$\boxed{E = h\nu}$

The Origin of Quantum Theory

Electromagnetic radiation is quantized; (discrete amounts) comes and comes in quanta

Blackbody Radiation Spectra

A blackbody is any object that absorbs all the radiation that falls on it. For instance, if the radiation is in the visible range, and all the component colours are absorbed, the object will look black since it is not reflecting any of the incident radiation. A true blackbody absorbs all em radiation from all sections of the em spectrum (from radiowaves to gamma rays) RMIVUXG

A blackbody at a specific temp. (in thermal equilibrium with its environment) is also emitting radiation; so absorbing and emitting radiation of all frequencies

The emission spectrum from a blackbody:

- Intensity depends only on temp. (for real materials, the type of materials will also contribute. Blackbody is idealized).
- Has a peculiar shape (distribution intensities of the various frequencies)

* The higher the temp, the higher the intensity of total radiation (Stefan's law) $I \propto T^4$ $I = \sigma T^4$

* The higher the temp, the higher the frequency at which maximum (peak) occurs (Wien's law)

* The intensity after attaining the peak eventually nose-dive to zero (from planck's eqn)

The higher the temp, the greater the amount of radiation and the higher the frequency at which the maximum emission occurs

The eqn observed from classical physics considerations for Blackbody radiation (obtained by the pair of Lord Rayleigh and James Jeans)

$$u(\nu)d\nu = \bar{\epsilon}G(\nu)d\nu = \frac{8\pi KT}{c^3}\nu^2 d\nu \quad \text{--- (1)}$$

Rayleigh-Jean formula

The energy density {u} as a function of frequency ν [$u(\nu)$] determines the radiation rate from the blackbody. This Rayleigh-Jeans formula which gives radiation rate increasing proportionately as square of ν (frequency) implies that by the time we reach UV (ultraviolet) frequency range, we already have radiation rates tending to infinity. This phenomenon is called the "ultraviolet catastrophe". It does not occur in real life ∴ the formula is wrong.

Max Planck, in 1900, first obtained an eqn that fits the shape very accurately:

$$u(\nu)d\nu = \frac{8\pi h}{c^3}\cdot\frac{\nu^3 d\nu}{e^{h\nu/KT}-1} \; ; \; h = 6.626\times10^{-34}\,\text{Js} \quad \text{--- (ii)}$$

We can obtain planck's formula by making the substitution

$$\bar{\epsilon}/KT \rightarrow \frac{h\nu}{e^{h\nu/KT}-1}$$ (Actual average energy per standing wave)

At low frequencies ν, R-J eqn agrees with Planck's (low wavelength)

At high frequencies, $h\nu \gg KT$ and $e^{h\nu/KT} \rightarrow \infty$

which means that $u(\nu)d\nu \rightarrow 0$. No more UV catastrophe

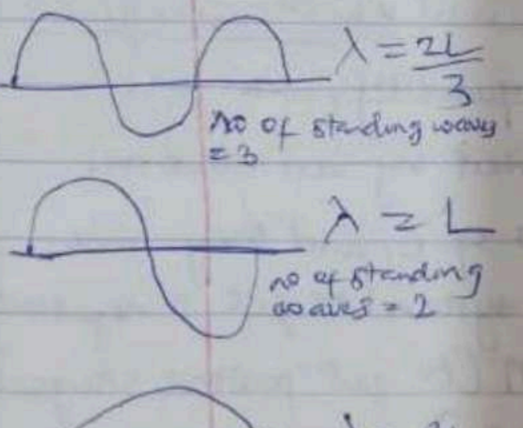
At low frequencies, planck's formula becomes

$$u(\nu)d\nu = \frac{8\pi h}{c^3}\nu^3\left(\frac{KT}{h\nu}\right)d\nu \approx \frac{8\pi KT}{c^3}\nu^2 d\nu$$

which means, the UV catastrophe has been resolved

BB can be approximated by a cylinder with a tiny hole in one side - All energies incident can enter, and they are largely all trapped within the cavity. Radiation from the "Cavity Radiator" (BB) results from interaction btw radiation within the cavity and radiation emitted by oscillating atoms on its walls. We know that e.m radiation can in principle, have a continuous distribution of frequencies. Hence these atomic oscillators can have any value for their frequencies (within certain range, of course depending on temp.)

Within a certain frequency range ($\nu + d\nu$), there are a specific no of states that radiation of a particular frequency can exist in. For example, on a given length of rope fixed at both ends, there are certain no of waves (of different wavelengths) that can be set on. These are called modes.



The higher the value of frequency, the more the no of standing waves that can be accomodated

e.g for the same container size, you can put in more no of pebbles (smaller wavelength → higher frequency) than big stone)

If the atomic oscillators actually oscillate (and ∴ radiate energy) at every frequency permitted in principle, then the higher the frequency, the higher the "energy density" of the emission spectrum. This is the UV Catastrophe predicted by classical physics

Planck → an oscillator cannot oscillate at every frequency permitted in principle but only at certain frequencies

Oscillator energies $\boxed{E_n = nh\nu}$, $n = 0, 1, 2 \ldots$

wavelength peak — constant of proportionality

Wien's Displacement Law ($\lambda_{max} = \frac{b}{T}$) → Temperature

States that the wavelength at which the emitted radiation has peak intensity is inversely proportional to temp. When a piece of metal is heated, it first becomes "red hot" → this is the longest visible spe wavelength. On further heating, it moves from red to orange and then yellow, at it's hottest, the metal will be seen to be glowing white. This is the shorter wavelengths dominating the radiation. This relationship can thus be used in thermometry

ELECTRONS AND QUANTA

Two of the basic ppties of matter are mass and electric charge. Each charge is a good example of quantized quantity, with the smallest charge possible being the charge carried by an electron. Other quantities of charge come in multiples of e, and they are usually obtained either by adding electrons to a system (to get a negatively charged system) or removing electrons from them (to get positive charges)

$Q = ne$ — electronic charge (-1.602×10^{-19} coulombs), $n = 1, 2, 3 \ldots$

* Some particles called hadrons can be modeled as comprising of substructures (called quarks) which have fractional electronic charges)

Thermionic Emission: Liberation of electrons from an electrode by virtue of its temp (releasing of energy supplied)

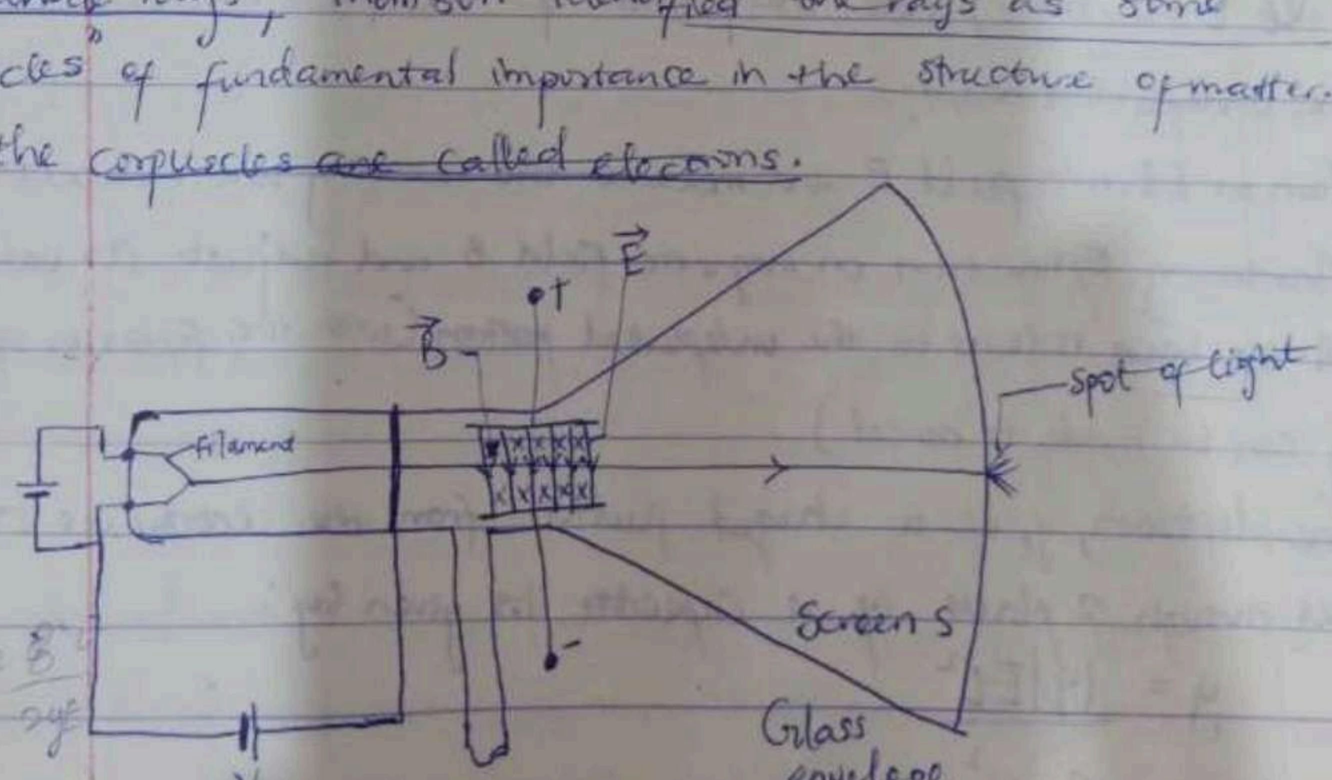
How do you get electrons?

