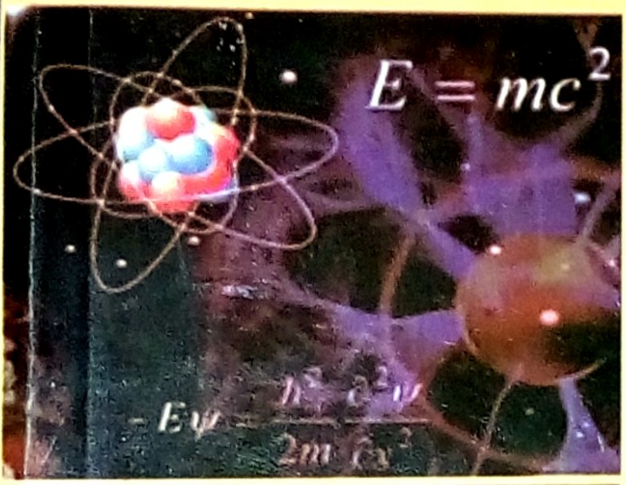

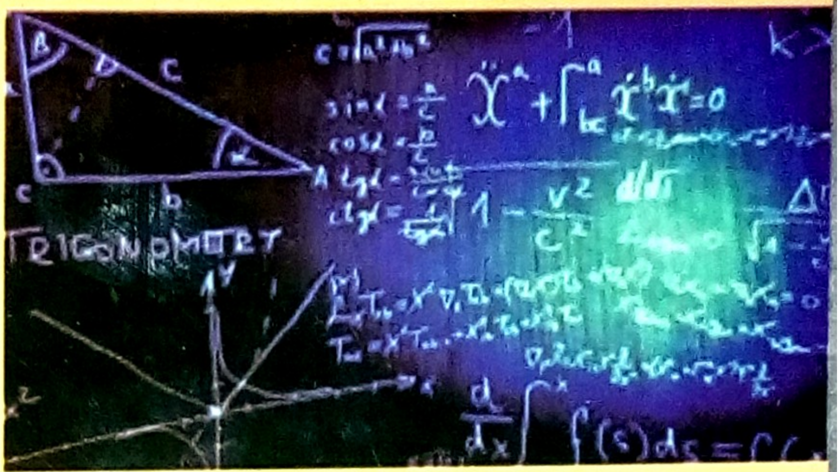

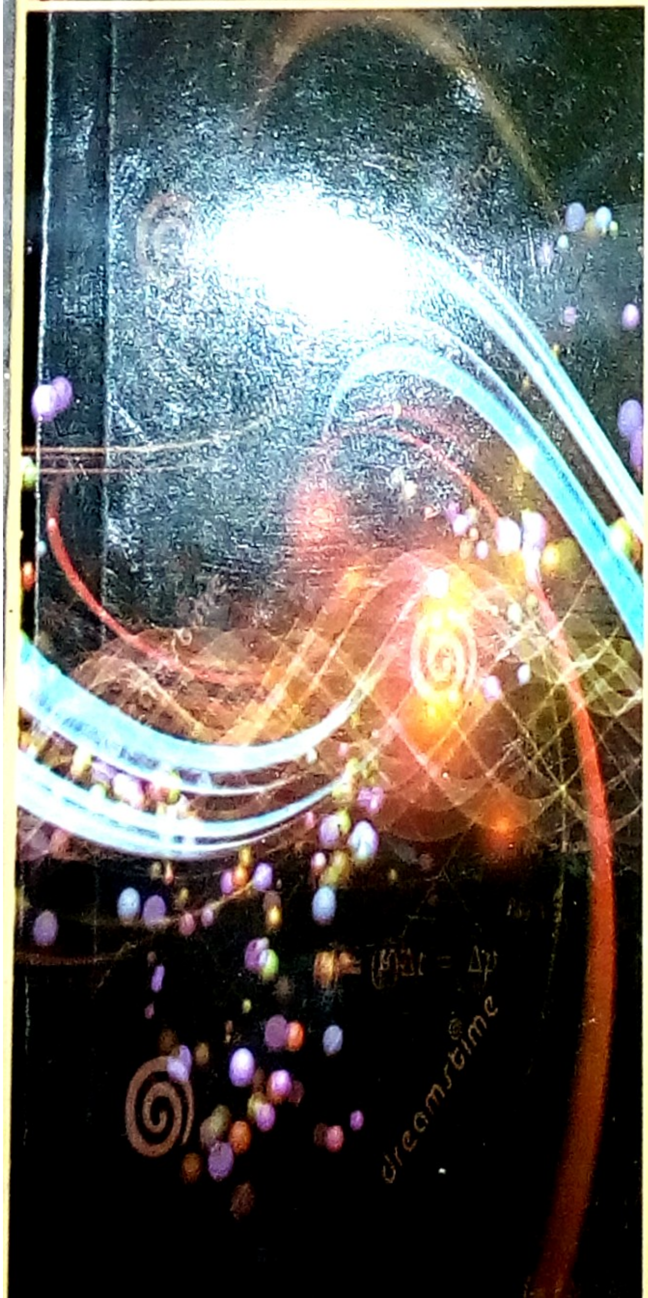

# Physics for University Beginners Volume One

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Department of Physics
University of Lagos

LAJOM CONSULTING
LAGOS, NIGERIA

JOKTOL...

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Adekola O. Adewale

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LAGOS, NIGERIA

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## Introduction

The book, ***Physics for University Beginners, Volume One***, is a first year text on University Physics. It covers topics such as Mechanics, Properties of Matter and Thermal Physics. Physics for University Beginners Volume Two dealt with Waves, Optics, Electricity, Magnetism and Modern Physics.

In order to enhance students' understanding of the course, we have provided several questions and answers on the subject. The questions and answers include solutions to past questions from relevant examinations and the lesson note of the author.

In writing this book, due cognizance has been taken of the fact that a significant proportion of University first year students are not familiar with some aspects of elementary Mathematics. As such, the mathematical aspect of each solution has been carefully simplified for easy understanding.

This book has been written mainly as an aid to first year students of Nigerian Universities and JUPEB students of the University of Lagos who have been admitted into the following Faculties: Science, Medical Sciences, Engineering, Environmental Sciences and Education. Students studying for allied examinations like Joint University Preliminary Examination Board (JUPEB), Interim Joint Matriculation Board Examination, University Matriculation Examination, Polytechnics and Colleges of Education Examinations will also find this book very useful.

This book could not have been realised without the support of many people. I would like to thank Dr. Elijah Oyedola Oyeyemi in particular and all academic staff of the Department of Physics, University of Lagos for their support and many valuable comments. I would like to thank Adewoyin Adeyinka and staff of MOSRON communications for reviewing the manuscript.

The best is usually saved for the last: I would like to express my sincere gratitude to my wife, Omolara, who has always inspired me with endless love and unconditional support. I am deeply grateful to our lovely son, Ademitide, for filling our lives with joy.

Suggestions, corrections and criticisms are welcome. Please send to:

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**Dr A. O. Adewale**
**May 1, 2014**

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# CHAPTER ONE

# PHYSICAL QUANTITIES AND UNITS

## 1.0 Introduction

Physics is the branch of science concerned with the nature and properties of matter and energy. It is a natural science based on experiments, measurements and mathematical analysis with the purpose of finding quantitative physical laws for everything from the nanoworld of the microcosmos to the planets, solar systems and galaxies that occupy the macrocosmos. In performing experiments in Physics, we come across several quantities.

In Physics, physical quantities can be divided into two groups: fundamental and derived quantities. In this chapter we will discuss several physical quantities and their units.

## 1.1 Fundamental and Derived Quantities

As you may know, Physics is an experimental science which requires measurements. The results of measurements are described by the use of numbers. A physical quantity is usually used to quantitatively describe a physical phenomenon. In the laboratory, you will be required to take measurements and you are expected to record your result (the physical quantity being measured) using numbers.

Measurements in Physics are made in terms of agreed standards/units. There are several systems of units that have been used over the years. One of these agreed standards/units is *le systeme international d'unites*, which in English means "the international system of units" and is abbreviated as "S.I.". S.I. units can be divided into three classes: base units, derived units and supplementary units. These S.I. units are the most common of all the units and they are universally agreed system of units adopted internationally in 1960. In this system of units, the standard of length is the metre, the standard for time is the second and the standard for mass is the kilogram. In this book, we will use the S.I units. Another metric system is the CGS system. In this system, the standard of length is the centimetre, the standard for time is the second and the standard for mass is the gram. You will also come across the CGS system in this book.

There are seven fundamental/base units which are listed in Table 1.1 below with their corresponding physical quantities.

Table1.1: Fundamental or base quantities and units

| S/N | Physical Quantity | Unit | Symbol |
|---|---|---|---|
| 1 | Mass | kilogram | kg |
| 2 | Length | metre | m |
| 3 | Time | second | s |
| 4 | Electric current | ampere | A |
| 5 | Thermodynamic temperature | kelvin | K |
| 6 | Luminous intensity | candela | cd |
| 7 | Quantity of substance | mole | mol |

There are standards that define these fundamental units. These standards are chosen so that they can be readily reproducible.

i. One kilogram (kg) is the mass of a particular platinum-iridium cylinder, kept at the International Bureau of Weights and Measures, Sevres, France.

ii. The metre (m) is defined as the length equal to 1,650,763.73 wavelengths in vacuum of the orange light from Krypton-86 atom.

iii. The second (s) is defined as the time interval or duration of 9,192,631,770 periods or cycles of the radiation corresponding to transition between the two hyperfine levels of the ground state of the cesium-133 atom.

iv. The ampere (A) is a constant current that will produce a force equal to $2 \times 10^{-7}$ newton per metre of length when maintained in two straight parallel conductors of infinite length and of negligible circular cross-section, and placed one metre apart in vacuum.

v. The kelvin (K) is defined as the fraction 1/273.16 of the thermodynamic temperature of the triple point of water (273.16 K).

vi. We can define the candela (cd) as the luminous intensity, in the perpendicular direction, of a surface of $\frac{1}{6}\times10^{-5}\,m^2$ of a blackbody at the temperature of freezing platinum, under a pressure of 1 atmosphere.

vii. The mole (mol) is the amount of a substance of a system which contains as many as there are carbon atoms in 0.012kg of carbon-12.

Derived quantities are the quantities that are defined in terms of the seven fundamental quantities. Derived units are obtained by combining base units using S.I. system of units. Some derived units have special names and symbols which can themselves be used to obtain other derived units. Examples of derived units are given in Table 1.2.

Table1.2: Derived units with special names

| S/N | Physical Quantity | Unit | Special Name/Symbol |
|---|---|---|---|
| 1 | Area | $m^2$ | square metre |
| 2 | Volume | $m^3$ | cubic metre |
| 3 | Density | $kgm^{-3}$ | kilogram per cubic metre |
| 4 | Speed/Velocity | $ms^{-1}$ | metre per second |
| 5 | Acceleration | $ms^{-2}$ | metre per second square |
| 6 | Force | $kgms^{-2}$ | newton (N) |
| 7 | Impulse | $kgms^{-1}$ | newton second (Ns) |
| 8 | Momentum | $kgms^{-1}$ | newton second (Ns) |
| 9 | Surface tension | N/m | newton per metre (N/m) |
| 10 | Pressure | $Nm^{-2}$ | pascal (pa) |
| 11 | Energy/work | Nm | joule (J) |
| 12 | Power/radiant flux | J/s | watt (W) |
| 13 | Frequency | $s^{-1}$ | hertz (Hz) |
| 14 | Electric charge | As | coulomb (C) |
| 15 | Capacitance | C/V | farad |
| 16 | Electric resistance | V/A | ohm (Ω) |

| | | | |
|---|---|---|---|
| 17 | Conductance | A/V | Siemens (S) |
| 18 | Magnetic flux | Vs | weber (Wb) |
| 19 | Magnetic flux density | $Wb/m^2$ | tesla (T) |
| 20 | Inductance | Wb/A | henry (H) |
| 21 | Luminous flux | cd sr | lumen (lm) |
| 22 | Illumination | $lm\ m^{-2}$ | lux (lx) |

The third class of units is the supplementary units. They are given in Table 1.3. They have been adopted because of the link between geometrical and physical descriptions of events.

Table1.3: Supplementary Units

| S/N | Quantity | Unit | Symbol |
|---|---|---|---|
| 1 | Plane angle | radian | rad |
| 2 | Solid angle | steradian | sr |

## 1.2 Multiples and Submultiples of S.I. Units

Prefixes are used to form multiples and submultiples of S.I. units as shown in Table 1.4.

Table1.4: Multiples and prefixes for units

| S/N | Factor | Prefix | Symbol |
|---|---|---|---|
| 1 | $10^{24}$ | yotta | Y |
| 2 | $10^{21}$ | zetta | Z |
| 3 | $10^{18}$ | exa | E |
| 4 | $10^{15}$ | peta | P |
| 5 | $10^{12}$ | tera | T |
| 6 | $10^{9}$ | giga | G |
| 7 | $10^{6}$ | mega | M |
| 8 | $10^{3}$ | kilo | k |
| 9 | $10^{2}$ | hecto | h |
| 10 | 10 | deca | da |
| 11 | $10^{-1}$ | deci | d |
| 12 | $10^{-2}$ | centi | c |
| 13 | $10^{-3}$ | milli | m |
| 14 | $10^{-6}$ | micro | μ |

| | | | |
|---|---|---|---|
| 15 | $10^{-9}$ | nano | n |
| 16 | $10^{-12}$ | pico | p |
| 17 | $10^{-15}$ | femto | f |
| 18 | $10^{-18}$ | atto | a |
| 19 | $10^{-21}$ | zepto | z |
| 20 | $10^{-24}$ | yocto | y |

For example, using the prefixes we see that 5 MV = 5 000 000 V and 5Nm = 5 x $10^{-9}$ m. Note that the abbreviations for the multiples $10^6$ and greater are capitalised, whereas the abbreviations for the smaller multiples are in lower case.

## 1.3 Dimension

Dimension as a tool in Physics denotes the physical nature of a quantity. Dimension shows the way in which the derived quantity is related to the base quantities, i.e. dimensions of any other quantity involve one or more of the fundamental dimensions. Length, mass and time are the fundamental dimensions. You could measure the amounts of matter in an object and express the units as kilogram or gram, but the quantity would still have the dimension of mass. The dimensional quantities for length, mass and time are expressed respectively as L, M and T. Table 1.5 shows the dimensions of some derived quantities.

**Table1.5: Dimensions of derived units**

| S/N | Physical Quantity | Unit | Dimension |
|---|---|---|---|
| 1 | Area | $m^2$ | $L^2$ |
| 2 | Volume | $m^3$ | $L^3$ |
| 3 | Density | $kgm^{-3}$ | $ML^{-3}$ |
| 4 | Speed/Velocity | $ms^{-1}$ | $LT^{-1}$ |
| 5 | Acceleration | $ms^{-2}$ | $LT^{-2}$ |
| 6 | Force | $kgms^{-2}$ | $MLT^{-2}$ |
| 7 | Impulse | $kgms^{-1}$ | $MLT^{-1}$ |
| 8 | Momentum | $kgms^{-1}$ | $MLT^{-1}$ |
| 9 | Surface tension | N/m | $MT^{-2}$ |
| 10 | Pressure | $Nm^{-2}$ | $ML^{-1}T^{-2}$ |

| | | | |
|---|---|---|---|
| 11 | Energy/work | Nm | $ML^2T^{-2}$ |
| 12 | Power/radiant flux | J/s | $ML^2T^{-3}$ |
| 13 | Frequency | $s^{-1}$ | $T^{-1}$ |
| 14 | Moment | Nm | $ML^2T^{-2}$ |

Dimensional analysis is a procedure by which the dimensional uniformity of any equation may be checked. For an equation to be dimensionally uniform or consistent, the dimension of the left hand side of the equation must be equal to the dimension of the right hand side.

Let us use the equation of motion $x = v_o t + \frac{1}{2}at^2$ as an example, where $x$, $v_o$, $a$ and $t$ represent distance, speed, acceleration and time respectively.

Dimension of $x$, $[x] = L$

Dimension of $v_o t$, $[v_o t] = LT^{-1} \times T = L$

Dimension of $at^2$, $[at^2] = LT^{-2} \times T^2 = L$

Since the dimension of all parts of the equation is the same, the equation is dimensionally consistent.

Apart from using dimensional analysis to check the dimensional consistency of any equation, we can also use it to check whether an equation that has been derived or is being used in solving a problem has the correct form. For example, we can use dimensional analysis to check the correctness of the equation: $T = 2\pi\sqrt{\frac{L}{g}}$, where $T$ is the period of a simple pendulum, $L$ is the length of the string and $g$ is acceleration due to gravity.

Let

$$T = kL^x g^y \text{ , where } k \text{ is a constant}$$

Dimensionally, we can write

$$T = kL^x\left(LT^{-2}\right)^y = kL^{x+y}T^{-2y}$$

Comparing the powers on both sides

For $T$: $1 = -2y$

$$y = -\frac{1}{2}$$

For $L$: $0 = x + y = x - \frac{1}{2}$

i.e. $x = \frac{1}{2}$

Substitute for $x$ and $y$ in the above equation, we have:

$$T = kl^{1/2} g^{-1/2} = k\sqrt{\frac{l}{g}}$$

This shows that the equation $T = 2\pi\sqrt{\frac{L}{g}}$ is dimensionally correct.

However, the equation $T = k\sqrt{\frac{l}{g}}$ is not physically correct.

## Activity 1 Physical Quantities and Units

**1.1.** What are the correct dimensions of energy and force?

A. $MLT^{-1}$, $MLT$

B. $ML^2T^{-2}$, $MLT^{-2}$

C. $ML^{-3}T$, $MLT^2$

D. $ML^2T^{-2}$, $ML^2T^2$

**Solution**

Energy = mgh ≡ $kgms^{-2}m = kgm^2s^{-2} \equiv ML^2T^{-2}$

Force = ma ≡ $kgms^{-2} = MLT^{-2}$

The correct option is B.

**1.2.** Experiment shows that the frictional drag force $F$ of a car moving in the air depends on the velocity $v$ of its motion, density $\rho$ of the air through which it travels, and the cross-sectional area of the car. If the force $F = k\rho^x v^{2y} A^z$, where $x$, $y$, and $z$ are integer and $k$ is a dimensionless constant, find the values of $x$, $y$, and $z$.

A. 1, 1, 1 B. ½, ½, ½ C. 1, -1, 1 D. 1, 0, 1

**Solution**

We already know that:

$$F = k\rho^x v^{2y} A^z,$$

Dimensionally:

$$MLT^{-2} = k\left(ML^{-3}\right)^x \left(LT^{-1}\right)^{2y} \left(L^2\right)^z$$
$$MLT^{-2} = kM^x L^{-3x+2y+2z} T^{-2y}$$

Note that $k$ is a dimensionless constant. Comparing both sides:

For $M$: $x = 1$
For $T$: $-2 = -2y$
$y = 1$
For $L$: $1 = -3x + 2y + 2z$
$1 = -3 + 2 + 2z$
$z = 1$

The correct option is A.

**1.3.** Identify the correct dimensions of density and pressure from the following:

A. $ML^{-3}$, $ML^{-1}T^{-2}$
B. $ML^{-1}T^{-2}$, $ML^{-3}$
C. $ML^{-2}$, $ML^{-1}T^{-2}$
D. $ML^{3}$, $ML^{-1}T^{2}$

**Solution**

Density $\rho = M/V \equiv \text{kg/m}^3 = \text{kgm}^{-3} = ML^{-3}$

$$Pressure\,P = \frac{F}{A} \equiv \frac{kgms^{-2}}{m^2} = kgm^{-1}s^{-2} = ML^{-1}T^{-2}$$

The correct option is A.

**1.4.** The velocity $v$ of the wave set-up by plucking a stretched string is found to depend on the tension $T$ in the string, its length $l$ and its mass $m$, and is given by $v = kT^x l^y m^z$ where $x$, $y$, $z$ are unknown numbers and $k$ is a constant. Find the values of $x$, $y$, and $z$.
A. ½, -½, ½ B. -½, ½, ½ C. -½, ½, -½ D. ½, ½, -½

**Solution**

$v = kT^x l^y m^z$

Dimensionally:

$$LT^{-1} = k\left(MLT^{-2}\right)^x \times L^y \times M^z$$

$$LT^{-1} = kM^{x+z} \times L^{x+y} \times T^{-2x}$$

Comparing:

For $T$: $-1 = -2x$
$x = ½$
For $L$: $1 = x + y$
$y = 1 - ½ = ½$
For $M$: $0 = x + z$
$0 = ½ + z$
$z = -½$

$x = ½, y = ½, z = -½$

The correct option is D.

