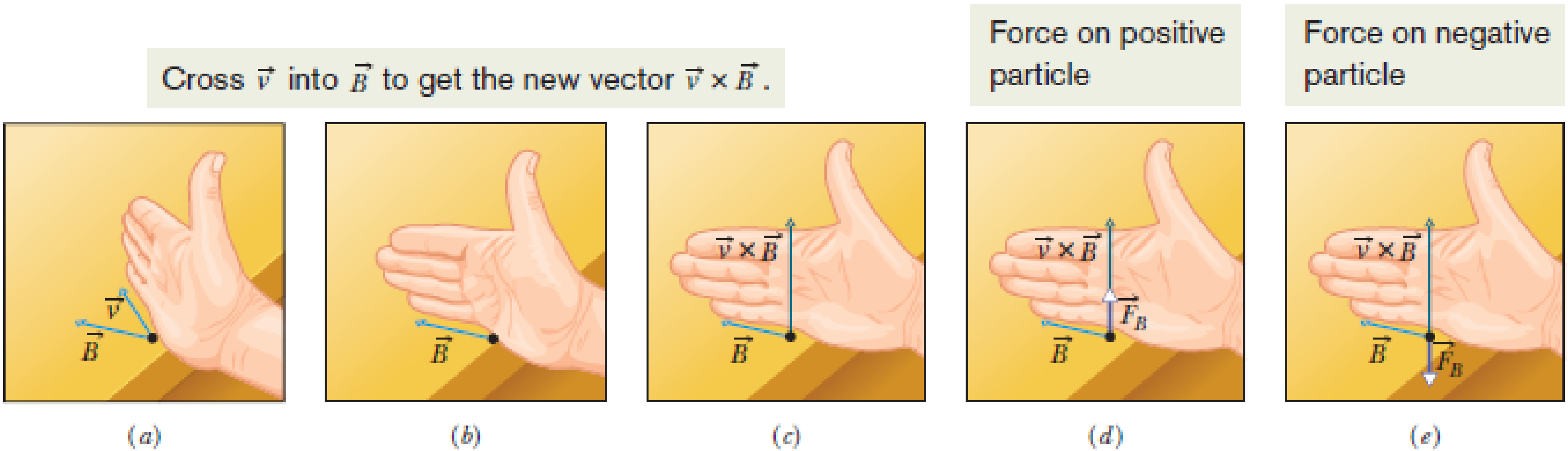


Fig. 28-2 (*a*) – (*c*) The right-hand rule (in which $\vec{v}$ is swept into $\vec{B}$ through the smaller angle ϕ between them) gives the direction of $\vec{v} \times \vec{B}$ as the direction of the thumb. (*d*) If q is positive, then the direction of $\vec{F}_B = q\vec{v} \times \vec{B}$ is in the direction of $\vec{v} \times \vec{B}$. (*e*) If q is negative, then the direction of $\vec{F}_B$ is opposite that of $\vec{v} \times \vec{B}$.

Mass Spectrometers

see https://images.app.goo.gl/uHKG3dwKTE2zGSy36

- If we have a uniform B field alone, and acting all over the chamber, from the right hand rule, the the electron is continually pushed off a rectilinear path into a circular path. i.e. it moves in a circle.
- If there is some gas in the glass tube, there is some ionization as the electron whirls around, and we can see the electron.
- (This is the basis for the cloud chamber used to visibly display the motion of electrons (and other charged particles we deal with in Modern Physics).

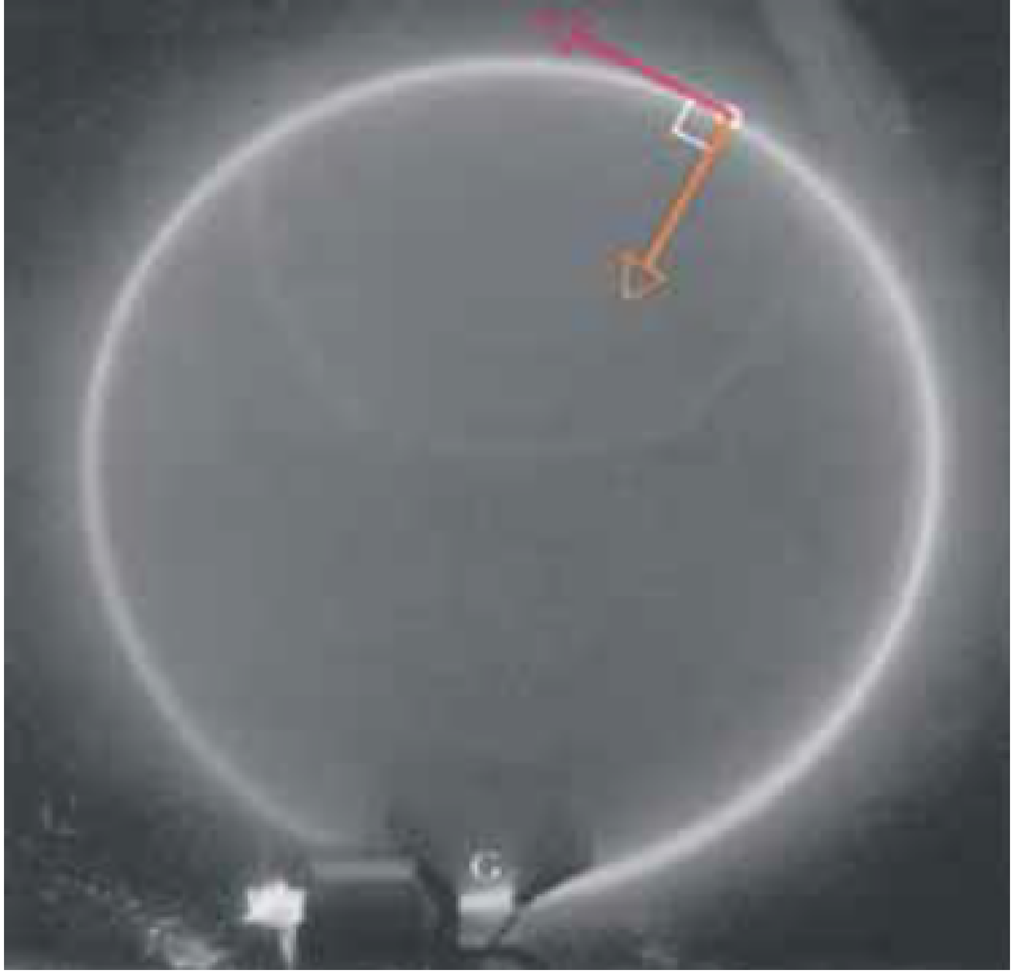


Fig. 28-10 Electrons circulating in a chamber containing gas at low pressure (their path is the glowing circle). A uniform magnetic field $\vec{B}$, pointing directly out of the plane of the page, fills the chamber. Note the radially directed magnetic force $\vec{F}_B$; for circular motion to occur, $\vec{F}_B$ *must* point toward the center of the circle. Use the right-hand rule for cross products to confirm that $\vec{F}_B = q\vec{v} \times \vec{B}$ gives $\vec{F}_B$ the proper direction. (Don't forget the sign of q.) *(Courtesy John Le P. Webb, Sussex University, England)*

From Magnetic Force $\mathbf{F_B} = q\mathbf{v}\mathrm{x}\mathbf{B}$

If the magnetic field is at right angle to direction of velocity v, then $\mathbf{F_B} = q\mathbf{vB}\sin\theta = qvB$

It is this force that is providing the centripetal force keeping the electron in a circular path of radius r

Hence, $\mathbf{F_B} = m\mathbf{v}^2/r$

It implies that the orbit r can be written as

$$r = \frac{mv}{|q|B} \quad \text{(radius).}$$

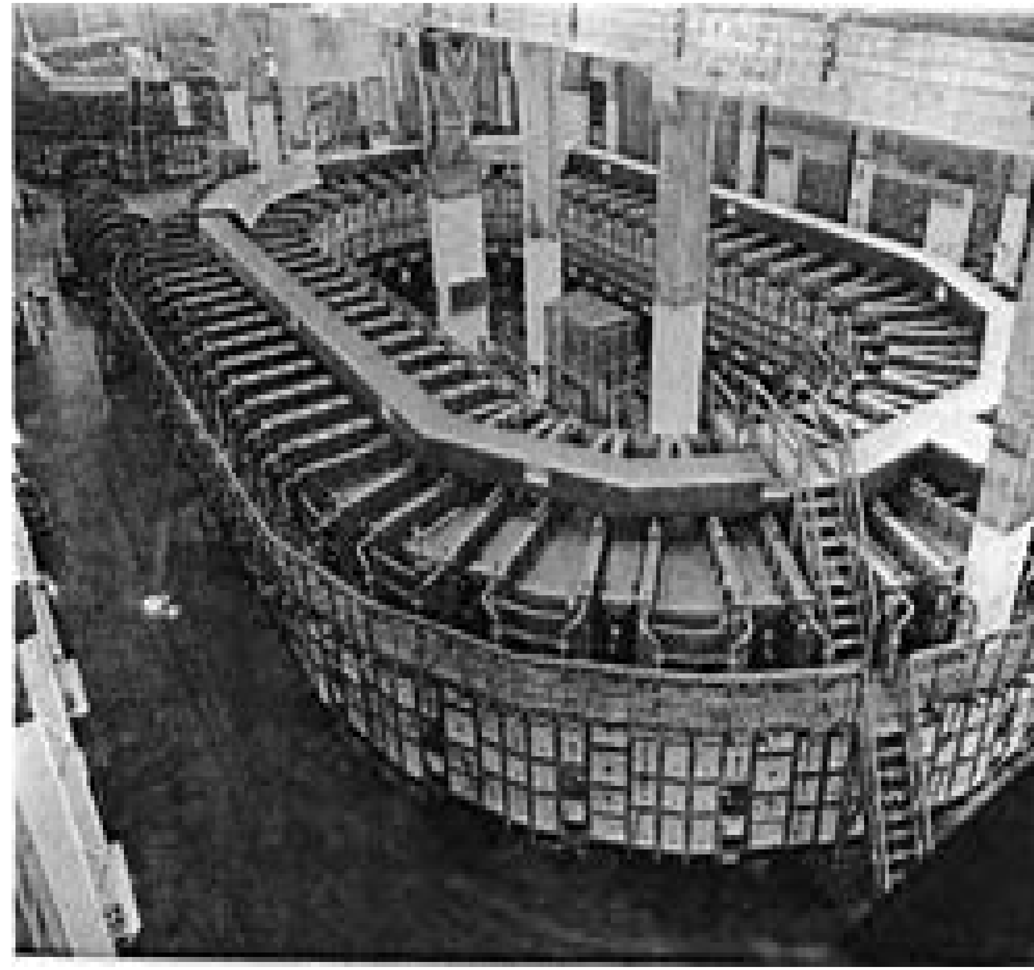
For another particle with a different m/q ratio, sent in with the same velocity v, into the same B field, the orbit is different.

The radius of the orbit is given by the same equation above.

(For positively charged particles, the direction of orbit is opposite that of the electron,) Hence if we have several charged particles/ions injected into the chamber with a constant B applied, the particles will start to move in circles of various radii, dictated by their peculiar q/m ratios according to the above equation.

The various particles can therefore be separated and identified on the basis of their q/m ratio which is the basis for the mass spectrometer.

Mass specs can range from table tops (we have an old non-functional MS at Room PY 234) to huge facilities. (see pix of the Calutron mass spec used in separating the Uranium isotopes used in the first atomic bomb)



Calutron mass spectrometers at the Y-12 Plant in Oak Ridge, Tennessee ca. 1945.

By Leslie R. Groves - Retrieved September 27, 2014 from Leslie R. Groves, Ed. (~1948) Manhattan District History, Manhattan Project, U.S. Army Corps of Engineers, Book V: Electromagnetic Project, Vol. 3: Design, Appendix C: Photograph No. 6: Alpha 1 Racetrack, declassified version, on Internet Archive, Public Domain, https://commons.wikimedia.org/w/index.php?curid=35699110

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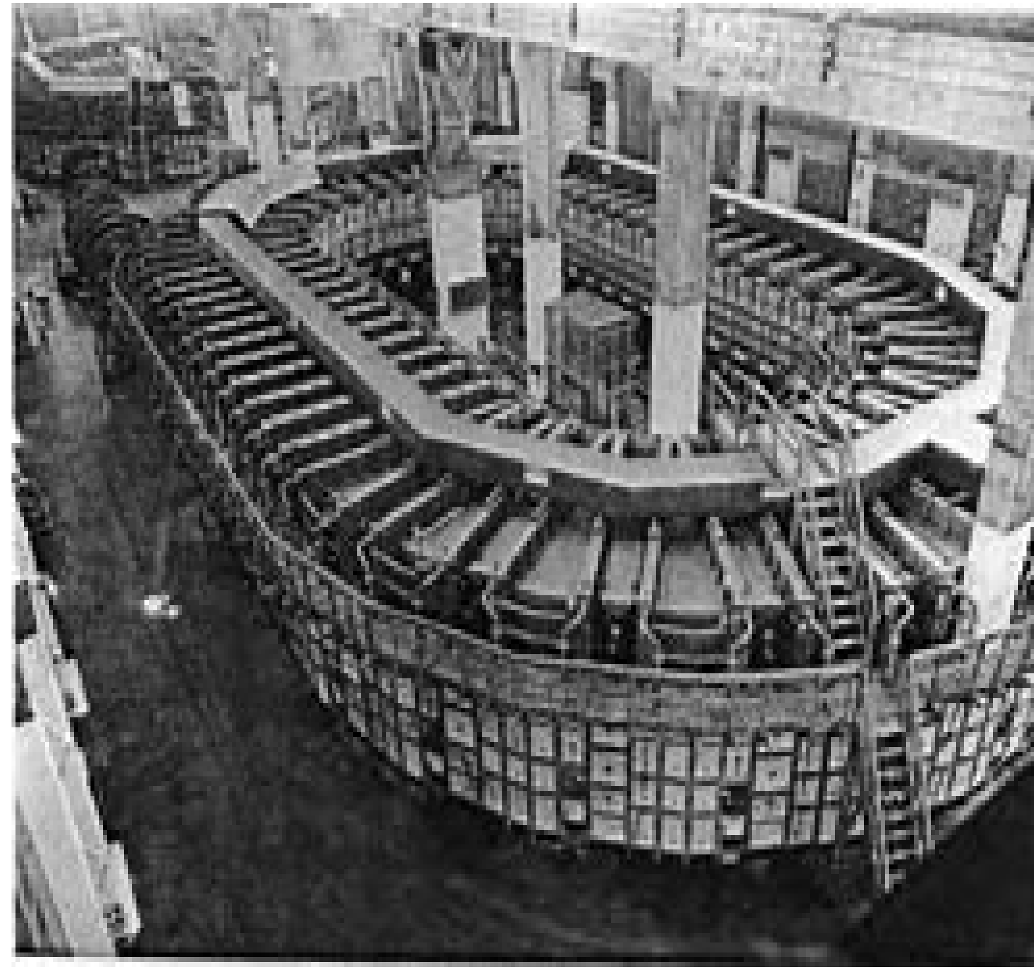

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Cyclotrons

- If we have an electron/ion moving under the influence of a perpendicular B field, and the energy is increased, it moves to a new orbit. Can be accelerated circularly like this to great energies whereafter it is released to bombard a target, either in medical therapy, production of radiopharmaceuticals, or nuclear physics experiments
- Such a device is called a cyclotron. (where charged particles are energized by moving them around in circles with periodic injection of energy via E-field thus transferring them to orbits with higher radii) See fig 28:13 of Halliday and Resnick. To accelerate a proton beyond 50 MeV, we have another device called the Synchrotron which takes care of the relativistic effects and can take particles to energies in the GeV range and beyond. . The proton synchrotron at the Fermi
- National Accelerator Laboratory (Fermilab) in Illinois has a circumference of 6.3km and can produce protons with energies of about 1 TeV (10^{12} eV). See details in Halliday

Linear Accelerators (Linacs)

- This is another interesting and common application where electrons and other charged particles are accelerated to great energies for use, as in a cyclotron.
- Here only an Electric field. So the electron/charged particle moves linearly with increasing velocity.
- Again, this can range from modest devices used in medical therapies to huge research facilities spanning several kilometers.
- There is a working particle accelerator at the Centre for Energy Research and Development of the OAU, Ile-Ife. The only one, so far, in black Africa.

How the electron was characterized, mass and charge – Millikan Experiment

- From 1909 to 1913, Robert Andrews Millikan (1868–1953) performed a brilliant set of experiments at the University of Chicago in which he measured the elementary charge *e* of the electron and demonstrated the quantized nature of the electronic charge. The apparatus he used, contains two parallel metal plates (see Chapter 15:7 of Serway and video previously posted)

Millikan’s Oil Drop Experiment

Oil droplets that have been charged by friction in an atomizer are allowed to pass through a small hole in the upper plate. A horizontal light beam is used to illuminate the droplets, which are viewed by a telescope with axis at right angles to the beam. The droplets then appear as shining stars against a dark background, and the rate of fall of individual drops can be determined.

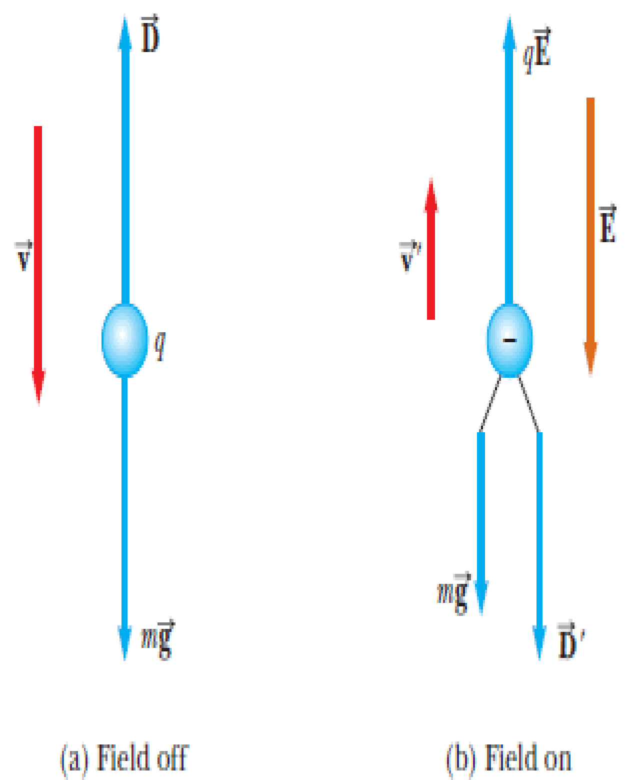
- We assume that a single drop having a mass of *m* and carrying a charge of *q* is being viewed and that its charge is negative.
- If no electric field is present between the plates, the two forces acting on the charge are the force of gravity, acting downward, and an upward viscous drag force (Fig. 15.22a). The drag force is
- proportional to the speed of the drop. When the drop reaches its terminal speed, *v*, the two forces balance each other ($mg = D$).