There are many ways a metal (cathode) can be made to give off electrons.

- Heating: Thermionic emission
- Light: Photoelectric emission
- Kinetic energy from other electrons: Secondary Emission (Scattering, Auger etc)
- Electrical voltage: Field emission
- Chemical potential difference (e.g in a battery/electric cell)

Crossed Fields and the Discovery of the electron

J.J ~~Th~~ompson tried to solve some riddles associated with cathode ray tubes by placing both a magnetic field and an electric field at right angles (perpendicularly) across the path of the "cathode rays", Thomson identified the rays as some "corpuscles" of fundamental importance in the structure of matter. Today the corpuscles ~~are called electrons~~.



An electric field E is established by connecting a battery across the deflecting-plate terminals. The magnetic field B is set up by means of a current in a system of coils (not shown). The magnetic field shown is into the plane of the figure, as represented by the array of Xs (which resemble the feathered ends of arrows).

Cathode rays released by thermionic emission from the filament stream in from the left and eventually hits the screen S on the far right. The Glass envelope is evacuated to allow free motion of the electron via scattering and ionization. Motion is now completely controlled by the applied E and B fields.

Steps taken by J.J in unmasking the identity of the electron

1.) Set $E=0$, $B=0$, note the position of the spot on screen S due to the undeflected beam.

2.) Turn on Electric field E and measure the resulting beam direction.

3.) Maintaining E, now turn on magnetic field B and adjust its value until the beam returns to the undeflected position (with the forces in opposition, they can be made to cancel.)

The deflection, y, of a charged particle from the centre, as it passes through 2 plates of a capacitor is given by:

$$y = \frac{|q|EL^2}{2mv^2}$$

v = particle's speed, m = mass, q = charge, L = length of the plates providing the electric field E

When the two fields are adjusted so that the two deflecting forces cancel (step 3). Electric force equals Magnetic Force

$$|q|E = |q|vB\sin(90^\circ)$$

$$|q|E = |q|vB$$

$$v = \frac{E}{B}$$

$$y = \frac{|q|EL^2}{2mv^2}\ ; \quad \frac{m}{|q|} = \frac{EL^2}{2yv^2}$$

$$\frac{m}{|q|} = \frac{EL^2}{2y\left(\frac{E^2}{B^2}\right)}$$

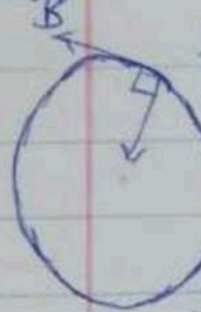
$$\therefore \frac{m}{|q|} = \frac{B^2L^2}{2yE}$$

Thus, the crossed fields allow us to measure the ~~ration~~ $(m/|q|)$ of the particle moving through Thomson's apparatus. He claimed that these particles are found in all matter. He also claimed that they are lighter than the lightest known atom (hydrogen) by a factor of more than 1000. Exact ~~ration~~ provided later to be 1836.15

The right hand rule (in which $\vec{v}$ is swept into $\vec{B}$ through the smaller angle ϕ bt them) gives the direction of $\vec{v} \times \vec{B}$ as the direction of the thumb.
If q is +ve, then the direction of $\vec{F}_B = q\vec{v} \times \vec{B}$ is in the direction of $\vec{v} \times \vec{B}$
If q is −ve, then ~~....~~ F_B is opposite that of $\vec{v} \times \vec{B}$.

Mass Spectrometers

If we have a uniform B field alone, and acting all over the chamber, from the right hand rule, the electron is continually pushed off a rectilinear path into a circular path. If there is some gas in the glass tube, there is some ionization as the electron whirls around, and we can see the electron. This is the basis for cloud chamber



Electrons circulating in a chamber containing gas at low pressure (their path is the glowing circle) A uniform magnetic field $\vec{B}$ pointing directly out of the plane of the page fills the chamber. Note the radially directed $\vec{F_B}$, for circular motion to occur, $\vec{F_B}$ must point towards the center of the circle - Use the right-hand rule for cross product to confirm that $\vec{F_B} = q\vec{V} \times \vec{B}$ gives $\vec{F_B}$ the proper ~~function~~ direction (Don't forget the sign of q)

From magnetic force, $F_B = qv \times B$

If the magnetic field is at right angle to direction of velocity, V, then

$F_B = qvB\sin\theta = qvB$

It is this force that ~~keeps~~ is providing the centripetal force keeping the electron in a circular path of radius r, Hence, $F_B = \frac{mv^2}{r}$

$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{|q|B} \text{ (radius)}$$

For another particle with a different m/q ratio, sent in with the same velocity v, into the same B field, the orbit is different. The radius of the orbit is given by the same eqn above. For fully charged particles, the direction of orbit is opposite that of the electron.

Hence if we have several charged particles/ions injected into the chamber with a constant speed B applied, the particle will start to move in circles of various radii, dictated by their peculiar q/m ratios. The various particles can $\therefore$ be separated and identified on the basis of their q/m ratio which is the basis for the mass spectrometer.

Cyclotrons

If we have an electron/ion moving under the influence of a perpendicular B field, and the energy is increased, it moves to a new orbit. Can be accelerated circularly like this to great energies whereafter it is released to bombard a target, either in medical therapy, production of radiopharmaceuticals, or nuclear physics experiments.

Such a device is called a cyclotron (where charged particles are energized by moving them around in circles with periodic injection of energy via E-field thus transferring them to orbits with higher radii. To accelerate a proton beyond 50MeV, we have another device called the Synchrotron which takes care of the relativistic effects and can take particles to energies in the GeV range and beyond. The proton synchrotron at the Fermi.

Linear Accelerators (Linacs)

This is another interesting and common application where electrons and other charged particles are accelerated to great energies for use as in a cyclotron.

Here, only an electric field so the electron/charged particle moves linearly with increasing velocity. Again, this can range from modest devices used in medical therapies to huge research facilities spanning several kilometers.

Robert Millikan → Charge of an electron.

Forms of Electrons

1.) Bound electrons :- These electrons move round a positive nucleus so they are bound electrons.

2.) Conduction electrons :- Electrons (same electrons) can exist in free state, not bound to any nucleus. They are responsible for bonding.

3) Free electrons :- Some electrons are totally independent and are not associated with any material.

~~Wave~~ — Particle Behaviour of EM Radiation

~~Wave~~ Particle Duality

In classical physics, energy is transported either by wave or particle. They are like observing water waves carrying energy over the water surface or bullet transferring energy from gun to the target.
Such experiences enables physics to be a wave model from certain microscopic phenomenon on the particle model from the others. They explain sound propagation in terms of the wave model and the pressure of gases in terms of particle model. Also in particle model (K.E). They concluded that all entity are either particle or wave; that is why to understand radiation, there is a need to invoke a particle model in some situation as in the quantum model or as a wave model in other situation e.g ~~as~~ in a diffraction of x-ray, the charge to mass ratio of the electron and its ionization trait in matter suggests a particle model but electron diffraction suggests a wave model. It is important to note that both models are used under the same circumstances.