**1.5.** Use dimensional analysis to determine the value of $y$ in the relation $T = ka^x \rho^y \gamma^z$, where $T$ is the period of vibration, $a$ is the radius, $\rho$ is the density and $\gamma$ is the surface tension.
A. -½ B. ½ C. 3/2 D. –3/2

**Solution**

$T = ka^x \rho^y \gamma^z$

Dimensionally:

$$T = kL^x \times \left(ML^{-3}\right)^y \times \left(MT^{-2}\right)^z$$
$$T = kM^{y+z} \times L^{x-3y} \times T^{-2z}$$

Comparing:
For $T$: $1 = -2z$
$z = -\frac{1}{2}$
For $M$: $0 = y + z$
$0 = y + (-\frac{1}{2})$
$y = \frac{1}{2}$

The correct option is B.

**1.6.** Given that the period of oscillation of a pendulum is given by $T = km^x l^y g^z$ where $k$ is a constant. Which of the following is correct?

A. $T = k\sqrt{\frac{l}{g}}$ B. $T = \frac{2}{\pi}\sqrt{\frac{l}{g}}$ C. $T = 2\pi\sqrt{\frac{l}{g}}$ D. $T = \frac{2}{\pi}\sqrt{\frac{g}{l}}$

**Solution**

$T = km^x l^y g^z$

Dimensionally:

$$T = kM^x \times L^y \times \left(LT^{-2}\right)^z$$
$$T = kM^x L^{y+z} T^{-2z}$$

Comparing:

For $T$: $1 = -2z \Rightarrow z = -\frac{1}{2}$

For $M$: $0 = x$

For $L$: $0 = y + z \Rightarrow y = \frac{1}{2}$

$$T = km^0 l^{\frac{1}{2}} g^{-\frac{1}{2}}$$

$$T = k\sqrt{\frac{l}{g}}$$

The correct option is A.

**1.7.** Which of the following relates period of vibration ($T$) with the following: radius $a$, density $\rho$ and surface tension $\gamma$.

A. $T = ka^{\frac{3}{2}}\rho^{\frac{1}{2}}\gamma^{-\frac{1}{2}}$

B. $T = ka^{-\frac{3}{2}}\rho^{\frac{1}{2}}\gamma^{\frac{1}{2}}$

C. $T = ka^{-\frac{3}{2}}\rho^{\frac{1}{2}}\gamma^{\frac{1}{2}}$

D. $T = ka^{\frac{3}{2}}\rho^{-\frac{1}{2}}\gamma^{-\frac{1}{2}}$

E. $T = ka^{-\frac{3}{2}}\rho^{-\frac{1}{2}}\gamma^{\frac{1}{2}}$

**Solution**

Let $T = ka^x \rho^y \gamma^z$

Dimensionally:

$$T = kL^x \left(ML^{-3}\right)^y \left(MT^{-2}\right)^z$$

$$T = kM^{y+z} L^{x-3y} T^{-2z}$$

Comparing:

For $T$: $1 = -2z \Rightarrow z = -\frac{1}{2}$

For $M$: $0 = y + z \Rightarrow y = \frac{1}{2}$

For $L: 0 = x - 3y \Rightarrow x = \frac{3}{2}$

$$T = ka^{3/2}\rho^{1/2}\gamma^{-1/2}$$

The correct option is A.

**1.8.** A sphere, mass 100g of radius $a$ = 2cm falling vertically through air of density = 1.2 $kgm^{-3}$, at a place where $g = 9.81ms^{-2}$ attains a steady velocity of $v = 30ms^{-1}$. If the retarding force on the sphere is given by $F = ka^x\rho^y v^z$, where $k$ is a non-dimensional coefficient, use the method of dimensions to find the values of $x$, $y$, $z$ and $k$.

A. $F = 2.3a\rho v$ B. $F = 1.3a^2\rho v^2$ C. $F = 2.3a^2\rho v^2$
D. $F = 1.3a\rho v$

**Solution**

$$F = ka^x\rho^y v^z$$

Dimensionally:

$$MLT^{-2} = kL^x \times (ML^{-3})^y \times (LT^{-1})^z$$
$$MLT^{-2} = kM^y \times L^{x-3y+z} \times T^{-z}$$

Comparing:

For $T$: $z = 2$
For $M$: $y = 1$
For $L$: $1 = x - 3y + z$
$1 = x - 3 + 2$
$x = 2$

Hence, $F = ka^2\rho v^2$
$mg = ka^2\rho v^2$
$0.1 \times 9.8 = k \times (2 \times 10^{-2})^2 \times 1.2 \times (30)^2$
$0.98 = k \times 4 \times 10^{-4} \times 1.2 \times 900$
$k = 2.269$
$k = 2.3$
$F = 2.3a^2\rho v^2$

The correct option is C.

**1.9.** Identify which of the quantities listed from (i) to (vii) are derived quantities:
(i) mass (ii) length (iii) electric current (iv) density (v) Temperature (vi) velocity (vii) energy

A. (i), (ii) (iii) & (v)
B. (iv), (v), (vi) & (vii)
C. (iv), (vi) & (vii)
D. all

**Solution**

The correct option is C.

**1.10.** The period $T$ of a loaded spring oscillator depends on its mass $m$ and spring constant $k$, as follows: $T = ck^x m^y$ where $x$ and $y$ are integers and $c$ is a dimensionless constant. Find the values of $x$ and $y$.

A. $\frac{1}{2}, -\frac{1}{2}$ B. $\frac{1}{2}, \frac{1}{2}$ C. $-\frac{1}{2}, -\frac{1}{2}$ D. $-\frac{1}{2}, \frac{1}{2}$

**Solution**

$$T = ck^x m^y$$

Dimensionally, we have:

$$T = c\left(MT^{-2}\right)^x M^y$$
$$T = cM^{x+y}T^{-2x}$$

Comparing:

For $T$: $1 = -2x$

$$x = -\frac{1}{2}$$

For $M$: $0 = x + y$

$$0 = -\frac{1}{2} + y$$

$$y = \frac{1}{2}$$

The correct option is D.

**1.11.** Which of the following unit is equivalent to $kgm^{-1}s^{-2}$?

A. $Ns^{-1}$ B. Pa C. J D. $Nm^{-1}$ E. $Js^{-1}$

**Solution**

The correct option is B.

**1.12.** Which of the following are the dimensions of pressure and torque respectively?

A. $ML^2T^{-3}$ and $MLT^{-2}$ B. $MLT^{-2}$ and $MLT^{-2}$

C. $ML^{-1}T^{-2}$ and $ML^2T^{-1}$ D. $ML^{-1}T^{-2}$ and $ML^2T^{-2}$

**Solution**

Pressure $P = \frac{F}{A}$

$$[P] = \frac{kgms^{-2}}{m^2} = kgm^{-1}s^{-2} \equiv ML^{-1}T^{-2}$$

Torque $\tau$ = force × perpendicular distance

$$[\tau] = kgms^{-2} \times m = kgm^2s^{-2} \equiv ML^2T^{-2}$$

The correct option is D.

## Summary of Chapter 1

In chapter 1, you have learned that:

1. Physics is an experimental science which requires measurements. The results of measurements are described by the use of numbers.
2. One of the agreed standards/units is the *le systeme international d'unites*, which in English means "the

international system of units" and is abbreviated as "S.I.". S. units can be divided into three classes: base units, derive units, and supplementary units.

3. There are seven fundamental/base quantities/units: mass (kg), length (m), time (s), electric current (A), thermodynamic temperature (K), luminous intensity (cd).

4. There are standards that define all of the fundamental units. These standards are chosen so that they can be readily reproducible.

5. Derived units are obtained by combining base units using S.I. system of units. Some derived units have special names and symbols which can themselves be used to obtain other derived units.

6. The third class of units is the supplementary units. They have been adopted because of the link between geometrical and physical descriptions of events.

7. The dimensional quantities for length, mass and time are expressed respectively as $L$, $M$ and $T$. Dimensional analysis is a procedure by which the dimensional uniformity of any equation may be checked and also to check whether an equation that has been derived or is being used in solving a problem has the correct form.

## Self-Assessment Questions (SAQs) for Chapter 1

**1.1.** Find, using dimensional analysis, the relation between the frequency $f$ of the vibration of a stretched string and its mass per unit length $\mu$, its tension $T$ and its length $l$.

**1.2.** Experiment shows that the viscous force $F$ on a spherical body of radius $r$ moving with angular velocity $\omega$ through a fluid of viscosity $\eta$ is $F = kr^a\eta^b\omega^c$, where $k$ is a dimensionless constant. Using the method of dimensional analysis, determine the values of $a$, $b$ and $c$.

1.3. Use dimensional analysis to check if the expression $p=\frac{1}{2}\rho v^2+\rho gh$ is correct. Given that $p$ is pressure in pascal, $\rho$ is density in $kgm^{-3}$, $v$ is velocity in m/s, $g$ is acceleration due to gravity in $m/s^2$ and $h$ is the height in m.

**1.4.** Which of the following is the internationally agreed system of units (S.I.) for physical measurements?
A. lb, ft, sec B. g, m, sec C. kg, m, sec D. cm, g, sec

**1.5.** Which of the following units is/are the unit(s) of latent heat? (I) Ns (II) Nm (III) $N/m^2$ (IV) $JK^{-1}$ (V) $Jkg^{-1}$.
A. I B. II C. III and IV D. V

**1.6.** What are the units of thermal conductivity?
A. $kgms^{-2}$ B. $Js^{-1}m^{-1}K^{-1}$ C. $JK^{-1}$ D. $Ns^{-1}m^{-1}K^{-1}$

**1.7.** Which of the following derived units is NOT a unit of power?
A. Joule/second
B. ampere volt
C. ampere/volt
D. $volts^2$/ohms

**1.8.** Which of the underlisted quantities has the dimension $ML^2T^{-2}$?
A. moment of a force
B. work
C. acceleration
D. moment of a force and work

**1.9.** What is the unit of quantity of electricity?
A. the coulomb
B. the volt
C. the ampere

D. the ohm

**1.10.** Which of the following is NOT a fundamental S.I. unit?
A. Metre B. Ampere C. Mole D. Newton

**1.11.** In a gas experiment, the pressure of the gas is plotted against the reciprocal of the volume of the gas at a constant temperature. The unit of the slope of the resulting curve is
A. newton B. newton/m C. joule
D. newton/$m^3$

**1.12.** Which of the following is a derived unit?
A. Kilogramme B. Metre C. Kelvin D. Newton

**1.13.** Which of the following units is equivalent to kgm/s?
A. J/s B. N/s C. Ns D. Nms

**1.14.** Which of the following are the correct S.I. units of the quantities indicated? I. Ns (Impulse) II. Nm (Torque) III. Watt/s (Power) IV. kgm/$s^2$ (Momentum).
A. I and II only
B. I, II and III only
C. I, II and IV only
D. I and III only

**1.15.** The product *PV* where *P* is pressure and *V* is volume has the same unit as
A. density B. power C. momentum D. energy

**1.16.** Which of the following are derived units? (I) Second (II) Newton (III) Kilogram (IV) Ampere (V) Joule.
A. I and III only
B. II and V only
C. II, IV and V only
D. I only

**1.17**. Which of the following are derived quantities? (I) Force (II) Temperature (III) Area (IV) Pressure

A. I and IV only
B. II, III and IV only
C. I and III only
D. I, III and IV only

**1.18**. Which of the following is/are supplementary unit(s)? (I) Kelvin (II) Newton (III) Second (IV) Radian

A. I and III only
B. IV only
C. I and II only
D. I, II and IV only

**1.19**. The unit of momentum is

A. J/s  B. N/s  C. kgm/s  D. Nms

**1.20**. Which of the following is the dimension of power?

A. $ML^2T^{-3}$  B. $MLT^{-2}$  C. $ML^2T^{-2}$  D. $ML^{-2}T^3$

**1.21**. In which of the following physical quantities are the units correctly indicated? (I) Weight [N] (II) Power [J/s] (III) Momentum [kgm/s] (IV) Acceleration [N/kg]

A. I and II only
B. III and IV only
C. I, II and III only
D. I, II, III and IV

**1.22**. The watt is equivalent to

A. Nm/s  B. Js  C. $kgm^2s^2$  D. N/s

**1.23**. Which of the following quantities has the same unit as the kilowatt - hour?

A. Force × time
B. Force × distance
C. Force × velocity
D. Force × acceleration

CHAPTER TWO

# SCALARS AND VECTORS

## 2.0 Introduction

Apart from classifying physical quantities into fundamental, derived and supplementary quantities, we can also classify them into either scalar or vector quantities. Scalar quantities are quantities that can be described completely by their magnitudes without directions. Examples of scalars include: mass, area, volume, density, time, temperature, work, energy, power, etc. Vector quantities are quantities that can be described by their magnitudes and directions. Examples of vectors include: displacement, velocity, acceleration, force, weight, momentum, etc. As an illustration, we can consider the difference between speed and velocity. The velocity of a boat in motion is determined by a magnitude which is the speed (scalar) and the direction of its motion. It is always important that you represent all vectors in terms of its magnitude and direction. The representation of a vector in terms of magnitude alone is incomplete and unacceptable.

## 2.1 Vector Representation

In this book we represent vectors in boldface letter, such as **A**, or use of an arrow over a letter, such as $\vec{A}$. The magnitude of the vector **A** is written as either A or $|A|$. We can also use a line with an arrowhead at its tip to represent vectors, as shown in Figure 2.1. The length of the line shows the magnitude of the vector and the direction of the arrow shows the vector's direction.

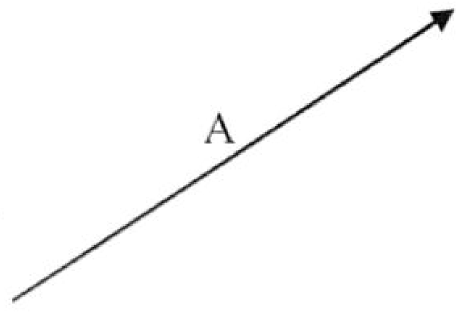



Figure 2.1: Vector representation

## 2.2 Components of a Vector

Consider a vector **A** lying in the $x$-$y$ plane and making an arbitrary angle $\theta$ with positive $x$ axis, as shown in Figure 2.2.

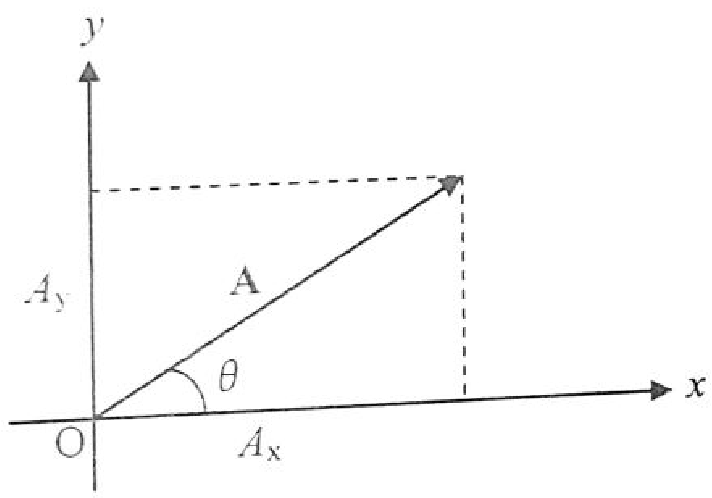



Figure 2.2: Vector **A** making an angle $\theta$

$A_x$ is the component of **A** along the $x$ axis and $A_y$ is the component along the $y$ axis. From Figure 2.2 and the definitions of sine and cosine, we see that:

$$\cos\theta = \frac{A_x}{A} \text{ and } \sin\theta = \frac{A_y}{A}.$$

Hence the components of **A** are:

$$A_x = A\cos\theta \qquad 2.1$$

$$A_y = A\sin\theta \qquad 2.2$$

These components form two sides of a right triangle with a hypotenuse of length $A$, as shown in Figure 2.3.

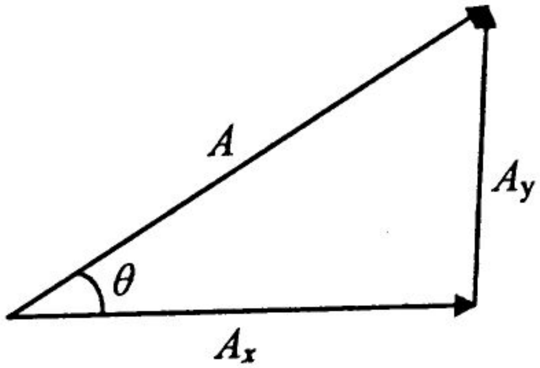



Figure 2.3: Components of vector **A**

Thus, it follows from Figure 2.3 that the magnitude and direction of **A** are related to its components through the expressions (recall Pythagoras theorem),

$$A = \sqrt{A_x^2 + A_y^2} \qquad \text{- magnitude} \qquad 2.3$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \qquad \text{- direction} \qquad 2.4$$

The direction is usually stated in terms of the positive $x$-axis.

## 2.3 Addition and Subtraction of Vectors

The addition and subtraction of a vector is quite different from scalar or numerical addition and subtraction, which you should be familiar with. You may be familiar, for instance, with 5kg + 2kg = 7kg. However, 5N + 2N may not add up to 7N. A single vector that would have the same effect as two or more vectors taken together is called a *resultant vector* (**R**). Several methods are employed in the addition of vectors. One of them is the polygon method. Consider three vectors **A**, **B** and **C** shown in Figure 2.4a. Starting from any convenient point, each vector in Figure 2.4a is drawn to scale and in proper directions with the head of one touching the tail of another. The resultant vector **R** is drawn with its tail end at the starting point and its tip at the tip of the last vector added, as shown in Figure 2.4b.

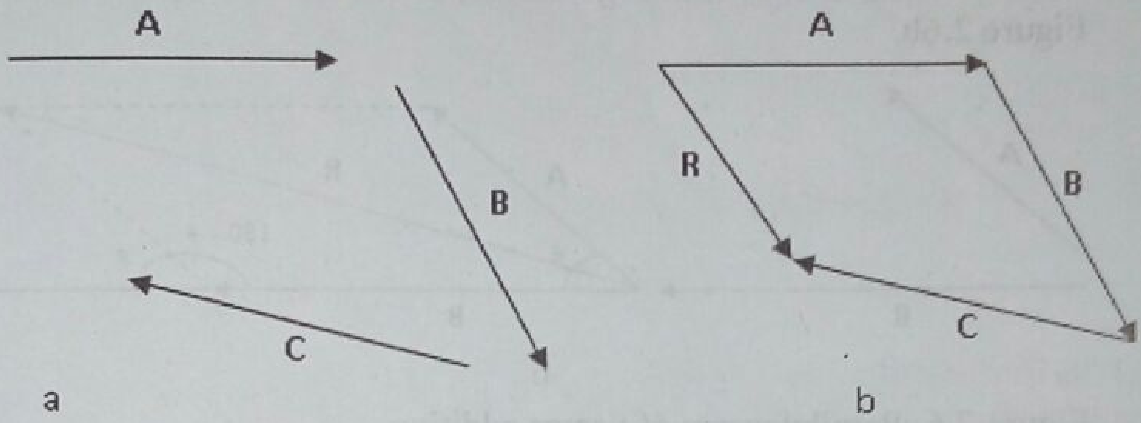



Figure 2.4: Polygon method of vector addition

The resultant vector is given as

$$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} \qquad 2.5$$

- Using the polygon method, perform graphically the following vector additions and subtractions, where **A**, **B** and **C** are the vectors shown in Figure 2.5a: (i) **A** + **B** (ii) **A** – **B**

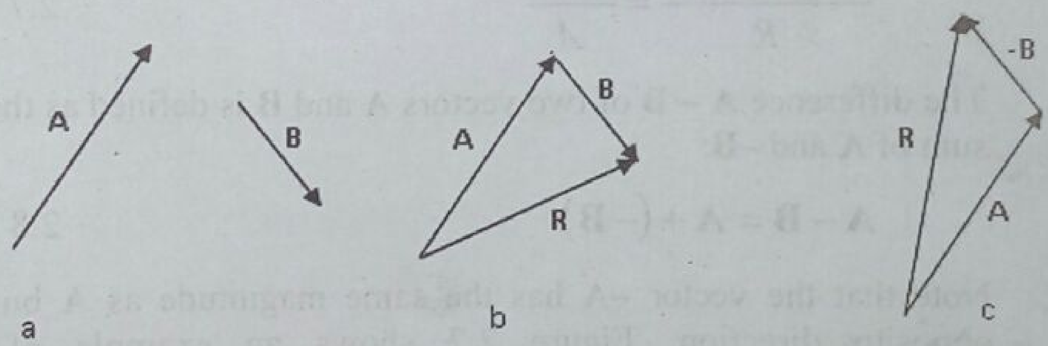



Figure 2.5: (a) individual vectors **A** and **B** (b) **R** = **A** + **B** (c) **R** = **A** + **B**

- (i) The resultant vector **R** = **A** + **B** is shown in Figure 2.5b.
  (ii) The resultant vector **R** = **A** – **B** is shown in Figure 2.5c, where –**B** is equal in magnitude but opposite to **B**.

