DEPARTMENT OF MATHEMATICS
B.Sc. (Mathematics) Degree Examination
Harmattan Semester, 2021/2022 Session
MTH 201- Mathematical Methods I

Time Allowed - 3hrs

Type 4

Instructions: Use HB pencils ONLY. Write and Shade your Names and Registration Number in the spaces provided on the OMR sheet. SHADE THE QUESTION TYPE. Attempt all questions. Shade the option E if none of the options A-D is correct.
All notations have their usual meanings as contained in the materials given.

1. Expand $xy+1$ in powers of $x+1$ and $y+1$.
   (A) $2-(x+1)-(y+1)+(x+1)(y+1)$
   (B) $2-(x+1)-(y+1)+2(x+1)(y+1)$
   (C) $2+(x+1)+(y+1)+(x+1)(y+1)$
   (D) $2+(x-1)+(y-1)-2(x+1)(y+1)$.

2. Which of the following equations is /are not exact?
   (I) $ydx-xdy=0$
   (II) $2xydx+(1+x^2)dy=0$
   (III) $(x+\sin y)dx+(x\cos y-2y)dy=0$
   (IV) $y^2dt+(2yt+1)dy=0$.
   (A) I only (B) I, III and IV only (C) II and III only (D) II, III and IV only.

3. If $u^2-v=x+y$ and $v^2-u=x-y$, find $\frac{\partial u}{\partial x}$.
   (A) $\frac{2v-1}{4uv-1}$ (B) $\frac{2v+1}{4uv-1}$ (C) $\frac{-2v-1}{4uv-1}$ (D) $\frac{-2v+1}{4uv-1}$, where $4uv-1\neq 0$.

4. The order and degree of the differential equation
$$\left(\frac{d^2y}{dt^2}\right)^2+\left(\frac{d^3y}{dt^3}\right)^2+y=0$$
are, respectively,
   (A) 3, 2 (B) 3, 3 (C) 2, 3 (D) 3, not defined.

5. $H=\tan^{-1}\frac{x}{y}, x=u+v, y=u-v$, find $\frac{\partial H}{\partial v}$.
   (A) $\frac{u}{u^2+v^2}$
   (B) $\frac{v}{u^2+v^2}$
   (C) $\frac{-u}{x^2+y^2}$
   (D) $\frac{-2v}{x^2+v^2}$.

6. For what values of $x\in\mathbb{R}$ does the series $\sum_{n=1}^{\infty}\frac{x^n}{n}$ converge?
   (A) $(-\infty,\infty)$ (B) $[-1,1)$ (C) $\emptyset$ (D) $(-\infty,1)$.

7. Given $U=(x^2+y^2+z^2)^{-1/2}$. Find $\frac{\partial^2U}{\partial x^2}+\frac{\partial^2U}{\partial y^2}+\frac{\partial^2U}{\partial z^2}$.
   (A) 1.5
   (B) -1
   (C) 0
   (D) 1.

8. Suppose the iterative sequence
$$x_{n+1}=\frac{1}{2}\left(x_n+\frac{4}{x_n}\right),\quad x_n>0\ \forall n\in\mathbb{N},$$
converges. Find its limit.
   (A) 4 (B) 2 (C) -4 (D) -2.

9. Under the trapezoidal rule with 3 subintervals, which expression gives the approximate area under the curve $y=e^x$ between $x=1$ and $x=4$?
   (A) $\frac{1}{4}(e^1+2e^2+2e^3+e^4)$
   (B) $\frac{1}{4}(e^1+e^2+e^3+e^4)$
   (C) $\frac{1}{2}(e^1+2e^2+2e^3+e^4)$

(D) $\frac{1}{2}(e^1 + e^2 + e^3 + e^4)$.

10. Which of the following statements is false regarding point $(0,0)$ and surface $z = x^3 + xy + y^2$?
(A) $(0,0)$ is a maximum point of z
(B) $(0,0)$ is a saddle point of z
(C) $(0,0)$ is not a minimum point of z
(D) $(0,0)$ is a stationary point of z.

11. Suppose the function $f : (-\infty, \infty] \longrightarrow \mathbb{R}$ is defined as follows:

$$f(x) = \begin{cases} -5x + 7, & x < 3 \\ x^2 - 16, & x \geq 3. \end{cases}$$

Find the $\lim_{x \longrightarrow 3^+} f(x)$.
(A) Doesn't exist
(B) -8
(C) -7
(D) 6.