Niel Bohr summarised the situation in the principle of complementality. The link between wave model or a particle model is provided by the probability interpretation of the wave-particle duality. For a particle moving in the x-direction with a precise value of linear momentum and energy, for example the wave function can be given as a sinusodial function of amplitude A.

$$\psi_{(x,t)} = A \sin 2\pi\left(\frac{x}{d} - \upsilon t\right)$$

This eqn is analogous to

$$\mathcal{E}(x,t) = A \sin 2\pi \left(\frac{x}{\lambda} - \nu t\right)$$

This expression is for the electric field of a sinusoidal electromagnetic wave of wavelength λ and frequency ν, moving in the positive x-direction. This expression ψ^2 plays a role for water wave analogous to $\overline{\mathcal{E}}^2$ that plays a role for wave radiation. After superposition of two electric field;

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$$

Similarly, for water wave we have

$$\psi = \psi_1 + \psi_2$$

This is the principle of superposition applied ~~with~~ to matter as well as to radiation.

Interaction of Radiation with Matter

There are basically three processes involving the scattering or absorption of radiation in matter.

1) The photoelectric effect
2) The Compton effect
3) The pair production.

BREMSSTRAHLUNG and the pair annihilation are involved in the production of radiation.

1) The Photoelectric effect

Is the ejection of electron from surface by the action of light in the cathode ray tube for example the ejection of electron from the cathode is accelerated through a potential difference V to the anode. If the p.d is reversed in sign and is made to be large enough, a value V_0 called stopping

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$\xrightarrow{\text{Increasing}}$ frequency/photon energy
Decreasing λ

potential is a reached at which the photoelectric current drops to zero. eV_0 gives $K.E_{max}$ for the fastest ejected photoelectron. To eject an electron, we apply

$K.E_{max} = eV_0$ $= \mathcal{E}V_0$ V_0 = stopping potential

e = charge of the photoelectron

$K.E_{max}$ = maximum K.E which is independent of the intensity of the light

There are three major characteristics of photoelectric effect that cannot be explained in terms of the classical wave theory of light.

A) The wave theory require that the oscillatory electric field vector E of the light wave increase in amplitude as the intensity of the light is increased. Since the force applied to the electron is qE ($F = qE$, $e = q$, $f = eE$), then this suggests that the kinetic energy of the photoelectron should also increase as the light is made more intense.

B) According to the wave theory, the photoelectric effect that should occur for any frequency of light provided that only the light is intense enough to keep give the energy needed to ~~get~~ eject the photoelectron. For frequency less than the characteristic cut-off frequency V_0 the photoelectric effect do not occur no matter how intense the illumination. (frequencies below critical frequency V_0, no electrons are emitted)

C) If the energy acquired by a photoelectron is absorbed from the wave incident on the metal plate, then the effective (intensity) target area for an electron in the metal is limited and probably not much more than that of a circuit having about an atomic diameter. In the classical theory, the light energy is uniformly distributed over the wave front. Thus, if the light is weak enough there should be a measurable time lapse btw the time ~~taken~~ when

$E = hf = h\nu = \frac{hc}{\lambda}$
$= nhf$

light starts to hit the surface and ejection of the photoelectron during this interval, the electron will be absorbing energy from the beam until it has accumulated enough to escape.

Application

A potassium ~~photographic~~ plate is placed 1m from a weak light source whose power is 1watt Assuming an ejected photoelectron recollects its energy from a circular plate whose radius is 10^{-10}m. The energy required to remove an electron through the potassium surface is about 2eV. How long will it take such a target to absorb this much energy from the light source. (Assume the light energy is to be spread uniformly over the wave front)

Soln

$$\text{Target area} = \pi r^2 = \pi (10^{-10})^2 = 10^{-20}\pi \; ; \; 1eV = 1.602\times10^{-19}$$

$$\text{Area of sphere} = 4\pi r^2 = 4\times\pi\times1^2$$
$$= 4\pi$$

$$\text{Power} = \frac{I \times \text{Target area}}{\text{Area of sphere}} = 1\times \quad \text{intensity} = 1$$

$$\frac{1\times10^{-20}\pi}{4\pi}$$

$$= 2.5\times10^{-21}\ J/s$$

$$2.1eV = 3.36\times10^{-19} \text{ from } P = E/t$$

$$t = \frac{3.36\times10^{-19} J}{2.5\times10^{-21} J/s}$$

$$= 134.4\ sec$$
$$= 0.04hr$$

* The greater the work function of a metal, the more energy is needed for an electron to leave its surface and the higher the critical frequency for photoelectric emission to occur.

Einstein's Quantum Theory of the Photoelectric effect.

Recall Planck's requirement that the energy content of the electromagnetic wave of frequency ν in a radian source can only be zero, $h\nu$, $nh\nu$

Einstein agreed that such a vol of energy is initially localised in a small vol of space and that it remain localised as it moved away from the source with velocity. That energy is given by, $E = h\nu$

He also assumed that in the photoelectric P.E process, one photon is absorbed by one electron in the photocathode. When electron Be emitted from the surface of the metal, it's K.E is given by

$$K.E_{max} = h\nu - \omega$$

$h\nu$ = energy of the absorbed incident photons

ω = work required to remove the electron from the metal

Since some electrons are bound more tightly than other, the photoelectron will emerge with a K.E given by

$$K.E_{max} = h\nu - \omega_0 \quad \text{(IV)}$$

ω_0 = work function and it is the minimum energy needed by an electron to pass through the metal surface and escape the attractive forces that normally bounds the electron to the metal.

Einstein's Photo hypothesis

Let einstein's photo hypothesis meet the three objections raised against the wave theory interpretation of the photoelectric effect, namely:

Objective 1 (the maximum K·E does not depend on intensity of illumination). There is agreement of the photon theory with experiment by doubling the light intensity merely also doubled the no of photon and thus double the photoelectric current. This does not change the energy $h\nu$ of the individual photons or the nature of the individual photoelectric ~~effect~~ process

Objective 2 (existence of cut off frequency). Is removed at once by eqn (IV) i.e if $K \cdot E_{max} = 0$

$$h\nu_0 = w_0 \quad \text{---} \quad (V)$$

which confirms that a photon of frequency ν_0 is just enough to eject the electron and known electrons to appear as K·E.

Objective 3 (absence of a time lag). Is eliminated in the photon theory cuz the required energy is supplied in ~~equa~~ concentrated bundle. It is not spread uniformly over a large area which is based on the assumption that the classical wave theory is true. If there is any illumination at all incident on the cathode then there will be atleast one photon that hits it. This photon will be immediately absorbed by some atoms leading to the immediate emmission of the photoelectrons. By subtracting eV_0 from the $K \cdot E_{max}$ in the Einstein's photoelectric eqn (iv). This will yield

$$V_0 = \frac{h\nu}{e} - \frac{w_0}{e}$$

Or

∴ einstein's theory predict the linear relationship b/w the stopping potential V_0 and the frequency ν

Application:

Find the work function of sodium is ν_0 is $4.39 \times 10^{14} s^{-1}$

$$h\nu_0 = w_0$$

$$6.63 \times 10^{-34} \times 4.39 \times 10^{14} = w_0$$

$$w_0 = 1.82 eV$$

2B) Compton effect

Corpuscular (Particle like ~~Light~~)

Nature of radiation acieved dramatic in 1923 when Compton experiment ~~Compton effect~~ will allow the beam of x-ray of wavelength λ to fall on the graphite target. The scattered x-ray have two wavelength; λ as the incident one; λ' being largest and exceed λ with and amount of $\Delta\lambda$

It is so called compton shift

$\boxed{\Delta\lambda = \lambda' - \lambda}$, it varies with a scattered angle. For x radiation of frequency ν, the energy of the incident photon

$$E = h\nu$$

Taking the idea of a photon as a localised bundle of energy, we consider it to be a particle of energy E and momentum, P. Using the relativistic energy

E_o(net), $E_o = m_oC^2$

Energy given us is at rest.