The second method is the parallelogram method. This method is used for addition of two vectors. If two vectors are inclined to each other at an angle $\theta$, their resultant can be represented in magnitude and direction by the diagonal of a parallelogram whose adjacent sides represent the two vectors in magnitude and direction. Consider vectors **A** and **B** shown in Figure 2.6a, their resultant **R**, is shown in Figure 2.6b.

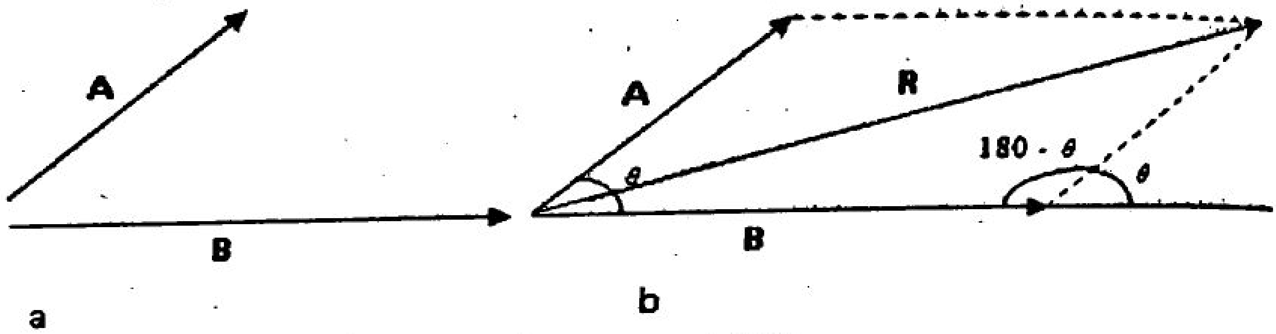



Figure 2.6: Parallelogram of vector addition

To determine the resultant vector **R** we can use the cosine rule:

$$R^2 = A^2 + B^2 - 2AB\cos(180 - \theta) \qquad 2.6$$

If the resultant **R** makes an angle α with the positive x-axis, we can use sine rule as follows:

$$\frac{\sin(180 - \theta)}{R} = \frac{\sin\alpha}{A} \qquad 2.7$$

The difference **A** – **B** of two vectors **A** and **B** is defined as the vector sum of **A** and –**B**:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \qquad 2.8$$

Note that the vector –**A** has the same magnitude as **A** but in the opposite direction. Figure 2.7 shows an example of vector subtraction.

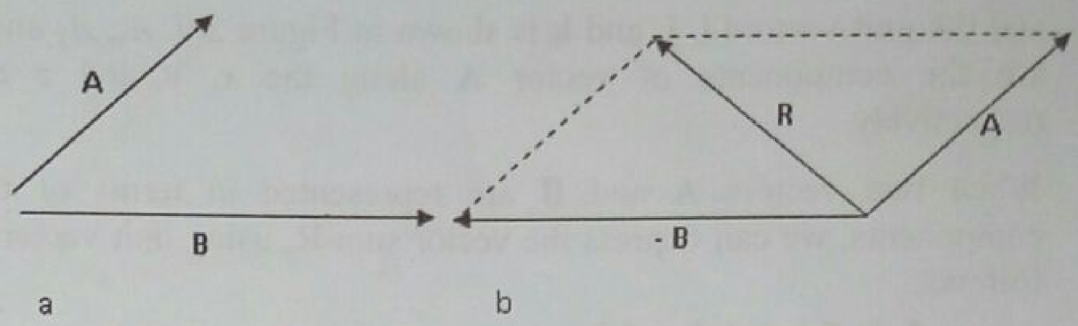



Figure 2.7: (a) individual vectors **A** and **B** (b) **R** = **A** – **B**

## 2.4 Unit Vectors

A unit vector is a dimensionless vector that has a magnitude of 1. It describes a direction in space and has no other physical significance. In the *x*-*y* coordinate system, the unit vector **i** is defined as a vector of magnitude 1 whose direction is towards the positive *x*-axis and the unit vector **j** is defined as a vector of magnitude 1 whose direction is towards the positive *y*-axis (Figure 2.8). The components of the vector **A** in Figure 2.2 can then be written as:

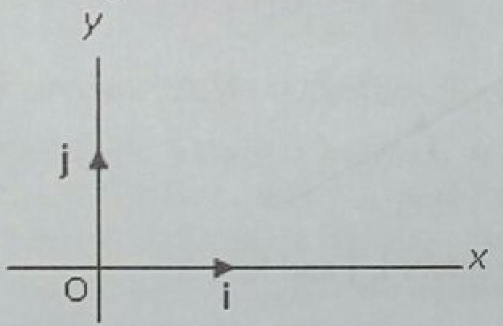



Figure 2.8: The unit vectors **i** and **j**

$$\mathbf{A}_x = A_x\mathbf{i}$$
$$\mathbf{A}_y = A_y\mathbf{j} \qquad 2.9$$

Hence the vector **A** can be written as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} \qquad 2.1($$

If the vector **A** lies in the *xyz* plane, then:

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \qquad 2.11$$

and the unit vectors **i**, **j**, and **k** is shown in Figure 2.9. $A_x$, $A_y$ and $A_z$ are the components of vector **A** along the $x$, $y$ and $z$ axes respectively.

When two vectors **A** and **B** are represented in terms of their components, we can express the vector sum **R**, using unit vectors as follows:

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

$$\mathbf{R} = \left(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\right) + \left(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}\right)$$

$$\mathbf{R} = \left(A_x + B_x\right)\mathbf{i} + \left(A_y + B_y\right)\mathbf{j} + \left(A_z + B_z\right)\mathbf{k} \qquad 2.12$$

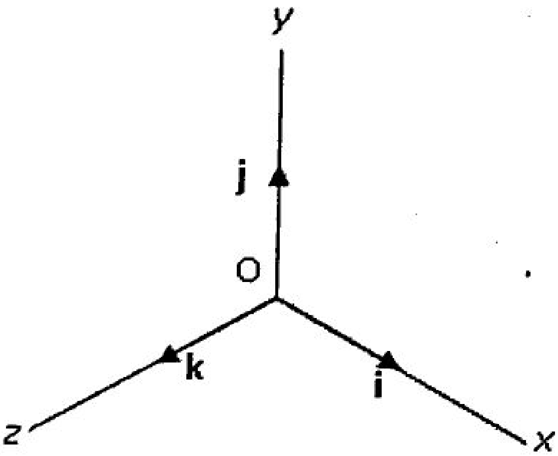



Figure 2.9: The unit vectors **i**, **j** and **k**

## 2.5 Multiplication of Vectors

There are two kinds of multiplication of vectors namely: scalar product and vector product. While the scalar product of two vectors yields a scalar quantity, the vector product of two vectors yields another vector.

The scalar product of two vectors **A** and **B**, denoted as **A**.**B** is defined as

$$\mathbf{A} \cdot \mathbf{B} = |A||B|\cos\theta \qquad 2.13$$

where $\theta$ is the angle between the two vectors (Figure 2.10).

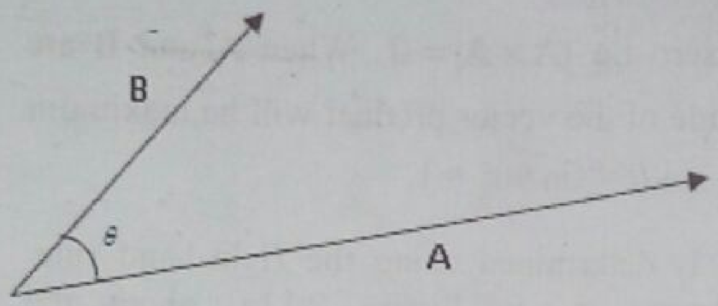



Figure 2.10: Vector **A** making an angle $\theta$ with vector **B**

We can also express scalar or dot product in terms of components as follows:

$$\mathbf{A}\cdot\mathbf{B} = A_x B_x + A_y B_y + A_z B_z \qquad 2.14$$

Combining Equations 2.13 and 2.14, we have:

$$\cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{|A||B|} \qquad 2.15$$

The scalar product $\mathbf{A.B} = |A||B|\cos\theta$ can be positive, negative or zero, depending on the angle between **A** and **B**. When $\theta$ is between 0° and 90°, $\cos\theta > 0$ (i.e $\cos\theta$ is positive) and the scalar product is positive. When $\theta$ is between 90° and 180°, $\cos\theta < 0$ (i.e. $\cos\theta$ is negative) and the scalar product is negative. When $\theta = 90°$, $\cos\theta =$ 0 and the scalar product is equal to zero. Finally, when **A** and **B** are parallel, $\theta = 0°$ or 180°, $\cos\theta = \pm 1$ and $\mathbf{A}\cdot\mathbf{B} = \pm|A||B|$.

The magnitude of the vector product of two vectors **A** and **B**, also called the cross product and denoted by **A** x **B**, is defined as:

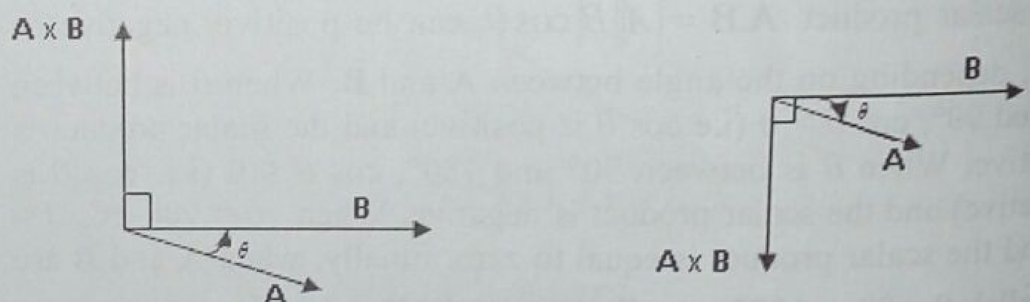
$$|\mathbf{A}\times\mathbf{B}| = |A||B|\sin\theta \qquad 2.16$$

The angle $\theta$ is measured from **A** towards **B** and it is taken to be the smaller of the two possible angles, so $\theta$ ranges from 0° to 180°. When the two vectors are parallel or anti-parallel, i.e. when $\theta = 0°$ or 180°, $|\mathbf{A}\times\mathbf{B}| = 0$. That is, the vector product of two parallel or anti-parallel vectors is always zero. In particular, the vector product of any vector with itself is zero i.e $|\mathbf{A} \times \mathbf{A}| = 0$. When **A** and **B** are perpendicular, the magnitude of the vector product will be maximum i.e. $|\mathbf{A} \times \mathbf{B}| = |A||B|$, since $\sin\theta = \sin 90° = 1$.

The direction of **A** x **B** is determined using the right-hand rule. Imagine rotating the vector **A**, in Figure 2.11a, about the perpendicular line until it is aligned with **B**, choosing the smaller of the two possible angles between **A** and **B**. Curl the fingers of your right hand around the perpendicular line so that the fingertips point in the direction of rotation; your thumb will then point in the direction of **A** x **B**.

Similarly, we can determine the direction of **B** x **A** by rotating **B** into **A** as shown in Figure 2.11b. This figure shows that the vector product is not commutative. That is $\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$. We can also express the vector product in terms of the components as follows:

$$\mathbf{A} \times \mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}) \qquad 2.17$$




(a) The vector product A x B determined by the right-hand rule

(b) B x A = -A x B

Figure 2.11: The vector product **A** x **B** and **B** x **A**.

Expanding the expression and using the multiplication identity for unit vectors, we have:

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} + (A_zB_x - A_xB_z)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k} \qquad 2.18$$

The vector product can also be expressed in determinant form as:

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix} \qquad 2.19$$

Expanding Equation 2.19, we have:

$$\mathbf{A}\times\mathbf{B}=\mathbf{i}\begin{vmatrix}A_y & A_z\\ B_y & B_z\end{vmatrix}-\mathbf{j}\begin{vmatrix}A_x & A_z\\ B_x & B_z\end{vmatrix}+\mathbf{k}\begin{vmatrix}A_x & A_y\\ B_x & B_y\end{vmatrix} \qquad 2.20a$$

$$\mathbf{A}\times\mathbf{B}=\left(A_yB_z-A_zB_y\right)\mathbf{i}+\left(A_zB_x-A_xB_z\right)\mathbf{j}+\left(A_xB_y-A_yB_x\right)\mathbf{k} \qquad 2.20b$$

## Activity 2 Vectors

**2.1.** An athlete ($A$) swims with a velocity of 4m/s perpendicularly across a gently flowing river ($R$) at 3m/s.
Find the resultant velocity of the athlete relative to the river.
A. 4m/s, 53° to $R$-direction
B. 5m/s, 53° to $R$-direction
C. 4m/s, 53° to $A$-direction
D. 5m/s, 53° to $A$-direction

**Solution**

Using parallelogram law of vector addition depicted in Figure 2.12:

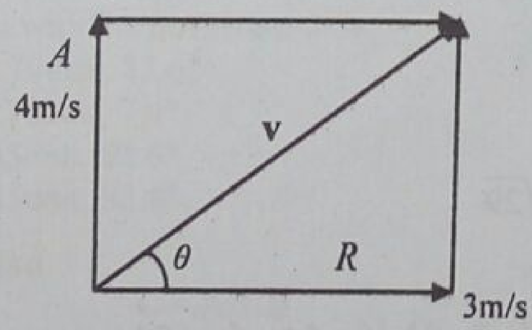



Figure 2.12: Activity 2.1

Using Pythagoras' theorem,

$v = \sqrt{4^2 + 3^2} = 5m/s$

$\tan\theta = \frac{v_y}{v_x} = \frac{4}{3} = 1.333$

$\theta = 53^o$

The correct option is B.

**2.2.** Vectors **A** and **B** are given as **A** = 3**i** + 4**j** -2**k** and **B** = 2**i** – 6**j** + 3**k**. Evaluate **A** x **B**.

A. 24**i** - 5**j** + 20**k**
B. -13**j** - 26**k**
C. 24**i** + 5**j** - 4**k**
D. 24**i** - 5**j** - 4**k**

**Solution**

$$\mathrm{A} \times \mathrm{B} = \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ 3 & 4 & -2 \\ 2 & -6 & 3 \end{vmatrix} = (12-12)\mathrm{i} - (9+4)\mathrm{j} + (-18-8)\mathbf{k} = -13\mathbf{j} - 26\mathbf{k}$$

The correct option is B .

**2.3.** Vectors **A** and **B** are given as **A** = 3**i** + 4**j** -2**k** and **B** = 2**i** – 6**j** + 3**k**. What is the angle between vectors **A** and **B**?

A. 134.4°
B. 73.4°
C. 45.6°
D. 129.5°

**Solution**

$|A| = \sqrt{3^2 + 4^2 + (-2)^2} = \sqrt{29}$

$|B| = \sqrt{2^2 + (-6)^2 + 3^2} = 7$

$\mathbf{A} \cdot \mathbf{B} = (3 \times 2) + (4 \times -6) + (-2 \times 3) = 6 - 24 - 6 = -24$

$\mathbf{A} \cdot \mathbf{B} = |A||B| \cos\theta$

$-24 = 7\sqrt{29}\cos\theta$

$\therefore \cos\theta = -0.6367$

$\theta = 50.5^o \ or \ 129.5^o$

The correct option is D.

**2.4.** Two vectors **A** and **B** are given as follows: **A** = 3.0**i** – 4.5**j** and **B** = -2.5**i** + 5.0**j**. Find the magnitude of their vector sum and the angle it makes with the positive *x*-axis.

A. 0.71N, 90.0°
B. 0.71N, 45.0°
C. 120.5N, 60.0°
D. 120.5N, 35.4°

**Solution**

Let **C** = **A** + **B** = vector sum

**C** = 3.0**i** – 4.5**j** + (-2.5**i** + 5.0**j**) = 0.5**i** + 0.5**j**

$$|C| = \sqrt{0.5^2 + 0.5^2} = \sqrt{0.5} = 0.71$$

$$\tan\theta = \frac{y}{x} = \frac{0.5}{0.5} = 1$$

$$\theta = 45^o$$

The correct option is B.

**2.5.** The resultant of two vectors **A** and **B** is vector **C** = 2.2**i** + 3.4**j**. If vector **A** = 1.5**i** - 2.0**j**, find the magnitude of **B** and the angle it makes with the positive *x*-axis.

A. 29.70unit, 82.6°
B. 29.70unit, 7.4°
C. 2.45unit, 82.6°
D. 5.45unit, 82.6°

**Solution**

$$\mathbf{C} = \mathbf{A} + \mathbf{B}$$

$$2.2\mathbf{i} + 3.4\mathbf{j} = 1.5\mathbf{i} - 2.0\mathbf{j} + \mathbf{B}$$

$$\mathbf{B} = 0.7\mathbf{i} + 5.4\mathbf{j}$$

$$|B| = \sqrt{0.7^2 + 5.4^2} = \sqrt{0.49 + 29.16} = 5.45\,unit$$

$$\tan\theta = \frac{y}{x} = \frac{5.4}{0.7} = 7.714$$

$$\theta = \tan^{-1}(7.714) = 82.6^o$$

The correct option is D.

**2.6.** Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are acting on a block as shown in the Figure 2.13. Find the resultant force in unit vector notation.

A. 6.5**i** + 7.0**j** B. 5.0**i** + 3.5**j**
C. 6.5**i** – 7.0**j** D. 7.0**i** + 6.5**j**

**Solution**

Resolve the forces into *x* and *y* components as follows:
$F_{1x} = 4\cos 30^o = 3.464N$, $F_{1y} = 4\sin 30^o = 2.0N$,
$F_{2x} = 0$, $F_{2y} = 5N$
$F_{3x} = 3N$, $F_{3y} = 0$

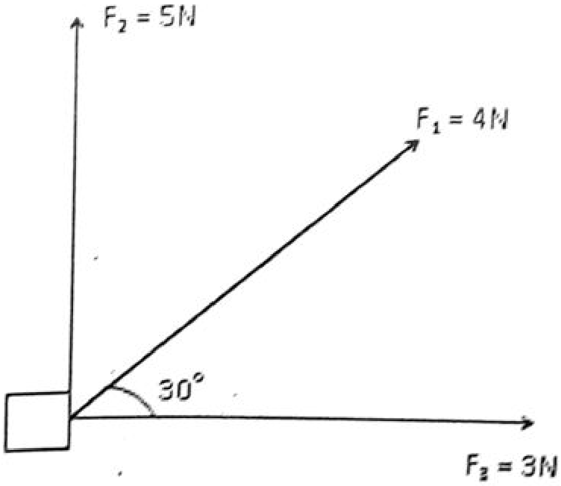



Figure 2.13: Activity 2.6

**Total *x* component $F_x = F_{1x} + F_{2x} + F_{3x} = (3.464 + 0 + 3)N = 6.464N$**
**Total *y* component $F_y = F_{1y} + F_{2y} + F_{3y} = (2 + 5 + 0)N = 7N$**
**Resultant force $\mathbf{F}_R = F_x\mathbf{i} + F_y\mathbf{j} = (6.5\mathbf{i} + 7.0\mathbf{j})N$**
**The correct option is A.**

**2.7.** Given two vectors **A** = 2**i** + 3**j** + 4**k** and **B** = **i** – 2**j** + 3**k**. What is the magnitude of each vector?

A. 4, 7  B. 6.4, 4.0  C. 5.4, 3.7  D. 29, 14

**Solution**

$\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$

$|A| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4+9+16} = \sqrt{29} = 5.4$

$\mathbf{B} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$

$|B| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14} = 3.7$

The correct option is C.

**2.8.** A body moves in an *x-y* plane with velocity *v*m/s. The components of $v$ are $v_x$ = 5m/s and $v_y$ = 7m/s along *x* and *y* axes respectively. What is the magnitude and direction of the velocity?

A. 9.6m/s, 54.5°
B. 8.4m/s, 54.5°
C. 8.6m/s, 54.5°
D. 9.6m/s, 20.0°

**Solution**

Magnitude:

$$v = \sqrt{v_x^2 + v_y^2} = \left(\sqrt{5^2 + 7^2}\right) m/s = 8.6 m/s$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{7}{5}$$

$$\theta = \tan^{-1}\left(\frac{7}{5}\right) = 54.5^o$$

The correct option is C.