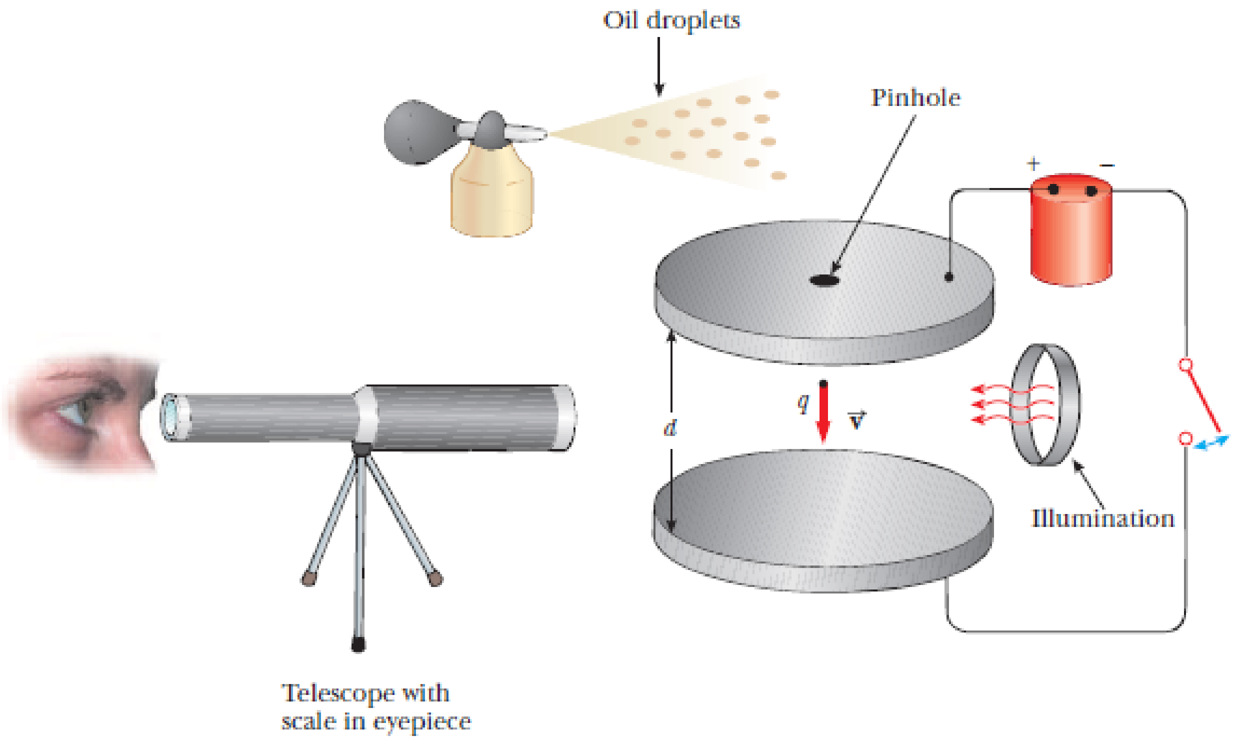
- Now suppose that an electric field is set up between the plates by a battery
- connected so that the upper plate is positively charged. In this case, a third force, q**E**, acts on the charged drop. Because *q* is negative and **E** is downward, the electric force is *upward* as in Figure 15.22b. If this force is great enough, the drop moves upward and the drag force acts downward.

- When the upward electric force, , balances the sum of the force of gravity and the drag force, both acting downward, the drop reaches a new terminal speed *v′*.

- With the field turned on, a drop moves slowly upward, typically at a rate of *hundredths* of a centimeter per second. The rate of fall in the absence of a field is comparable. Hence, a single droplet with constant mass and radius can be followed for hours as it alternately rises and falls, simply by turning the electric field on and off.

The forces on a charged oil droplet in Millikan's experiment





ACTIVE FIGURE 15.21
A schematic view of Millikan's oil-drop apparatus.

For further studies see https://courses.lumenlearning.com/introchem/chapter/millikans-oil-drop-experiment/

- After making measurements on thousands of droplets, Millikan and his co-workers found that, to within about 1% precision, every drop had a charge equal to some positive or negative integer multiple of the elementary charge *e*,
- $q = ne \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots$ **[15.7, Serway]**

- where $e = 1.60 \text{ x} 10^{-19}$ C.
- It was later established that positive integer multiples of *e* would arise when an oil droplet had lost one or more electrons. Likewise, negative integer multiples of *e* would arise when a drop had gained one or more electrons.
- Gains or losses in integral numbers provide conclusive evidence that charge is quantized. In 1923, Millikan was awarded the Nobel prize in physics for this work.

Various forms of electrons

- Bound electrons (inner shell, outer shell,)

As we shall learn in details under the models for the atoms, all atoms contain electrons. Same type of electrons whatever the element. These electrons move round a positive nucleus. So they are bound electrons

- Conduction electrons

Electrons (same electrons) can also exist in free state, not bound to any nucleus. Loosely moving electrons are responsible for binding (covalent, metallic) where a particular electron is not associated with a particular atom in a material.

- Free electrons

Some electrons are totally independent and are not associated with any material. – as we saw in the cathode ray tube.

- They can be accelerated as we have discussed before to get fast electrons which can be used for several applications, including brachytherapy. At this juncture, these fast electrons are regarded as ioniizing radiation.
- Some particle called beta radiation /rays have been found to emerge from within the nucleus in some radioactive materials undergoing Beta decay). These highly energetic particles (with energies typically in MeV) were later found to have exactly the same mass and charge as the electron.
- Auger electrons are electrons from inner shell that are released in some processes and interactions.

This ends our story about electrons.

Module 3: **Particle behavior of electromagnetic radiation**

Photoelectric effect
X-rays
Compton effect
Pair production and annihilation

Introduction

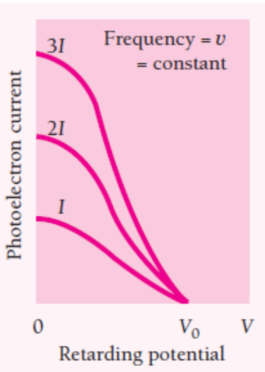
- This topic dealing with the behaviours of electromagnetic (em) radiation is very critical in practically every aspect of modern life. EM radiation has very wide applications in medicine, agriculture, industry, communications, etc; and understanding the peculiar behavior of this radiation enables us to maximize the benefits whilst doing that safely. It turns out that we cannot sufficiently understand the interactions of electromagnetic radiation with matter unless we bring in quantum physics, which knowledge we began acquiring only from 1900 forward.
- The summary is that electromagnetic radiation has dual nature, behaving in some instances like a wave, (with no definite location in space) and in other instances, like a particle (with particular locations in space associated with it). Einstein combined the wave and particle nature of light with the word Wave Packet.
- The wave behavior is responsible for such phenomena as interference, diffraction, polarization, etc, while the particle behavior of electromagnetic radiation is responsible for the Photoelectric effect, X-rays, Compton Effect, and Pair production and Pair annihilation phenomena. These we study in this Module.

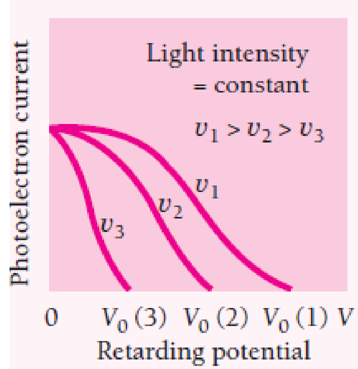
3.1. Photoelectric Effect.

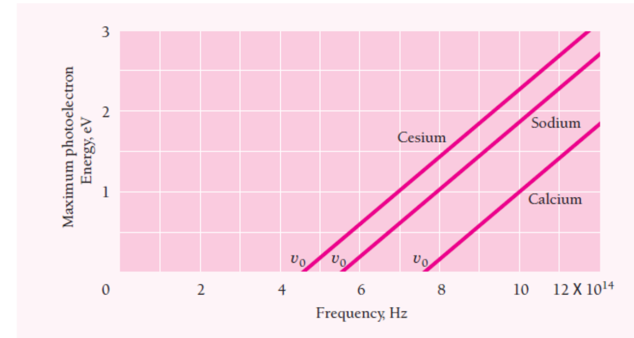
- This refers to the well-known phenomenon where light (photo) incident on a material leads to the release of electrons from that material. (Hence Photo → Electric). The materials that emit electrons following an incident beam of photons are called photocathode. (cf Module 2 on electron emission methods). Photoelectric emission is the basis for several solar electric devices, where light from the sun leads to ejection of electrons (electricity) from an appropriate material.
- Watch this useful animation on Youtube: https://www.youtube.com/watch?v=0b0axfyJ4oo

- Figure 2.9 [Arthur Beiser] shows how the photoelectric effect was studied. An evacuated tube contains
- two electrodes connected to a source of variable voltage, with the metal plate whose surface is irradiated as the anode. Some of the photoelectrons that emerge from this surface have enough energy to reach the cathode despite its negative polarity, and they constitute the measured current.
- The slower photoelectrons are repelled before they get to the cathode. When the voltage is increased to a certain value $V0$, of the order of several volts, no more photoelectrons arrive, as indicated by the current dropping to zero. This extinction voltage corresponds to the maximum photoelectron kinetic energy.
- Figure 2.10 shows the result (V_0 that yields zero photoelectrons) when Light/em radiation of SAME FREQUENCY but DIFFERENT INTENSITIES impinge on the anode material.

THREE OBSERVERATIIONS THAT CAN NOT BE EXPLAINED BY THE CLASSICAL EM W RY.







1 Within the limits of experimental accuracy (about 10^{-9}s), there is no time interval between the arrival of light at a metal surface and the emission of photoelectrons. However, because the energy in an em wave is supposed to be spread across the wavefronts, a period of time should elapse before an individual electron accumulates enough energy (several eV) to leave the metal. Calculations show that over a month would be needed for an atom to accumulate energy of the magnitudethat photoelectrons from a sodium surface are observed to have.

2 A bright light yields MORE photoelectrons than a dim one of the same frequency, but the electron ENERGIES REMAIN THE SAME (Fig. 2.10). The em theory of light, on the contrary, predicts that the more intense the light, the greater the energies of the electrons.

3 The higher the frequency of the light, the more energy the photoelectrons have (Fig. 2.11). Blue light results in faster electrons than red light. At frequencies below a certain critical frequency ν_0, which is characteristic of each particular metal, no electrons are emitted. Above ν_0 the photoelectrons range in energy from 0 to a maximum value that

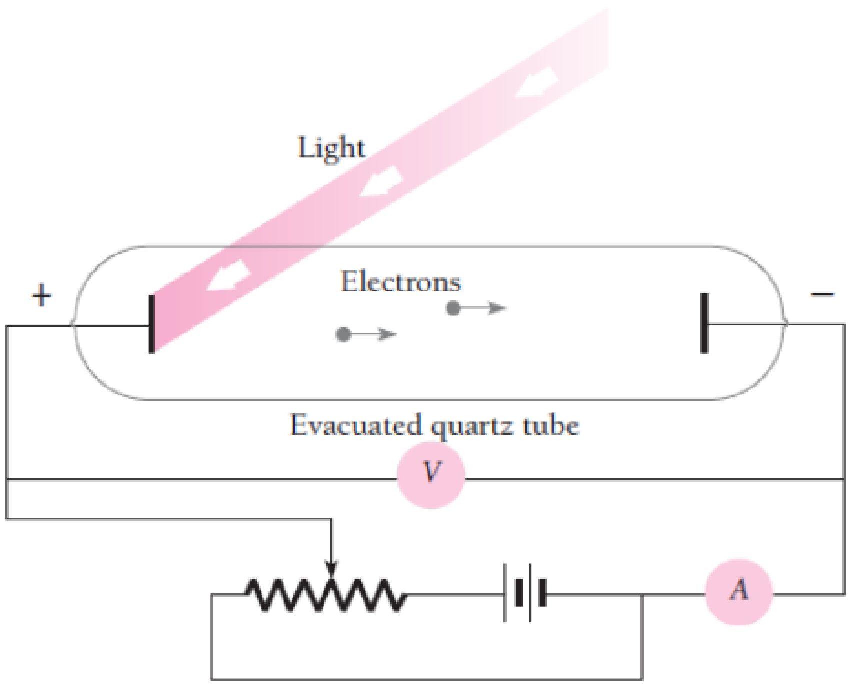


Figure 2.9 Experimental observation of the photoelectric effect.

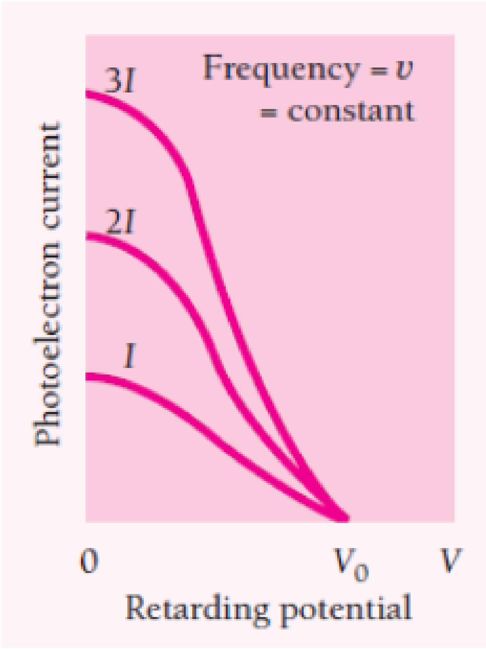


Figure 2.10 Photoelectron current is proportional to light intensity I for all retarding voltages. The stopping potential V_0, which corresponds to the maximum photoelectron energy, is the same for all intensities of light of the same frequency ν.

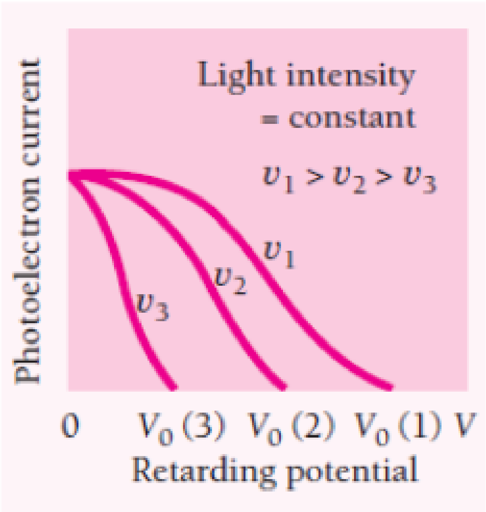


Figure 2.11 The stopping potential V_0, and hence the maximum photoelectron energy, depends on the frequency of the light. When the retarding potential is $V = 0$, the photoelectron current is the same for light of a given intensity regardless of its frequency.

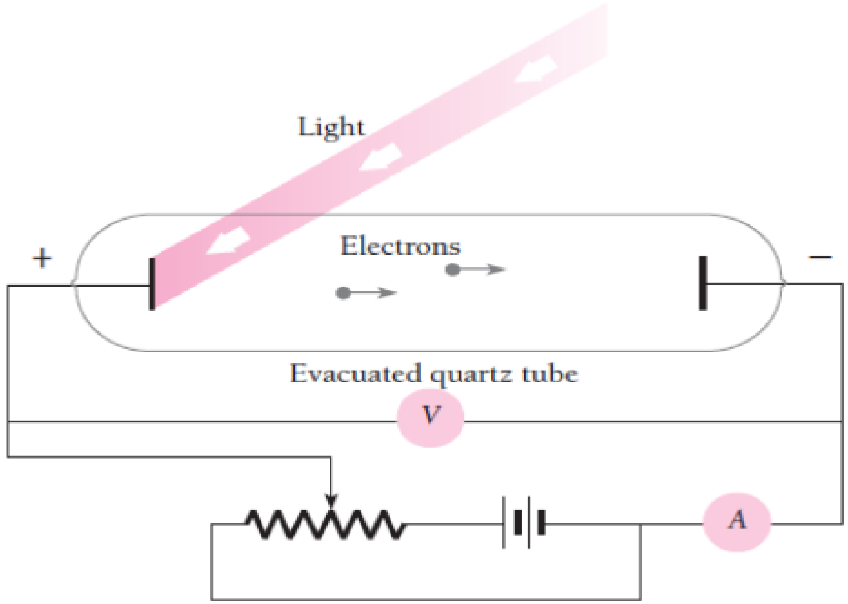


Figure 2.9 Experimental observation of the photoelectric effect.