In motion, $m = \frac{m_o}{\sqrt{1-v^2/c^2}}$

So $K.E = mc^2 - m_oC^2$

$$= \frac{m_oC^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_oC^2 = m_oC^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$$

$$= \left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} - 1\right]m_oC^2$$

$E_{total} = mC^2$ Classically $\frac{v}{c} << 1$

Since $\frac{v}{c} << 1$

$$\left(1+\frac{1}{2}\frac{v^2}{c^2} - 1\right)m_oC^2, \quad KE = \frac{1}{2}\frac{V^2}{C^2}m_oC^2$$

$$K.E = \frac{1}{2}m_oV^2$$

For relativistic energy, $K.E = KE + m_oC^2$

It is convenient to have an expression for the total relativistic energy. E that involve the momentum P - Such can be obtained by evaluating the quantity

$$m^2c^4 - m_o^2C^4 = m_o^2C^4\left[\frac{1}{1-\frac{v^2}{c^2}} - 1\right]$$

$$= m_o^2C^4\left(\frac{v^2/c^2}{1-\frac{v^2}{c^2}}\right)$$

$$\frac{m_o^2C^2V^2}{1-v^2/c^2} = C^2m^2V^2 = c^2P^2 = E^2$$

Thus $m^2c^4 = c^2p^2 + m_0^2c^4$

Or $E^2 = p^2c^2 + m_0^2c^4$

For a photon, energy at rest = 0 (mass = 0)

$E = m_0^2c^4 = 0$

$p = \frac{E}{c} = \frac{h\nu}{c}$; $\lambda = \frac{c}{\nu}$, $\nu = \frac{c}{\lambda}$

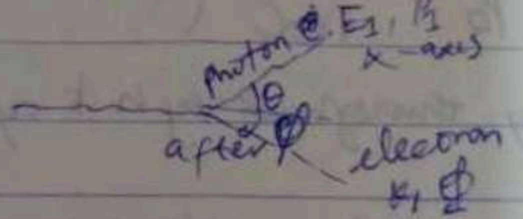
$\therefore \frac{h\frac{c}{\lambda}}{c}, p = \frac{h}{\lambda}$

Let's consider a collision btw a photon and free stationary electron ~~free stationary~~

Photon e^- y-axis

E_0, P_0 before

$P_{before} = P_{after}$



Conservation of moment :-

(x,y) $P_{before} = P_{after}$ (x,y)

x-axis: $P_0 = P_1 \cos\theta + \Phi \cos\phi$

y-axis $0 = P_1 \sin\theta - \Phi \sin\phi$

$P_1^2 \sin^2\theta = \Phi^2 \sin^2\phi$

Adding, ~~we~~ find ; $P_0^2 + P_1^2 - 2P_0P_1\cos\theta = \Phi^2$ — (9)

We can also use the conservation of total relativistic energy;

$E_0 + m_0c^2$ = Total energy — Before

$E_1 + KE + m_0c^2$ — After

$E_0 + m_0c^2 = E_1 + KE + m_0c^2$

$\Rightarrow KE = E_0 - E_1$ — (10)

Since $P = \frac{E}{c} = \frac{h\nu}{c}$

Eqn 10 $\Rightarrow$ $c(p_0 - p_1) = KE$

Eqn 6 $\Rightarrow$ $(K.E + m_0c^2)^2 = c^2p^2 + (m_0c^2)^2$

Simplify $KE^2 + 2KE \cdot m_0c^2 = c^2p^2$

or $\frac{KE^2}{c^2} + 2KE \cdot m_0 = p^2$

By evaluating p^2 from eqn (9) and K.E from eqn (10)

$(p_0 - p_1)^2 + 2m_0c(p_0 - p_1) = p_0^2 + p_1^2 - 2p_0p_1\cos\theta$

reduces to $m_0c(p_0 - p_1) = p_0p_1(1-\cos\theta)$

or $\frac{1}{p_1} - \frac{1}{p_0} = \frac{1}{m_0c}(1-\cos\theta)$

By multiplying throughout by h & applying eqn (8), we obtain compton eqn.

Compton eqn ; $\Delta\lambda = (\lambda_1' - \lambda_0) = \lambda_c(1-\cos\theta)$ (12)

where λ_c is called compton wavelength $= \frac{h}{m_0c} = 2.43\times10^{-12}$

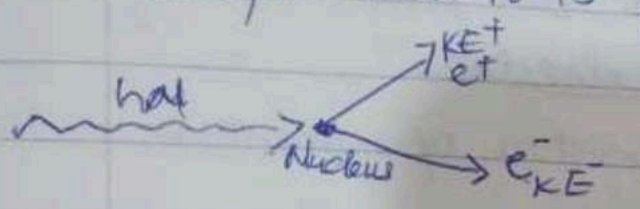
$= 0.0243 A^\circ$

Ass

Let's consider an x-ray beam with $\lambda = 1.00A^\circ$ and also a gamma-ray beam from Cs-137 with $\lambda = 1.88\times10^{-2}A^\circ$. If radiation scatter from free electron is view at 90° with the incident beam. (i) What is the compton wavelength shift ($\Delta\lambda$) in each case. (ii) What K.E is given to the recoiling electron in each case (iii) What percentage of incident energy is lost in the collision in each case

3. Pair Production & Pair Annihilation

In addition to photoelectric effect and compton effect, there is another process whereby the photon looses their energy in interaction with the matter, the process of pair production. Here, an high energy photon looses all it's energy $h\nu$ in an encounter with a nucleus creating an electron (e^-) and positron pair (e^+) and both acquires a K.E. The positron is identical to the electron except that it is positively charged



$$h\nu_{total} = E^+ + E^-$$

$$h\nu = m_0c^2 + KE^+ + m_0c^2 + KE^- \quad — (15)$$

$$= 2m_0c^2 + KE^+ + KE^-$$

From eqn (15), it is clear that the minimum or threshold energy needed by a photon to create a pair is $2m_0c^2$ which is like about 1.02MeV i.e $2m_0c^2 = 1.02MeV = 0.012Å$ (in wavelength)

Absorption of photon in the interaction with matter occur principally by the photoelectric process at a low energy and by the compton effect at the medium energy and by pair production at high energy. Electron–positron pair are produced in nature by cosmic ray photon and in the lab by bremsstrahlung photon from particle accelerator.

Pair Annihilation

Is the reverse process of pair production. Here, an electron and a positron initially ~~at rest~~ with one another and united and then are annihilated. The matter disappear and in it's place, we got radiant energy. Since initial momentum is zero and must be conserved, we cannot have only one photon cuz a single photon cannot have zero momentum. The most probable process is the creation of atleast two photon moving with equal momentum in the opposite direction.

$$\dot{e}^{+}\ \dot{e}^{-} \quad \xleftarrow[h\nu_1]{P_1} \quad \xrightarrow[h\nu_2]{P_2}$$ Conservation of momentum

$0 = P_1 + P_2$; $\Rightarrow P_1 = -P_2$, Hence $P_1 = P_2$

$$\frac{h\nu_1}{c} = \frac{h\nu_2}{c}$$

(oppositely directed but equal in magnitude

Hence $P_1 = P_2$ or $h\nu_1 = h\nu_2$

Using the conservation of total relativistic energy;

Energy of e^{+} + energy of e^{-} = energy after

$m_0c^2 + m_0c^2 = h\nu + h\nu$

$\Rightarrow h\nu = m_0c^2 = 0.511 MeV$

$\lambda = 0.024 \mathring{A}$

In passing through the matter the positron (e^{+}) loses energy in successive collision until it combines with electron to form a bounded system called Positronium. The Positronium atom is a short-lived, decaying into photon within about 10^{-9} seconds

Application: In a reference frame an electron-positron initially at rest unite to give 2 annihilation photons moving along x-axis in the opposite direction. Find the wavelength (λ) of these photons in terms of m_0 of the electrons or positrons

$$P = \frac{E}{C} = \frac{h\nu}{c} = \frac{m_0c^2}{c} = m_0C$$

$$\lambda = h/p = \frac{h}{m_0c}$$

Now, consider the same annihilation event to be observed in the frame x' moving relative to x with the same velocity (ν) to the left. What λ ~~of~~ this observer (moving) will record for the annihilation photons

Soln

Here, the pair has initial total relativistic energy $2mc^2$ rather than the rest mass energy $2m_0c^2$

$$2mc^2 = P_1'C + P_2'C$$

$$2mv = P_1' - P_2'$$

$$P_1' = m(c+v) = \frac{m_0(c+v)}{\sqrt{1-v^2/c^2}}$$

$$= m_0C\sqrt{\frac{c+v}{c-v}}$$

$$P_1' = h/\lambda_1'$$

$$\lambda_1' = \frac{h}{P_1'} = \frac{h}{m_0C}\sqrt{\frac{c-v}{c+v}} = \lambda\sqrt{\frac{c-v}{c+v}}$$

Similar manner, by subtracting the second eqn from the first

$$\lambda_2' = \frac{h}{P_2'} = \frac{h}{m_oC}\sqrt{\frac{C+V}{C-V}} = \lambda\sqrt{\frac{C+V}{C-V}}$$

THOMPSON'S MODEL OF ATOM

Here, the negatively charged e^- were located within a continuous distribution of +ve charge. The +ve charged distribution was suggested to be spherical in shape, with a radius of about 10^{-10}m. Owning to their mutual repulsion, the electron would be uniformly distributed throughout the sphere of +ve charge. In excited atom, the electrons will vibrate about their equilibrium position since a classical EM theory predicts that an accelerated charged body such as a vibrating electron emits E.M radiation, it was possible to understand qualitatively the emission of such radiation by excited atoms on the basis of Thompson's model.