**2.9.** Given that a vector **A** of magnitude 4 units, lying in the *x-y* plane makes an angle of 30° with the *x*-axis, as shown in Figure 2.14. Calculate its components along the *x*, *y* and *z* axes ($A_x$, $A_y$, $A_z$).

A. $2\sqrt{3}, 2, 0$

B. $0, 2, 4\sqrt{3}$

C. $4, 0, 4\sqrt{3}$

D. $3\sqrt{4}, 2, 0,$

E. $\sqrt{4}, 2\sin 30, 0$

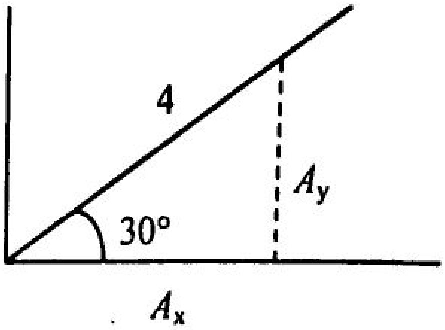



Figure 2.14: Activity 2.9

**Solution**

$$\sin 30 = \frac{A_y}{8}$$

$$A_y = 8 \sin 30 = 4$$

$$\cos 30 = \frac{A_x}{8}$$

$$A_x = 8 \cos 30 = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$$

$$A_z = 0$$

The correct option is A.

**2.10.** Find the angle between two vectors **A** = 2**i** + 3**j** + 4**k** and **B** = **i** – 2**j** + 3**k**.

A. 46.6° B. 56.0° C. 39.7° D. 0.397° E. 66.6°

**Solution**

$$\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}, \ \mathbf{B} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$$

$$|A| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$$

$$|B| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$$

$$\mathbf{A} \cdot \mathbf{B} = |A|\,|B| \cos\theta$$

$$(2 \times 1) + (3 \times -2) + (4 \times 3) = \sqrt{29} \times \sqrt{14} \cos\theta$$

$$2-6+12=\sqrt{406}\cos\theta$$

$$\frac{8}{\sqrt{406}}=\cos\theta$$

$$\cos\theta=0.39703$$

$$\theta=66.61''$$

The correct option is E.

**2.11.** Vector **A** has magnitude 8 units in the direction of the positive $x$–axis, vector **B** has magnitude $6\sqrt{3}$ units and lies in the $x$-$y$ plane, making an angle 30° with the positive $x$-axis and 60° with the positive $y$-axis. Find the vector product **A** x **B**.

A. $24\sqrt{3}\mathbf{k}$ B. $24\sqrt{3}\mathbf{i}$ C. 24**k** D. 24**i** E. 24**i** + 24**k**

**Solution**

Resolve the vectors into two perpendicular components: $A_x = 8$ units, $A_y = 0$

$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} = 8\mathbf{i} + 0\mathbf{j}$

$$B_x = 6\sqrt{3}\cos 30'' = 9 \text{ units}$$

$$B_y = 6\sqrt{3}\sin 30'' = 3\sqrt{3} \text{ units}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} = 9\mathbf{i} + 3\sqrt{3}\mathbf{j}$$

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0 & 0 \\ 9 & 3\sqrt{3} & 0 \end{vmatrix} = 24\sqrt{3}\mathbf{k}$$

The correct option is A.

**2.12.** When a body moves in the $x$-$y$ plane, its velocity may be expressed in terms of the components $v_x$ and $v_y$. What is the angle between them? If $v_x$ is 5m/s and $v_y$ is 7m/s, what is the value of the angle between them?

A. $\theta = \tan v_y/v_x$, 0.02°

B. $\theta = \cos^{-1} v_x/v_y$, 44.4°
C. $\theta = \tan^{-1} v_x/v_y$, 35.5°
D. $\theta = \sin^{-1} v_x/v_y$, 45.6°
E. $\theta = \tan^{-1} v_y/v_x$, 54.5°

**Solution**

$\tan \theta = v_y/v_x$

$$\theta = \tan^{-1} v_y/v_x$$

$$\theta = \tan^{-1} v_y/v_x = \tan^{-1}(7/5) = 54.5°$$

The correct option is E.

**2.13.** If **A** = 2**i** +3**j**, **B** = 3**i** + 5**j** and **C** 4**i** - 6**j**, find **A** x **B** x **C**.
A. -114**i** - 76**j**
B. 6**i** - 4**j**
C. 114**i** + 76**j**
D. 6**i** + 4**j**

**Solution**

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 0 \\ 3 & 5 & 0 \end{vmatrix} = \mathbf{i}\begin{vmatrix} 3 & 0 \\ 5 & 0 \end{vmatrix} - \mathbf{j}\begin{vmatrix} 2 & 0 \\ 3 & 0 \end{vmatrix} + \mathbf{k}\begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} = \mathbf{k}$$

$$(\mathbf{A}\times\mathbf{B})\times\mathbf{C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1 \\ 4 & -6 & 0 \end{vmatrix} = 6\mathbf{i} + 4\mathbf{j}$$

The correct option is D.

## Summary of Chapter 2

In chapter 2, you have learned that:

1. Scalar quantities are quantities that can be described completely by their magnitudes without directions. Examples of scalars include: mass, area, volume, density, time, temperature, work, energy, power, etc. Vector quantities are

quantities that can be described by their magnitudes and directions. Examples of vectors include: displacement, velocity, acceleration, force, weight, momentum, etc.

2. Vectors can be represented with boldface letter such as **A**, use of an arrow over a letter such as $\vec{A}$ or a line with an arrowhead at its tip.

3. A vector **A** lying in the *x-y* plane and making an arbitrary angle $\theta$ with positive $x$ axis has components:

   $A_x = A\cos\theta$ and $A_y = A\sin\theta$, with $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$.

4. A single vector that would have the same effect as two or more vectors taken together is called a resultant vector (**R**). Several methods are employed in the addition of vectors: polygon method, parallelogram method. A unit vector is a dimensionless vector that has a magnitude of 1.

5. A vector **A** can be written as $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$; the unit vectors are **i**, **j**, and **k**; $A_x$, $A_y$ and $A_z$ are the components of vector **A** along the $x$, $y$ and $z$ axes respectively.

6. The scalar product of two vectors **A** and **B**, denoted as **A.B**, is defined as $\mathbf{A}\cdot\mathbf{B} = |A||B|\cos\theta$ where $\theta$ is the angle between the two vectors.

7. We can also express scalar or dot product in terms of components as follows: $\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$.

8. The magnitude of the vector product of two vectors **A** and **B** is defined as $|\mathbf{A}\times\mathbf{B}| = |A||B|\sin\theta$.

9. The vector product can also be expressed in determinant form as:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

## Self-Assessment Questions (SAQs) for Chapter 2

**2.1.** A car is travelling eastward at a speed of 40m/s. But a 30m/s wind is blowing southward. What is the direction and speed of the car relative to its original direction?

**2.2.** Two forces act on a block of mass 2kg as follows: 100N at 170.0° and 100N at 50.0°. Find the resultant force.

**2.3.** Find the magnitude of $\mathbf{R} = 8\mathbf{i} - 12\mathbf{j}$ and the angle it makes with the positive $x$ axis.

**2.4.** A man walks 1 km due east and then 1 km due north. His displacement is:

A. 1km N15°E
B. 1km N30°E
C. √2km N45°E
D. √2km N60°E

**2.5.** Given two vectors $\mathbf{A} = 6\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{B} = 4\mathbf{i} - 5\mathbf{j} + 8\mathbf{k}$, find the magnitude of the vector $2\mathbf{A} - 3\mathbf{B}$.

**2.6.** Find the angle between the two vectors $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = -4\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

**2.7.** Three vectors are given by $\mathbf{A} = 3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $\mathbf{B} = -\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$ and $\mathbf{C} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$. Find $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$.

**2.8.** Which of the following physical quantities is NOT a vector?
A. velocity

B. speed
C. force
D. electric field intensity

**2.9.** Which of the following is NOT a vector quantity?
A. Force
B. Acceleration
C. Weight
D. Distance

**2.10.** Which of the following is a vector quantity?
A. Force
B. Altitude
C. Mass
D. Distance

**2.11.** Which of the following is a scalar quantity?
A. Force
B. Altitude
C. Weight
D. Displacement

**2.12.** Which of the following is a set of vectors?
A. Force, mass and moment
B. Acceleration, velocity and momentum
C. Mass, weight and density
D. Mass, moment and density

**2.13.** Which of the following are vector quantities? I. Speed II. Displacement III. Acceleration IV. Electric field intensity V. Magnetic induction
A. I, II and III only
B. II, III and IV only
C. III, IV and V only
D. II, III, IV and V only

**2.14.** Which of the following is a set of vectors?

A. Weight, displacement and momentum
B. Velocity, volume and upthrust
C. Density, capacitance and distance
D. Mass, force and impulse

**2.15.** Which of the following physical quantities is a vector?
A. displacement
B. current
C. work
D. area

# CHAPTER THREE

# KINEMATICS

## 3.0 Introduction

Mechanics, a branch of physics, is usually divided into two parts: kinematics and dynamics. Kinematics deals with the mathematical description of the motion of objects without consideration of what causes the motion. Dynamics, on the other hand, study the causes of the motion. We will begin our study of mechanics with the kinematics of particles. A particle is a body of negligible size and internal structure.

## 3.1 Motion in one dimension

Motion in one dimension refers to the motion of an object in a straight line along the $x$, $y$ or $z$ axis. Discussed below are some terms used in describing motion.

### 3.1.1 Displacement

Consider a particle moving in a straight line, say along the $x$-axis, from an initial position $x_i$ to a final position $x_f$; the distance $\Delta x$ covered by the particle would be:

$$\Delta x = x_f - x_i \qquad 3.1$$

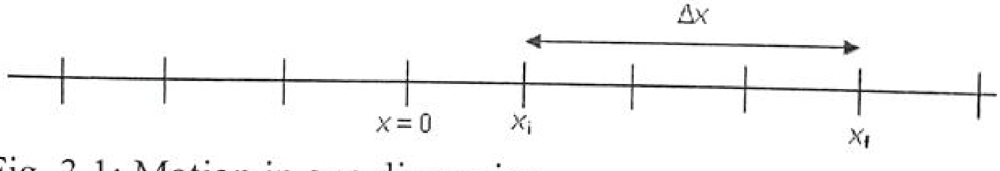



Fig. 3.1: Motion in one dimension

We can define the displacement of the particle as the straight-line distance between two points, along with the direction from the starting point to the final position. The displacement is a vector of magnitude equal to $\Delta x$, and in the simple case of one-dimensional motion, the direction of the displacement is in the direction of the x-axis along which the motion is allowed.

### 3.1.2 Speed/Velocity

Speed is a scalar quantity. Speed can be defined as the rate of change of distance with time. There are two ways in which the speed of a particle is defined: average speed and instantaneous speed. Suppose that a particle travels a distance $\Delta x$ in a time interval $\Delta t$, the average speed denoted as $v_{av}$ of the particle is defined as:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad 3.2$$

The instantaneous speed $v(t)$ of the particle is defined as the limit of $v_{av}$ as the time interval $\Delta t$ tends to zero or:

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx(t)}{dt} \qquad 3.3$$

Velocity is a vector quantity. Velocity is defined as the rate of change of displacement with time. That is,

$$\vec{v}_{av} = \frac{total\ displacement}{time\ taken} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} \qquad 3.4$$

where $\vec{r}_f$ *and* $\vec{r}_i$ are position vectors at time $t_f$ and $t_i$ respectively. Note that average speed is not the magnitude of average velocity.

The instantaneous velocity $\vec{v}(t)$ of a particle is defined as the limit of $\vec{v}_{av}$ as the time interval $\Delta t$ tends to zero, or

$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}(t)}{dt} \qquad 3.5$$

The terms "velocity" and "speed" have distinct definitions in Physics and care must be taken not to confuse one for the other. Instantaneous speed measures how fast a particle is moving at a particular time; instantaneous velocity measures how fast a particle is moving in a specific direction at a particular time.

### 3.1.3 Acceleration

Acceleration is defined as the time rate of change of velocity. It is a vector quantity. The average acceleration can be written as

$$a_{av} = \frac{change\ in\ velocity}{time\ taken} \qquad 3.6$$

If the velocity of a particle changes from $v_i$ to $v_f$ in time interval $\Delta t$, the magnitude of the average acceleration is:

$$a_{av} = \frac{v_f - v_i}{\Delta t} \qquad 3.7$$

The instantaneous acceleration $a(t)$ is the limit of the average acceleration as the time interval approaches zero. That is:

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}(t)}{dt} = \frac{d^2 \vec{r}(t)}{dt^2} \qquad 3.8$$

The magnitude of $\vec{a}(t)$ is written as:

$$a(t) = \left|\vec{a}(t)\right| = \left|\frac{d\vec{v}(t)}{dt}\right| \qquad 3.9$$

From Equation 3.5 and 3.8, we can write:

$$r(t) = \int_{t_i}^{t_f} v(t)dt + c \qquad 3.10$$

and

$$v(t) = \int_{t_i}^{t_f} a(t)dt + c \qquad \text{where } c \text{ is a constant.} \qquad 3.11$$

## 3.2 Motion with Constant Acceleration in a Straight Line

A particle is said to be undergoing a constant acceleration motion if the rate of change of acceleration is constant throughout the motion. Consider a particle moving with constant acceleration $a$ in a straight line, say the $x$-axis, with its velocity changing from $v_o$ to $v$ in a time interval $t$. The acceleration of the motion is given from Equation 3.7 as:

$$a_{av} = a = \frac{v - v_o}{t}$$ (Let $a_{av} = a$, since the acceleration is constant)

Rearranging, we have:

$$v = v_o + at \qquad 3.12$$

The distance, $x$ travelled by the particle is given as:

$$x = \left(\frac{average\ speed}{2}\right) \times t$$

$$x = \left(\frac{v + v_o}{2}\right) \times t \qquad 3.13$$

Substitute Equation 3.12 into Equation 3.13, we have:

$$x = \left(\frac{v_o + at + v_o}{2}\right) \times t$$

$$x = v_o t + \frac{1}{2}at^2 \qquad 3.14$$

Eliminate $t$ from Equations 3.12 and 3.13, we have:

$$v^2 = v_o^2 + 2ax \qquad 3.15$$

Note that Equations 3.12, 3.13, 3.14 and 3.15 are for constant acceleration motion only.

## 3.3 Freely Falling Bodies

A body falling under the influence of the Earth's gravitational attraction is an example of motion with constant acceleration in a straight line, this time along the $y$ axis. According to experiments performed by scientists, all bodies at a particular location fall with the same downward acceleration, regardless of their size or weight. This kind of motion is known as free fall and the acceleration due to gravity ($g$) has an approximate magnitude of $g = 9.80\text{m/s}^2$.

It is customary to use $y$ to represent the vertical direction and to give downward motion a positive acceleration due to gravity. Since the acceleration due to gravity is always downward, any motion in the downward direction is in the direction of acceleration due to gravity and hence its acceleration is taken as positive and any motion in upward direction is taken as negative.

Suppose that a ball is dropped from a height of 30 m above the ground. The initial velocity $v_o = 0$, and the acceleration $a = g = 9.80\text{m/s}^2$. The time taken to reach the ground is calculated as follows:

$$y = v_o t + \frac{1}{2}at^2$$

$$30 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$$

$$t = 2.47s$$

If, however, the ball is thrown vertically upward with an initial velocity of $30\text{m/s}$, the acceleration $a = g = -9.8\text{m/s}^2$ and the velocity at maximum height reached is $v = 0$, since the body comes momentarily to rest at the maximum height. The time taken to reach the maximum height is calculated as follows:

$$0 = 30 - 9.8t$$

$$t = 3.06s$$

## 3.4 Motion in Two or Three Dimensions

In sections 3.2 and 3.3 we described the motion of an object in a straight line along one of the Cartesian axes ($x$ or $y$). To describe the motion of a particle in space, we must first be able to describe the

particle's position. The position vector $\vec{r}$ of a particle at a point $P$ at a certain instant is a vector that goes from the origin of the coordinate system to the point $P$ (Figure 3.2). Using the unit vector of Section (2.4), we can write

$$\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \qquad 3.16$$

where the $x$-, $y$-, and $z$-components of vector $\vec{r}$ is the Cartesian coordinates $x$, $y$, and $z$ of point $P$.

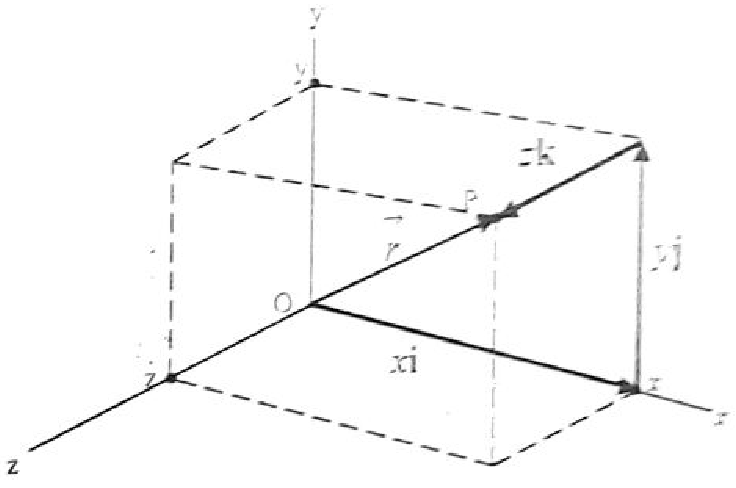



Figure 3.2: The position vector $\vec{r}$ from the origin to point $P$.

The change in position as the particle moves from $P_1$ to $P_2$ during a time interval $\Delta t$ is given as:

$$\Delta\vec{r} = \vec{r_2} - \vec{r_1} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k} \qquad 3.17$$

where $\vec{r_1}$ *and* $\vec{r_2}$ are the position vectors of $P_1$ and $P_2$, respectively.

The average velocity vector $\vec{v}_{av}$ during the time interval is defined as:

$$\vec{v}_{av} = \frac{total\ displacement}{time\ taken} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

$$\vec{v}_{av} = \frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\Delta t} \qquad 3.18$$

The instantaneous velocity $\vec{v}$ is given as:

$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}(t)}{dt}$$

$$\vec{v}(t) = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k}$$

$$\vec{v}(t) = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k} \qquad 3.19$$

The magnitude of the instantaneous velocity vector $\vec{v}$ - that is, the speed – is given as:

$$\left|\vec{v}\right| = v = \sqrt{v_x^2 + v_y^2 + v_z^2} \qquad 3.20$$

If the particle is moving in the *xy*-plane then the speed (the magnitude of $\vec{v}$ ) is:

$$v = \sqrt{v_x^2 + v_y^2} \qquad 3.21$$

and the direction of the instantaneous velocity $\vec{v}$ is given by the angle $\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right)$.

The instantaneous acceleration vector is:

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}(t)}{dt} = \frac{d^2 \vec{r}(t)}{dt^2}$$

$$\vec{a}(t) = \frac{dv_x}{dt}\mathbf{i} + \frac{dv_y}{dt}\mathbf{j} + \frac{dv_z}{dt}\mathbf{k} \qquad 3.22$$

Let us consider two-dimensional motion during which the acceleration remains constant in both magnitude and direction. Because $a$ is assumed constant, its components $a_x$ and $a_y$ also are constants. Therefore, we can apply the equations of kinematics to the $x$ and $y$ components of the velocity and the displacement vectors:

$$v_x = v_{ox} + a_x t \qquad 3.23$$

$$v_y = v_{oy} + a_y t \qquad 3.24$$

$$x = \left(\frac{v_x + v_{ox}}{2}\right)t \qquad 3.25$$

$$y = \left(\frac{v_y + v_{oy}}{2}\right)t \qquad 3.26$$

$$x = v_{ox}t + \frac{1}{2}a_x t^2 \qquad 3.27$$

$$y = v_{oy}t + \frac{1}{2}a_y t^2 \qquad 3.28$$

$$v_x^2 = v_{ox}^2 + 2a_x x \qquad 3.29$$

$$v_y^2 = v_{oy}^2 + 2a_y y \qquad 3.30$$

where $v_x$, $v_{ox}$, $a_x$ and $x$ represent $x$-component of velocity after time $t$, $x$-component of initial velocity, $x$-component of acceleration and displacement along the $x$ axis, respectively; $v_y$, $v_{oy}$, $a_y$ and $y$ represent $y$-component of velocity after time $t$, $y$-component of initial velocity, $y$-component of acceleration and displacement along the $y$ axis, respectively.