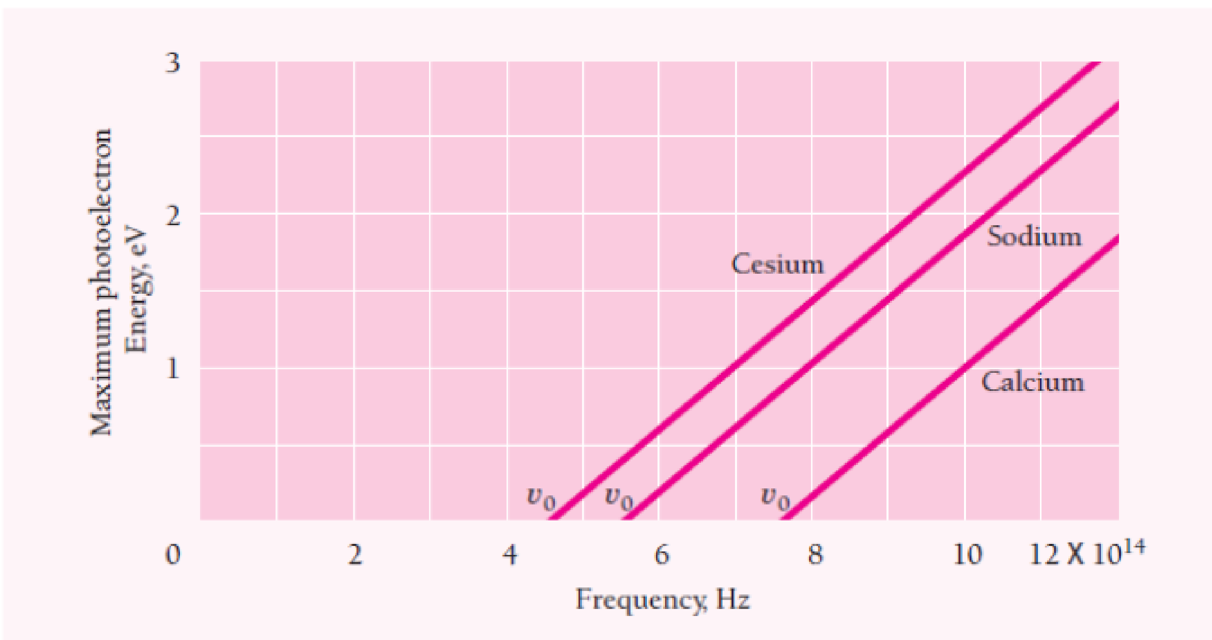


Figure 2.12 Maximum photoelectron kinetic energy KE_{max} versus frequency of incident light for three metal surfaces.

Explanation in Quantum Physics

In 1905, Einstein realized that the photoelectric effect could be understood if the energy in light is not spread out over wavefronts but is concentrated in small packets, or **photons.** (The term photon was coined by the chemist Gilbert Lewis in 1926.)

Each photon of light of frequency ν has the energy $h\nu$, the same as Planck's quantum energy (Remember Blackbody radiation, the birth of Quantum Physics). Planck had thought that, although energy from an electric oscillator apparently had to be given to em waves in separate quanta of $h\nu$ each, the waves themselves behaved exactly as in conventional wave theory.

Einstein's break with classical physics was more drastic: Energy was not only given to em waves in separate quanta but was also carried by the waves in separate quanta.

.

The three experimental observations listed above follow directly from Einstein’s hypothesis.

(1) Because em wave energy is concentrated in photons and not spread out, there should be no delay in the emission of photoelectrons

(2) All photons of frequency ν have the same energy, so changing the intensity of a monochromatic light beam will change the number of photoelectrons but not their energies.

(3) The higher the frequency ν, the greater the photon energy $h\nu$ and so the more energy the photoelectrons have.

work function ϕ

- What is the meaning of the critical frequency ν_0 below which no photoelectrons are emitted?
- There must be a minimum energy ϕ for an electron to escape from a particular metal surface or else electrons would pour out all the time. This energy is called the **work function** of the metal, and is related to ν0 by the formula
- The greater the work function of a metal, the more energy is needed for an electron to leave its surface, and the higher the critical frequency for photoelectric emission to occur.

Table 2.1 Photoelectric Work Functions

Metal	Symbol	Work Function, eV
Cesium	Cs	1.9
Potassium	K	2.2
Sodium	Na	2.3
Lithium	Li	2.5
Calcium	Ca	3.2
Copper	Cu	4.7
Silver	Ag	4.7
Platinum	Pt	6.4

Some examples of photoelectric work functions are given in Table 2.1.

To pull an electron from a metal surface generally takes about half as much energy as that needed to pull an electron from a free atom of that metal (see Fig. 7.10 – in Beiser); for instance, the ionization energy of cesium is 3.9 eV compared with its work function of 1.9 eV. Since the visible spectrum extends from about 4.3 to about 7.5 x 10^{14} Hz, which corresponds to quantum energies of 1.7 to 3.3 eV, it is clear from Table 2.1 that the photoelectric effect is a phenomen
ultraviolet

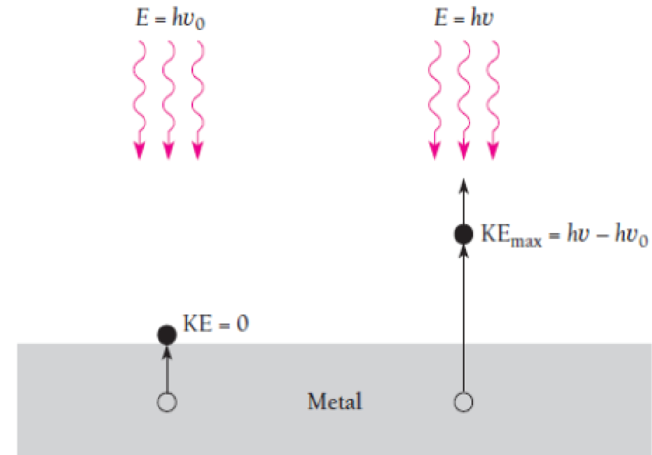


Figure 2.13 If the energy $h\nu_0$ (the work function of the surface) is needed to remove an electron from a metal surface, the maximum electron kinetic energy will be $h\nu - h\nu_0$ when light of frequency ν is directed at the surface.

Photoelectric effect $$h\nu = KE_{max} + \phi \tag{2.8}$$

In terms of electronvolts, the formula $E = h\nu$ for photon energy becomes

Photon energy $$E = \left(\frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{1.602 \times 10^{-19}\ \text{J/eV}}\right)\nu = (4.136 \times 10^{-15})\nu\ \text{eV} \cdot \text{s} \tag{2.10}$$

If we are given instead the wavelength λ of the light, then since $\nu = c/\lambda$ we have

Photon energy $$E = \frac{(4.136 \times 10^{-15}\ \text{eV} \cdot \text{s})(2.998 \times 10^{8}\ \text{m/s})}{\lambda} = \frac{1.240 \times 10^{-6}\ \text{eV} \cdot \text{m}}{\lambda} \tag{2.11}$$

Example 2.2

Ultraviolet light of wavelength 350 nm and intensity 1.00 W/m^2 is directed at a potassium surface. (*a*) Find the maximum KE of the photoelectrons. (*b*) If 0.50 percent of the incident photons produce photoelectrons, how many are emitted per second if the potassium surface has an area of 1.00 cm^2?

Solution

(*a*) From Eq. (2.11) the energy of the photons is, since 1 nm = 1 nanometer = 10^{-9} m,

$$E_p = \frac{1.24 \times 10^{-6}\ \text{eV} \cdot \text{m}}{(350\ \text{nm})(10^{-9}\ \text{m/nm})} = 3.5\ \text{eV}$$

Table 2.1 gives the work function of potassium as 2.2 eV, so

$$\text{KE}_{\max} = h\nu - \phi = 3.5\ \text{eV} - 2.2\ \text{eV} = 1.3\ \text{eV}$$

(*b*) The photon energy in joules is 5.68×10^{-19} J. Hence the number of photons that reach the surface per second is

$$n_p = \frac{E/t}{E_p} = \frac{(P/A)(A)}{E_p} = \frac{(1.00\ \text{W/m}^2)\,(1.00 \times 10^{-4}\ \text{m}^2)}{5.68 \times 10^{-19}\ \text{J/photon}} = 1.76 \times 10^{14}\ \text{photons/s}$$

The rate at which photoelectrons are emitted is therefore

$$n_e = (0.0050)n_p = 8.8 \times 10^{11}\ \text{photoelectrons/s}$$

X-RAYS

They consist of high-energy photons

The photoelectric effect provides convincing evidence that photons of light can transfer energy to electrons. Is the inverse process also possible? That is, can part or all of the kinetic energy of a moving electron be converted into a photon? As it happens, the inverse photoelectric effect not only does occur but had been discovered (though not understood) before the work of Planck and Einstein.

In 1895 Wilhelm Roentgen found that a highly penetrating radiation of unknown nature is produced when fast electrons impinge on matter. These **x-rays** were soon found to

- travel in straight lines,
- to be unaffected by electric and magnetic fields,
- To pass readily through opaque materials,
- to cause phosphorescent substances to glow, and
- to expose photographic plates.

The faster the original electrons, the more penetrating the resulting x-rays, and the greater the number of electrons, the greater the intensity of the x-ray beam.

Not long after this discovery it became clear that x-rays are em waves. Electromagnetic theory predicts that an accelerated electric charge will radiate em waves, and a rapidly moving electron suddenly brought to rest is certainly accelerated. Radiation produced under these circumstances is given the German name **bremsstrahlung** ("braking radiation").

Energy loss due to bremsstrahlung is more important for electrons than for heavier particles because electrons are more violently accelerated when passing near nuclei in their paths. The greater the energy of an electron and the greater the atomic number of the nuclei it encounters, the more energetic the bremsstrahlung.

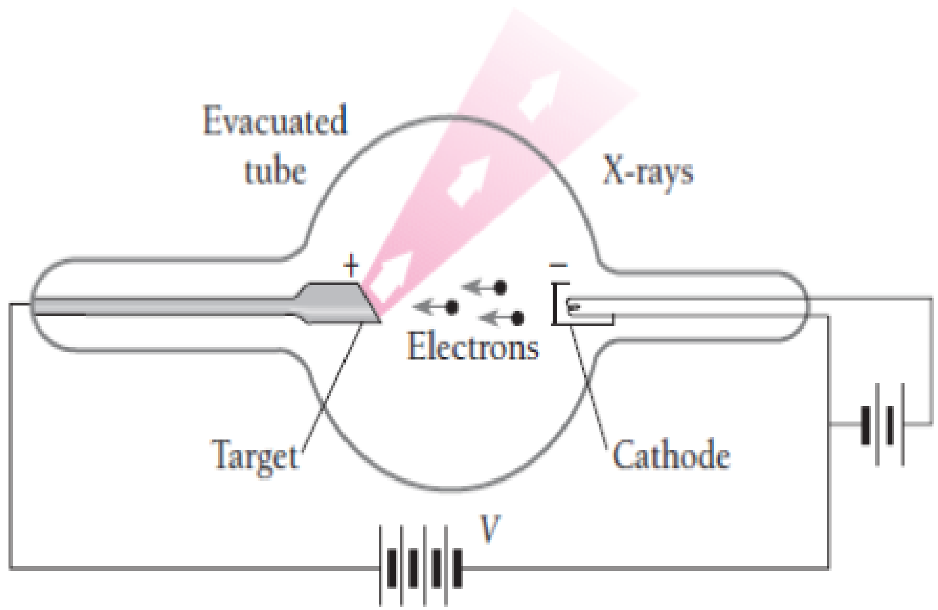


Figure 2.15 An x-ray tube. The higher the accelerating voltage V, the faster the electrons and the shorter the wavelengths of the x-rays.

- Figure 2.15 is a diagram of an x-ray tube. A cathode, heated by a filament through which an electric current is passed, supplies electrons by thermionic emission.
- The high potential difference V maintained between the cathode and a metallic target accelerates the electrons toward the latter. The face of the target is at an angle relative to the electron beam, and the x-rays that leave the target pass through the side of the tube. The tube is evacuated to permit the electrons to get to the target unimpeded.

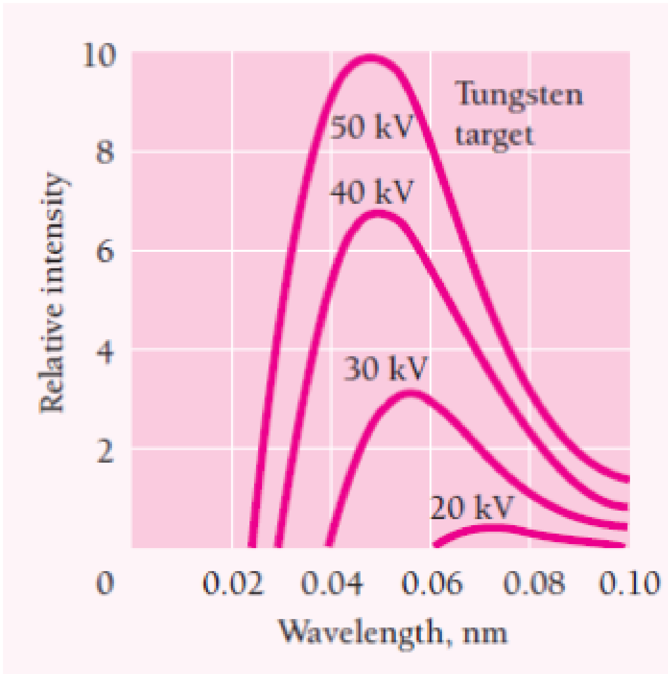


Figure 2.16 X-ray spectra of tungsten at various accelerating pote

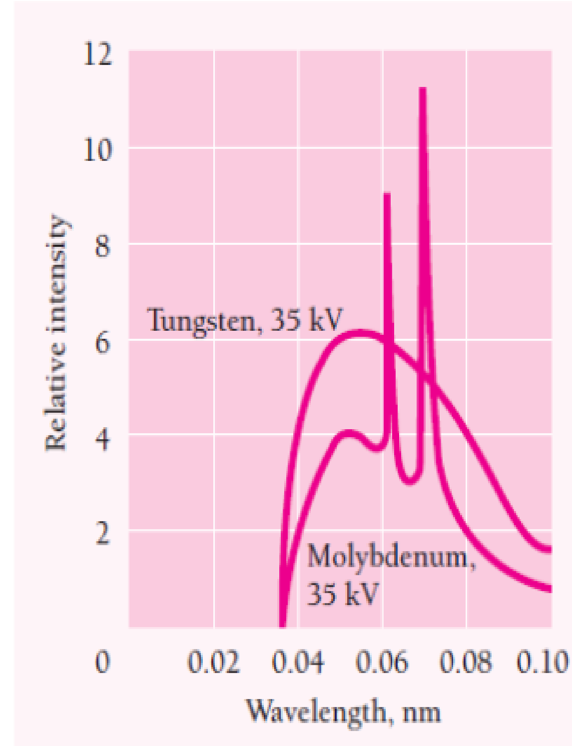


Figure 2.17 X-ray spectra of tungsten and molybdenum at 35 kV accelerating potential.

Figures 2.16 and 2.17 show the x-ray spectra that result when tungsten and molybdenum targets are bombarded by electrons at several different accelerating potentials. The curves exhibit two features electromagnetic theory cannot explain:

- **1** In the case of molybdenum, intensity peaks occur that indicate the enhanced production of x-rays at certain wavelengths. These peaks occur at specific wavelengths for each target material and originate in rearrangements of the electron structures of the target atoms after having been disturbed by the bombarding electrons. The presence of x-rays of specific wavelengths, in addition to a continuous x-ray spectrum. Is a decidedly nonclassical effect,
- **2** The x-rays produced at a given accelerating potential *V* vary in wavelength, but none has a wavelength shorter than a certain value λmin. Increasing *V* decreases λmin. At a particular *V*, λmin is the *same* for both the tungsten and molybdenum targets. Duane and Hunt found experimentally that λmin is inversely proportional to *V* their precise re

X-ray production

$$\lambda_{min} = \frac{1.24 \times 10^{-6}}{V} \text{ V} \cdot \text{m} \qquad (2.12)$$

- The second observation fits in with the quantum theory of radiation. Most of the electrons that strike the target undergo numerous glancing collisions, with their energy going simply into heat. (This is why the targets in x-ray tubes are made from high melting-point metals such as tungsten, and a means of cooling the target is usually employed.)
-
- A few electrons, though, lose most or all of their energy in single collisions with target atoms. This is the energy that becomes x-rays.
-
- X-rays production, then, except for the peaks mentioned in observation 1 above, represents an inverse photoelectric effect. Instead of photon energy being transformed into electron KE, electron KE is being transformed into photon energy. A short wavelength means a high frequency, and a high frequency means a high photon energy $h\nu$.

Since work functions are only a few electronvolts whereas the accelerating potentials in x-ray tubes are typically tens or hundreds of thousands of volts, we can ignore the work function and interpret the short wavelength limit of Eq. (2.12) as corresponding to the case where the entire kinetic energy KE = Ve of a bombarding electron is given up to a single photon of energy $h\nu_{max}$. Hence

$$Ve = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{Ve} = \frac{1.240 \times 10^{-6}}{V} \text{ V} \cdot \text{m}$$

which is the Duane-Hunt formula of Eq. (2.12)—and, indeed, the same as Eq. (2.11) except for different units. It is therefore appropriate to regard x-ray production as the inverse of the photoelectric effect.

Two features for the x-ray spectrum: bremsstrahlung PLUS characteristic radiation

Watch this youtube video for more clarifications: https://www.youtube.com/watch?v=5RjBsIO0pxg

Example 2.3

Find the shortest wavelength present in the radiation from an x-ray machine whose accelerating potential is 50,000 V.

Solution

From Eq. (2.12) we have

$$\lambda_{\min} = \frac{1.24 \times 10^{-6}\ \text{V}\cdot\text{m}}{5.00 \times 10^{4}\ \text{V}} = 2.48 \times 10^{-11}\ \text{m} = 0.0248\ \text{nm}$$

This wavelength corresponds to the frequency

$$\nu_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.00 \times 10^{8}\ \text{m/s}}{2.48 \times 10^{-11}\ \text{m}} = 1.21 \times 10^{19}\ \text{Hz}$$

Quiz

- Through what minimum potential difference must an electron in an x-ray tube be accelerated so that it can produce x rays with a wavelength of 0.100 nm?

- When electrons bombard a molybdenum target, they produce both continuous and characteristic x rays. If the accelerating potential is increased from 35.0 keV to 50.0 keV,
- (a) what is the change in the mean value of λ_{min}, (the most energetic x-ray produced)?
- (b) do the wavelengths of the K_a and K_b lines increase, decrease,or remain the same?

2.7 COMPTON EFFECT

Further confirmation of the photon model

According to the quantum theory of light, photons behave like particles except for their lack of rest mass. How far can this analogy be carried? For instance, can we consider a collision between a photon and an electron as if both were billiard balls?