Application: Assume that there is one electron of charge e^- inside a spherical region of uniform positive charge of density, ρ, show that its motion, if it has the K.E, can be simple harmonic oscillation about the centre of the sphere.

Solution

Using Coulomb's law:

$$F = \frac{-1}{4\pi\varepsilon_0}\left(\frac{4}{3}\pi a^3\rho\right)\frac{e}{a}$$

$$F = -\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{a^2}$$

We assume the charge density; $q_1 = \frac{4}{3}\pi a^3\rho$

$r = +1$

$\lambda = F$

Then, $F = -\frac{1}{4\pi\varepsilon_0}\left(\frac{4}{3}\pi a^3\rho\right)\frac{e}{a^2}$

$$= -\frac{\rho e a}{3\varepsilon_0}$$

where $k = \frac{\rho e}{3\varepsilon_0}$

$\therefore F = -ka$ (It is a simple harmonic motion).

Questions!

If the electron at A is free with no initial velocity, this force will produce a simple harmonic motion along a diameter of a sphere, since it is always directed towards the center and has a strength which is proportional to the displacement of the center.

Let the total positive charge have a magnitude of ~~one~~ $10e^-$ electron charge and let it be distributed over a sphere of radius, 10^{-10}m. Then find the force constant, k and frequency of motion of e^-

Soln

$$\rho = \frac{e}{\frac{4}{3}\pi r^3}, \quad k = \frac{\rho e}{3\varepsilon_0}; \quad k = \frac{e^2}{4\pi\varepsilon_0 r^3} = \left(\frac{1}{4\pi\varepsilon_0} = 9\times10^{9}\right) \Rightarrow \frac{9\times10^{9} e^2}{r^3}$$

$$e = (10\times1.6\times10^{-19})$$

$$= 2.3\times10^{2}\ \mathrm{N/m}$$

Frequency of S.H.M

$$v = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2.3\times10^{2}}{9.11\times10^{-31}}}$$

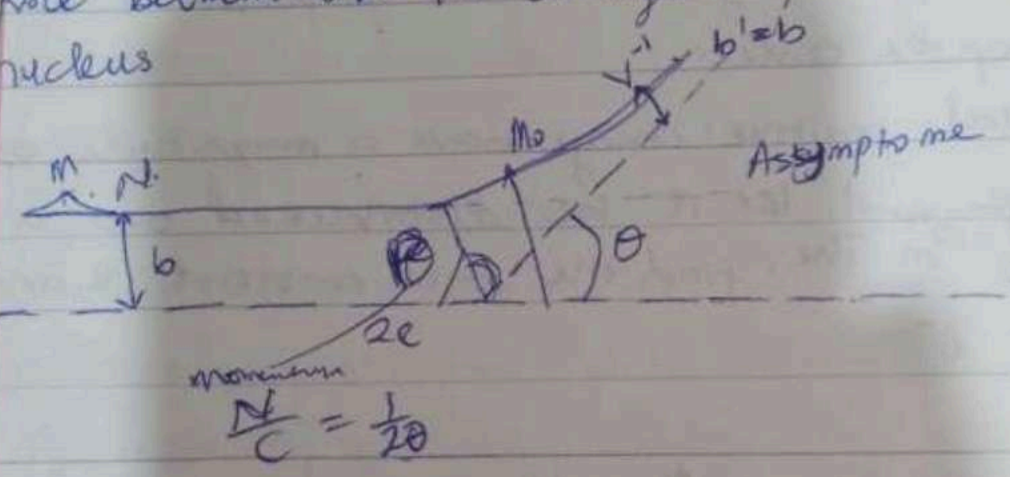
$$= 2.5\times10^{15}\ s^{-1}$$

$$\lambda = \frac{c}{v} = \frac{3\times10^{8}}{2.5\times10^{15}}$$

$$= 1.2\times10^{-7}\ m$$

RUTHERFORD'S MODEL

The Rutherford's model of the structure of an atom, explains that all the positively charge are concentrated in the nucleus. If the dimension of the nucleus is small enough, then an alpha particle passing very near can be scattered by a strong or coulomb repulsive or through a large angle in the transversal of a single atom. $\theta \simeq 1rad$, $r = 10^{-14}m$, instead of $r = 10^{-10}m$ for $\theta = 10^{-4}rad$ as observed in the atom. The scattering here is due to the repulsive coulomb force between the +ve charge α-particles and the +ve charge nucleus



The scattering of the an α-particle of total charge of +2e, M is passing near the region of charge +2e. The position of the particle relative to the medium is classified as radial "r" on the polar angle. The perpendicular distance from that axis to the line of the initial motion is called the impact diameter "d"

Application! The force acting on the particle and being the coulomb force is always in a radial direction. Hence the angular momentum of the particle or the origin has a constant value "Z". It is observed that $mvb = mv'b' = C$. Also, the K.E is observed i.e $\frac{1}{2}mv^2 = \frac{1}{2}mv'^2$

$v = v'$; $b = b'$

By using practical mechanics, the repulsive coulomb force $\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{2Ze^2}{r^2}\right)$

We can obtain the following eqn for the trajectory of clarity of an α-particle

$$\frac{1}{r} = \frac{1}{b}\sin\psi + \frac{\Delta}{2b^2}(\cos\psi - 1) \rightarrow \text{hyperbola polar coordinate}$$

Here, $\Delta = \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{\frac{mv^2}{2}}$

It is the convenient parameter equal to the distance of closest approach ~~with~~ in the nucleus. In an head-on collision of b, since Δ is a distance at which a potential energy is equal to K.E

$$PE = \frac{1}{4\pi\varepsilon_0} \times \frac{2Ze^2}{\Delta = R} = K.E \ \frac{1}{2}mv^2$$

The radial coordinate will be equal to R when

$$\phi = \frac{(\pi - \theta)}{2}$$

Evaluate R which is the distance of closest approach of the particle to the centre of the nucleus radial "r" coordinate

Soln

$r = R$

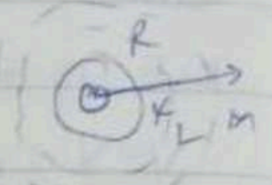
$$a = \frac{(\pi - \theta)}{2}$$

$$\frac{1}{R} = \frac{1}{b}\sin\frac{(\pi - \theta)}{2} + \frac{\Delta}{2b^2}\left[\frac{\cos(\pi - \theta)}{2} - 1\right]$$

$$b = \frac{\Delta}{2}\cot\frac{\theta}{2} = \frac{\Delta}{2}\tan\frac{(\pi - \theta)}{2}$$

$$\Rightarrow R = \frac{\Delta}{2}\left[1 + \frac{1}{\cos\frac{(\pi-\theta)}{2}}\right]$$

$$= \frac{\Delta}{2}\left[1 + \frac{1}{\sin\theta/2}\right]$$



SIZE OF THE NUCLEUS

The size of an atom is somewhat difficult to define owing to the fact that the atomic electron cloud does not have a well defined outer edge. The electron from time to time may move closer or far from the nucleus. The atomic size is the average distance from the nucleus that the outermost electron is to be found, except for a few of the lightest atom. This average radius is almost the same for all atoms i.e 2×10^{-8} cm. The nucleus like the atom also does not have a sharp outer boundary. It's surface is diffused with approximation, the nucleus is assumed to be a sphere with a radius given $R = 1.25\times10^{-23}A^{1/3}$ (cm) $[A = \text{atomic mass}]^{1/3}$. Since, the volume of the sphere is proportional to the radius and the radius is proportional to "A" i.e $V \propto R \propto A \Rightarrow V \propto A$

$\frac{A}{V}$ (Uniform density: no of nucleon per unit volume) = Constant

This uniform density of nuclear matter suggests that the nuclides are similar to liquid drop which also have the same density whether they are large or small. This liquid-drop model of the nucleus are accounted from many of the physical pptties of the nucleus as nuclear forces do not act on an electron. It's scattering is due to the Coulomb interaction with the nucleus charge distribution.