12. Suppose

$$f(x) = \begin{cases} -x, & x < 1 \\ x^2, & x \geq 1. \end{cases}$$

Find the $\lim_{x \longrightarrow 1} f(x)$.
(A) Doesn't exist
(B) -1
(C) 0
(D) 1.

13. Suppose a continuous function $f(x)$ does not have a root in the interval $[a, b]$. Which of the following statements is true?
(A) $f(a) \cdot f(b) < 0$
(B) $f(a) \cdot f(b) > 0$
(C) $f(a) \cdot f(b) = 0$
(D) $f(a) > f(b)$.

14. If $U = e^{xyz}$, find $\frac{\partial^3 U}{\partial x \partial y \partial z}$ at the point $(1, 1, 1)$.
(A) $3e$
(B) $5e$
(C) $4e$
(D) $2e$.

$\frac{dy}{dx} = yze^{xyz}$

15. Which of the following equations is satisfied by $z = xy \ln xy$?
(A) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$
(B) $y\frac{\partial z}{\partial x} = x\frac{\partial z}{\partial y}$
(C) $x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$
(D) $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$.

$z = xy \ln xy$

16. Compute $\lim_{x \longrightarrow 3^-} \frac{|4x - 12|}{x - 3}$.
(A) Doesn't exist
(B) 4
(C) -4
(D) 0.

$\frac{4|x-3|}{x-3}$ -4

17. The necessary and sufficient condition for $M(x,y)dx + N(x,y)dy = 0$ to be exact is
(A) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
(B) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$
(C) $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$
(D) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$.

18. The differential equation $e^x\frac{dy}{dx} + 3y = x^3y$ is
(A) Neither Separable nor Linear
(B) Both Linear and Separable
(C) Linear but not Separable
(D) Separable but not Linear.

19. The differential equation $x^2\left(\frac{d^2y}{dx^2} + 2\right) = xy - y$ is
(A) Linear but non-homogeneous
(B) Both Linear and homogenous
(C) Not linear but homogeneous
(D) both not linear and non-homogeneous.

20. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^3}$ is
(A) Conditionally convergent
(B) both absolutely convergent and convergent
(C) convergent but not absolutely
(D) is divergent.

21. Which of the following statements is true regarding the function f(x) given by

$$f(x) = \begin{cases} 2x - 1, & x < -1 \\ x^2 + 1, & -1 \leq x \leq 1 \\ x + 1, & x > 1. \end{cases}$$

$2(-1) - 1 =$

f is continuous
(A) everywhere
(B) everywhere except at $x = -1$ and $x = 1$
(C) everywhere except at $x = -1$
(D) everywhere except at $x = 1$.

22. Which of the following series is divergent?
(A) $\sum_{n=1}^{\infty} \frac{1}{n}$
(B) $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$
(C) $\sum_{n=1}^{\infty} \frac{1}{n^2}$
(D) $\sum_{n=1}^{\infty} \frac{n}{(n+1)!}$

23. Suppose that a function $f : [2, 6] \longrightarrow \mathbb{R}$ satisfies the hypotheses of Mean Value Theorem (MVT) and the equation $f(2) - f(6) = 12$. By MVT, there exists $\xi \in (2, 6)$ such that
(A) $f'(\xi) = -1$, (B) $f'(\xi) = -3$
(C) $f'(\xi) = 1$
(D) $f'(\xi) = -2$.

24. Given $f(x, y) = \frac{x^2 + y^2}{4xy} + \frac{y}{x}\sin\frac{x}{y}$, $x, y > 0$.
Evaluate $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + x + y$.
(A) 0 (B) $1 + x$ (C) $1 + y$ (D) $x + y$

25. Find an approximate value of $\int_0^1 \frac{1}{1+x}dx$ by using Simpson's one-third rule with $h = 0.5$.
(A) 0.6735 (B) 0.6945 (C) 0.6664
(D) 0.6865.

26. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y} - xe^y$ is
(A) $e^x - e^y - \frac{x^2}{2} = C$
(B) $e^x + e^y + \frac{x^2}{2} = C$
(C) $e^x + e^{-y} + \frac{x^2}{2} = C$
(D) $e^x + e^{-y} - \frac{x^2}{2} = C$,
where C is any arbitrary constant.

27. The solution to the differential equation
$$\frac{dy}{dx} - \frac{y}{x} = x^{-2},$$
where C is any arbitrary constant, is
(A) $y = \frac{2Cx^2 + 1}{2x}$
(B) $y = \frac{2Cx^2 - 1}{2x}$
(C) $y = \frac{2x^2 + C}{2x}$
(D) $y = \frac{x}{C - x}$.