Figure 2.22 shows such a collision: an x-ray photon strikes an electron (assumed to be initially at rest in the laboratory coordinate system) and is scattered away from its original direction of motion while the electron receives an impulse and begins to move

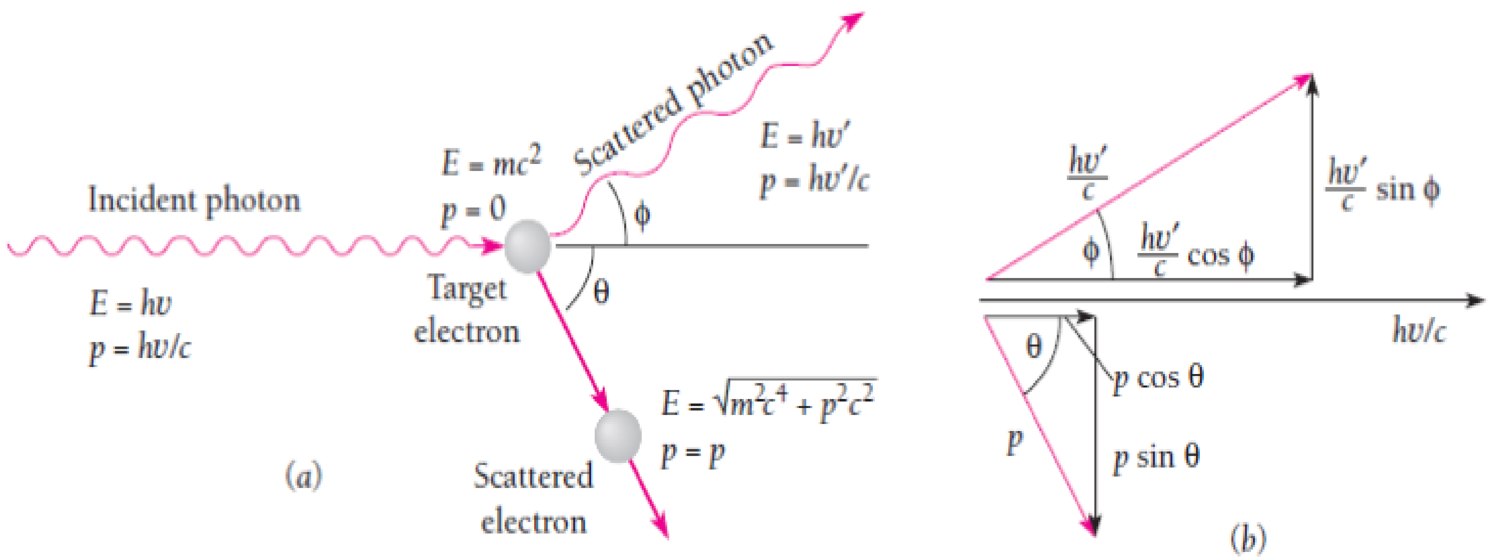


Figure 2.22 (*a*) The scattering of a photon by an electron is called the Compton effect. Energy and momentum are conserved in such an event, and as a result the scattered photon has less energy (longer wavelength) than the incident photon. (*b*) Vector diagram of the momenta and their components of the incident and scattered photons and the scattered electron.

Loss in photon energy = gain in electron energy

$$h\nu - h\nu' = KE \tag{2.14}$$

Since the energy of a photon is $h\nu$, its momentum is

Photon momentum

$$p = \frac{E}{c} = \frac{h\nu}{c} \tag{2.15}$$

The initial photon momentum is $h\nu/c$, the scattered photon momentum is $h\nu'/c$, and the initial and final electron momenta are respectively 0 and p. In the original photon direction

Initial momentum = final momentum

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta \tag{2.16}$$

and perpendicular to this direction

Initial momentum = final momentum

$$0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta \tag{2.17}$$

Compton effect $$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi) \quad (2.21)$$

Compton effect $$\lambda' - \lambda = \lambda_C(1 - \cos\phi) \quad (2.23)$$

Compton wavelength $$\lambda_C = \frac{h}{mc} \quad (2.22)$$

m is the mass of the scattering particle. For electrons, therefore, the Compton wavelength = $\lambda_C = 2.426 \times 10^{-12}$ m, which is 2.426 pm (1 pm = 1 picometer = 10^{-12} m).

Example 2.4

X-rays of wavelength 10.0 pm are scattered from a target. (*a*) Find the wavelength of the x-rays scattered through 45°. (*b*) Find the maximum wavelength present in the scattered x-rays. (*c*) Find the maximum kinetic energy of the recoil electrons.

Solution

(*a*) From Eq. (2.23), $\lambda' - \lambda = \lambda_C(1 - \cos\phi)$, and so

$$\begin{aligned} \lambda' &= \lambda + \lambda_C(1 - \cos 45^\circ) \\ &= 10.0 \text{ pm} + 0.293\lambda_C \\ &= 10.7 \text{ pm} \end{aligned}$$

(*b*) $\lambda' - \lambda$ is a maximum when $(1 - \cos\phi) = 2$, in which case

$$\lambda' = \lambda + 2\lambda_C = 10.0 \text{ pm} + 4.9 \text{ pm} = 14.9 \text{ pm}$$

(*c*) The maximum recoil kinetic energy is equal to the difference between the energies of the incident and scattered photons, so

$$KE_{max} = h(\nu - \nu') = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

where λ' is given in (*b*). Hence

$$\begin{aligned} KE_{max} &= \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^{8} \text{ m/s})}{10^{-12} \text{ m/pm}}\left(\frac{1}{10.0 \text{ pm}} - \frac{1}{14.9 \text{ pm}}\right) \\ &= 6.54 \times 10^{-15} \text{ J} \end{aligned}$$

which is equal to 40.8 keV.

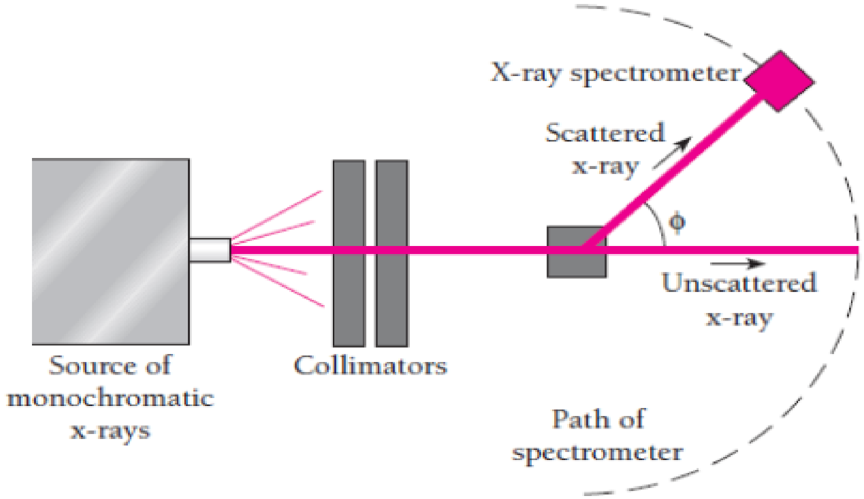


Figure 2.23 Experimental demonstration of th

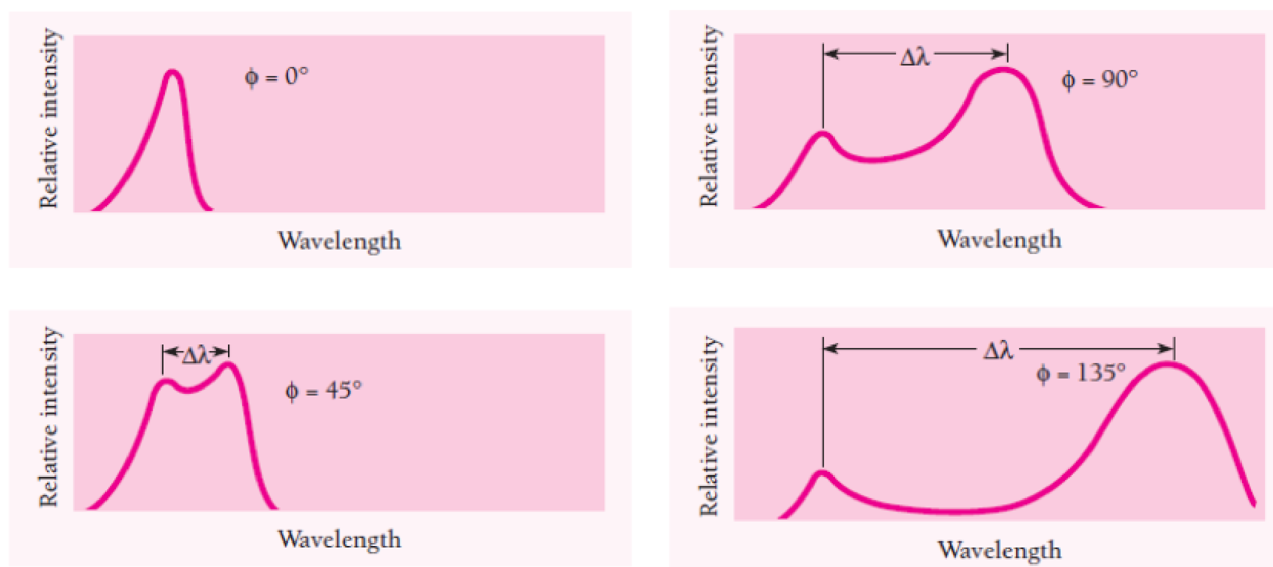


Figure 2.24 Experimental confirmation of Compton scattering. The greater the scattering angle, the greater the wavelength change, in accord with Eq. (2.21).

In deriving Eq. (2.21) it was assumed that the scattering particle is able to move freely, which is reasonable since many of the electrons in matter are only loosely bound to their parent atoms. Other electrons, however, are very tightly bound and when struck by a photon, the entire atom recoils instead of the single electron.
In this event the value of m to use in Eq. (2.21) is that of the entire atom, which is tens of thousands of times greater than that of an electron, and the resulting Compton shift is accordingly so small as to be undetectable.
Watch an animation on Compton scattering at https://www.youtube.com/watch?v=meYllYgHROQ

PAIR PRODUCTION

Energy into matter

- As we have seen, in a collision a photon can give an electron all of its energy (the photoelectric effect) or only part (the Compton effect). It is also possible for a photon to materialize into an electron and a positron, which is a positively charged electron. In this process, called **pair production,** electromagnetic energy is converted into matter

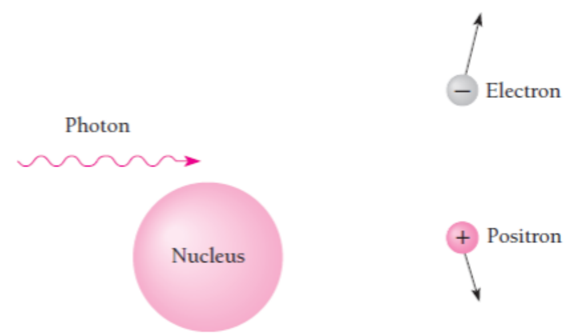
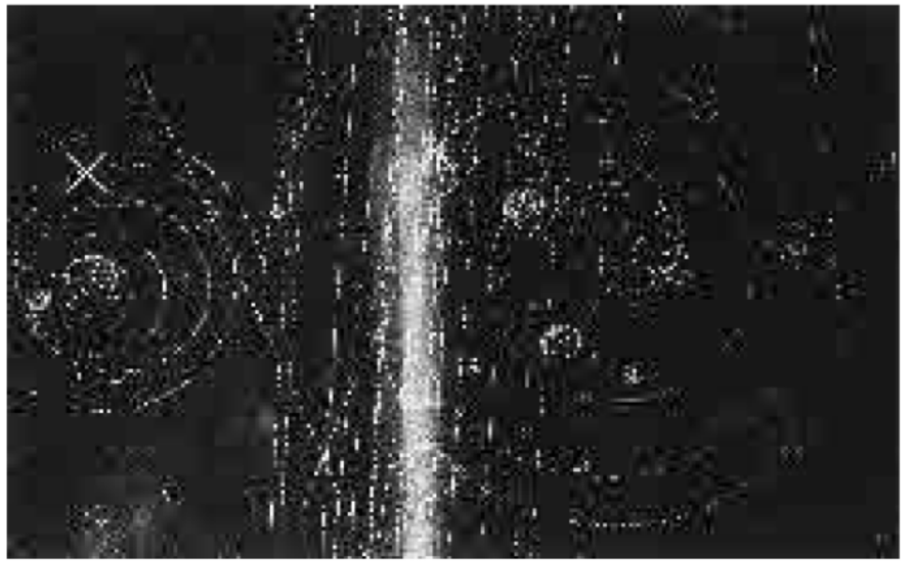


Figure 2.25 In the process of pair production, a photon of sufficient energy materializes into an electron and a positron.

energy, including rest energy, of the electron and positron equals the photon energy;

and linear momentum is conserved with the help of the nucleus, which carries away enough photon momentum for the process to occur. Because of its relatively enormous mass, the nucleus absorbs only a negligible fraction of the photon energy.

(Energy and linear momentum could not both be conserved if pair production were to occur in empty space, so it does not occur there.)



Bubble-chamber photograph of electron-positron pair formation. A magnetic field perpendicular to the page caused the electron and positron to move in opposite curved paths, which are spirals because the particles lost energy as they moved through the chamber. In a bubble chamber, a liquid (here, hydrogen) is heated above its normal boiling point under a pressure great enough to keep it liquid. The pressure is then released, and bubbles form around any ions present in the resulting unstable superheated liquid. A charged particle moving through the liquid at this time leaves a track of bubbles that can be photographed.

The rest energy mc^2 of an electron or positron is 0.51 MeV, hence pair production requires a photon energy of at least 1.02 MeV. Any additional photon energy becomes kinetic energy of the electron and positron. The corresponding maximum photon wavelength is 1.2 pm. Electromagnetic waves with such wavelengths are called **gamma rays,** symbol γ, and are found in nature as one of the emissions from radioactive nuclei and in cosmic rays.

The inverse of pair production occurs when a positron is near an electron and the two come together under the influence of their opposite electric charges. Both particles vanish simultaneously, with the lost mass becoming energy in the form of two gamma-ray photons:

The total mass of the positron and electron is equivalent to 1.02 MeV, and each photon has an energy $h\nu$ of 0.51 MeV plus half the kinetic energy of the particles relative to their center of mass. The directions of the photons are such as to conserve both energy and linear momentum, and no nucleus or other particle is needed for this **pair annihilation** to take place.

Photon Absorption

The three chief ways in which photons of light, x-rays, and gamma rays interact with matter are summarized in Fig. 2.27. In all cases photon energy is transferred to electrons which in turn lose energy to atoms in the absorbing material.

At low photon energies the photoelectric effect is the chief mechanism of energy loss. The importance of the photoelectric effect decreases with increasing energy, to be succeeded by Compton scattering. The greater the atomic number of the absorber, the higher the energy at which the photoelectric effect remains significant. In the lighter elements, Compton scattering becomes dominant at photon energies of a few tens of 1 MeV are reached (Fig. 2.28).

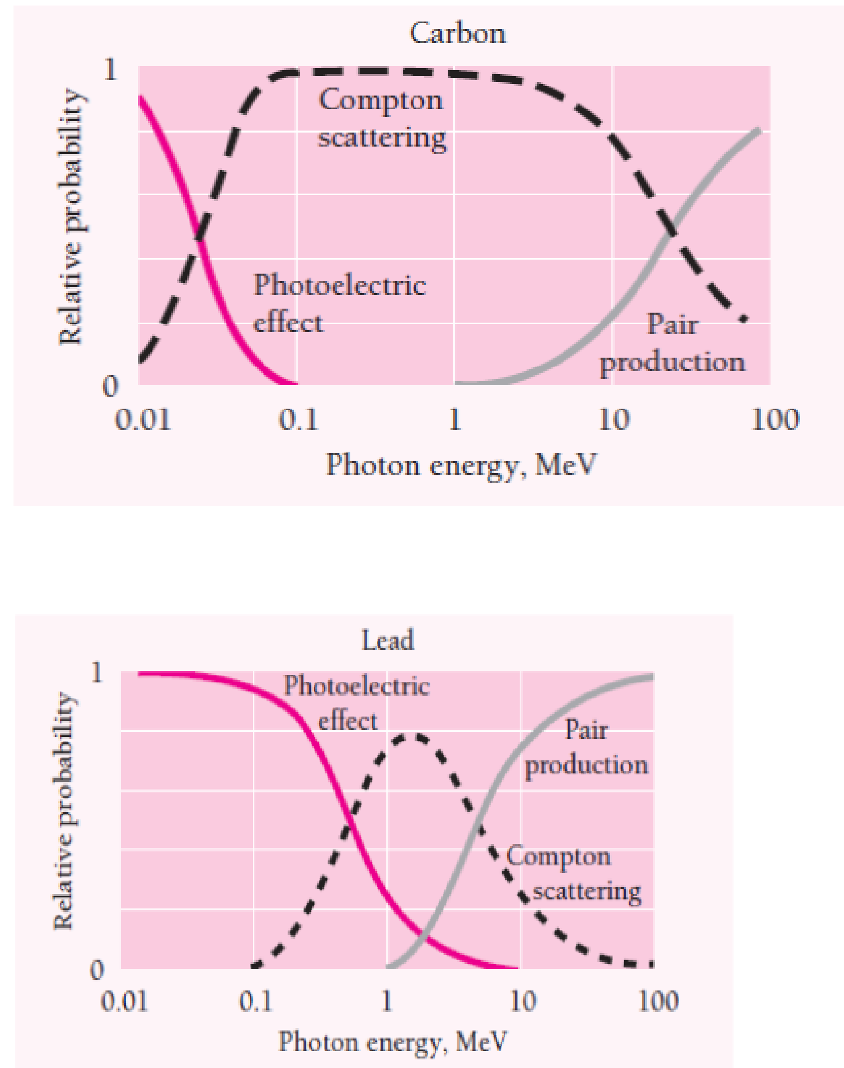


Figure 2.28 The relative probabilities of the photoelectric effect, Compton scattering, and pair production as functions of energy in carbon (a light element) and lead (a heavy element).