An electron scatter through appreciable angle as at the single close encounter with the nucleus. ∴ measurement of electron scattering should be able to provide information about the nuclear charge distribution such as the size. This charge distribution is that of a proton in the nucleus but there is also an indication that the neutron atom has approximately the same distribution as the proton.

Qualitatively, we know that the separation angle btw adjacent minima of the diffraction pattern θ is given by $\theta = \frac{\lambda}{r}$

λ = electron de Broglie wavelength; r = radius of the charged distribution

Application: A $500 K_{eV}$ K.E electron has scattered from a target of nuclei of charge distribution r into a diffraction pattern that has minimal with an average separation of $\theta = 30°$. Evaluate r.

Soln

In the extreme relativistic limit, the electron momentum is given by

$$P = E/C = KE/C = \frac{500 keV}{3 \times 10^8 ms^{-1}} \times \frac{1 J}{6.2 \times 10^{12} meV} = 2.7 \times 10^{-19} kgm/s$$

$$\lambda = \frac{h}{P}; \ \lambda = \frac{6.6 \times 10^{-34} Js}{2.7 \times 10^{-19} kgm/s} = 2.4 \times 10^{-15} m$$

when $\theta = 30° = 0.53 rad$

$$r = \frac{\lambda}{\theta} = \frac{2.4 \times 10^{-15} m}{0.53} = 4.5 \times 10^{-15} m = 4.5 fm \ (1 f (Fermi) = 10^{-15} m)$$

ISOTOPES, ISOTONES & ISOBARS

Identify a nucleus by Z or A, then the term nuclide and nuclei having the term Z and N and hence the same A, it is symbolized by $^A_Z X$. Among special nucleids found in the nucleus either naturally or artificially, some are stable whereas others are unstable or radioactive. In the light nuclei; neutron and proton are equal ($Z = N$), however, in the heavier nuclei, ($N > Z$) there must be an excess of neutron to produce a stabilizing effect or through nuclear interaction. This extra electron balance the ~~destructive~~ repulsive effect of the coulomb repulsion btw proton cuz of the great variety of nuclide, we will classify them into three basic categories.

1) Isotopes: (~~different~~ same atomic no ~~same~~ different mass no)

They have same protons, Z, but different N and [$A = Z + N$] different A. They all belong to the same element. e.g $^{35}_{17}Cl$, $^{36}_{17}Cl$, $^{37}_{17}Cl$; $^{1}_{1}H$, $^{2}_{1}H$, $^{3}_{1}H$; $^{16}_{8}O$; $^{17}_{8}O$, $^{18}_{8}O$

2) Isotones: (same neutron no, different mass no & atomic no)

They have same N but different Z, hence different A. They obviously do not belong to the same chemical element e.g $^{13}_{6}C$ and $^{14}_{7}N$, neutron no = 7 in both elements.

In nature, some chemical elements are found only with one variable of nucleus or isotope such as fluorine $^{19}_{9}F$ others have several natural isotope such as carbon ($^{12}_{6}C$, $^{13}_{6}C$, $^{14}_{6}C$), hydrogen ($^{1}_{1}H$, $^{2}_{1}H$, $^{3}_{1}H$) including those produced artificially. For example $^{12}_{6}C$ represent about 98.89% of all carbons.

The analysis of the ppties of isotope and isobar is important in that it discloses a several characteristics of nuclear structure this enable us to predict the stability of a nucleus. The significance will be important in radioactivity.

3) Isobar:
They have thesame A and different Z and N e.g $^{14}_{6}C$ and $^{14}_{7}N$

$^{14}_{6}C$	$^{14}_{7}N$
A: 14	14
Z: 6	7
N: 8	7

OLUBOTUN STEPHEN FELICITY
Yellow house Room 32

4/5/2022 WAVE BEHAVIOUR OF MATTER
Matter can behave as a particle or wave
Parameters in describing wave: *frequency, wavelength, *intensity *phase velocity.
Wave spreads out and occupies relatively large region of space

Matter as a particle: * mass, * Velocity * momentum/energy.
Particle occupies a definite position in space
Phenomenon that light exhibits: *Black-body radiation, * Photoelectric effect * Interference * Diffraction

BB and Photoelectric effect can be explained considering light as a stream of particles while Interference and diffraction can be explained by considering the wave nature of light.

De Broglie's Hypothesis

If radiation which is basically a wave can exhibit particle nature under certain circumstances, and since nature loves symmetry, then entities which exhibit particle nature ordinarily, should also exhibit wave nature under suitable circumstances. This hypothesis is crucial.

If light can act like a wave sometimes and like a particle other times, then all matter usually thought off as particles should exhibit wave-like behaviours

De Broglie's wavelength; $\lambda = \frac{h}{P}$ → wave, P → particle

$P = mv$

$$K \cdot E = \frac{1}{2}mv^2 = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{P^2}{2m}$$

$$P = \gamma mv \; ; \; \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\lambda = \frac{h}{\gamma mv} \rightarrow \text{De Broglie's Eqn}$$

Question:

Find the K·E of a proton whose De Broglie's wavelength is 10^{-17} m which is roughly the proton diameter

Soln

$$\lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{10^{-17}} = 6.63\times10^{-17}$$

$$K.E = \frac{p^2}{2m} = \frac{(6.63\times10^{-17})^2}{2\times\cancel{9.11\times10^{-31}}\ 1.67\times10^{-27}}$$

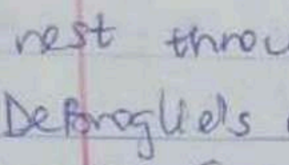
$$= 1.31\times10^{-6}J$$

2) Find the De Broglie's wavelength of a 46g ball with a velocity of 30m/s. $\left(\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}}{46\times30} = \frac{6.63\times10^{-34}\,J\text{-}s}{0.046\times30}\right.$ $= 4.8\times10^{-34}m$

3) A particle of charge q and mass, m is accelerated from rest through a small potential difference, V, (a) Find its De Broglie's wavelength assuming the particle is non-relativistic (i.e is not moving towards the speed of light, $\gamma = 1$) (b) Cal. λ if the particle is an electron and P.d is 30volts ~~Soln~~

Soln

3)



$$\lambda = h/p$$

$$K.E = \tfrac{1}{2}mv^2 = qV = p^2/2m$$

$$\cancel{p = mv};\ \cancel{\tfrac{1}{2}pv = qV,\ pv = 2qV}$$

$$p = \sqrt{2mqV}$$

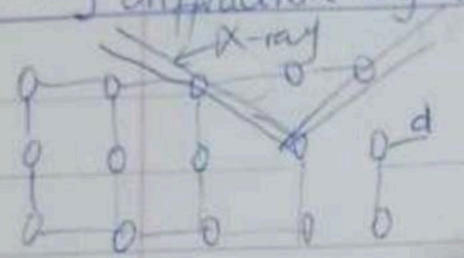
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times30\times1.602\times10^{-19}}}$$

$$= 2.24\times10^{-10}m = 2.24\text{Å}$$

ELECTRON DIFFRACTION

In 1927, it was established that e^- undergo diffraction at x-ray. X-ray is a wave so it's normal for it to exhibit diffraction.

X-ray diffraction by Crystal



Diffraction occurs as a result of elastic scattering btw x-ray and crystal.

Bragg's law: $n\lambda = 2d\sin\theta$ (for a constructive interference)

d = spacing btw the diffracting plates

θ = incident angle, n = integer, λ = wavelength

X-ray diffraction:

Crystals are regular arrays of atoms, while x-ray is electromagnetic wave. Crystal atoms scatter incident x-ray primarily through interaction with the atoms electrons. Such interaction is elastic scattering. The regular array of atoms produce array of spherical wave. Some cancel out through destructive interference while some would add up constructively in a specific direction as determined by Bragg's law.