## 3.5 Projectile Motion

A projectile is a body that is given an initial velocity and then follows a path determined completely by the effects of gravitational acceleration and air resistance. A golf ball struck by a club, a thrown football and a bullet shot from a rifle are all projectiles. The path followed by a projectile is called its trajectory. A projectile motion is two-dimensional and we can treat the *x*- and *y*- coordinates separately. The *x*-component of acceleration due to gravity *g* is zero since *g* is acting vertically downward; the *y*-component is constant and equal to -*g*. Since the *x*-acceleration and *y*-acceleration are both constant, we can use Equations 3.23 -3.30 for projectile motion.

Consider a particle launched at point O ($x_o = 0$, $y_o = 0$) with velocity $v_{ox}$ and $v_{oy}$ at time $t = 0$. The initial velocity $v_o$ makes an angle $\alpha_o$ with the horizontal, as shown in Figure 3.3. The components of acceleration are $a_x = 0$, $a_y = -g$. For the horizontal or *x*-motion, we have,

$$v_x = v_{ox} \qquad 3.31$$

$$x = v_{ox}t \qquad 3.32$$

For the vertical or *y*-motion, we have:

$$v_y = v_{oy} - gt \qquad 3.33$$

$$y = v_{oy}t - \frac{1}{2}gt^2 \qquad 3.34$$

In projectile motion, the horizontal component of the velocity, $v_x$, is constant and the vertical component of the velocity, $v_y$, changes by equal amounts in equal times. Can you explain these using Equations 3.23 and 3.24?

Resolving the initial velocity into *x*- and *y*-components, we have:

$$v_{ox} = v_o\cos\alpha_o \qquad v_{oy} = v_o\sin\alpha_o \qquad 3.35$$

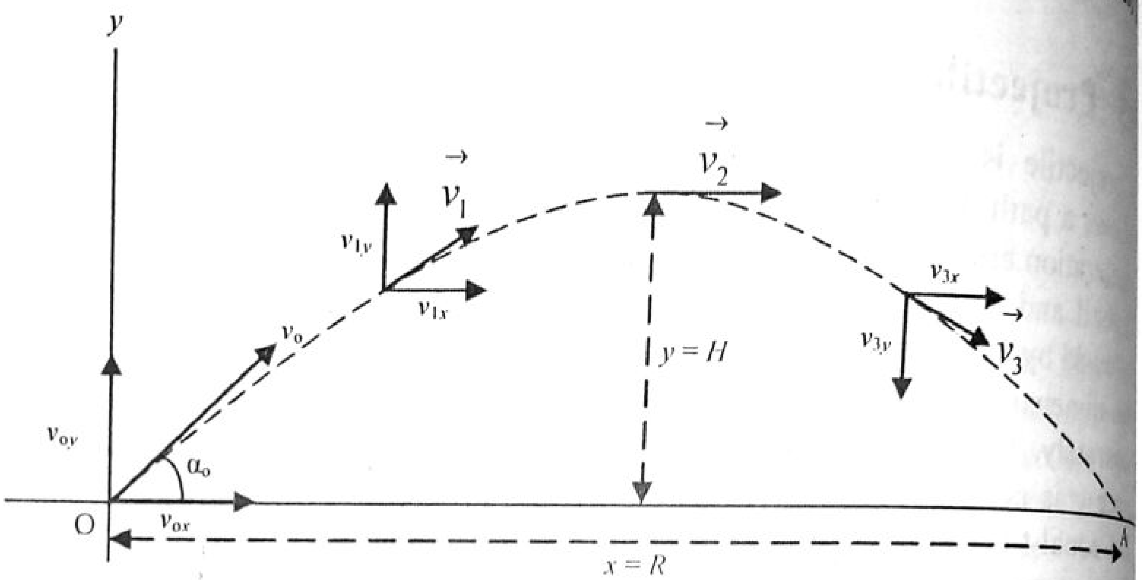



Figure 3.3: Projectile motion

At the highest point in the projectile's path, $v_2 = v_x = v_y = 0$, using Equation 3.33,

$$0 = v_o \sin\alpha_o - gt$$

$$t = \frac{v_o \sin\alpha_o}{g} \qquad 3.36$$

Equation 3.36 is the expression of the time taken to reach maximum height, $H$. The time taken for the projectile to travel from O to A is usually referred to as the time of flight, $T = 2t$. Substituting Equation 3.36 into 3.34,

$$y = H = v_o \sin\alpha_o \times \frac{v_o \sin\alpha_o}{g} - \frac{1}{2} g \times \left(\frac{v_o \sin\alpha_o}{g}\right)^2$$

$$H = \frac{v_o^2 \sin^2\alpha_o}{2g} \qquad 3.37$$

To derive an expression for the maximum horizontal distance, $R$, we substitute Equation 3.36 into 3.32:

$$x = R = v_o \cos\alpha_o \times \frac{2v_o \sin\alpha_o}{g}$$

$$R = \frac{2v_o^2 \sin\alpha_o \cos\alpha_o}{g} = \frac{v_o^2 \sin 2\alpha_o}{g} \quad 3.38$$

We can derive an equation for the trajectory's shape in terms of $x$ and $y$. From Equations 3.32 and 3.35, we have,

$$t = \frac{x}{v_o \cos\alpha_o} \quad 3.39$$

Substitute Equation 3.39 into 3.34:

$$y = v_o \sin\alpha_o \frac{x}{v_o \cos\alpha_o} - \frac{1}{2} g \times \left(\frac{x}{v_o \cos\alpha_o}\right)^2$$

$$y = (\tan\alpha_o)x - \frac{g}{2v_o^2 \cos^2\alpha_o} x^2 \quad 3.40$$

Equation 3.40 can be written simply as $y = bx - ax^2$, where $a$ and $b$ are constants since $v_o$, $\tan\alpha_o$, $\cos\alpha_o$ and $g$ are constants. This is the equation of a parabola. In projectile motion, the trajectory is always a parabola.

## 3.6 Horizontal Projectile Motion

Suppose that an object is thrown horizontally with an initial velocity $v_{ox}$ (Figure 3.4). In this motion, like the one described in Section 3.5, the horizontal acceleration ($a_x = 0$) and the vertical acceleration ($a_y = +g = 9.80\text{m/s}^2$). The acceleration due to gravity is taken as positive since the object is travelling downward. Throughout the object's path, the horizontal velocity remains constant: $v_x = v_{ox} = v_{1x} = v_{2x}$ and the initial vertical velocity $v_{oy} = 0$ since the initial velocity has no component along the $y$-axis.

For the horizontal or $x$-motion, we have:

$$v_x = v_{ox} \quad 3.41$$

$$x = v_{ox} t \quad 3.42$$

For the vertical or $y$-motion, we have:

$$v_y = gt \qquad 3.43$$

$$y = \frac{1}{2}gt^2 \qquad 3.44$$

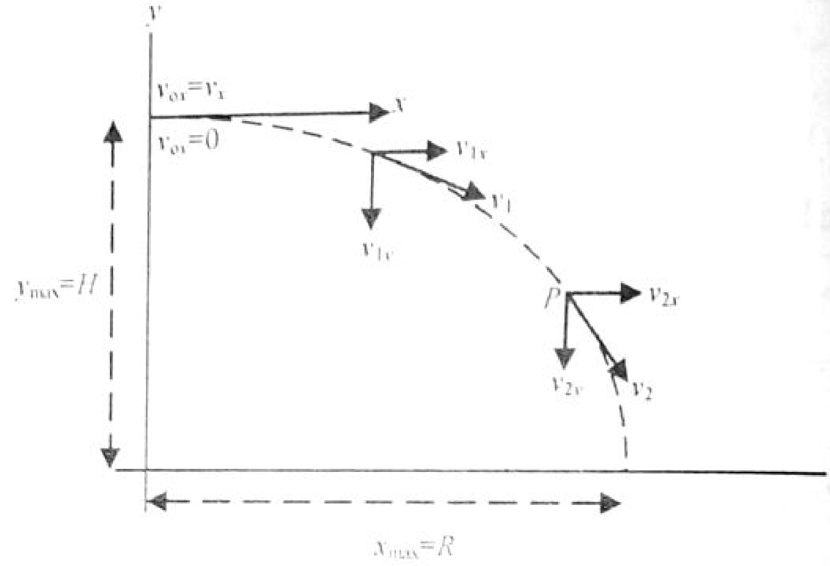



Figure 3.4: Horizontal projectile motion

For instance, if the projectile reaches point $P$ after time $t = t_2$, the the position and velocity of the projectile at $P$ is as follows:

The horizontal component of the displacement after time $t = t_2$ is:

$$x_2 = v_{ox}t_2$$

The vertical component of the displacement after time $t = t_2$ is:

$$y_2 = \frac{1}{2}gt_2^2$$

Hence the displacement is:

$$r = \sqrt{x_2^2 + y_2^2} \text{ and } \alpha = \tan^{-1}\left(\frac{y_2}{x_2}\right)$$

The horizontal component of the velocity after time $t = t_2$ is:

$$v_{2x} = v_{ox}$$

For the vertical or $y$-motion, we have:

$$v_y = gt \quad 3.43$$

$$y = \frac{1}{2}gt^2 \quad 3.44$$

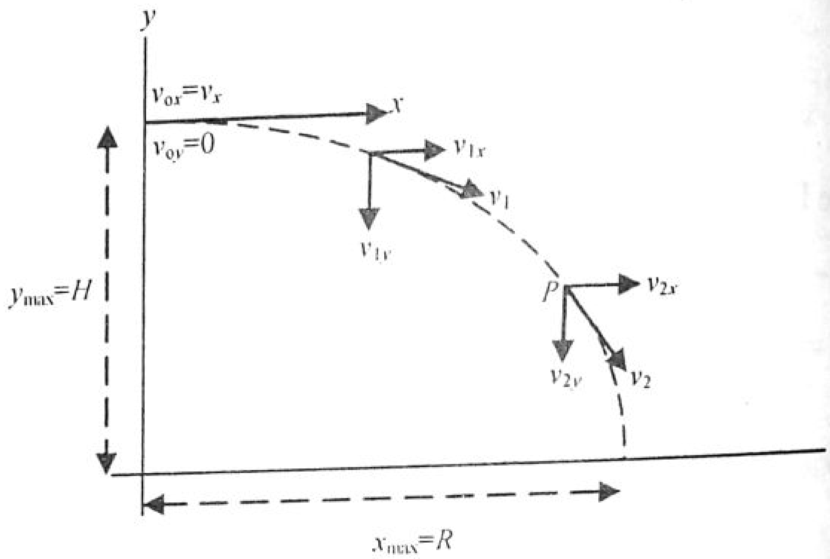



Figure 3.4: Horizontal projectile motion

For instance, if the projectile reaches point $P$ after time $t = t_2$, then the position and velocity of the projectile at $P$ is as follows:

The horizontal component of the displacement after time $t = t_2$ is:

$$x_2 = v_{ox}t_2$$

The vertical component of the displacement after time $t = t_2$ is:

$$y_2 = \frac{1}{2}gt_2^2$$

Hence the displacement is:

$$r = \sqrt{x_2^2 + y_2^2} \text{ and } \alpha = \tan^{-1}\left(\frac{y_2}{x_2}\right)$$

The horizontal component of the velocity after time $t = t_2$ is:

$$v_{2x} = v_{ox}$$

The vertical component of the velocity after time $t = t_2$ is:

$$v_{2y} = gt_2$$

Therefore the velocity is:

$$v = \sqrt{v_{2x}^2 + v_{2y}^2} \text{ and } \alpha = \tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right)$$

## Activity 3 Kinematics

**3.1.** A train changes its position ($x$) as a function of time ($t$) as follows: $x = 20 + 5.0\,t^2$, with $x$ in meters. (a) Find the displacement of the train between $t_1$ = 1.0s and $t_2$ = 2.0s. (b) Find the average velocity during the same interval. (c) Find the instantaneous velocity at time $t_1$ = 3.0s.

**Solution**

(a) At time $t_1$ = 1.0s the displacement is

$$x_1 = \left(20 + 5.0t^2\right)m = \left(20 + 5.0 \times 1^2\right)m = 25m$$

At time $t_2$ = 2.0s the displacement is

$$x_2 = \left(20 + 5.0t^2\right)m = \left(20 + 5.0 \times 2^2\right)m = 40m$$

The displacement during the time interval is

$$\Delta x = x_2 - x_1 = (40 - 25)m = 15m$$

(b) The average velocity during this interval is:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(40-25)m}{(2-1)s} = 15m/s$$

(c) Instantaneous velocity is:

$$v = \frac{dx}{dt} = 10t$$

At $t = 3.0s$, $v = 10 \times 3.0 m/s = 30 m/s$

**3.2.** The acceleration with which a particle moves is given as $a(t) = \left(-3 + 2t^2\right)m/s^2$. Find its velocity at time $t = 3s$, given that the velocity at $t = 0$ is zero.

A. 15m/s B. -3m/s C. 9m/s D. 5m/s

**Solution**

Velocity, $v = \int a dt = \int \left(-3 + 2t^2\right)dt = -3t + \frac{2t^3}{3} + c$

At $t = 0$, $v = 0$,

$0 = 0 + 0 + c$

$c = 0$

Hence, $v = -3t + \frac{2t^3}{3}$

At $t = 3s$

$$v = \left(-3 \times 3 + \frac{2 \times 3^3}{3}\right)m/s = 9m/s$$

The correct option is C.

**3.3.** The displacement $s(t)$ of a car as a function of time $t$ is given as $s(t) = \left(3t^2 - 2t + 1\right)m$. Find its velocity at time $t = 4s$. At what time will the velocity of the body become zero?

A. 12m/s, 0 B. 22m/s, 0.333s C. 12m/s, 0.333s D. 10m/s, 0

**Solution**

Velocity, $v = \frac{ds}{dt} = 6t - 2$

At time $t = 4s$, $v = \frac{ds}{dt} = (6 \times 4 - 2)m/s = 22m/s$

Time at which the velocity of the body becomes zero:

$v = 6t - 2 = 0$

$t = 0.333s$

The correct option is B.

**3.4.** The position of a particle is given by $x(t) = (t^3 - 3t^2 + 5)m$ where $x$ is in meters and $t$ is in seconds. Calculate the position of the particle at $t$ = 4s and the average velocity for the time interval $t$ = 1s to 6s.

A. 4m, 8m/s B. 5m, 10m/s C. 8m, 44m/s D. 21m, 22m/s

**Solution**

The position at 4s

$$x(t) = (t^3 - 3t^2 + 5)m = (4^3 - 3 \times 4^2 + 5)m = 21m$$

Average velocity

$$v = \frac{total\ displacement}{time\ taken}$$

$$v = \left(\frac{x(6) - x(1)}{6 - 1}\right)m/s = \left(\frac{113 - 3}{5}\right)m/s = 22m/s$$

The correct option is D.

**3.5.** The position of a projectile travelling in two dimensional space is given by $x(t) = (2t^2 + 4t + 3)m$ and $y(t) = (3t^2 - 5t + 2)m$. Calculate the magnitude of the projectile's displacement and the average velocity between time interval $t$ = 2s and $t$ = 5s.

A. 58.0m, 29m/s B. 72.2m, 24.1m/s

C. 58.0m, 10.31m/s D. 29.0m, 19.31m/s

**Solution**

Displacement along $x$

$$s_x = x(5) - x(2)$$

$$s_x = (2(5)^2 + 4(5) + 3 - [2(2)^2 + 4(2) + 3])m = 54m$$

$$s_y = y(5) - y(2)$$

$$s_y = \left(3(5)^2 - 5(5) + 2 - \left[3(2)^2 - 5(2) + 2\right]\right)m = 48m$$

Hence, object displacement is:

$$s = \sqrt{s_x^2 + s_y^2} = \sqrt{54^2 + 48^2}\,m = 72.2m$$

Average velocity along $x$:

$$v_x = \frac{x(5) - x(2)}{5-2} = \frac{54}{3}m/s = 18m/s$$

Average velocity along y:

$$v_y = \frac{y(5) - y(2)}{5-2} = \frac{48}{3}m/s = 16m/s$$

Average velocity, $v = \sqrt{18^2 + 16^2}\,m/s = \sqrt{580}\,m/s = 24.1m/s$

The correct option is B.

**3.6.** The acceleration of a moving object is equal to the:
A. gradient of a displacement-time graph
B. gradient of a velocity-time graph
C. area below a speed-time graph
D. area below a displacement-time graph

**Solution**

The correct option is B.

**3.7.** The position of a particle moving along y-axis is given by $y(t) = (t^3 - 4t^2 + 6t)m$, where $y$ is in meters and $t$ is in seconds. What is the object's displacement and average velocity for the time interval from $t = 3$s to $t = 5$s?

A. 46.0m, 5.75m/s
B. 46.0m, 23.0m/s
C. 2.0m, 1.0m/s
D. 2.0m, 0.25m/s

**Solution**

$y(t) = (t^3 - 4t^2 + 6t)m$
$y(3) = (3^3 - 4\times 3^2 + 6\times 3)m = 9m$
$y(5) = (5^3 - 4\times 5^2 + 6\times 5)m = 55m$

Displacement: $y = y(5) - y(3) = (55 - 9)m = 46m$

Average velocity:

$$v = \frac{y(5) - y(3)}{5-3} m/s = \left(\frac{55-9}{2}\right) m/s = \frac{46}{2} m/s = 23.0 m/s$$

The correct option is B.

**3.8.** The position of a particle moving along x-axis is given by $x(t) = (2t^3 - 6t + 5)m$, where $x$ is in meters and $t$ is in seconds. Find the acceleration of the particle at $t$ = 2.0s. Is the velocity constant or changing with time?

A. 24.0m/s$^2$, changing
B. 6.0m/s$^2$, changing
C. 24.0m/s$^2$, constant
D. 6.0m/s$^2$, constant

**Solution**

$x(t) = (2t^3 - 6t + 5)m$

Velocity:

$$v = \frac{dx}{dt} = (6t^2 - 6.0)m/s$$

$$a = \frac{dv}{dt} = (12t)m/s^2 = 24m/s^2$$

The velocity is changing since the velocity is a function of time.
The correct option is A.

**3.9.** The acceleration of a particle is given by $a(t) = (2m/s^4)t^2$. If the particle is at rest at $t = 0$, find the velocity of the particle after $t = 2$s.

A. 5.0m/s B. 4.0m/s C. 8.0m/s D. 12.5m/s

**Solution**

$a(t) = (2m/s^4)t^2$

$v(t) = \int a(t)dt == \int 3t^2 dt = \frac{3t^3}{3} + c = t^3 + c$

$v_o = 0 \ at \ t = 0$

$0 = 0 + c$

$c = 0$

Hence, $v(t) = t^3$

At t = 2s, $v = 2^2 m/s = 4m/s$

The correct option is B.

**3.10**. The acceleration of a car is given as a function of time as $a(t) = (2.0 - 0.1t)m/s^2$. If the position and velocity at time $t = 0$ s $x_o = 0$ and $v_o = 10ms^{-1}$ respectively, what is the velocity of the car after $t = 2.0$s?

A. $1.8ms^{-1}$ B. $13.6ms^{-1}$ C. $40ms^{-1}$ D. $80ms^{-1}$

**Solution**

$a(t) = (2.0 - 0.1t)m/s^2$

$v(t) = \int adt = \int (2.0 - 0.2t)dt$

$v(t) = 2t - 0.1t^2 + c$

At $t = 0$, $v_o = 10$m/s, hence c = 10m/s. Therefore

$v(t) = 2t - 0.1t^2 + 10$

At $t = 2.0$s

$v = (2 \times 2 - 0.1 \times 2^2 + 10)m/s$

$v = 13.6m/s$

The correct option is B.

**3.11.** An elementary particle is projected into space and travelled as $r = (t^3 + 2t)\mathbf{i} - 3e^{-2t}\mathbf{j} + 2\sin 5t\mathbf{k}$. Find its acceleration at $t = 0$.

A. $-12\mathbf{j}$ B. $-12\mathbf{i}$ C. $12\mathbf{k}$ D. $-12\mathbf{k}$

**Solution**

$r = (t^3 + 2t)\mathbf{i} - 3e^{-2t}\mathbf{j} + 2\sin 5t\mathbf{k}$

Velocity, $v = \dfrac{dr}{dt} = (3t^2 + 2)\mathbf{i} + 6e^{-2t}\mathbf{j} + 10\cos 5t\mathbf{k}$

Acceleration, $a = \dfrac{dv}{dt} = \dfrac{d^2v}{dt^2} = 6t\mathbf{i} - 12e^{-2t}\mathbf{j} - 50\sin 5t\mathbf{k}$

At $t = 0$, $a = 6 \times 0\mathbf{i} - 12e^{o}\mathbf{j} - 50\sin 0\mathbf{k} = -12\mathbf{j}$

The correct option is A.