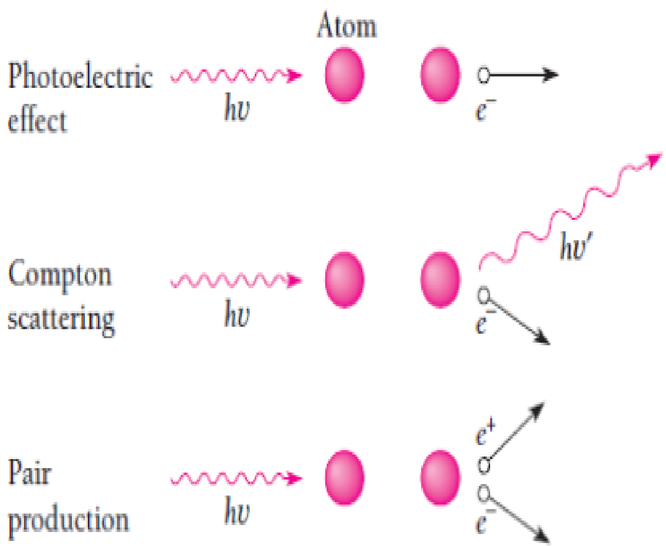


Figure 2.27 X- and gamma rays interact with matter chiefly through the photoelectric effect, Compton scattering, and pair production. Pair production requires a photon energy of at least 1.02 MeV.

- VIDEOS
  Photoelectric effect: https://www.youtube.com/watch?v=0b0axfyJ4oo
  Xray: https://www.youtube.com/watch?v=5RjBsIO0pxg
  Compton scattering: https://www.youtube.com/watch?v=meYllYgHROQ
-

Module 4: **The atomic nucleus**

Thomson's model
Rutherford's model
The size of the nucleus

- The atom is the smallest indivisible unit of an element. We have seen that electrons, which are negatively charged, can be emitted from atoms. Since the atom is electrically neutral, overall, it is evident that there are other constituents of an atom that are positively charged, neutralizing the electric charge of the electrons.

Some terminology

- Nucleus: The central positively charged portion of the atom, around which the electrons orbit.
- Nucleon: A generic name for a particle that is found in the nucleus of atoms. (e.g proton, neutron are nucleons. cf An Undergraduate is found in the University!)
- Nuclide: A particular set of nucleons, with characteristic identity. E.g. Carbon-12, Carbon-14 (Samuel Ajayi is a student in PHY305 class!)

Some Nuclear Terminology

Nuclei are made up of protons and neutrons. The number of protons in a nucleus (called the **atomic number** or **proton number** of the nucleus) is represented by the symbol Z; the number of neutrons (the **neutron number**) is represented by the symbol N. The total number of neutrons and protons in a nucleus is called its **mass number** A; thus

$$A = Z + N. \tag{42-1}$$

Neutrons and protons, when considered collectively, are called **nucleons.**

We represent nuclides with symbols such as those displayed in the first column of Table 42-1. Consider ^{197}Au, for example. The superscript 197 is the mass

Table 42-1

Some Properties of Selected Nuclides

Nuclide	Z	N	A	Stability[a]	Mass[b] (u)	Spin[c]	Binding Energy (MeV/nucleon)
^{1}H	1	0	1	99.985%	1.007 825	$\frac{1}{2}$	—
^{7}Li	3	4	7	92.5%	7.016 004	$\frac{3}{2}$	5.60
^{31}P	15	16	31	100%	30.973 762	$\frac{1}{2}$	8.48
^{84}Kr	36	48	84	57.0%	83.911 507	0	8.72
^{120}Sn	50	70	120	32.4%	119.902 197	0	8.51
^{157}Gd	64	93	157	15.7%	156.923 957	$\frac{3}{2}$	8.21
^{197}Au	79	118	197	100%	196.966 552	$\frac{3}{2}$	7.91
^{227}Ac	89	138	227	21.8 y	227.027 747	$\frac{3}{2}$	7.65
^{239}Pu	94	145	239	24 100 y	239.052 157	$\frac{1}{2}$	7.56

[a]For stable nuclides, the **isotopic abundance** is given; this is the fraction of atoms of this type found in a typical sample of the element. For radioactive nuclides, the half-life is given.

[b]Following standard practice, the reported mass is that of the neutral atom, not that of the bare nucleus.

[c]Spin angular momentum in units of $\hbar$.

- Thomson postulated that the positive charges are distributed in the atom more or less like guava seeds, that are sandwiched in the atom.
- Rutherford gave us our current model. The positive charges are actually concentrated in a tiny central nucleus.
- In this concluding lecture in Section A of PHY 205, we see how Rutherford arrived at his conclusion, as well as estimated the size of the nucleus.

Discovering the Nucleus

(Haliday and Resnick, Chapter 42-2)

In the first years of the 20th century, not much was known about the structure of atoms beyond the fact that they contain electrons. The electron had been discovered (by J. J. Thomson) in 1897, and its mass was unknown in those early days.

Thus, it was not possible even to say how many negatively charged electrons a given atom contained. Scientists reasoned that because atoms were electrically neutral, they must also contain some positive charge, but nobody knew what form this compensating positive charge took. One popular model was that the positive and negative charges were spread uniformly in a sphere.

In 1911 Ernest Rutherford proposed that the positive charge of the atom is densely concentrated at the centre of the atom, forming its **nucleus**, and that, furthermore, the nucleus is responsible for most of the mass of the atom. Rutherford’s proposal was no mere conjecture but was based firmly on the results of an experiment suggested by him and carried out by his collaborators, Hans Geiger (of Geiger counter fame) and Ernest Marsden, a 20-year-old student who had not yet earned his bachelor’s degree.

In Rutherford’s day it was known that certain elements, described as **radioactive,** transform into other elements spontaneously, emitting particles in the process.

One such element is radon, which emits alpha (α) particles that have an energy of about 5.5 MeV. We now know that these particles are helium nuclei.

Rutherford’s idea was to direct energetic alpha particles at a thin target foil and measure the extent to which they were deflected as they passed through the foil. Alpha particles, which are about 7300 times more massive than electrons, have a charge of $+2e$.

Figure 42-1 shows the experimental arrangement of Geiger and Marsden.

Their alpha source was a thin-walled glass tube of radon gas.

The experiment involves counting the number of alpha particles that are deflected through various scattering angles φ.

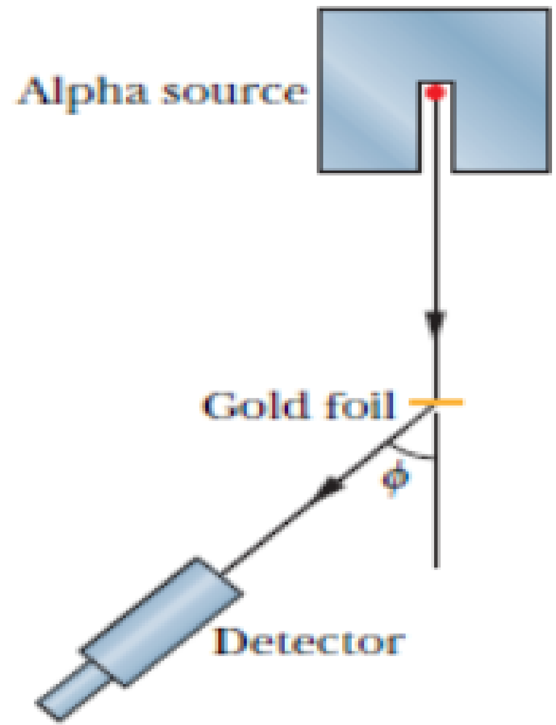
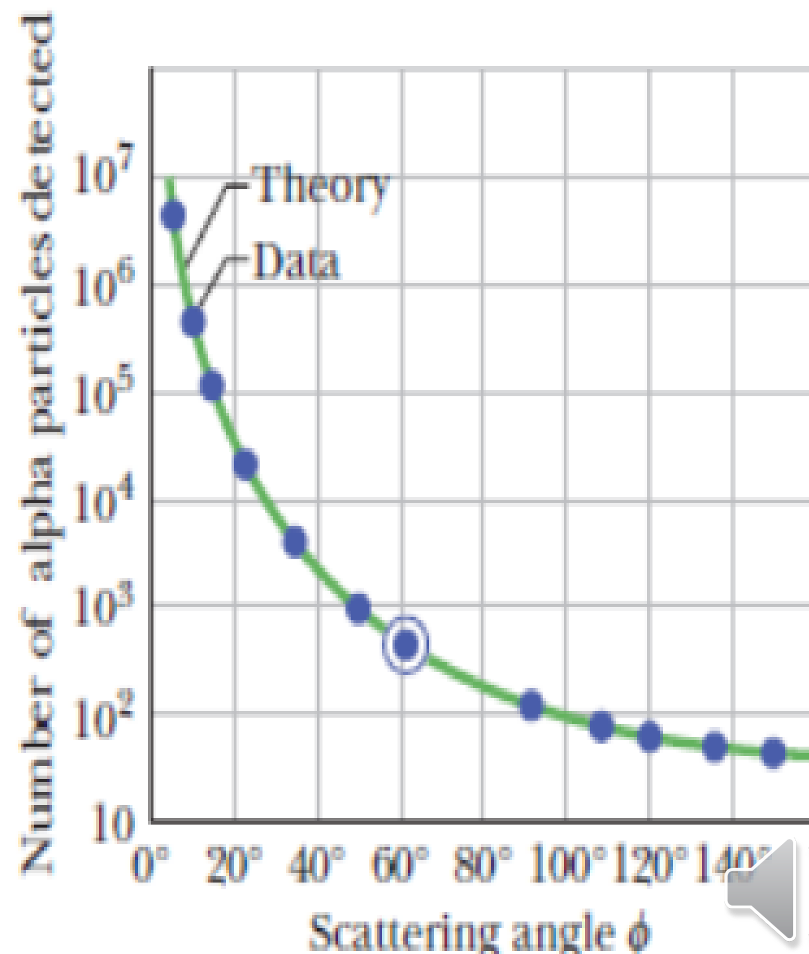


Fig. 42-1 An arrangement (top view) used in Rutherford's laboratory in 1911–1913 to study the scattering of α particles by thin metal foils. The detector can be rotated to various values of the scattering angle ϕ. The alpha source was radon gas, a decay product of radium. With this simple "tabletop" apparatus, the atomic nucleus was discovered.

Results of Geiger and Marsden Scattering Experiment:.
Most of the particles are scattered through rather small angles, but—and this was the big surprise—a very small fraction of them are scattered through very large angles, approaching 180°. In Rutherford's words:
"IT WAS QUITE THE MOST INCREDIBLE EVENT THAT EVER HAPPENED TO ME IN MY LIFE. IT WAS ALMOST AS INCREDIBLE AS IF YOU HAD FIRED A 15-INCH SHELL AT A PIECE OF TISSUE PAPER AND IT [THE SHELL] CAME BACK AND HIT YOU."

Fig. 42-2 The dots are alpha-particle scattering data for a gold foil, obtained by Geiger and Marsden using the apparatus of Fig. 42-1. The solid curve is the theoretical prediction, based on the assumption that the atom has a small, massive, positively charged nucleus. The data have been adjusted to fit the theoretical curve at the experimental point that is enclosed in a circle.



Why was Rutherford so surprised? At the time of these experiments, most physicists believed in the so-called plum pudding model of the atom, which had been advanced by J. J.Thomson. In this view the positive charge of the atom was thought to be spread out through the entire volume of the atom.

The electrons (the “plums”) were thought to vibrate about fixed points within this sphere of positive charge (the “pudding”).

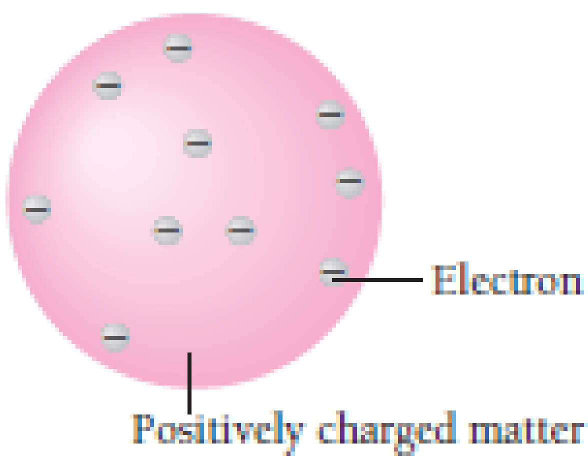


Figure 4.1 The Thomson model of the atom. The Rutherford scattering experiment showed it to be incorrect.

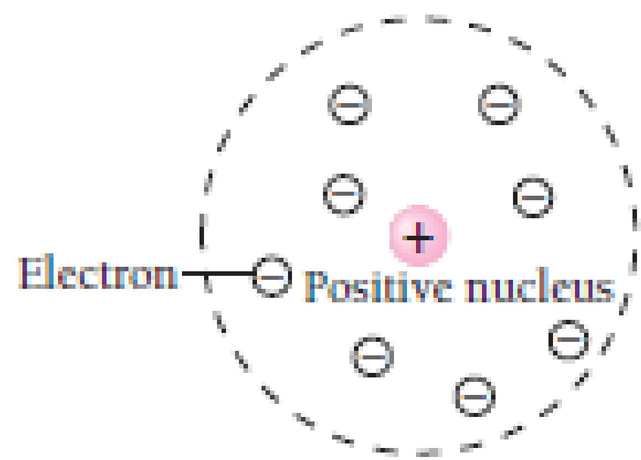


Figure 4.3 The Rutherford model of the atom.

- The maximum deflecting force that could act on an alpha particle as it passed through such a large positive sphere of charge would be far too small to deflect the alpha particle by even as much as 1°. (The expected deflection has been compared to what you would observe if you fired a bullet through a sack of snowballs.) The electrons in the atom would also have very little effect on the massive, energetic alpha particle. They would, in fact, be themselves strongly deflected, much as a swarm of gnats would be brushed aside by a stone thrown through them.

- Rutherford saw that, to deflect the alpha particle backward, there must be a large force; this force could be provided if the positive charge, instead of being spread throughout the atom, were concentrated tightly at its centre. Then the incoming alpha particle could get very close to the positive charge without penetrating it; such a close encounter would result in a large deflecting force

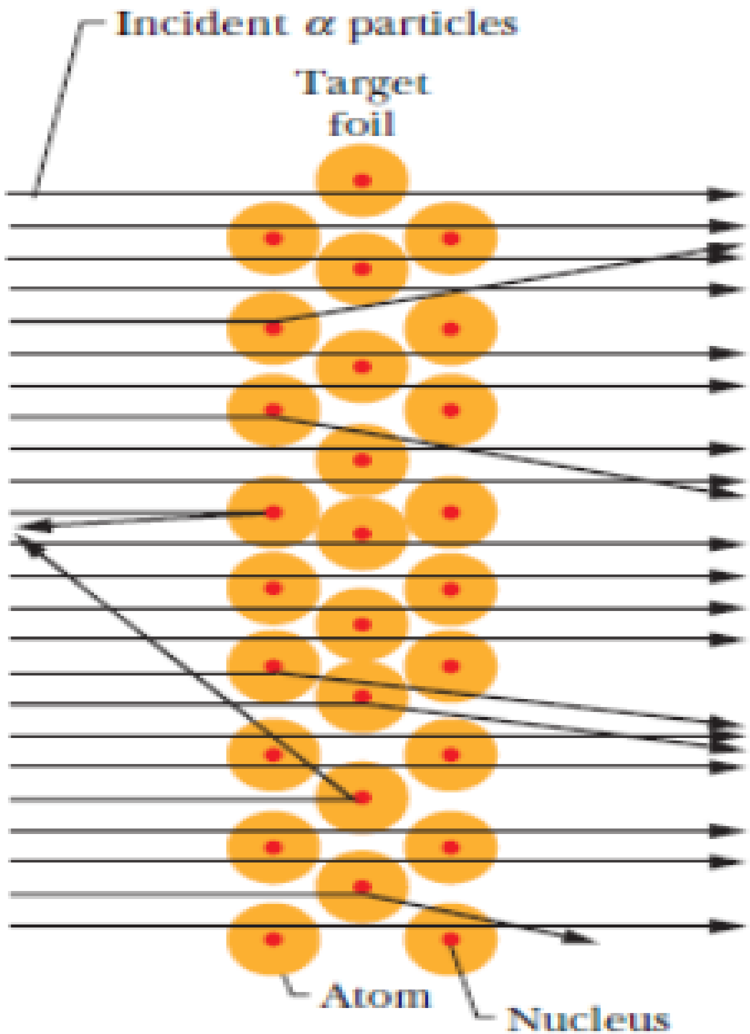


Fig. 42-3 The angle through which an incident alpha particle is scattered depends on how close the particle's path lies to an atomic nucleus. Large deflections result only from very close encounters.

- Figure 42-3 shows possible paths taken by typical alpha particles as they pass through the atoms of the target foil. As we see, most are either undeflected or only slightly deflected, but a few (those whose incoming paths pass, by chance, very close to a nucleus) are deflected through large angles. From an analysis of the data, Rutherford concluded that the radius of the nucleus must be smaller than the radius of an atom by a factor of about 10^4.
- **In other words, the atom is mostly empty space.**

Rutherford scattering of an alpha particle by a gold nucleus

An alpha particle with kinetic energy K_i = 5.30 MeV happens, by chance, to be headed directly toward the nucleus of a neutral gold atom (Fig. 42-4*a*). What is its *distance of closest approach d* (least center-to-center separation) to the nucleus? Assume that the atom remains stationary.

KEY IDEAS

(1) Throughout the motion, the total mechanical energy E of the particle–atom system is conserved. (2) In addition to the kinetic energy, that total energy includes electric potential energy U as given by Eq. 24-43 ($U = q_1q_2/4\pi\varepsilon_0 r$).

Calculations: The alpha particle has a charge of $+2e$ because it contains two protons. The target nucleus has a charge of $q_{\text{Au}} = +79e$ because it contains 79 protons. However, that nuclear charge is surrounded by an electron "cloud" with a charge of $q_e = -79e$, and thus the alpha particle initially "sees" a neutral atom with a net charge of $q_{\text{atom}} = 0$. The electric force on the particle and the initial electric potential energy of the particle–atom system is $U_i = 0$.

Once the alpha particle enters the atom, we say that it passes through the electron cloud surrounding the nucleus. That cloud then acts as a closed conducting spherical shell and, by Gauss' law, has no effect on the (now internal) charged alpha particle. Then the alpha particle "sees" only the nuclear charge q_{Au}. Because q_α and q_{Au} are both positively charged, a repulsive electric force acts on the alpha particle, slowing it, and the particle–atom system has a potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{Au}}}{r}$$

that depends on the center-to-center separation r of the incoming particle and the target nucleus (Fig. 42-4*b*).