Electron diffraction:

- Electron have mass and do not travel in the speed of light i.e $\gamma = 1$ [although it can be made to do so by applying electric field]
- K.E for non-relativistic particle (not moving with speed of light)

$$K.E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

- If an electron is accelerated through a potential difference "V", it gains a K.E eV, so we can write: $K.E = eV = \frac{p^2}{2m}$

$$\therefore P = \sqrt{2meV}$$

$$\therefore \lambda = \frac{h}{p} = h/\sqrt{2meV} = \frac{h}{\sqrt{2mqV}}$$

- Hence the λ of an electron accelerated through a p.d (V) is given above.
- Cuz electrons behave as a wave due to diffraction pattern that emerges, we can make an analogy with the diffraction of x-ray by a crystal

$$n\lambda = 2d\sin\theta$$

Q. What K.E in eV should neutrons have if they are to be ~~deported~~ diffracted from crystals ; ~~Frequency~~ $\lambda = 1 \times 10^{-10} m$

$\lambda = \frac{h}{p}$; $p = \frac{h}{\lambda}$

$p = \frac{6.63 \times 10^{-34}}{1 \times 10^{-10}}$

$= 6.63 \times 10^{-24}$ kgm/s

$K.E = \frac{p^2}{2m_{of\ neutron}} = \frac{(6.63 \times 10^{-24})^2}{2 \times 1.66 \times 10^{-27}}$

$= 1.32 \times 10^{-20} J$

$=$ ~~0.060825eV~~ $0.0825eV$

2Q Cal. the deBroglie wavelength of an electron accelerated through 50V

$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 50 \times 1.6 \times 10^{-19}}}$

$= 174 \times 10^{-12} m$

$= 1.74 \times 10^{-10} m$

Wave Particle Duality

Light exhibits wave particle duality cuz it exhibits both waves and particle properties. Wave particle duality is not confined only to light. Every matter believe to exhibit wave particle duality. However, behaviour of relatively large objects is dominated by their particle nature. In any case it is impossible to measure both the wave and particle properties simultaneously.

The Heisenbergs Uncertainty Principle

It is impossible to know simultaneously and with exactness both the position and the momentum of the fundamental particles.

If a measurement of a position is made with

Precision (Δx) and simultaneous measurement of momentum in the direction of x is made with precision ΔP, then the product of the two uncertainties can never be smaller than $\hbar/2$.

$$\Delta P \cdot \Delta x \geq \frac{\hbar}{2}$$

where $\hbar = h/2\pi$

$$\Delta E \cdot \Delta t = \frac{\hbar}{2} = \frac{h}{4\pi}$$

To express this in terms of de Broglie wavelength

$\lambda = \frac{h}{P}$ (corresponding wave number K)

$$K = \frac{2\pi}{\lambda} = \frac{2\pi P}{h}$$

Hence ~~the uncertainty~~ ΔK in the wave number of the de Broglie ~~wavelength~~ associated with the particle results in an uncertainty ΔP i.e in the particle's momentum according to the formula below

$$\Delta P = \frac{h\Delta K}{2\pi} = \Delta K \hbar$$

Examples! A typical atomic nucleus is about 5.0×10^{-15} m in the radius. Use the uncertainty principle to place a lower limit on the energy an electron must have if it is to be part of the nucleus

Soln

$$\Delta x = 5.0 \times 10^{-15} m$$

$$\Delta P \geq \frac{\hbar}{2\Delta x} \geq \frac{(h/2\pi)}{2\Delta x}$$

$$\Delta P \geq \frac{1.054 \times 10^{-34} J\cdot s}{2 \times 5.0 \times 10^{-15} m}$$

$$\Delta P \geq 1.1 \times 10^{-20} kgm/s$$

$$K.E = PC \geq (1.1 \times 10^{-20}) \times (3.0 \times 10^{8})$$

$$\geq 3.3 \times 10^{-12} J$$

$$\geq 20625000 eV$$

2) A hydrogen atom is ~~5.3×10~~ 5.3×10^{-11} m in radius, use the uncertainty principle to estimate the minimum energy an electron can have on the atom

Soln

$\Delta x = 5.3 \times 10^{-11} m$

$\Delta p \geq \frac{\hbar}{2\Delta x} \geq \frac{(h/2\pi)}{2\Delta x}$

$\Delta p \geq \frac{(6.63 \times 10^{-34}/2\pi)}{2 \times 5.3 \times 10^{-11}}$

$\Delta p \geq 9.95 \times 10^{-25} kgm/s$

$KE = \frac{p^2}{2m} \geq \frac{(9.95 \times 10^{-25})^2}{2 \times 9.11 \times 10^{-31}}$

$\geq 5.4 \times 10^{-19} J$

$\geq 3.375 eV$

ATOMIC SPECTRA

In an atom, electrons have discrete and specific energies and when an electron transit from energy level to another, it emits light with specific wavelength. The emitted light can be observed as a series of coloured lines with dark spaces in between. The series of coloured lines is called ATOMIC SPECTRA.

NOTE:

No two elements emit the same spectra, each element produce a unique set of atomic spectra.

∴ elements can be identified by their atomic spectrum.

The wavelength in the spectrum of an element were found to fall into sets called SPECTRA SERIES

In 1885, J.J Balmer discovered the first set of such series of the hydrogen spectrum.

He discovered that the line

with the longest wavelength is 656.3nm

* And the next to the longest wavelength is 486.3nm and so on.

* As the wavelength decreases, the lines are found closer together and weaker in intensity until the series limit at 364.6nm is reached.

Balmer's formular for the λ of the series = $$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where $n = 3, 4, 5, 6 \ldots$

$R = 1.097 \times 10^{-7} m^{-1}$ (Rydberg Constant)

16/3/2022

* The value of wavelength at infinity is the shortest.

656.3nm corresponds to $n = 3$ and 364nm corresponds to $n = \infty$ which is the series limit.

Balmer's series contains λ in the ~~visible region~~ portion of the hydrogen spectrum. The spectral lines of hydrogen in the ultraviolet and infra-red regions fall into several series.

In the ultraviolet region, we have Lyman's series.

- Lyman's formular

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right); \; n = 2, 3 \ldots$$

while in the infrared region, we have three spectral series:

i) Paschen's series

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right); \; n = 4, 5, 6 \ldots$$

- Brackett's series:

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right); \; n = 5, 6, 7 \ldots$$

- P-fund series:

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right); \; n = 6, 7, 8 \ldots$$

Examples:

1) Calculate the longest and shortest wavelengths in the Lyman's series of hydrogen atom spectrum.

2) Find the longest wavelength in the P-fund series

Soln

1) The longest wavelength in the Lyman's series is

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^{-7}\left(\frac{1}{1} - \frac{1}{2^2}\right)$$

$\lambda = 1.22 \times 10^{-7} m$

Shortest: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$

$\frac{1}{\lambda} = 1.097 \times 10^{-7} \times 1$

$\lambda = 9.12 \times 10^{8} m = 91.2 nm$

2) Longest in P-fund: 6

$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{6^2}\right)$

$\frac{1}{\lambda} = 1.097 \times 10^{-7}\left(\frac{1}{25} - \frac{1}{36}\right)$

$\lambda = 8.28 \times 10^{7} m$

$= 828 nm$

Rydberg formular for hydrogen

$\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$n_2 > n_1$

- Lyman's series: $n_1 = 1$, $n_2 = 2, 3, 4 \ldots$ the series limit is 91.13nm (shortest)
- Balmer's series: $n_1 = 2$; $n_2 = 3, 4, 5 \ldots$ it's limit is **364.5nm** (shortest wavelength)
- Paschen's series: $n_1 = 3$, $n_2 = 4, 5, 6 \ldots$ limit = 820.14nm
- Brackett's series: $n_1 = 4$; $n_2 = 5, 6, 7 \ldots$ limit = 1458.03nm
- P-fund series: $n_1 = 5$; $n_2 = 6, 7, 8 \ldots$ Limit = 2278.17nm

THE BOHR'S MODEL

In 1915, Bohr came up with his postulate which was based partially on classical mechanism & partially on quantum mechanics. The Bohr's postulates are:

- Electrons move about the nucleus in circular orbit determined by Coloumbs and Newton's law.
- Only certain orbit are stable. The electron does not radiate electromagnetic energy in this orbit because the energy is constant in time, these are called STATIONARY STATE
- A sepctra line of frequency 'f' is emitted when an electron jumps from an initial energy E_i to a final orbit of energy E_f

$hf = E_i - E_f$

- The sizes of the stable electron orbits are determined by requiring the electron angular momentum to be an integral multiple of $\hbar$

$m_e v_r = n\hbar$; $n = 1, 2, 3, 4 \ldots$

These postulates leads to quantized orbit and quantized energies for a simple electron orbiting a nucleus with charge $+Ze$

$$r_n = \frac{n^2\hbar^2}{m_e K e^2} \longrightarrow \text{for atomic number}$$

m_e = mass of electron

r = radius of an atom

K = Coloumb's constant $= \frac{1}{4\pi\varepsilon_0}$

when $n=1$; the radius is called Bohr's radius and its been denoted as a_0 instead of r_1

$$a_0 = \frac{\hbar^2}{m_e k e^2} \text{ all are constants}$$

$a_0 = 0.529 A^\circ = 0.0529 nm$

generally, $r_1 = \frac{n^2 a_0}{Z}$

Z is the atomic no of the proton.