**3.12.** A particle moving along the $x$ axis is located at $x_i = 2$ m at $t_i = 1$s and at $x_f = 8$ m at $t_f = 3$ s. Find the displacement and average velocity during this time interval.

A. -6 m, -3m/s
B. -10 m, -5m/s
C. 10 m, 5m/s
D. 6 m, 3m/s

**Solution**

Displacement $r = x_f - x_i = (8 - 2)$ m $= 6$ m

Average velocity:

$$v = \frac{x_f - x_i}{x_f - t_i} = \frac{6}{2} m/s = 3m/s$$

The correct option is D.

**3.13.** A car decelerates with 10m/s$^2$ from 50m/s to 20m/s. Calculate the distance travelled by the car.

A. 150 m B. 105 m C. 10 m D. 75 m

**Solution**

$a$ = -10m/s$^2$, $v$ = 20m/s and $v_o$ = 50m/s

Using:

$$v^2 = v_o^2 + 2ax$$

$$20^2 = 50^2 + (2\times -10\times x)$$

$$20^2 = 50^2 - 20x$$

$$x = 105m$$

The correct option is B.

**3.14.** Calculate the time taken and the distance covered by a train moving with a velocity of 15m/s to accelerate uniformly at the rate of 2m/s$^2$ to reach a velocity of 20m/s.

A. 2.5 s, 23.75 m
B. 17.5 s, 43.75 m
C. 13.5 s, 33.75 m
D. 2.5 s, 43.75 m

**Solution**

$v_o$ = 15m/s, $v$ = 20m/s and $a$ = 2m/s$^2$

From $v = v_o + at$, we have $t = \frac{v - v_o}{a} = \left(\frac{20-15}{2}\right)s = 2.5s$

Using:

$$x = v_o t + \frac{1}{2}at^2 = \left(15\times 2.5 + \frac{1}{2}\times 2\times 2.5^2\right)m = (37.5 + 6.25)m = 43.75$$

The correct option is D.

**3.15.** A stone is thrown vertically upwards with an initial velocity of 20m/s. Find the time taken for the stone to reach its highest point and the distance travelled.

A. 5s, 50m B. 4s, 40m C. 3s, 30m D. 2s, 20m

**Solution**

$v_o$ = 20m/s, $v$ = 0 at the maximum, $a$ = -$g$ = -10m/s$^2$

$$t = \frac{v - v_o}{a} = \left(\frac{0 - 20}{-10}\right)s = 2s$$

Using

$$y = v_o t + \frac{1}{2}a_y t^2 = \left(20 \times 2 + \frac{1}{2} \times -10 \times 2^2\right)m = (40 - 20)m = 20m$$

The correct option is D.

**3.16.** A ball is thrown vertically upwards. The quantity which remains the same is:

A. displacement
B. speed
C. kinetic energy
D. acceleration

**Solution**

The correct option is D.

**3.17.** An object on the surface of the moon has a velocity of 10m/s at a height of 120 m above the moon's surface. Taking the moon's gravitational acceleration as $1.6 m/s^2$, with what speed does the object strike the moon?

A. 2m/s B. 22m/s C. 9.6m/s D. 46m/s

**Solution**

$v_o = 10 m/s$, $y = 120$ m, $a_y = 1.6 m/s^2$

$$v^2 = v_o^2 + 2ay = 10^2 + (2 \times 1.6 \times 120) = (100 + 384) = 484$$

$$v = 22m/s$$

The correct option is B.

**3.18.** A train travels with an initial velocity of 20m/s. If it accelerates uniformly at $2 m/s^2$ and travels 100m, calculate the final velocity.

A. 18.0m/s B. 20.8m/s C. 28.3m/s D. 57.6m/s

**Solution**

$v_0 = 20\text{m/s}, a = 2\text{m/s}^2, x = 100$ m

$v^2 = v_o^2 + 2ax = 20^2 + (2 \times 2 \times 100) = 800$

$v = 28.3m/s$

The correct option is C.

**3.19.** A man throws a ball vertically upward with an initial speed of 40m/s. What is the velocity of the ball on striking the ground?

A. 30m/s B. 40m/s C. 10m/s

D. Insufficient information

**Solution**

At maximum height, $v = 0$, using $v = v_0 + a_y t$

$0 = 40 - 10t$

$t = 4.0$ s

From maximum height, $v_0 = 0$, $a_y = 10\text{m/s}^2$

$v = v_0 + a_y t$

$v = (0 + 10 \text{ x } 4)\text{m/s} = 40\text{m/s}$

The correct option is B.

**3.20.** A stone is thrown horizontally with an initial velocity of 20m/s from the top of a building 90m high. Find the horizontal range of the stone. Calculate also the velocity with which the stone strikes the ground.

A. 84.8 m; 46.9m/s, 64.7°

B. 45.0 m; 5.83m/s, 6.1°

C. 30.0 m; 43.67m/s, 30°

D. 23.5 m; 24.7m/s, 20°

**Solution**

From the figure below, the $x$ and $y$ components of the initial velocity are $v_{ox} = 20\text{m/s}$ and $v_{oy} = 0$. For the vertical motion:

$$y = v_{oy}t + \frac{1}{2}a_y t^2$$

$$90 = 0 \times t + \frac{1}{2} \times 10 \times t^2$$

$$t = 4.24s$$

For horizontal motion:

$a_x = 0$

$$x = v_{ox}t + \frac{1}{2}a_x t^2 = v_{ox}t$$

$$= 20 \times 4.24m = 84.8m$$

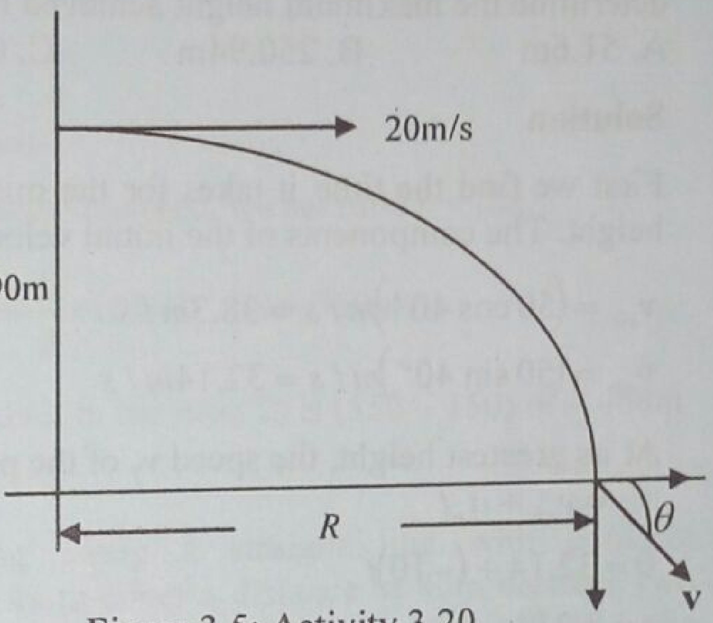



Figure 3.5: Activity 3.20

The velocity **v** with which the stone hits the ground is shown to be inclined at an angle of $\theta$ with the horizontal. Let $v_x$ and $v_y$ be the horizontal and vertical component of **v**, respectively.

For horizontal motion:

$$v_x = v_{ox} + a_x t = (20 + 0 \times t)m/s = 20m/s$$

For vertical motion:

$$v_y = v_{oy} + a_y t = (0 + 10 \times 4.24)m/s = 42.4m/s$$

Therefore, using Pythagoras' theorem we obtain the magnitude of the velocity:

$$v = \left(\sqrt{20^2 + 42.4^2}\right)m/s = 46.9m/s$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{42.4}{20} = 2.12$$

$$\theta = 64.7^o$$

The correct option is A.

**3.21.** A missile is launched with a speed of 50m/s at an angle 40° above the surface of a warship. Ignoring the effects of air resistance, determine the maximum height achieved by the missile.

A. 51.6m B. 250.94m C. 0.50m D. 50.00m

**Solution**

First we find the time it takes for the missile to reach its maximum height. The components of the initial velocity:

$$v_{ox} = (50\cos 40^o)m/s = 38.3m/s$$
$$v_{oy} = (50\sin 40^o)m/s = 32.14m/s$$

At its greatest height, the speed $v_y$ of the projectile is zero. Then:

$$v_y = v_{oy} + a_y t$$
$$0 = 32.14 + (-10)t$$
$$t = 3.214s$$

Therefore, the greatest height $y_{max}$ is given by:

$$y_{max} = v_{oy}t + \frac{1}{2}a_y t = \left(32.14 \times 3.214 + \frac{1}{2}(-10)(3.214)^2\right)m = 51.6m$$

Alternatively, we can use:

$$H = \frac{v_o^2 \sin^2 \alpha_o}{2g} = \left(\frac{50^2 \times \sin^2 40}{2 \times 10}\right)m = 51.6m$$

The correct option is A.

**3.22.** A car accelerates uniformly in a straight line with acceleration $10m/s^2$ and travels 150m in a time interval of 5s. How far will it travel in the next 5s?

A. 150m B. 400m C. 300m D. 500m

**Solution**

To calculate the initial velocity, we use:

$$x = v_o t + \frac{1}{2}at^2$$

$$150 = v_o \times 5 + \frac{1}{2} \times 10 \times 5^2$$

Solving, we have, $v_o = 5\text{m/s}$.

To calculate the total distance covered, we set time $t = 10\text{s}$,

$$x = v_o t + \frac{1}{2}at^2 = \left(5 \times 10 + \frac{1}{2} \times 10 \times 10^2\right)m = 550m$$

Therefore, the distance travel in the next 5s is $(550 - 150)$ m = 400m
The correct option is B.

**3.23.** A train travelling along a straight line with constant acceleration takes a time 4s to cover a distance of 80m between two points. If the speed as it passes the second point is 30m/s, what is the acceleration of the train? Calculate the initial speed of the train.

A. $5\text{m/s}^2$, 10m/s B. $10\text{m/s}^2$, 5m/s
C. $40\text{m/s}^2$, 10m/s D. $15\text{m/s}^2$, 15m/s

**Solution**

Using, $x = vt - \frac{1}{2}at^2$ with $t = 4$ s, $v = 30\text{m/s}$ and $x = 80$ m

$$80 = 30 \times 4 - \frac{1}{2} \times a \times 4^2$$

$$-40 \times 2 = -16a$$

$$\therefore a = 5m/s^2$$

Using

$$v = v_o + at$$

$$30 = v_o + 5 \times 4$$

$$v_o = 10m/s$$

The correct option is A.

**3.24.** A mango is dropped from a tree 80m high, with zero initial speed; 1.0s later another coconut is thrown vertically downward (with non-zero initial speed). If both fruits strike the ground at the same time, calculate the initial speed with which the second coconut is thrown.

A. 22.8m/s B. 21.9m/s C. 11.7m/s D. 45.6m/s

**Solution**

For the first coconut, $v_o = 0$, $a_y = g = 10m/s^2$, $y = 80m$. To calculate the time of flight, we use:

$$y = v_{oy}t + \frac{1}{2}a_y t^2$$

$$80 = 0 \times t + \frac{1}{2} \times 10t^2$$

$$t = 4s$$

For the second coconut, $a_y = g = 10m/s^2$, $y = 80$ m, $t = (4.0 - 1.0)$ s = 3.0 s. Using:

$$y = v_{oy}t + \frac{1}{2}a_y t^2$$

$$80 = v_{oy} \times 3.0 + \frac{1}{2} \times 10 \times 3.0^2$$

$$80 = 3.0v_{oy} + 45$$

$$3.0v_{oy} = 35$$

$$v_{oy} = 11.7m/s$$

The correct option is C.

**3.25.** A car starts from rest with constant acceleration covering a distance of 1200m in 40s. Calculate its velocity at this time.

A. 40m/s B. 20m/s C. 30m/s D. 60m/s

**Solution**

$v_o = 0$, $t = 40$ s, $x = 1200$ m

Using:

$$x = \frac{(v_o + v) \times t}{2}$$

$$1200 = \frac{(0 + v)}{2} \times 40$$

$$v = \frac{1200}{20} m/s = 60 m/s$$

The correct option is D.

**3.26.** A stone of mass 20kg is thrown vertically upward to attain a maximum height of 20m. Calculate the initial speed with which the stone was thrown.

A. 40m/s B. 20m/s C. 30m/s D. 60m/s

**Solution**

Since the ball is thrown upward, $a_y = -g = -10 m/s^2$, $y = 20m$, $v = 0$ at maximum height.

Using:

$$v^2 = v_o^2 + 2ay$$

$$0 = v_o^2 + 2 \times -10 \times 20$$

$$v_o^2 = 400$$

$$v_o = 20 m/s$$

The correct option is B.

**3.27.** A car starts from rest and travels 250m in 25s, at constant acceleration. Calculate the final speed of the car.

A. 20m/s B. 40m/s C. 2.0m/s D. 60m/s

**Solution**

Initial speed, $v_o = 0$ (since the car starts from rest), $x = 250m$, $t = 25s$

(a) We use:

$$x = v_o t + \frac{1}{2} a t^2$$

$$250 = 0 \times 25 + \frac{1}{2} \times a \times 25^2$$

$$500 = 625a$$

$$a = \frac{500}{625} m/s^2 = 0.8 m/s^2$$

(b) $v = v_o + at = (0 + 0.8 \times 25) m/s = 20 m/s$

The correct option is A.

**3.28.** A ball was thrown from the ground and its velocity was 10m/s at an altitude of 15m above the ground. Calculate the velocity at the point of throw and the maximum height reached.
A. 15m/s, 7.1m
B. 20m/s, 20.0m
C. 25.0m/s, 10.0m
D. 15m/s, 15.0m

**Solution**

At $x = 15$ m, $v = 10$m/s, $a = -g = -10.0$m/s$^2$

Using

$$v^2 = v_o^2 + 2ay$$

$$10^2 = v_o^2 - 2 \times 10 \times 15$$

$$v_o^2 = 100 + 300 = 400$$

$$v_o = 20 m/s$$

At maximum height, $v = 0$

$$v^2 = v_o^2 + 2ay$$

$$0^2 = 20^2 - 2 \times 10 \times y$$

$$20y = 400$$

$$y = 20m$$

The correct option is B.

**3.29.** A boy throws a stone of mass 5kg vertically downward with an initial speed of 12.0m/s from a height 30.0m above the ground. How long does it takes the stone to reach the ground and what is the speed of the stone just before it hits the ground?

A. 2.47 s, 27.3m/s
B. 1.53 s, 27.3m/s
C. 2.47 s, 24.25m/s
D. 1.54 s, 24.25m/s

**Solution**

$v_o = 12.0\text{m/s}$, $x = 30.0$ m, $a = 10\text{m/s}^2$

Using:

$v^2 = v_o^2 + 2ay$

$v^2 = 12^2 + 2 \times 10 \times 30 = 144 + 600 = 744$

$v = 27.3m/s$

$v = v_o + at$

$27.3 = 12.0 + 10t$

$t = 1.53$ s

The correct option is B.

**3.30.** A ball is thrown vertically upward from the ground with a speed of $20\text{ms}^{-1}$. How long does it take to reach its highest point and how high does the ball rise?

A. 2.4s; 29.8m B. 2.0s; 20.0m C. 2.5s; 40m D. 2.4s; 31m

**Solution**

At the highest point, $v = 0$

$v = v_o + at$

$0 = 20 - 10t$

$10t = 20$

$$t = \frac{20}{10}s = 2.0s$$

Using

$$v^2 = v_o^2 + 2ay$$

$$0 = 20^2 - 2 \times 10 \times y$$

$$y = 20m$$

The correct option is B.

## Summary of Chapter 3

In chapter 3, you have learned that:

1. Mechanics, a branch of physics, is usually divided into t... parts: kinematics and dynamics.
2. Speed can be defined as the rate of change of distance w... time. There are two ways in which the speed of a particle ... defined: average speed and instantaneous speed.
3. Velocity is defined as the rate of change of displacement wi... time.
4. Acceleration is defined as the time rate of change of velocity. It is a vector quantity.
5. All bodies at a particular location fall with the same downwar... acceleration, regardless of their size or weight.
6. A projectile motion is two-dimensional and we can treat the x- and $y$- coordinates separately. The $x$-component o... acceleration due to gravity $g$ is zero since $g$ is acting vertically downward; the $y$-component is constant and equal to $-g$.

## Self-Assessment Questions (SAQs) for Chapter 3

**3.1.** A stone of mass 2kg is thrown vertically upward with a speed of 20m/s. Calculate the maximum height reached.

**3.2.** A baseball is thrown vertically upward with an initial speed of 20m/s. Calculate how fast it was travelling on its way down when caught 5.0 m above where it was thrown.

**3.3.** Calculate the average speed of a car travelling from rest and covering 1200m in one minute.

**3.4.** A car accelerates uniformly at $3m/s^2$ covering a distance of 70m in 5s. Calculate the initial and final velocities.

**3.5.** A ball is thrown straight downward with non-zero initial speed from a height of 50m. Calculate the initial speed if it takes the ball 2s to strike the ground. With what speed will it strike the ground?

**3.6.** A cliff is 20m above the ground. A boy rolls a ball off the edge with unknown initial velocity and it strikes the ground a distance of 3m horizontally away. Calculate (a) the ball's initial velocity, (b) its time of flight, and (c) the magnitude and the direction of its velocity when it strikes the ground.

**3.7.** A train moving with a velocity of 10m/s accelerates uniformly at $1m/s^2$ until it reaches a velocity of 15m/s. Calculate (i) the time taken, (ii) the distance travelled during the acceleration, (iii) the velocity reached 100 m from the place where the acceleration began.

**3.8.** A car is accelerating uniformly as it passes two points *A* and *B* which are 30m apart. The time taken between the two points is 4.0s, and the car's speed at point *A* is 5.0m/s. Find the car's acceleration and its speed at point *B*.

**3.9.** A wooden box slides down an incline plane with uniform acceleration. It starts from rest and attains a speed of 10m/s in 3.0s. Find (a) the acceleration and (b) the distance moved in the first 6.0s.

**3.10.** A projectile is launched with 30.0m/s at an angle of $60.0°$ with the horizontal. (a) Find the position of the projectile, and the magnitude and direction of its velocity at $t = 2.0$ s (b) Find the time when the ball reaches the highest point. (c) Find the horizontal range.

**3.11.** An elementary particle with initial velocity of $1.50 \times 10^5 m/s$ enters a region where it is electrically accelerated, with its velocity reaching 5.70 $\times 10^6$m/s. Calculate its constant acceleration after travelling for 1.0cm.

**3.12.** A car moving with constant acceleration covered the distance between two points $X$ and $Y$, 100.0m apart in 5.0s. Its speed as it passes the second point was 50.0m/s. What was the speed at the first point and what was the acceleration?

**3.13.** The speed of a car travelling due west is uniformly reduced from 20.0ms$^{-1}$ to 10.0ms$^{-1}$ in a distance of 80.0m. What is the magnitude of the deceleration and how much time has elapsed during this deceleration?

**3.14.** A man throws a ball vertically upward with an initial speed of 60.0m/s. What is the maximum height reached by the ball and how long does it take to return to the point it was thrown?

**3.15.** A projectile is launched with an initial velocity of 60m/s at an angle 60° to the vertical. What is the magnitude of its displacement after 5s?

**3.16.** A missile was to be launched at an angle 15° to the horizontal at an initial velocity $v_o$ to hit a target 300m away. Calculate $v_o$ and the maximum height $H$.

**3.17.** An object of mass 50g is projected at angle 60° to the horizontal with an initial speed of 20m/s. Calculate the horizontal range and the time taken to reach maximum height.

**3.18.** A footballer lobs a football at an angle of 30° to the horizontal with an initial speed of 20ms$^{-1}$. What are the greatest height attained and the time of flight?

**3.19.** A car moving at a speed of 30m/s decelerates at a constant rate of 2.0m/s$^2$. How far does it goes before stopping?

reaching $5.70 \times 10^6$m/s. Calculate its constant acceleration after travelling for 1.0cm.

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**3.17.** An object of mass 50g is projected at angle 60° to the horizontal with an initial speed of 20m/s. Calculate the horizontal range and the time taken to reach maximum height.

**3.18.** A footballer lobs a football at an angle of 30° to the horizontal with an initial speed of 20ms$^{-1}$. What are the greatest height attained and the time of flight?

**3.19.** A car moving at a speed of 30m/s decelerates at a constant rate of 2.0m/s$^2$. How far does it goes before stopping?