As the repulsive force slows the alpha particle, energy is transferred from kinetic energy to electric potential energy. The transfer is complete when the alpha particle momentarily stops at the distance of closest approach d to the target nucleus (Fig. 42-4*c*). Just then the kinetic energy is $K_f = 0$ and the particle–atom system has the electric potential energy

$$U_f = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{Au}}}{d}.$$

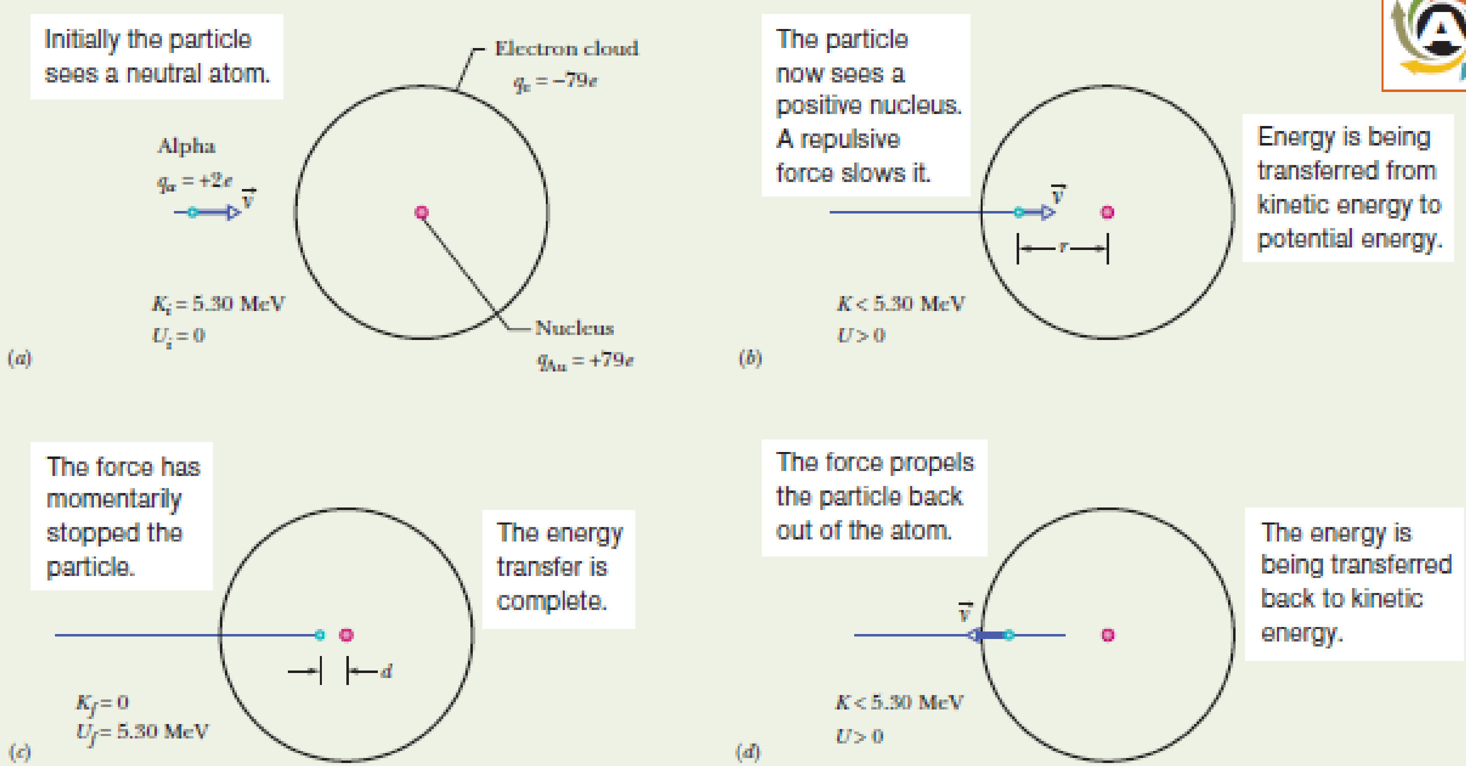


Fig. 42-4 An alpha particle (*a*) approaches and (*b*) then enters a gold atom, headed toward the nucleus. The alpha particle (*c*) comes to a stop at the point of closest approach and (*d*) is propelled back out of the atom.

To find d, we conserve the total mechanical energy between the initial state i and this later state f, writing

$$K_i + U_i = K_f + U_f$$

and

$$K_i + 0 = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{Au}}{d}.$$

(We are assuming that the alpha particle is not affected by the force holding the nucleus together, which acts over only a short distance.) Solving for d and then substituting for the charges and initial kinetic energy lead to

$$d = \frac{(2e)(79e)}{4\pi\varepsilon_0 K_\alpha}$$

$$= \frac{(2 \times 79)(1.60 \times 10^{-19}\ \text{C})^2}{4\pi\varepsilon_0(5.30\ \text{MeV})(1.60 \times 10^{-13}\ \text{J/MeV})}$$

$$= 4.29 \times 10^{-14}\ \text{m}. \qquad \text{(Answer)}$$

This distance is considerably larger than the sum of the radii of the gold nucleus and the alpha particle. Thus, this alpha particle reverses its motion (Fig. 42-4*d*) without ever actually "touching" the gold nucleus.

Additional examples, video, and practice available at *WileyPLUS*

Rutherford Scattering Formula

The formula that Rutherford obtained for alpha particle scattering by a thin foil on the basis of the nuclear model of the atom is

Rutherford scattering formula

$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\epsilon_0)^2 r^2 \, \mathrm{KE}^2 \sin^4(\theta/2)} \tag{4.1}$$

This formula is derived in the Appendix to this chapter. The symbols in Eq. (4.1) have the following meanings:

$N(\theta)$ = number of alpha particles per unit area that reach the screen at a scattering angle of θ
N_i = total number of alpha particles that reach the screen
n = number of atoms per unit volume in the foil
Z = atomic number of the foil atoms
r = distance of the screen from the foil
KE = kinetic energy of the alpha particles
t = foil thickness

The predictions of Eq. (4.1) agreed with the measurements of Geiger and Marsden, which supported the hypothesis of the nuclear atom. This is why Rutherford is credited

Nuclear Dimensions

In his derivation of Eq. (4.1) Rutherford assumed that the size of a target nucleus is small compared with the minimum distance R to which incident alpha particles approach the nucleus before being deflected away. Rutherford scattering therefore gives us a way to find an upper limit to nuclear dimensions.

Let us see what the distance of closest approach R was for the most energetic alpha particles employed in the early experiments. An alpha particle will have its smallest R when it approaches a nucleus head on, which will be followed by a 180° scattering. At the instant of closest approach the initial kinetic energy KE of the particle is entirely converted to electric potential energy, and so at that instant

$$\mathrm{KE}_{\mathrm{initial}} = \mathrm{PE} = \frac{1}{4\pi\epsilon_0}\frac{2Ze^2}{R}$$

since the charge of the alpha particle is $2e$ and that of the nucleus is Ze. Hence

Distance of closest approach

$$R = \frac{2Ze^2}{4\pi\epsilon_0 \mathrm{KE}_{\mathrm{initial}}} \tag{4.2}$$

The maximum KE found in alpha particles of natural origin is 7.7 MeV, which is 1.2×10^{-12} J. Since $1/4\pi\epsilon_0 = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$,

$$R = \frac{(2)(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})^2 Z}{1.2 \times 10^{-12} \text{ J}}$$

$$= 3.8 \times 10^{-16} Z \text{ m}$$

The atomic number of gold, a typical foil material, is $Z = 79$, so that

$$R \text{ (Au)} = 3.0 \times 10^{-14} \text{ m}$$

The radius of the gold nucleus is therefore less than 3.0×10^{-14} m, well under 10^{-4} the radius of the atom as a whole.

Quiz: Imagine that the nucleus of the gold nucleus above were scaled up to become 1cm in radius. What would be the size of the atom (i.e the radius of the orbit of the farthest electron)?

Nuclear Radii

A convenient unit for measuring distances on the scale of nuclei is the *femtometer* ($= 10^{-15}$ m). This unit is often called the *fermi;* the two names share the same abbreviation. Thus,

$$1 \text{ femtometer} = 1 \text{ fermi} = 1 \text{ fm} = 10^{-15} \text{ m}. \tag{42-2}$$

We can learn about the size and structure of nuclei by bombarding them with a beam of high-energy electrons and observing how the nuclei deflect the incident electrons. The electrons must be energetic enough (at least 200 MeV) to have de Broglie wavelengths that are smaller than the nuclear structures they are to probe.

The nucleus, like the atom, is not a solid object with a well-defined surface. Furthermore, although most nuclides are spherical, some are notably ellipsoidal. Nevertheless, electron-scattering experiments (as well as experiments of other kinds) allow us to assign to each nuclide an effective radius given by

$$r = r_0 A^{1/3}, \tag{42-3}$$

in which A is the mass number and $r_0 \approx 1.2$ fm. We see that the volume of a nucleus, which is proportional to r^3, is directly proportional to the mass number A and is independent of the separate values of Z and N. That is, we can treat most nuclei as being a sphere with a volume that depends on the number of nucleons, regardless of their type.

PHY205 (Introductory Modern Physics)

Wave Behavior of Matter

A wave is described by frequency ν, wavelength λ, phase velocity u and intensity I. It's spread out and occupies a relatively large region of space.

A particle is specified by mass m, velocity v, momentum p, and energy E. it's occupies a definite position in space. Nature of Light.

Phenomenal like Blackbody radiation and Photoelectric effect can be explained considering light as a stream of particles while Interference and Diffraction experiments showed the wave nature of light.

Brainstorm: So, is light a wave or a particle?

De Broglie hypothesis

In the Year 1924 Louis de Broglie made the bold suggestion

" If radiation which is basically a wave can exhibit particle nature under certain circumstances, and since nature likes symmetry, then entities which exhibit particle nature ordinarily, should also exhibit wave nature under suitable circumstances"

The reasoning he used might be paraphrased as follows

- ✓ **Nature loves symmetry**
- ✓ **Therefore the two great entities, matter and energy, must be mutually symmetrical**
- ✓ **If energy (radiant) is undulatory and/or corpuscular, matter must be corpuscular and/or undulatory**

The de Broglie Hypothesis

- ➢ **If light can act like a wave sometimes and like a particle at other times, then *all matter*, usually thought of as particles, should exhibit wave-like behaviour**

- ➢ **The relation between the momentum and the wavelength of a photon can be applied to material particles also**

de Broglie Wavelength

wavelength

$$\lambda = \frac{h}{p} \qquad (1.1)$$

momentum

where h is a constant (Planck's constant, h ≈ 6.63 x 10^{-34} Js)

Equation 1.1 relates a particle-like property (*p*) to a wave-like property (*λ*)

The momentum of a particle of mass *m* and velocity *v* is

$$p = \gamma m v \qquad 1.2$$

and its **de Broglie wavelength** is accordingly

$$\lambda = \frac{h}{\gamma m v} \qquad 1.3$$

where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ which is the relativistic factor

The greater the particle's momentum, the shorter its wavelength

Examples from Concepts of Modern Physics by Arthur Beiser

Find the de Broglie wavelengths of (*a*) a 46-g golf ball with a velocity of 30 m/s, and (*b*) an electron with a velocity of 10^7 m/s.

Solution

(*a*) Since $v \ll c$, we can let $\gamma = 1$. Hence

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(0.046\ \text{kg})(30\ \text{m/s})} = 4.8 \times 10^{-34}\ \text{m}$$

The wavelength of the golf ball is so small compared with its dimensions that we would not expect to find any wave aspects in its behavior.

(*b*) Again $v \ll c$, so with $m = 9.1 \times 10^{-31}$ kg, we have

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(9.1 \times 10^{-31}\ \text{kg})(10^7\ \text{m/s})} = 7.3 \times 10^{-11}\ \text{m}$$

The dimensions of atoms are comparable with this figure—the radius of the hydrogen atom, for instance, is 5.3×10^{-11} m. It is therefore not surprising that the wave character of moving electrons is the key to understanding atomic structure and behavior.



Find the kinetic energy of a proton whose de Broglie wavelength is 1.000 fm = 1.000 × 10^{-15} m, which is roughly the proton diameter.

Solution

A relativistic calculation is needed unless pc for the proton is much smaller than the proton rest energy of $E_0 = 0.938$ GeV. To find out, we use Eq. (3.2) to determine pc:

$$pc = (\gamma m v)c = \frac{hc}{\lambda} = \frac{(4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(2.998 \times 10^{8}\ \text{m/s})}{1.000 \times 10^{-15}\ \text{m}} = 1.240 \times 10^{9}\ \text{eV}$$

$$= 1.2410\ \text{GeV}$$

Since $pc > E_0$ a relativistic calculation is required. From Eq. (1.24) the total energy of the proton is

$$E = \sqrt{E_0^2 + p^2c^2} = \sqrt{(0.938\ \text{GeV})^2 + (1.2340\ \text{GeV})^2} = 1.555\ \text{GeV}$$

The corresponding kinetic energy is

$$\text{KE} = E - E_0 = (1.555 - 0.938)\ \text{GeV} = 0.617\ \text{GeV} = 617\ \text{MeV}$$

Example from Modern Physics by RAYMOND A. SERWAY

A particle of charge q and mass m is accelerated from rest through a small potential difference V. (a) Find its de Broglie wavelength, assuming that the particle is non-relativistic.

Solution When a charge is accelerated from rest through a potential difference V, its gain in kinetic energy, $\frac{1}{2}mv^2$, must equal the loss in potential energy qV. That is,

$$\tfrac{1}{2}mv^2 = qV$$

Because $p = mv$, we can express this in the form

$$\frac{p^2}{2m} = qV \qquad \text{or} \qquad p = \sqrt{2mqV}$$

Substituting this expression for p into the de Broglie relation $\lambda = h/p$ gives

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

(b) Calculate λ if the particle is an electron and $V = 50$ V.

Solution The de Broglie wavelength of an electron accelerated through 50 V is

$$\lambda = \frac{h}{\sqrt{2m_e qV}}$$

$$= \frac{6.63 \times 10^{-34}\,\mathrm{J \cdot s}}{\sqrt{2(9.11 \times 10^{-31}\,\mathrm{kg})(1.6 \times 10^{-19}\,\mathrm{C})(50\,\mathrm{V})}}$$

$$= 1.7 \times 10^{-10}\,\mathrm{m} = 1.7\,\text{Å}$$

EXERCISE

1. If you double the kinetic energy of a nonrelativistic particle, what happens to its de Broglie wavelength? What if you double its speed?
2. In a tube television electrons are accelerated through a 25.0 kV potential difference. If they are nonrelativistic, what is their de Broglie wavelength?
3. Compute the de Broglie wavelength of the following: (a) a smoke particle of mass 10^{-9} moving at 1 cm/s (b) An electron with a kinetic energy of (i) 1 eV, (ii) 100 MeV
4. Find the de Broglie wavelength of a proton moving with a speed of 1×10^{-27} m/s

Electron diffraction

The de Broglie hypothesis was confirmed by Bell Labs physicists (Davisson and Germer) in 1927 that electrons undergo diffraction when interacting with crystalline matter, just like x rays. This proved the wave nature of electrons

Note:

1. Electrons have mass and do not travel at the speed of light, they DO NOT obey the relation $\lambda f = c$, *i.e.* $\gamma = 1$.

2. Kinetic energy for non-relativistic particles can be written

$$K = \tfrac{1}{2}mv^2 = \frac{p^2}{2m} \qquad 1.4$$

3. If an electron is accelerated through a potential difference V, it gains a kinetic energy eV, so we can write:

$$K = eV = \frac{p^2}{2m} \quad \Rightarrow \quad p = \sqrt{2meV} = \frac{h}{\lambda} \qquad 1.5$$

4. Hence, the wavelength of an electron accelerated through potential V is given as

$$\lambda = \frac{h}{\sqrt{2meV}} \qquad 1.6$$

5. Because **electrons behaved as a wave due to** diffraction pattern that emerged. We can make an analogy with the diffraction of x-rays by a crystal;

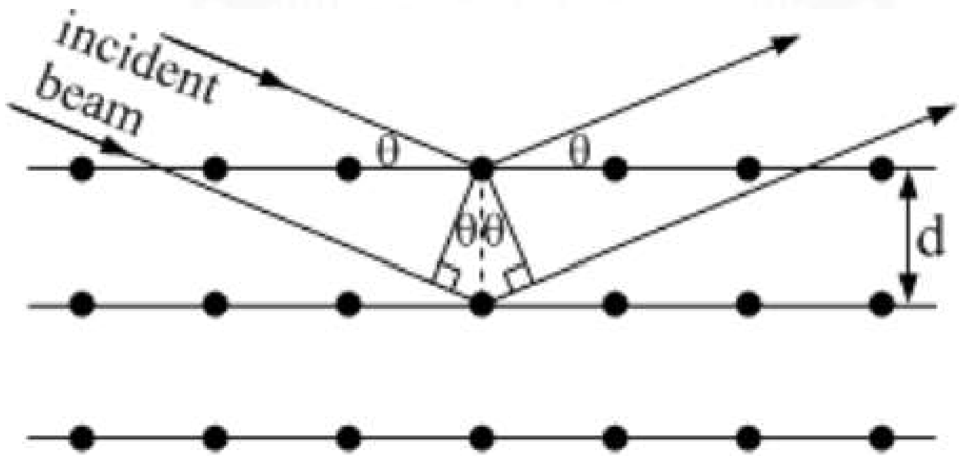


Fig. 1: Electron Waves Reflected from Atomic Planes

For constructive interference of waves Bragg's Law must be satisfied i.e.

$$n\lambda = 2d\sin\theta \qquad 1.7$$

where **n** is an integer and is the order of diffraction



Example from Modern Physics by RAYMOND A. SERWAY

What kinetic energy (in electron volts) should neutrons have if they are to be diffracted from crystals?