28. Find the values of α if
$$f(x) = \begin{cases} x^2 + 3x, & x < \alpha \\ 4, & x \geq \alpha \end{cases}$$
is continuous every where on reals.
(A) all real numbers
(B) $\alpha = -1$ and $\alpha = -4$ only
(C) $\alpha = 1$ and $\alpha = -4$ only
(D) $\alpha = -1$ and $\alpha = 4$ only.

29. Which of the following statements is true about the P-series $\sum_{n=1}^{\infty} \frac{1}{n^P}$?
(A) Converges if $P = 1$
(B) Converges if $P > 1$
(C) Converges if $P < 1$
(D) Converges if $P \leq 1$.

30. Consider the function
$$f(x) = \begin{cases} 2x^2 + 3, & x < 3 \\ 3x + B, & x \geq 3. \end{cases}$$
Find the the value of B such that $f(x)$ is continuous at $x = 3$.
(A) Doesn't exist
(B) 21
(C) −13
(D) 12.

31. Suppose that the total naira value of an enterprise at time t is denoted by $u(t)$ and that the rate of increase is directly proportional to the value. If the initial value of the enterprise is

10 *million* Naira, find the growth of the enterprise (in million Naira) at time t.

(A) $10e^{10t}$
(B) $10e^{Ct}$
(C) $10e^{-Ct}$
(D) $10e^{-10t}$.

32. Find the value of k such that the parabolas $y = c_1x^2 + k$ are the orthogonal trajectories of the family of ellipses $x^2 + 2y^2 - y = c_2$.

(A) $\frac{-1}{2}$
(B) $\frac{1}{4}$
(C) $\frac{3}{4}$
(D) -2.

33. The solution of the initial value problem $9y'' - 12y' + 4y = 0$, $y(0) = 2$, $y'(0) = 1$, is

(A) $(6 - x)e^{2x/3}$
(B) $\frac{1}{3}(6 - x)e^{2x/3}$
(C) $\frac{1}{3}(6 + x)e^{2x/3}$
(D) $-\frac{2}{3}(6 + x)e^{2x/3}$.

34. Find a particular solution of the initial value problem $y'' - 2y' - 3y = 36e^{5x}$, $y(0) = 9$, $y'(0) = 25$.

(A) $2e^{5x}$
(B) $4e^{5x}$
(C) e^{5x}
(D) $3e^{5x}$.

35. An appropriate guess for a particular solution of the differential equation $y'' - 2y' - 3y = 28e^{3x}$ is

(A) Axe^{3x}
(B) Ax^3e^{3x}
(C) Ax^2e^{3x}
(D) Ae^{3x}.

36. Given that it is possible to use the method of undetermined coefficients to find a particular solution of the differential equation $y'' - 4y' + 5y = 5sin^2x$. An appropriate undetermined coefficients function is

(A) nonexistent
(B) $B\cos(2x) + C\sin(2x)$
(C) $A + B\cos(x) + C\sin(x)$
(D) $A + B\cos(2x) + C\sin(2x)$.

37.
$$\sum_{n=1}^{\infty} \frac{2}{3}\frac{1}{3(3n-1)} - \sum_{n=1}^{\infty} \frac{2}{3}\frac{1}{3(3n+2)}$$
equals

(A) $\frac{2}{3}$
(B) $\frac{1}{3}$
(C) $\frac{3}{4}$
(D) $\frac{1}{4}$.

38. Find the radius of convergence of the series
$$\sum_{n=1}^{\infty} \frac{6^n}{n}(4x-1)^{n-1}.$$

(A) $\frac{1}{4}$
(B) $\frac{1}{4}$
(C) $\frac{1}{24}$
(D) $\frac{1}{12}$.

39. By using alternating series test, which of the following series converges?

(A) $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n+1}$
(B) $\sum_{n=1}^{\infty}(-1)^n \left(1 - \frac{1}{n}\right)$
(C) $\sum_{n=1}^{\infty}(-1)^n \sin\frac{1}{n}$
(D) $\sum_{n=1}^{\infty}(-1)^n \frac{n^n}{8^n}$.

40. Compute $\lim_{x\to 0}\left(\frac{2x^2 - 3x + 4}{x} + \frac{5x - 4}{x}\right)$.

(A) 5
(B) 4
(C) 3
(D) 2.