**3.20.** A car moving at 20m/s slows uniformly to a speed of 5m/s in a time of 10s. Determine the distance travelled in the fifth second.

# CHAPTER FOUR

# NEWTON'S LAWS OF MOTION

## 4.1 Introduction

In chapter three, we discussed kinematics – the mathematical description of the motion of objects without consideration of what causes the motion. Now we need to know more about the dynamics of motion, that is, what causes motion and changes in motion. Force causes motion. A force can be a push or a pull and it produces a change in the velocity of the object on which it acts. There are different types of forces. We can place a variety of force types into two broad categories on the basis of whether the force resulted from the contact or non-contact of the two interacting objects. Examples of contact forces are: frictional force, tension force, normal force, air resistance force, spring force. Non-contact forces are gravitational, electric and magnetic forces. Many scientists have studied force and motion but Isaac Newton (1642 – 1727) summarised the various relationships and principles of the early scientists into three statements, popularly known as Newton's laws of motion. These laws sum up the concepts of dynamics.

## 4.2 Newton's First Law of Motion: Concept of Inertia

Newton's first law of motion states that every particle continues to be in a state of rest and if already in motion, it will continue in motion with uniform velocity unless a resultant force acts on it. Newton's first law expresses the concept of inertia. Inertia is the natural tendency of an object to maintain a state of rest or to remain in uniform motion in a straight line. As an illustration of this concept, passengers in a bus move forward when the bus stops suddenly. You may have experienced this before. They tend to continue in their state of motion until brought to rest by friction or collision. This is why it is advisable to use a seat-belt while driving.

Newton related the concept of inertia to mass. Mass is a measure of the inertia of a body. The implication of this statement is that a massive object has more inertia, or more resistance to a change in motion, than a less massive object does.

## 4.3 Concept of Momentum and Impulse

When an object is moving, it is said to have an amount of momentum. The momentum is defined as the product of the mass of the object and the velocity. That is,

$$\text{Momentum } P = \text{mass x velocity} \qquad 4.1$$

The unit of momentum is kilogram metre per second ($kgms^{-1}$).

Impulse of a force is defined as the product of the force and the time with which it acts. That is:

$$\text{Impulse } I = \text{force x time} \qquad 4.2$$

The unit of impulse is Newton second (Ns).

## 4.4 Newton's Second Law of Motion

Newton's second law of motion states that the time rate of change of momentum is directly proportional to the applied resultant force and the momentum change takes place in the direction of the force. Mathematically, Newton's second law can be written as follows,

$$F_R \alpha \frac{change\ of\ momentum}{time}$$

which implies:

$$F_R = \frac{k(mv - mv_o)}{t} \qquad 4.3$$

The constant of proportionality $k$, can be shown to be equal to 1.

In calculus Notation, Equation 4.3 can be written as follows:

$$F_R = \frac{d(mv)}{dt} \qquad 4.4$$

If the mass $m$ is constant, we can have:

$$F_R = m\frac{dv}{dt} = ma \qquad 4.5$$

where $a$ is the acceleration of the motion.

Equation 4.5 is also regarded as the Newton's second law of motion. The newton (N) is the unit of force. One newton is the amount of the force that gives an acceleration of $1 m/s^2$ to a body of mass 1kg.

$$1 \text{ newton} = 1kg \times 1m/s^2$$

or $1N = 1kgm/s^2$

Suppose a constant force $F_R$ is applied at different times to two bodies of masses $m_1$ and $m_2$ and producing acceleration $a_1$ and $a_2$ respectively on the two bodies. Then, according to Equation 4.5:

$$F_R = m_1 a_1 = m_2 a_2$$

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} \qquad 4.6$$

However, if the mass and velocity are both functions of time, Equation 4.4 can be written as:

$$F_R = m\frac{dv}{dt} + v\frac{dm}{dt} \qquad 4.7$$

It can be shown from Equation 4.3 that the impulse of a force is equal to the change in momentum. That is:

$$F \times t = mv - mv_o = \text{momentum change} \qquad 4.8$$

From Equation 4.3 we can also write:

$$F_R = \frac{mass}{time} \times velocity\ change$$

So: $F_R =$ mass per second x velocity change 4.9

In vector form, Newton's second law of motion can be written as:

$$\sum F_x = ma_x,\ \sum F_y = ma_y\ \sum F_z = ma_z \qquad 4.10a$$

$\sum F_i$ = vector sum of the $i$-component of all the forces acting on the body of mass $m$, where $i = x$, $y$, or $z$. That is:

$$\vec{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} \qquad 4.10b$$

Each component of the net force equals the mass multiplied by the corresponding acceleration.

## 4.5 Newton's Third Law of Motion

Newton's third law states that to every action there is an equal and opposite reaction. This law implies that if a body $A$ exerts a force (action) on a body $B$, then $B$ will exert an equal and opposite force (reaction) on $A$. The mathematical statement of Newton's third law is:

$$\vec{F}_{AB} = -\vec{F}_{BA} \qquad 4.11$$

where $\vec{F}_{AB}$ is the force exerted on body $A$ by body $B$, and $\vec{F}_{BA}$ is the force exerted on body $B$ by body $A$ (Figure 4.1). The negative sign in Equation 4.11 shows that $\vec{F}_{AB}$ and $\vec{F}_{BA}$ act in opposite direction.

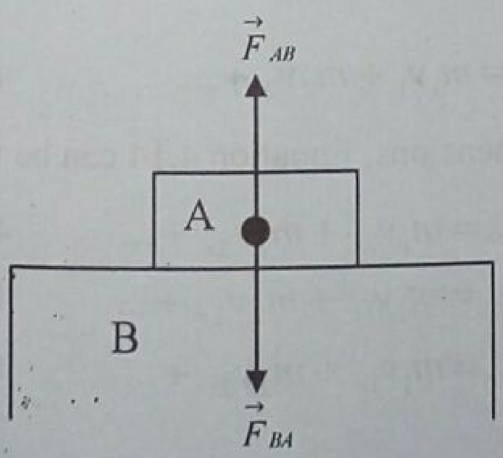


Figure 4.1: Action and reaction forces

## 4.6 The Law of Conservation of Linear Momentum

The linear momentum has been defined as $p = Mv$, where $v$ is the velocity of the body. The law of conservation of linear momentum states that during collisions in which the colliding objects and the product bodies are not acted upon by an externally applied force, the sum of linear momentum before collision is equal to the sum of linear momentum after the collision.

We define the total initial momentum $p_i$ of a system of $m$ bodies, of masses $M_1, M_2, \ldots, M_m$, moving with initial uniform velocities $u_1, u_2, \ldots, u_m$; respectively, as:

$$p_i = \sum_{i=1}^{m} M_i u_i \qquad 4.12$$

During collisions, some of the colliding bodies may break up and others may join together to produce a system of $n$ bodies $m_1, m_2, \ldots, m_n$. The total final momentum $p_f$ of the system is:

$$p_f = \sum_{i=1}^{n} m_f v_f \qquad 4.13$$

where the bodies are assumed to move with final velocities $v_1, v_2, \ldots, v_n$, respectively for masses $m_1, m_2, \ldots, m_n$.

The law of conservation of linear momentum implies that $p_i = p_f$, that is:

$$M_1u_1 + M_2u_2 + \ldots = m_1v_1 + m_2v_2 + \ldots \qquad 4.14$$

If the motion is in three dimensions, Equation 4.14 can be written as:

$$M_1u_{1x} + M_2u_{2x} + \ldots = m_1v_{1x} + m_2v_{2x} + \ldots \qquad 4.15a$$

$$M_1u_{1y} + M_2u_{2y} + \ldots = m_1v_{1y} + m_2v_{2y} + \ldots \qquad 4.15b$$

$$M_1u_{1z} + M_2u_{2z} + \ldots = m_1v_{1z} + m_2v_{2z} + \ldots \qquad 4.15c$$

The law of conservation of linear momentum can be derived from Newton's laws and it holds in phenomena, such as in quantum physics, relativity and electromagnetic field theory, in which Newton's laws do not apply. In other words, because it is valid in a wider range of phenomena than Newton's laws, the law of conservation of linear momentum is considered more fundamental than Newton's laws.

## 4.7 Elastic and Inelastic Collisions

The collision of two or more bodies can be classified as either elastic or inelastic. The law of conservation of momentum is conserved in both classes of collision.

In elastic collision, the kinetic energy is conserved; that is, the sum of initial kinetic energy is equal to the sum of final kinetic energy. If several particles are involved in the collision such that the particle of mass $M_i$ has initial speed $u_i$ before the collision and speed $v_i$ after the collision, we have:

$$\sum_i \frac{1}{2} M_i u_i^2 = \sum_i \frac{1}{2} M_i v_i^2 \qquad 4.16a$$

In an elastic collision, the colliding particles do not disintegrate or join together.

However, in an inelastic collision kinetic energy is not conserved because some of this kinetic energy may be converted into other forms of energy such as heat, sound, radiation, etc. A completely inelastic collision between two objects is a collision in which the two objects stick together after the collision and move off together as one body. The implication of this is that:

$$\sum_i \frac{1}{2} M_i u_i^2 \neq \sum_i \frac{1}{2} M_i v_i^2 \qquad 4.16b$$

The coefficient of restitution $e$ for a particular collision describes the extent to which the collision is elastic or completely inelastic. The coefficient of restitution is defined only for one-dimensional collisions of two bodies and is given by:

$$e = \frac{v_r}{u_r} = \frac{v_2 - v_1}{u_2 - u_1} \qquad 4.17$$

where $v_r$ is the relative speed of the objects after the collision, and $u_r$ is the relative speed before the collision. The maximum value of $e$ is 1 for an elastic collision and zero for a completely inelastic collision. For other one-dimensional two-body collisions, $0 < e < 1$.

## 4.8 Friction

Whenever two bodies are in contact with each other, there is always a resistance to motion experienced by the surfaces of the two bodies. This resistance to motion is referred to as force of friction, or simply friction. Friction occurs for solid, liquid and gases but this section is concerned with friction between solid surfaces. Basically, two kinds of friction exist between two solid surfaces. They are: static and sliding (kinetic). Static friction ($f_s$) refers to the frictional force that occurs when there is no relative motion between two bodies in contact. Consider a moveable body $A$ resting on a body $B$ which is stationary as shown in Figure 4.2. Imagine that body $A$ does not move after a small horizontal force $F$ is applied to it. With no acceleration, the Net force on the body $A$ is zero, that is $F - f_s = 0$, or $F = f_s$; that is, the applied force is equal to the static frictional force. Suppose that the horizontal force is increased to $F_1$ and the body $A$ still does not move. Then $f_s$ must now be larger, since $F_1 = f_s$ and $F_1$ is greater than $F$. Finally, if the applied force is large enough to overcome the static friction, the body $A$ then tends to move. The value of the horizontal force when this sudden motion occurs is the maximum or limiting frictional force $f_{smax}$ for the two surfaces in contact, when they are at relative rest. Experiments show that for all types of surfaces in contact, and which are not in relative motion, $f_s$ is proportional to the normal force N. That is:

$$f_{smax} \propto N$$

$$f_{s\max} = \mu_s N \qquad 4.18$$

where $\mu_s$ is a constant of proportionality known as the coefficient of static friction.

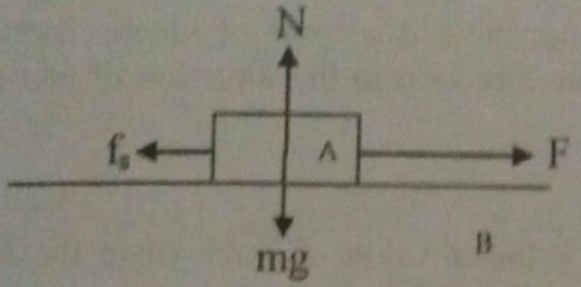


Figure 4.2: Static friction

The magnitude of $\mu_s$ depends on the nature of the surfaces but is independent of the area in contact, provided that the normal force is unchanged.

If the horizontal surface $B$ is inclined at an angle $\theta$ as shown in Figure 4.3, then the weight $mg$ will act at an angle to the plane. The component of $mg$ normal to $B$ is the normal force:

$$N = mg\cos\theta \qquad 4.19$$

The inclination $\theta$ of $B$ may be increased from a low value to a maximum value $\theta_m$ at which the body $A$ just moves. At that point, we have:

$$f_{s\max} = mg\sin\theta_m \qquad 4.20$$

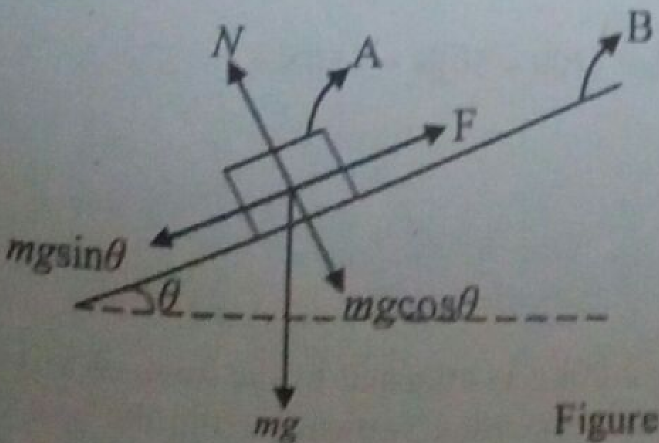


Figure 4.3: Kinetic friction

Substitute Equations 4.19 and 4.20 into 4.18:

$$mg\sin\theta_m = \mu_s mg\cos\theta_m$$

$$\mu = \tan\theta_m \qquad 4.21$$

Once the body $A$ is sliding, there is a force of kinetic friction ($f_k$) acting on it in the opposite direction to the direction of motion and has a magnitude of:

$$f_k = \mu_k N \qquad 4.22$$

It is found that it takes a lower value of force (than the limiting frictional force $f_s$) to keep two surfaces moving with respect to one another. The opposing force $f_k$ between two surfaces, already in relative motion is known as the kinetic or dynamic frictional force and $f_k$ is always less than $f_s$, which means that the coefficient of kinetic friction is less than the coefficient of static friction ($\mu_k < \mu_s$) for two surfaces. $\mu_k$ is independent of the relative velocity of the surfaces. Just like $\mu_s$, $\mu_k$ is independent of the area of the surfaces in contact, provided that the normal force $N$ remains unchanged.

## Activity 4 Dynamics

**4.1.** A force of 200N pulls a block of mass 50kg and overcomes a constant frictional force of 50N. What is the acceleration of the block?

A. $2.8m/s^2$ B. $3.0m/s^2$ C. $3.3m/s^2$ D. $4.0m/s^2$

**Solution**

Resultant force $F_R = F - f_k = (200 - 50)N = 150N$
From $F_R = ma$
$150 = 50 \times a$
$a = 3.0m/s^2$

The correct option is B.

**4.2.** An object of mass 2.00kg is attached to the hook of a spiral-balance and the balance is suspended vertically from the roof of a lift. What is the reading on the spring-balance when the lift is going up with an acceleration of $0.5 m/s^2$?

A. 19.8N B. 20.4N C. 21.0N D. 10.0N

**Solution**

The object is acted on by two forces:
(a) the tension $T$ in the spring-balance which acts upwards;
(b) its weight $mg$ or 20N which acts downwards.
Since the object accelerates upwards, $T$ is greater than 20N. So the net force, acting on the object = $T - 20$. Now:

$F = ma$

$T - 20 = 2 \times 0.5$

$T = 21.0N$

The correct option is C.

**4.3.** An object of mass 3.00kg is attached to the hook of a spiral-balance, and the balance is suspended vertically from the roof of a lift. What is the reading on the spring balance when the lift goes down with an acceleration of $0.1 m/s^2$?

A. 19.8N B. 20.4N C. 20N D. 10N

**Solution**

When the lift descends with an acceleration of $0.1 m/s^2$, its weight 30N is greater than $T$, the tension in the spring-balance. Resultant force:

$F = 30 - T_1 = ma = 2 \times 0.1$

$T_1 = 29.8N$

The correct option is A.

**4.4.** The frictional force on a car of mass 1000kg moving up a hill inclined at $30°$ to the horizontal is 1000N. Calculate the force $F$ due to the engine when the car is accelerating at $2 m/s^2$.

A. 6000N B. 1000N C. 0N D. 8000N

**Solution**

lift. What is the reading on the spring-balance when the lift is going up with an acceleration of $0.5m/s^2$?

A. 19.8N B. 20.4N C. 21.0N D. 10.0N

**Solution**

The object is acted on by two forces:

(a) the tension $T$ in the spring-balance which acts upwards;

(b) its weight $mg$ or 20N which acts downwards.

Since the object accelerates upwards, $T$ is greater than 20N. So the net force, acting on the object = $T - 20$. Now:

$F = ma$

$T - 20 = 2 \times 0.5$

$T = 21.0N$

The correct option is C.

**4.3.** An object of mass 3.00kg is attached to the hook of a spiral-balance, and the balance is suspended vertically from the roof of a lift. What is the reading on the spring balance when the lift goes down with an acceleration of $0.1m/s^2$?

A. 19.8N B. 20.4N C. 20N D. 10N

**Solution**

When the lift descends with an acceleration of $0.1m/s^2$, its weight 30N is greater than $T$, the tension in the spring-balance. Resultant force:

$F = 30 - T_1 = ma = 2 \times 0.1$

$T_1 = 29.8N$

The correct option is A.

**4.4.** The frictional force on a car of mass 1000kg moving up a hill inclined at $30^0$ to the horizontal is 1000N. Calculate the force $F$ due to the engine when the car is accelerating at $2m/s^2$.

A. 6000N B. 1000N C. 0N D. 8000N

**Solution**

There are three forces on the car – its weight $mg$, $F$ and the frictional force.
Weight of car = $mg$ = 10 000N
Component downhill = $mg\cos\theta$ = 10 000 cos 60°N = 5000N
Resultant force uphill, $F_R = F - 5000 - 1000$
From $F_R = ma$,
$F - 5000 - 1000 = 1000 \times 2$
$F = 8000$N
The correct option is D.

**4.5.** Calculate the force $F$ due to the engine when the car in Q4.4 is moving with a steady velocity of 20m/s.
A. 6000N B. 1000N C. 0N D. 8000N

**Solution**

Since the velocity is steady, acceleration $a = 0$, so resultant force $F_R$ = 0. Then:

$F = (5000 + 1000)$N = 6000N

The correct option is A.

**4.6.** A car of mass 1000kg is accelerating at 3m/s$^2$. If the resistance to the motion is 1000N, what is the force due to the engine?
A. 1000N B. 2000N C. 4000N D. 3000N

**Solution**

Resultant force, $F_R = ma = 1000 \times 3$N = 3000N

Let the force due to the engine be $F$, then:
$F_R = F - 1000 = 3000$
$P = (3000 + 1000)$N = 4000N

The correct option is C.

**4.7.** What is the tension in a vertical rope pulling a block of mass 50kg with an acceleration of 1m/s$^2$?
A. 5.5N B. 55N C. 0.5N D. 550N

**Solution**

According to Newton's second law of motion:

$F_R = ma$

$F_R = T - mg = ma$

$T = ma + mg$

$T = 50(1+10)N = 550N$

The correct option is D.

**4.8.** A lift moves (i) up and (ii) down with an acceleration of $2m/s^2$. In each case, calculate the reaction of the floor on an object of mass 50kg place on the floor of the lift.

A. 600N, 400N  B. 400N, 600N
C. 600N, 600N  D. 400N, 400N

**Solution**

(i) When the lift moves upward:

$F_R = ma$

$F_R = T - mg = ma$

$T = m(a+g)$

$T = 50(2+10) = 600N$

(ii) When the lift moves downward:

$F_R = ma$

$F_R = mg - T = ma$

$T = m(g-a)$

$T = 50(10-2) = 400N$

The correct option is A.

**4.9.** A force of magnitude 100N is applied to a body of mass 2kg at an angle 30° to the horizontal. Calculate the acceleration of the body. Neglect friction.

**Solution**

Assuming the body is moving along the horizontal directi component of the force responsible for the motion is the hori component. Therefore:

$$F_R = F_x = 100\cos 30'' = ma$$

$$a = \frac{100 \times 0.866}{2} m/s^2 = 43.3 m/s^2$$

**4.10.** Calculate the constant force that will accelerate a body of m $5 \times 10^3$ kg from rest to a speed of 50m/s in 30s.