Solution Appreciable diffraction will occur if the de Broglie wavelength of the neutron is of the same order of magnitude as the interatomic distance. Taking $\lambda = 1.00$ Å,

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}\ \mathrm{J\cdot s}}{1.00 \times 10^{-10}\ \mathrm{m}} = 6.63 \times 10^{-24}\ \mathrm{kg\cdot m/s}$$

The kinetic energy is given by

$$K = \frac{p^2}{2m_\mathrm{n}} = \frac{(6.63 \times 10^{-24}\ \mathrm{J\cdot s})^2}{2(1.66 \times 10^{-27}\ \mathrm{kg})}$$

$$= 1.32 \times 10^{-20}\ \mathrm{J} = 0.0825\ \mathrm{eV}$$

Note that these neutrons are nonrelativistic because K is much less than the neutron rest energy of 940 MeV, and so our use of the classical expression $K = p^2/2m_{\mathrm{n}}$ is justified. Because the average thermal energy of a particle in thermal equilibrium is $\frac{1}{2}k_{\mathrm{B}}T$ for each independent direction of motion, neutrons at room temperature (300 K) possess a kinetic energy of

$$K = \tfrac{3}{2}k_{\mathrm{B}}T = (1.50)(8.62 \times 10^{-5}\ \mathrm{eV/K})(300\ \mathrm{K})$$
$$= 0.0388\ \mathrm{eV}$$

Wave-Particle Duality

Some aspects of light behavior, such as interference and diffraction, are explained by treating light as a wave, while other aspects are explained by treating light as being made up of particles.

Light exhibits wave-particle duality, because it exhibits both waves and particles properties.

Wave-particle duality is not confined only to light, every matter believed to exhibit wave-particle duality. However, behavior of relatively large objects, like baseballs, is dominated by their particle nature.

In any case, it is impossible to measure both the wave and particle properties simultaneously

This dilemma of the dual particle+wave nature of light, which is called *wave-particle duality*, cannot be resolved with a simple explanation. The best we can do is to say that neither the wave nor the particle picture is wholly correct all of the time, that both are needed for a complete description of physical phenomena, and that in fact the two are *complementary* to one another.

The Heisenberg's uncertainty principle

- In the example of a free particle, we see that if its momentum is completely specified, then its position is completely unspecified
- When the momentum p is completely specified we write:

$$\Delta p = 0 \quad (\text{because: } \Delta p = p_1 - p_2 = 0)$$

and when the position x is completely unspecified we write:

$$\Delta x \to \infty$$

- In general, we always have: $\Delta x \cdot \Delta p \geq$ a constant

This constant is known as:

(called *h-bar*) ⟵ $\hbar = \dfrac{h}{2\pi}$

h is the Planck's constant

$(h = 6.625 \times 10^{-34} J.s)$

From Dr. DO Xuan Hoi

So we can write: $\Delta x \cdot \Delta p \geq \hbar$ 1. 8

That is the **Heisenberg's uncertainty principle**

" it is impossible to know simultaneously and with exactness both the position and the momentum of the fundamental particles"

N.B.: • We also have for the particle moving in three dimensions

$\Delta x \cdot \Delta p_x \geq \hbar$ $\Delta y \cdot \Delta p_y \geq \hbar$ $\Delta z \cdot \Delta p_z \geq \hbar$ 1.9

• With the definition of the constant $\hbar$:

$$p = h / \lambda = hK / 2\pi \longrightarrow p = \hbar K$$

• Uncertainty for energy :

$\Delta E \cdot \Delta t \geq \hbar$ 1.10

From Dr. DO Xuan Hoi

Examples from Concepts of modern physics. — Arthur Beiser

Planck's constant h is so small that the limitations imposed by the uncertainty principle are significant only in the realm of the atom. On such a scale, however, this principle is of great help in understanding many phenomena. It is worth keeping in mind that the lower limit of $\hbar/2$ for $\Delta x\, \Delta p$ is rarely attained. More usually $\Delta x\, \Delta p \geq \hbar$, or even (as we just saw) $\Delta x\, \Delta p \geq h$.

A typical atomic nucleus is about 5.0×10^{-15} m in radius. Use the uncertainty principle to place a lower limit on the energy an electron must have if it is to be part of a nucleus.

Solution

Letting $\Delta x = 5.0 \times 10^{-5}$ m we have

$$\Delta p \geq \frac{\hbar}{2\Delta x} \geq \frac{1.054 \times 10^{-34}\ \text{J} \cdot \text{s}}{(2)(5.0 \times 10^{-15}\ \text{m})} \geq 1.1 \times 10^{-20}\ \text{kg} \cdot \text{m/s}$$

If this is the uncertainty in a nuclear electron's momentum, the momentum p itself must be at least comparable in magnitude. An electron with such a momentum has a kinetic energy KE many times greater than its rest energy mc^2. From Eq. (1.24) we see that we can let KE $= pc$ here to a sufficient degree of accuracy. Therefore

$$KE = pc \geq (1.1 \times 10^{-20}\ \text{kg} \cdot \text{m/s})(3.0 \times 10^{8}\ \text{m/s}) \geq 3.3 \times 10^{-12}\ \text{J}$$

Since 1 eV = 1.6×10^{-19} J, the kinetic energy of an electron must exceed 20 MeV if it is to be inside a nucleus. Experiments show that the electrons emitted by certain unstable nuclei never have more than a small fraction of this energy, from which we conclude that nuclei cannot contain electrons. The electron an unstable nucleus may emit comes into being at the moment the nucleus decays (see Secs. 11.3 and 12.5).

A hydrogen atom is 5.3×10^{-11} m in radius. Use the uncertainty principle to estimate the minimum energy an electron can have in this atom.

Solution

Here we find that with $\Delta x = 5.3 \times 10^{-11}$ m.

$$\Delta p \geq \frac{\hbar}{2\Delta x} \geq 9.9 \times 10^{-25}\ \text{kg} \cdot \text{m/s}$$

An electron whose momentum is of this order of magnitude behaves like a classical particle, and its kinetic energy is

$$KE = \frac{p^2}{2m} \geq \frac{(9.9 \times 10^{-25}\ \text{kg} \cdot \text{m/s})^2}{(2)(9.1 \times 10^{-31}\ \text{kg})} \geq 5.4 \times 10^{-19}\ \text{J}$$

For Further Readings:

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Bohr theory of atomic structure:

- **Atomic spectra**
- **Wilson-Sommerfeld quantization rules**
- **Sommerfeld's relativistic theory**
- **The correspondence principle**
- **Problems of the old quantum theory**

ATOMIC SPECTRA

- The existence of spectral lines is an important aspect of the atom that finds no explanation in classical physics
- Fig. 1 shows the **emission line spectra** of some elements. Every element displays a unique line spectrum.

➡ Each element has a characteristic line spectrum.

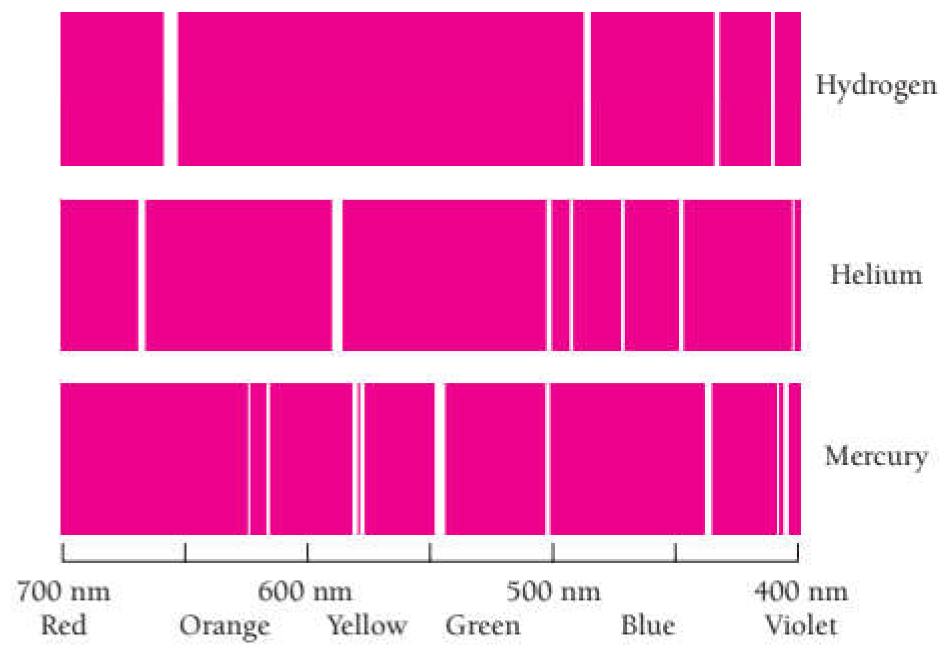


Fig.1: Some of the principal lines in the emission spectra of hydrogen, helium, and mercury

▪ The number, intensity, and exact wavelengths of the lines in the spectrum of an element depend upon temperature, pressure, the presence of electric and magnetic fields, and the motion of the source.

▪ The wavelengths in the spectrum of an element were found to fall into sets called **spectral series.**

▪ J. J. Balmer in 1885 discovered the first set of such series of the visible part of the hydrogen spectrum shown in Fig. 2.

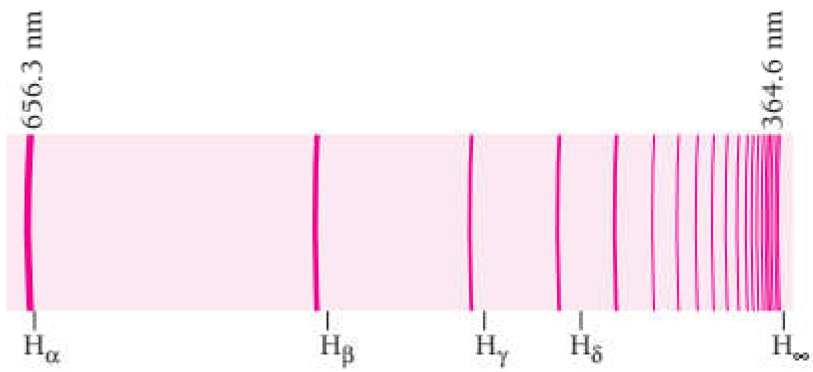


Fig.2: The Balmer series of hydrogen.

- The line with the longest wavelength, 656.3 nm, the next is 486.3 nm, and so on. As the wave-length decreases, the lines are found closer together and weaker in intensity until the **series limit** at 364.6 nm is reached, beyond which there are no further separate lines but only a faint continuous spectrum.
- Balmer's formula for the wavelengths of the series is

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad n = 3, 4, 5, \ldots$$

- The quantity R is known as the **Rydberg constant,** which is **Rydberg constant** R = 1.097 x10^7 m^{-1} = 0.01097 nm^{-1}

- The 656.3 nm line correspond to n = 3, the next is 486.3 nm is n = 4, and so on. The series limit corresponds to n = ∞ so that it occurs at a wavelength of 4/R, in agreement with experiment.

- The Balmer series contains wavelengths in the visible portion of the hydrogen spectrum. The spectral lines of hydrogen in the ultraviolet and infrared regions fall into several other series.

In the ultraviolet the **Lyman series gives** the wavelengths as

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad n = 2, 3, 4, \ldots$$

In the infrared, three spectral series have been give the wavelengths as

Paschen $$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \qquad n = 4, 5, 6, \ldots$$

Brackett $$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right) \qquad n = 5, 6, 7, \ldots$$

Pfund $$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right) \qquad n = 6, 7, 8, \ldots$$

These spectral series of hydrogen are plotted in terms of wavelength as shown in Fig. 3

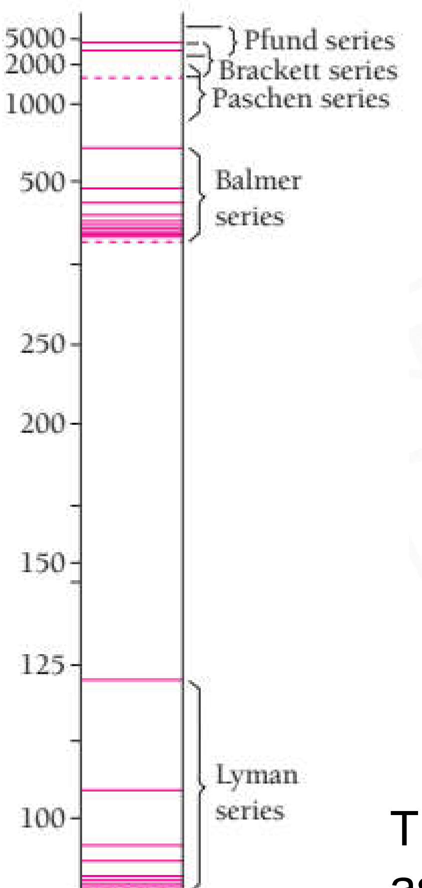


Fig. 3

The Bohr model

Rutherford model for the atom considered that the electrons reside outside the positively charge nucleus, thereby accelerated by the Coulomb force. Based on classical physics, these electrons must radiate electromagnetic waves and their energy loss would cause the electrons spiraling into the nucleus.

At this point Niels Bohr was thoughtfully considering how Planck's quantum nature of radiation ideas could be applied to atomic spectra, and particularly the Rydberg formula for hydrogen. Bohr pictured the electron in hydrogen orbiting the central atomic nucleus.

In 1915 Bohr came up with his postulated which was based partly on classical mechanics and partly on some startling new quantum ideas.

Bohr's postulates were:

❖ Electrons move about the nucleus in circular orbits determined by Coulomb's and Newton's laws;

❖ Only certain orbits are stable. The electron does not radiate electromagnetic energy in these special orbits, and because the energy is constant with time these are called stationary states;

❖ A spectral line of frequency *f* is emitted when an electron jumps from an initial orbit of energy E_i to a final orbit of energy E_f , where

$$hf = E_i - E_f$$

- The sizes of the stable electron orbits are determined by requiring the electron's angular momentum to be an integral multiple of $\hbar$:

$$m_e v r = n\hbar \qquad n = 1, 2, 3, \ldots$$

These postulates lead to quantized orbits and quantized energies for a single electron orbiting a nucleus with charge +*Ze*, given by

$$r_n = \frac{n^2 a_0}{Z} \qquad E_n = -\frac{ke^2}{2a_0}\frac{Z^2}{n^2} = -\frac{13.6\ Z^2}{n^2}\ \text{eV}$$

where n is an integer and $a_0 = (\hbar^2/m_e ke^2 = 0.529\ \text{Å} = 0.0529\ \text{nm}$ is the Bohr radius.

For more information on how these Equations were derived and their applications kindly read Chapter 4, Concepts of Modern Physics by Arthur Beiser or/and Chapter 4, Modern Physics by Raymond A. Serway

Ex.

For hydrogen, calculate the radius (r) and energy level (E_T) of the electron in the lowest energy state (ground state).

Calculate the wavelength of the spectrum of hydrogen atom when an electron falling from the fourth level (E = - 0.85) eV to the first level (E = - 13.6) eV

Calculate the longest and shortest wavelength in the Lyman series of hydrogen atom spectrum. Given: Planck's constant = 6.625×10^{-34} J.s.

Calculate the longest and shortest wavelength in the Lyman series of hydrogen atom spectrum. If electron energy at any energy level (n) in hydrogen atom $E_n = 13.6/n^2$ electron volt and Planck's constant = 6.625×10^{-34} J.s.

Calculate the voltage necessary to ionize hydrogen atom in the ground state.

Bohr's atomic model succeeded to a great extent in the following:

- It explained the hydrogen atom spectrum, which is the simplest electronic system;
- The first who introduced some concepts of quantum theory on the atom (idea of quantized energy state for electrons in the atom), which lead to determination of electron energy in each energy level and calculate the energy of each level;
- It emphasized that in the ground state electrons revolve around the nucleus in definite orbits, each orbit has certain energy and electrons revolve without radiation of energy as long as it move in its energy level, and accordingly, they will not fall back to the nucleus. Thereby, reconciled between Rutherford and Maxwell concepts

Bohr provided the modern concept of the atomic model.

Bohr's atomic model's limitations

- The quantitative calculation of Bohr's theory did not totally agree with all experimental data;
- It failed to explain the spectrum of any other element than hydrogen;
- It ignored the wave properties of electron but considered the electron as a negatively charged particle only;
- The postulation contradicts Heisenberg's uncertainty principle by upholding the possibility to determine both the location and the speed of an electron precisely at the same time;
- It postulated that electron moves in a circular, planer orbit, which means that the atom is planar whereas it was confirmed that the hydrogen atom has three – dimensional co-ordinates X, Y and Z



- The theory cannot be used for the quantitative study of chemical bonding;
- It was found that when electric or magnetic field is applied to the atom, each spectral line is splitted into several lines. The former one is called *Stark effect* while the later as *Zeeman effect*. Bohr’s theory fails to explain these effects.

Wilson-Sommerfeld quantization rules

The assumption made by Bohr that the angular momentum of the electron is quantized is a special case of **the Wilson-Sommerfeld quantization rules**. The quantization rules apply to any system with a coordinate *q* that is a periodic function.