If $\underset{\text{energy}}{E_n} = -\frac{Ke^2}{2a_0}\frac{Z^2}{n^2}$

$$E_n = -\frac{13.6 Z^2}{n^2} eV$$

Exercise! Cal. the energy of oxygen when $n = 1$?

$$E_n = \frac{-13.6 \times 8^2}{1^2}$$

$= -870.4 eV$

Ass

Cal. the frequency

2) Cal. the radius and energy level of the electron in the lowest energy state of hydrogen (i.e when $n = 1$)

3) Cal. the wavelength of the spectrum of hydrogen atom when an electron falling from the 4th level $e = 0.55 eV$ to the first level where $e = -13.6 eV$

4) Find the Bohr's atomic model limitation (Read)

* Read Chapter 4 of concept of modern physics

$$r = \frac{(\hbar)^2 n^2}{m_e k e^2} = \frac{(1.054 \times 10^{-19})^2 \times 1^2}{9.11 \times 10^{-31} \times 9 \times 10^{9} \times (1.6 \times 10^{-19})^2}$$

$$= 5.29 \times 10^{29}$$

$$\boxed{r = \frac{2Ze^2}{4\pi\varepsilon_0 K.E} = \frac{2 \times 1 \times (1.6 \times 10^{-19})^2 \times (9 \times 10^{9})}{13.6 \times 1.6 \times 10^{-19}}}$$

SCHRODINGER WAVE EQUATION AND SIMPLE APPLICATION

Wave function is a variable function that mathematically describe the wave characteristics of a particle.

The value of the wave function of a particle at a given point of space and time is related to the likelihood of the particle being there at that time.

The probability that a particle is somewhere in the volume element, dv is unity

$$\int \psi^2 d\psi = 1$$

Characteristics of Wave Function

1) It must be a complex function.

e.g $\psi = a + ib$

2) The wave function and its derivative must be single value and must be continuous $\psi, d\psi, d^2\psi$

For this purpose, a wave function was introduced which at every point which will contain wave and time information of the particle

$\psi(x, y, z, t)$

Schrodinger theory is an extension of de Broglie postulate. He could consider a particle of momentum P and energy E.

$$\lambda = \frac{h}{P} \quad \text{and } \nu = \frac{E}{P} \text{ or } f = \frac{E}{P}$$

wave number, $K = \frac{2\pi P}{h} = \frac{P}{\hbar}$

$= \frac{2\pi}{\lambda}$

Angular freq., $\omega = 2\pi f$

$\therefore \omega = E/\hbar$

For a free particle, energy of that particle can be expressed as $E = \frac{P^2}{2m}$

Thus K can be expressed as

$$K = \sqrt{\frac{2mE}{\hbar}} = \frac{\sqrt{2mE}}{\hbar}$$

If the particle travels in the x direction, the wave will also travel in the same direction. The wave function that will meet this condition

$$\psi(x,t) = e^{-i(\omega t - kx)} \quad (1)$$

$$\psi(x,t) = e^{-i\left[\frac{E}{\hbar}t - x\frac{\sqrt{2ME}}{\hbar}\right]} \quad (2)$$

differentiating with respect to t

$$\frac{\partial \psi(x,t)}{\partial t} = -i\frac{E}{\hbar}\psi \quad (3)$$

twice differentiating with respect to x

$$\frac{\partial^2\psi}{\partial x^2} = \frac{2ME}{\hbar^2}\psi \quad (4)$$

Comparing eqn (3) and (4) we arrive at this;

$$\boxed{\frac{-\hbar^2}{2M}\frac{\partial^2\psi(x,t)}{\partial x^2} = i\hbar\frac{\partial}{\partial t}\psi(x,t)} \quad (5)$$

time dependence schrodinger eqn in one direction for a free particle

~~The~~ a particle is restricted by a given potential energy $\psi(x,t)$

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2M}\frac{\partial^2\psi}{\partial x^2} + V\psi \quad (6)$$

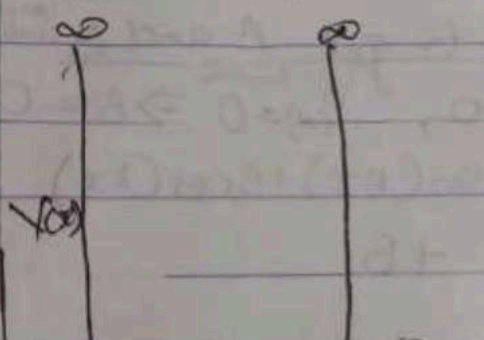
~~(for two dimension)~~

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\left[\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right] + V\psi \quad (7)$$ (three dimension)

Example!

A free electron trapped in a metal or a charge carrier trapped by the potential barrier can be approximately described or represented ~~experimentalized~~ by an infinite ~~1D~~ D-1 dimensional potential well

time independent 1D (Schrodinger eqn particle)



Conditions

1) $V = 0$ (P.E = 0), $0 < x < L$

2) P.E, $V = \infty$ from $L \le x \le 0$

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m[E-V]}{\hbar^2}\psi(x) = 0 \quad (8)$$

Normalization condition means finding the total integration of the volume where the particle is. If the particle is present space, particle absent

$$\frac{d^2\psi(x)}{dx^2} + \frac{2ME}{\hbar}\psi(x) = 0 \quad \text{--- (9)}$$

Recall that $K = \sqrt{\frac{2ME}{\hbar}}$, $K^2 = \frac{2ME}{\hbar^2}$

$$\therefore \frac{d^2\psi(x)}{dx^2} + K^2\psi(x) = 0 \quad \text{--- (10)}$$

The eqn (10) is a wave function for a free particle inside a well - The possible soln to this eqn is

$$\psi(x) = A\sin(Kx) + B\cos(Kx) \quad \text{--- (11)}$$

A and B are constants

<u>Condition to get A and B</u> by the boundary condition

1) At $x = 0$, $\psi(x) = 0$ (wave function) $\Rightarrow B = 0$

$$\psi(x) = A\sin(Kx) + B\cos(Kx)$$
$$0 = 0 + B$$
$$B = 0$$

2) At $x = L$, $\psi(x) = 0$

$$\psi(x) = A\sin(KL) \cancel{+ B\cos(KL)} = 0$$
$$A\sin(n\pi) = 0 \cancel{+ B\cos(n\pi)}$$

where $n = 1, 2, 3 \ldots$

$$KL = n\pi$$
$$K = \frac{n\pi}{L} \quad \text{--- (12)}$$

$$\psi(x) = A\sin\left(\frac{n\pi}{L}x\right) \quad \text{--- (13)}$$

$$K^2 = \frac{2ME}{\hbar^2}$$

$$K = \frac{n\pi}{L}$$

$$K^2 = \frac{n^2\pi^2}{L^2}$$

$$E_n = \frac{n^2\hbar^2\pi^2}{2ML^2} \quad \text{(energy of particle inside well)}$$

$$E_n = \frac{n^2h^2}{8ML^2} \quad \text{--- (14)}$$

To solve for A by applying normalization condition i.e integral from $0 \to L$

$$\int_0^L |\psi|^2 dx = 1$$

$$\int_0^L A^2\sin^2\left(\frac{n\pi x}{L}\right)dx = 1$$

$$A = \sqrt{2/L} \quad \text{--- (15)}$$

$$\psi_n(x) = \sqrt{2/L}\,\sin\left(\frac{n\pi}{L}x\right) \quad \text{--- 16}$$

The wave function and the corresponding energy which are often called ~~ending~~ Eigen function and Eigen value respectively describes the quantum state of the particle.