Note: a periodic function requires that the time dependence of a coordinate *q* repeats itself after well-defined periods of time (the period) but the time dependence of *q* does not have to be harmonic. For this type of motion, the Wilson-Sommerfeld quantization rule states that the linear momentum p_q associated with coordinate *q* must satisfy the following condition:

$$\oint_{\text{One period}} p_q \, dq = n_q h$$

where n_q is the quantum number (an integer)

The Wilson-Sommerfeld quantization rule can also explain the **hyperfine splitting** in the Hydrogen spectrum. Consider an electron in an elliptical orbit around a proton. The orbit can be described in terms of two position coordinates: the radius r and the polar angle θ. The two momenta associated with these position coordinates are the linear momentum p_r and the angular momentum L. The Wilson-Sommerfeld quantization rule requires that

$$\oint L\, d\theta = n_\theta h$$

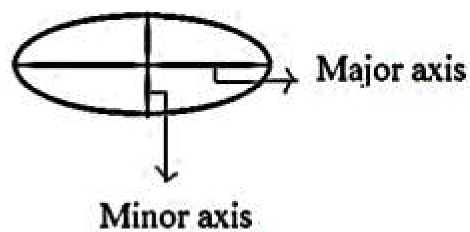
$$\oint p_r\, dr = n_r h$$

Sommerfeld's relativistic theory

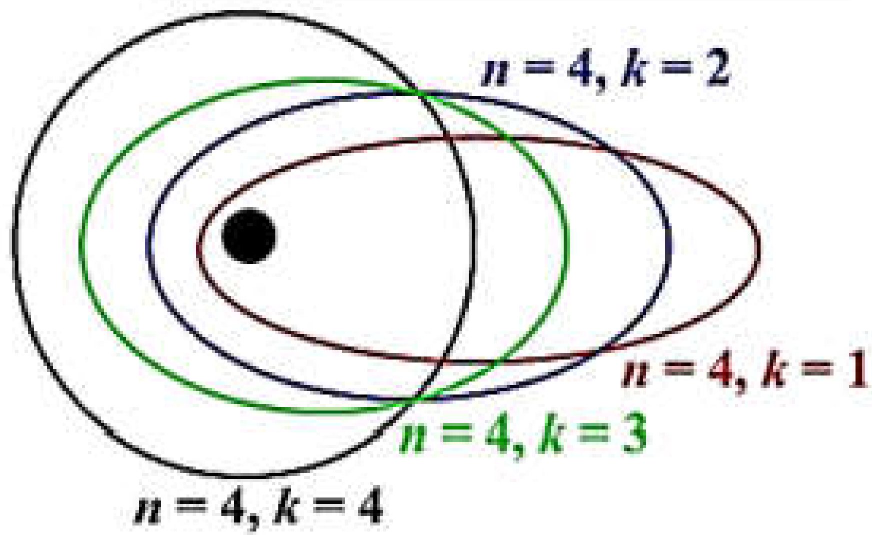
In order to explain the observed fine structure of spectral lines, Sommerfeld postulated an atomic model as follow:

- The orbits may be both *circular* and *elliptical;*

- When path is elliptical, then there are two axis – major axis and minor axis. When length of major & minor axis becomes equal then orbit is circular;



- The angular momentum of electron moving in an elliptical orbit is (kh/2π). Where k is an integer except zero. Value of k = 1,2,3,4….. . (n/k) = length of major axis / length of minor axis. With increase in value of k, ellipticity of the orbit decreases. When n= k, then orbit is circular;
-



- Sommerfeld suggested that orbits are made up of sub energy levels. These are s,p,d,f. which possess slightly different energies.

Bohr gave a quantum number ‘**n**’, which determines the energy of electron. Sommerfeld introduced a new quantum number called Orbital or Azimuthal Quantum number (*l*) which determines the orbital angular momentum of electron.

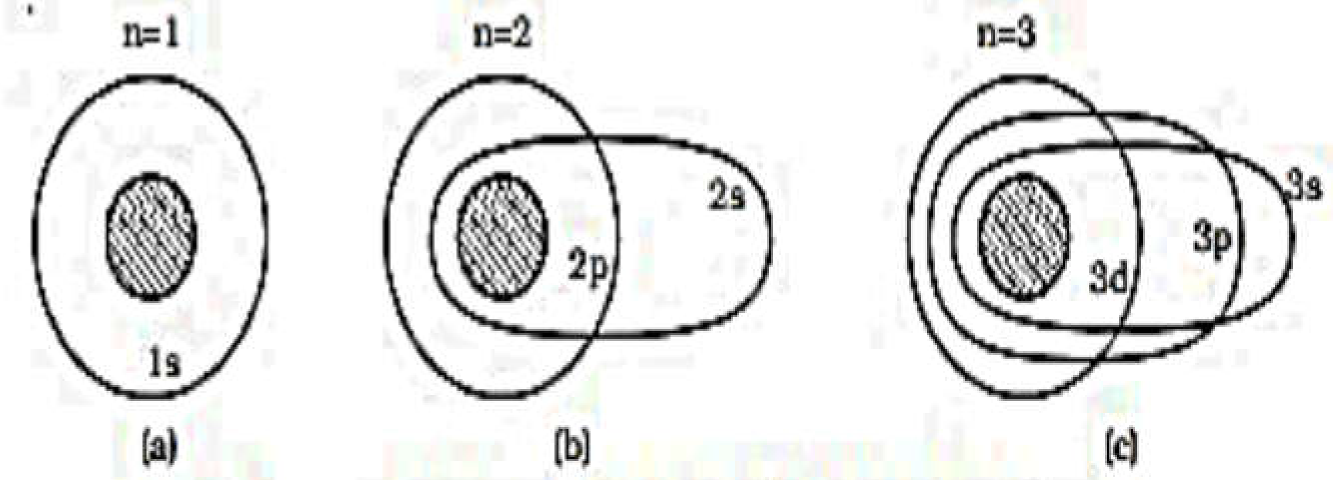
Values of l =0 to (n-1)

For, n=1 ; l=0 ; 1s sub shell

n=2 ; l=0,1 ; 2s , 2p sub shell

n=3 ; l=0,1 ,2 ; 3s , 3p , 3d sub shell

n=4 ; l=0, 1, 2 , 3 ; 4s , 4p , 4d , 4f sub shell



- When an electron jumps from one orbit to another orbit, the difference of energy (ΔE) depends upon sub energy levels. This explains the splitting of individual spectral lines of hydrogen and thus fine spectrum

Defects of Sommerfeld atomic model

It does not explain the behavior of system having more than one electron;

It does not explain the Zeeman & Stark effect;

It does not give any explanation for the intensities of the spectral lines.

The Correspondence Principle

Classical electrodynamics + Bohr's atomic model

Determine the properties of radiation

The need for a principle to relate the new modern results with classical results.

Bohr's correspondence principle

Bohr's correspondence principle: In the limits where classical and quantum theories should agree, the quantum theory must reduce to the classical result.

For more information and examples kindly read Chapter 4, Modern Physics For Scientists and Engineers by Stephen T. Thornton

The Correspondence Principle

•The frequency of the radiation emitted $\nu_{classical}$ is equal to the orbital frequency ν_{orb} of the electron around the nucleus.

$$\nu_{classical} = \nu_{orb} = \frac{\omega}{2\pi} = \frac{\mathrm{v}/r}{2\pi} = \frac{1}{2\pi}\left(\frac{e^2}{4\pi\varepsilon_0 m r^3}\right)^{1/2} = \frac{me^4}{4\varepsilon_0{}^2 h^3}\frac{1}{n^3}$$

•This should agree with the frequency of the transition from $n + 1$ to n (when n is very large):

$$\nu_{Bohr} = \frac{E_0}{h}\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

$$= \frac{E_0}{h}\left[\frac{n^2 + 2n + 1 - n^2}{n^2(n+1)^2}\right] = \frac{E_0}{h}\left[\frac{2n+1}{n^2(n+1)^2}\right]$$

For large n:

$$\nu_{Bohr} \approx \frac{2nE_0}{hn^4} = \frac{2E_0}{hn^3}$$

Substituting for E_0:

$$\nu_{Bohr} = \frac{me^4}{4\varepsilon_0{}^2 h^3}\frac{1}{n^3} = \nu_{classical}$$

Problems of the old quantum theory

The **old quantum theory** is a collection of results which predate modern quantum mechanics. The theory was never complete or self-consistent, but was rather a set of heuristic corrections to classical mechanics. The theory is now taken as the semi-classical approximation to modern quantum mechanics. The main tool of the old quantum theory was the Bohr–Sommerfeld quantization condition, a procedure for selecting out certain states of a classical system as allowed states: the system can then only exist in one of the allowed states and not in any other state.

Limitations old quantum theory are:

- The old quantum theory provides no means to calculate the intensities of the spectral lines.
- It fails to explain the anomalous Zeeman effect (that is, where the spin of the electron cannot be neglected).
- It cannot quantize "chaotic" systems, i.e. dynamical systems in which trajectories are neither closed nor periodic and whose analytical form does not exist. This presents a problem for systems as simple as a 2-electron atom.

However it can be used to describe atoms with more than one electron (e.g. Helium) and the Zeeman effect.

For Further Readings:

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ii. Modern Physics by Raymond A. Serway
iii. Modern Physics For Scientists and Engineers by Stephen T. Thornton

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Schrodinger wave equation and simple applications

Schrödinger's equation is the fundamental equation of quantum mechanics in the same sense with the second law of motion which is the fundamental equation of Newtonian mechanics

Recall classical wave equation:

$$\frac{\partial^2 y}{\partial t^2} = c^2 \left(\frac{\partial^2 y}{\partial x^2} \right)$$

Similarly **Schrödinger's equation** was constructed to solve problems involving electrons and other sub-atomic particles, for which classical laws were found to be deficient.

For this purpose, a new wave function was introduced, which at every point in space and time written as $\psi(x,y,z,t)$

The wave function possess the following characteristics:

- It is *complex* function, that is, it has a real part and an imaginary part;

- This means that the wave function is interpreted in terms of probabilities instead of certainties. Therefore the square of the wave function is interpreted as the probability (per unit volume) that the particle will be found at that point. The probability that a particle is somewhere (in volume element dV) is unity so that;

$$\int \Psi^2 dV = 1$$

- wave function is not a measure of any simple physical entity such as displacement. It carries all the information about the particle.
- Ψ and $\partial\Psi/\partial x, \partial\Psi/\partial y, \partial\Psi/\partial z$ must be continuous and single-valued everywhere.

Schrödinger's theory is therefore an extension of the de Broglie's postulate. Consider a particle of momentum *P* and energy *E*

$$\lambda = h/p \qquad \text{and} \qquad \upsilon = E/h$$

$$\text{Wave number } k = \frac{2\pi}{\lambda} = \frac{P}{\hbar}; \text{ Angular frequency } \omega = 2\pi v = \frac{E}{\hbar}$$

$$\text{For a free particle } E = P^2/(2m)$$

, **thus**

$$k = \frac{\sqrt{2mE}}{\hbar}$$

If the particle travels in the x-direction, the wave will also travel in the same direction. The wave function that meeting this condition is

$$\psi(x,t) = e^{-i(\omega t - kx)} \qquad \mathbf{1}$$



Finding out the wave equation that obeyed wave functions. We rewrite Eq. (1) by expressing ω and *k* in terms of *P*, that is,

$$\psi = e^{-i(\omega t - kx)} = e^{-i\left(\frac{Et}{\hbar} - \frac{x}{\hbar}\sqrt{2mE}\right)} \qquad 2$$

Differentiating Eqn. (2) once wrt time *(t)* and twice with respect to *x* to obtain Eq. (3) and (4), respectively

$$\frac{\partial \psi}{\partial t} = -i\frac{E}{\hbar}\psi \qquad 3; \qquad \frac{\partial^2 \psi}{\partial x^2} = -\frac{2mE}{\hbar^2}\psi \qquad 4$$

Comparing Eq.(3) and (4) to obtain that;

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t) \qquad 5$$

Eq.5 is the-time dependent Schrödinger Equation in 1D *for a free particle*

If the particle is restricted the presence of a force is represented by some given potential energy function (*U(x,t)*), Eq. 5 now takes the form;

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + U\Psi \qquad 6$$

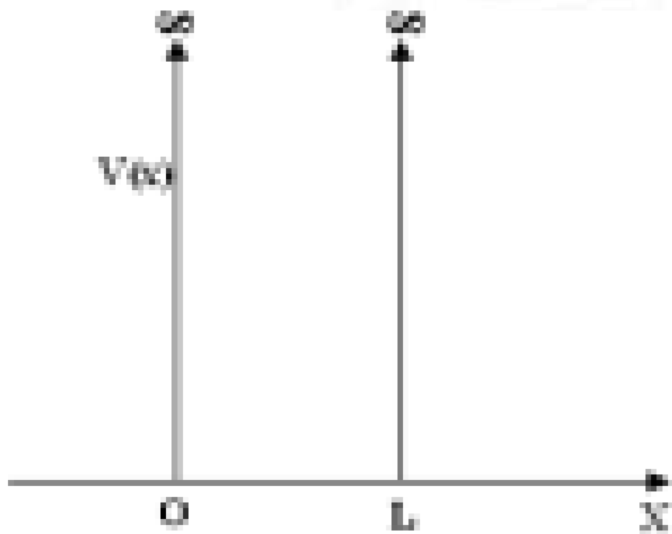
In three dimensions the time-dependent form of Schrödinger's equation is

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}\right) + U\Psi \qquad 7$$

where the particle's potential energy *U* is some function of *x*, *y*, *z*, and *t.*

Application of Schrodinger's Wave Equation- Particle in a one dimensional deep potential box:

A free electron trapped in a metal or charge carriers trapped by the potential barriers *V* can be approximated by an electron in an infinitely deep, one dimensional potential well. It is shown in figure.



Boundary Conditions:

$$\left.\begin{aligned} V(x) &= 0, \qquad 0 < x < L \\ V(x) &= \infty, \qquad L \le x \le 0 \end{aligned}\right\} \longrightarrow (9)$$

From Schrodinger's time independent wave equation

$$\frac{d^2\psi}{dx^2} + \frac{2m[E - V(x)]}{\hbar^2}\psi(x) = 0$$

Inside the potential well, Eq.(10) becomes

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi(x) = 0 \quad (or) \quad \frac{d^2\psi}{dx^2} + k^2\,\psi(x) = 0 \qquad 11$$

$$where\ k^2 = \frac{2mE}{\hbar^2} \qquad 12$$

Eq. 11 is a wave equation for a free particle inside a potential well and its possible solution is:

$$\psi(x) = A\sin(kx) + B\cos(kx) \qquad 13$$

where A & B are constants.

Applying B.Cs from Eq.(9) to Eq.(12), we get,

$$at\ x = 0, \psi(x) = 0 => B = 0 \qquad 14$$

$$at\ x = L, \psi(x) = 0 => A\,sinkL = 0 => kL = n\pi \quad (as\ A \neq 0) \qquad 15$$

$$k = \frac{n\pi}{L}$$ where $n = 1, 2, 3, \dots \dots \dots$ 16

Eq. 12 *becomes*: $\psi(x) = A\sin\left(\frac{n\pi}{L}\right)x$ 15

From Eqs. 12 and 16:

$$\frac{n^2\pi^2}{L^2} = \frac{2mE}{\hbar^2} \Rightarrow E_n = \frac{n^2\hbar^2\pi^2}{2mL^2} \Rightarrow E_n = \frac{n^2h^2}{8mL^2} \quad 17$$

Thus for each value of ‘n’ the possible energy of the particle is given by Eq. (17), which shows that the total energy of the system is quantized.

The value of A can be obtained by applying normalization condition. Since the particle is inside the box of length ‘L’, the probability that the particle is present inside the box is unity i.e.

$$\int_0^L |\psi|^2 dx = 1 \quad 18$$

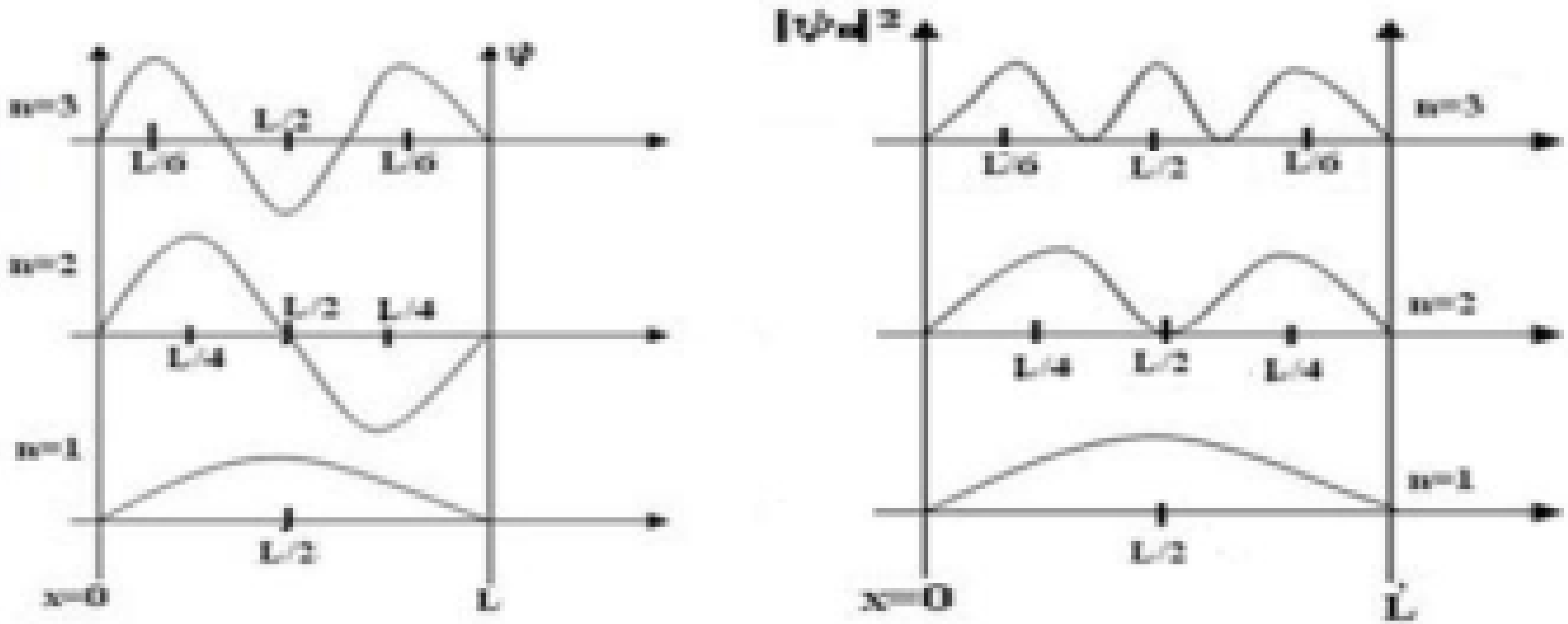
So,

$$\int_0^L A^2 \sin^2\left(\frac{n\pi}{L}\right)x\,dx = 1 \Rightarrow A = \sqrt{\left(\frac{2}{L}\right)} \rightarrow (10) \qquad 19$$

$$\therefore \textit{Normalised wave function}, \psi(x) = \sqrt{\left(\frac{2}{L}\right)}\sin\left(\frac{n\pi}{L}\right)x \qquad 20$$

The wave function ψ_n and the corresponding energies E_n, which are often called **Eigen functions** and **Eigen values** respectively, describes the quantum state of the particle.

The electron wave functions ψ_n and the corresponding probability density functions $|\psi_n|^2$, for the ground and first two excited states of an electron in a potential well are shown in figures below.



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