**Solution**

$v_0 = 0$, $m = 5 \times 10^3$ kg, $v = 50$m/s, $t = 30$ s
Using:

$$a = \frac{v - v_o}{t} = \left(\frac{50 - 0}{30}\right) m/s^2 = 1.67 m/s^2$$

Force $F = ma = 5 \times 10^3 \times 1.67\text{N} = 8.35 \times 10^3\text{N}$

**4.11.** A spherical ball is moved up an inclined plane which is a angle 30°, with an initial speed 5m/s. The coefficients of static and kinetic friction between the block and the surface are 0.44 and 0.3 respectively. Determine the distance moved by the ball before coming to rest.

A. 1.54m B. 3.08m C. 2.45m D. 9.80m

**Solution**

The frictional force $f_k = \mu_k N$ and the component of the weight of the ball acts on the ball to bring it to rest. Hence the resultant force, $F_R$ is given as:

$$F_R = \mu_k N + mg\sin\theta \qquad 1$$

Note that the frictional force and the component of the weight act in the same direction. The horizontal component of the weight always acts downward as shown in Figure 4.4 and the frictional force is acting downward because the ball is projected upward.

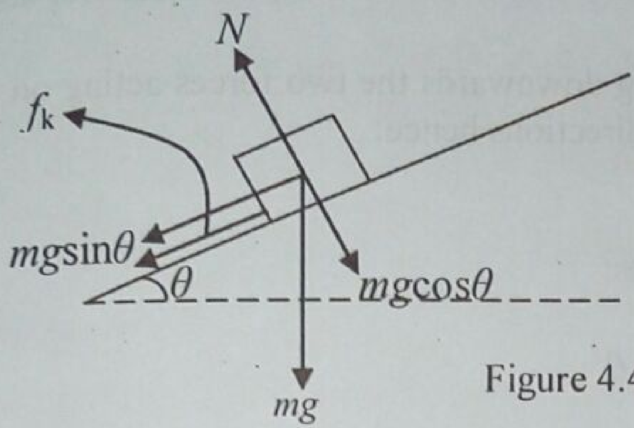


Figure 4.4: Activity 4.11

Since the ball is inclined at an angle $\theta$,

$$N = mg\cos\theta$$

From Newton's second law of motion:

$$F_R = ma \qquad 2$$

From Equations 1 and 2, we have:

$$a = \frac{F_R}{m} = \mu_k g\cos\theta + g\sin\theta$$

$$a = (0.36\times 10\cos 30 + 10\sin 30)m/s^2 = 8.117m/s^2$$

At rest, $v = 0$, $a = -8.117\text{m/s}^2$ (since the body is retarding), using:

$$v = v_o + at$$

$$0 = 5 - 8.117t$$

$$t = 0.616s$$

During this time the ball has travelled a distance of:

$$x = v_o t + \frac{1}{2}at^2 = \left(5\times 0.616 - \frac{1}{2}\times 8.117\times 0.616^2\right)m = 1.54m$$

The correct option is A.

**4.12.** At what speed will the spherical ball in **Q4.11** return to its starting point?

A. 5.0m/s B. 2.5m/s C. 5.785m/s D. 2.41m/s

**Solution**

Now that the ball is moving downwards the two forces acting on th ball are acting in opposite directions hence:

$F_R = mg\sin\theta - \mu_k N$

The acceleration *a*, becomes:

$$a = \frac{F_R}{m} = g\sin\theta - \mu_k g\cos\theta$$

$$a = 10(\sin 30 - 0.36\cos 30)m/s^2 = 1.882m/s^2$$

The initial velocity $v_o$ = 0 because the ball came to rest befor returning to the starting point. Using:

$$v^2 = v_o + 2ax = 0 + 2\times 1.882\times 1.54 = 5.797$$

$$v = 2.41m/s$$

The correct option is D.

**4.13.** A force 80N acts on a body initially at rest and moves 72m in 6 s. If the force is removed after the first 6s, calculate how far the body moves in the next 6s.

A. 144m B. 72m C. 56m D. 166m

**Solution**

Let us find the final velocity for the first 6s.

$$x = \frac{(v_o + v)t}{2}$$

$$72 = \frac{(0+v)\times 6}{2}$$

$$v = 24m/s$$

This velocity $v = 24$m/s will be the initial velocity for the next 6s; $a =$ 0 (since the force is removed).

$$x = v_o t + \frac{1}{2}at^2 = 24\times 6m = 144m$$

The correct option is A.

**4.14.** A block of mass 3.5kg is pushed by a force of magnitude 25N along a frictionless floor by means of a cord which makes angle 60° with the horizontal. Calculate the magnitude of the acceleration of the block.

A. $7.14\text{m/s}^2$ B. $6.19\text{m/s}^2$ C. $3.57\text{m/s}^2$ D. $10.00\text{m/s}^2$

**Solution**

The acceleration is caused by the horizontal component of the force.

$$F_x = ma$$
$$25\cos 60 = 3.5a$$
$$a = 3.57\text{m/s}^2$$

The correct option is C.

**4.15.** A man pulls a load of mass $m$ with a rope along a horizontal surface at constant velocity. If the coefficient of kinetic friction between the load and the surface is $\mu_k$ and the force $F$ applied to the rope by the man is along the horizontal, which of the following set of Newton's laws apply to this load? $F_f$ is the frictional force.

(i) $F_{net} = 0$ (ii) $F_{net} = ma$ (iii) $a = 0$ (iv) $F = F_f$

A. All
B. (i), (ii) & (iii)
C. (i), (iii) & (iv)
D. (ii), (iii) & (iv)

**Solution**

The correct option is C.

**4.16.** A body of mass 2kg moving on a horizontal frictionless plane with initial speed of 10m/s is pulled 4m by a force of magnitude 25N in the direction of the applied force. Calculate the final speed.

A. 10m/s B. 20m/s C. 14.1m/s D. 12.4m/s

**Solution**

The frictional force $f_k = 0$ since the plane is frictionless.

$F_R = ma$

$25 = 2 \times a$

$a = 12.5 m/s^2$

$v^2 = v_o^2 + 2ax$

$v^2 = 10^2 + 2 \times 12.5 \times 4 = 200$

$v = 14.1 m/s$

The correct option is C.

**4.17.** A force of magnitude 5N acts on a 2kg body moving initially in the direction of the force with speed of 4m/s. Calculate the distance travelled by the body if the final speed of the body is 6m/s?

A. 5 m B. 10 m C. 4 m D. 6 m

**Solution**

$F = ma$

$5 = 2 \times a$

$a = 2.5 m/s^2$

$v^2 = v_o^2 + 2ax$

$6^2 = 4^2 + 2 \times 2.5 \times x$

$36 - 16 = 5x$

$x = 4$ m

The correct option is C.

**4.18.** The coefficient of static friction between a box of mass 15.0kg and horizontal surface is 0.4. If a force of 20.0N is applied to the box, calculate the friction force and the limiting frictional force between the two surfaces.

A. 8.0N, 60.0N

B. 20.0N, 60.0N

C. 58.8N, 20.0N

D. 20.0N, 8.0N

**Solution**

Frictional force is equal to the applied force when the body is stationary, i.e. frictional force = 20.0N

Limiting frictional force:

$f_s = \mu_s mg = 0.4 \times 15.0 \times 10N = 60.0N$

The correct option is B.

**4.19.** A force of 30.0N is acted on a block of mass 15.0kg placed on a horizontal surface but the box does not move. Calculate the magnitude of the frictional force between the surfaces when the applied force is along the horizontal and when it makes angle 60° with the horizontal.
A. 20.0N, 10.0N
B. 10.0N, 17.32N
C. 30.0N, 15.0N
D. 20.0N, 20.0N

**Solution**

When the applied force is along the horizontal, frictional force $f_s$ = 30N. When the applied force makes angle 30° with the horizontal frictional force:
$f_s$ = component of the applied force along the $x$–axis = 30cos 60°N = 15N

The correct option is C

**4.20.** All these are true of friction except:
A. Friction and the applied force is zero
B. Friction and the applied force are equal in magnitude when acceleration is zero
C. Coefficient of static friction $\mu_s$ is less than coefficient of kinetic friction $\mu_k$
D. Limiting friction is proportional to normal reaction.

**Solution**

The correct option is C.

**4.21.** When a force acts on an object, it produces acceleration which may be given as $a = F/m$ where $F$ is force on the object and $m$ is the mass of the object. This is otherwise known as:
A. First law of motion
B. Newton's second law
C. Third law

D. All of the above

**Solution**

The correct option is B.

**4.22.** An upward force of $12 \times 10^3$N acts on an elevator of mass 2000kg. Calculate the acceleration of the elevator.

A. -6m/s² B. 6m/s² C. 3.0m/s² D. -4.0m/s²

**Solution**

Two forces act on the elevator: the upward force and the weight. Since the weight is acting downward, the two forces can be said to be acting in opposite directions. Hence, the resultant force on elevator:

$$F_R = ma$$

$$12 \times 10^3 - mg = ma$$

$$12 \times 10^3 - 2000 \times 10 = 2000 \times a$$

$$a = -4.0 m/s^2$$

The correct option is D.

**4.23.** According to Newton's first law, when a body is in equilibrium, the vector sum of the forces on it is?

A. zero B. positive C. Negative D. None

**Solution**

The correct option is A.

**4.24.** A body at rest is acted on by a constant net force of 80N and moves 72m in 6 s. What is the mass of the body?

A. 12kg B. 10kg C. 20kg D. 36kg

**Solution**

$v_0 = 0$, $x = 72$ m, $t = 6$ s

$$x = v_0 t + \tfrac{1}{2} at^2$$
$$72 = 0 \times 6 + \tfrac{1}{2} \times a \times 6^2$$
$$72 = 18a$$

$a_2 = 4m/s^2$

$F = ma$

$80 = m \times 4$

$m_2 = 20kg$

The correct option is C.

**4.25.** A load of 100kg is placed on an elevator of mass 700kg. Find the tension in the supporting cable when the elevator originally moving downward at $10ms^{-1}$ is brought to rest with constant acceleration in a distance of 30m.

A. 9360N B. 9440N C. 9540N D. 6640N

**Solution**

$v^2 = u^2 + 2as$

$0 = 10^2 - 2 \times a \times 30$

$60a = 100$

$a = 1.7m/s^2$

Since the elevator is moving downward;

$mg - T = ma$

$T = mg - ma$

$T = 800(10 - 1.7)N$

$T = 6640N$

The correct option is D.

(However, if the elevator is moving upward:

$T - mg = ma$

$T = ma + mg = 800(10 + 1.7) = 9360N$, which is option A)

**4.26.** A bullet of mass 0.5kg is shot into a 100kg block which is initially at rest on a frictionless horizontal surface. The bullet comes to rest within the block and both slide at 0.5m/s in the same direction as the incoming bullet. Calculate the initial speed of the bullet.

**Solution**

$m_1 = 0.5kg$, $u_1 = ?$, $u_2 = 0$, $m_2 = 100kg$, $v_1 = v_2 = v = 0.5m/s$

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$0.5 \times u_1 + 100 \times 0 = (0.5 + 100) \times 0.5$

$$u_1 = \frac{(0.5+100)\times 0.5}{0.5} m/s = 100.5 m/s$$

**4.27.** Two 2.0kg bodies, *A* and *B* collide. The velocities before collision are $u_A$ = 15**i** + 30**j**, $u_B$ = -10**i** + 5**j** and $v_A$ = -5**i** + 20**j** after the collision. All speeds are given in m/s. Calculate the final velocity $v_B$.

A. 30**i** + 15**j**
B. -10**i** + 55**j**
C. 5**i** + 10**j**
D. 10**i** + 15**j**

**Solution**

$m_Au_A + m_Bu_B = m_Av_A + m_Bv_B$

$2(15\mathbf{i} + 30\mathbf{j}) + 2(-10\mathbf{i} + 5\mathbf{j}) = 2(-5\mathbf{i} + 20\mathbf{j}) + 2v_B$

$15\mathbf{i} + 30\mathbf{j} - 10\mathbf{i} + 5\mathbf{j} = -5\mathbf{i} + 20\mathbf{j} + v_B$

$v_B = 5\mathbf{i} + 5\mathbf{i} + 35\mathbf{j} - 20\mathbf{j}$

$v_B = 10\mathbf{i} + 15\mathbf{j}$

The correct option is D.

**4.28.** A bullet of mass 0.2kg is fired with a velocity of 800m/s into a soft wood of mass 2kg, lying on a smooth surface. What is the final velocity if the collision is completely inelastic?

A. 72.7m/s
B. 8m/s
C. 80m/s
D. 0.800m/s

**Solution**

In a completely inelastic collision, the colliding bodies move with the same velocity *v* after the collision. Therefore:

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$0.2 \times 800 + 2 \times 0 = (0.2 + 2)v$

$v = 72.7 m/s$

The correct option is A.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$0.5 \times u_1 + 100 \times 0 = (0.5 + 100) \times 0.5$$

$$u_1 = \frac{(0.5 + 100) \times 0.5}{0.5} m/s = 100.5 m/s$$

4.27. Two 2.0kg bodies, $A$ and $B$ collide. The velocities before collision are $u_A = 15\mathbf{i} + 30\mathbf{j}$, $u_B = -10\mathbf{i} + 5\mathbf{j}$ and $v_A = -5\mathbf{i} + 20\mathbf{j}$ after the collision. All speeds are given in m/s. Calculate the final velocity $v_B$.

A. $30\mathbf{i} + 15\mathbf{j}$ B. $-10\mathbf{i} + 55\mathbf{j}$
C. $5\mathbf{i} + 10\mathbf{j}$ D. $10\mathbf{i} + 15\mathbf{j}$

**Solution**

$$m_Au_A + m_Bu_B = m_Av_A + m_Bv_B$$

$$2(15\mathbf{i} + 30\mathbf{j}) + 2(-10\mathbf{i} + 5\mathbf{j}) = 2(-5\mathbf{i} + 20\mathbf{j}) + 2v_B$$

$$15\mathbf{i} + 30\mathbf{j} - 10\mathbf{i} + 5\mathbf{j} = -5\mathbf{i} + 20\mathbf{j} + v_B$$

$$v_B = 5\mathbf{i} + 5\mathbf{i} + 35\mathbf{j} - 20\mathbf{j}$$

$$v_B = 10\mathbf{i} + 15\mathbf{j}$$

The correct option is D.

4.28. A bullet of mass 0.2kg is fired with a velocity of 800m/s into a soft wood of mass 2kg, lying on a smooth surface. What is the final velocity if the collision is completely inelastic?

A. 72.7m/s B. 8m/s C. 80m/s D. 0.800m/s

**Solution**

In a completely inelastic collision, the colliding bodies move with the same velocity $v$ after the collision. Therefore:

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$0.2 \times 800 + 2 \times 0 = (0.2 + 2)v$$

$$v = 72.7 m/s$$

The correct option is A.

**4.29.** A 1.0kg ball moving at 12m/s collides with a 2.0kg ball moving in the opposite direction at 24m/s. Determine the velocities of the balls after impact if (a) $e = 2/3$, (b) the balls stick together, and (c) the collision is perfectly elastic.

**Solution**

In all three cases, momentum is conserved:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$1.0\times 12 + 2.0\times(-24) = 1.0\times v_1 + 2.0\times v_2$$

$$-36 = v_1 + 2v_2 \qquad 1$$

(a) When e = 2/3, $e = \dfrac{v_2 - v_1}{u_1 - u_2}$

$$\frac{2}{3} = \frac{v_2 - v_1}{12-(-24)}. \text{ Hence:}$$

$$24 = v_2 - v_1 \qquad 2$$

Combining Equations (1) and (2) gives

$v_2 = -4.0m/s$ and $v_1 = -28m/s$

(b) In this case $v_2 = v_1 = v$ and so Equation (1) becomes

$$-36 = 3v \text{ or } v = -12m/s$$

(c) Here e = 1, so $1 = \dfrac{v_2 - v_1}{12-(-24)}$

Hence:

$$36 = v_2 - v_1 \qquad 3$$

Combining Equations (1) and (3) gives $v_2 = 0$ and $v_1 = -36m/s$

**4.30.** A horizontal force of 5.00N is applied to a 2.00kg block initially at rest on a rough horizontal surface. What is its acceleration if the coefficient of friction is 0.2?

A. $2.0m/s^2$ B. $1.96m/s^2$ C. $0.50m/s^2$ D. $3.5m/s^2$

**Solution**

**4.29.** A 1.0kg ball moving at 12m/s collides with a 2.0kg ball moving in the opposite direction at 24m/s. Determine the velocities of the balls after impact if (a) $e = 2/3$, (b) the balls stick together, and (c) the collision is perfectly elastic.

**Solution**

In all three cases, momentum is conserved:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$1.0 \times 12 + 2.0 \times (-24) = 1.0 \times v_1 + 2.0 \times v_2$$

$$-36 = v_1 + 2v_2 \qquad 1$$

(a) When e = 2/3, $e = \frac{v_2 - v_1}{u_1 - u_2}$

$$\frac{2}{3} = \frac{v_2 - v_1}{12 - (-24)}$$. Hence:

$$24 = v_2 - v_1 \qquad 2$$

Combining Equations (1) and (2) gives

$$v_2 = -4.0m/s \ and \ v_1 = -28m/s$$

(b) In this case $v_2 = v_1 = v$ and so Equation (1) becomes

$$-36 = 3v \ or \ v = -12m/s$$

(c) Here e = 1, so $1 = \frac{v_2 - v_1}{12 - (-24)}$

Hence:

$$36 = v_2 - v_1 \qquad 3$$

Combining Equations (1) and (3) gives $v_2 = 0 \ and \ v_1 = -36m/s$

**4.30.** A horizontal force of 5.00N is applied to a 2.00kg block initially at rest on a rough horizontal surface. What is its acceleration if the coefficient of friction is 0.2?

A. 2.0m/s$^2$ B. 1.96m/s$^2$ C. 0.50m/s$^2$ D. 3.5m/s$^2$

**Solution**

From Newton's second law of motion, resultant force $F_R = ma$, therefore:

$5.00 - f_k = 2.00 \times a$

$5.00 - \mu_k mg = 2.00 \times a$

$5.00 - 0.2 \times 2.00 \times 10 = 2.00 \times a$

$a = 1.50 m/s^2$

The correct option is C.

## Summary of Chapter 4

In chapter 4, you have learned that:

1. Newton's first law of motion states that every particle continues to be in a state of rest and if already in motion, it will continue in motion with uniform velocity unless a resultant force acts on it.
2. The momentum is defined as the product of the mass of the object and the velocity.
3. Impulse of a force is defined as the product of the force and the time with which it acts.
4. Newton's second law of motion states that the rate of change of momentum is directly proportional to the applied resultant force and the momentum change takes place in the direction of the force.
5. Newton's third law states that to every action there is an equal and opposite reaction.
6. The law of conservation of linear momentum states that during collisions in which the colliding objects and the product bodies are not acted upon by an externally applied force, the sum of linear momentum before collision is equal to the sum of linear momentum after the collision.
7. The collision of two or more bodies can be classified into either elastic or inelastic. The law of conservation of momentum is conserved in both classes of collision.

8. Whenever two bodies are in contact with each other, there is always a resistance to motion experienced by the surfaces of the two bodies. This resistance to motion is referred to as force of friction, or simply, friction. Friction occurs for solid, liquid and gases but this section is concerned with friction between solid surfaces. Basically, two kinds of friction exist between two solid surfaces. They are: static and sliding (kinetic).

## Self-Assessment Questions (SAQs) for Chapter 4

**4.1.** A body *A* of mass 1.5kg, travelling along the positive *x*-axis with speed 4.5m/s, collides with another body *B* of mass 3.2kg which, initially, is at rest. As a result of the collision, *A* is deflected and moves with a speed 2.1m/s in a direction which is at angle 30° below the *x*-axis. *B* is set in motion at an angle $\theta$ above the *x*-axis. Calculate the velocity of *B* after the collision.

**4.2.** A man of mass 80kg stands next to a stationary ball of mass 4kg on a frictionless surface. He kicks the ball forward along the surface with a speed 15m/s. Calculate the man's recoil speed.

**4.3.** A hose directs a horizontal jet of water, moving with a velocity of 20m/s, on to a vertical wall. The cross-sectional area of the jet is $5 \times 10^{-4}m^2$. If the density of water is $1000kgm^{-3}$, calculate the force on a wall assuming the water is brought to rest there.

**4.4.** A ball *A* of mass 0.1kg, moving with a velocity of 6m/s, collides head-on with a ball *B* of mass 0.2kg at rest. Calculate their common velocity if both balls move off together. If *A* had rebounded with a velocity of 2m/s in the opposite direction after collision, what would be the new velocity of *B*?

**4.5.** In the system shown in Figure 4.5, a force 80N accelerates the 5kg body to the right. Find the acceleration of the motion if the coefficient of kinetic friction is 0.44.

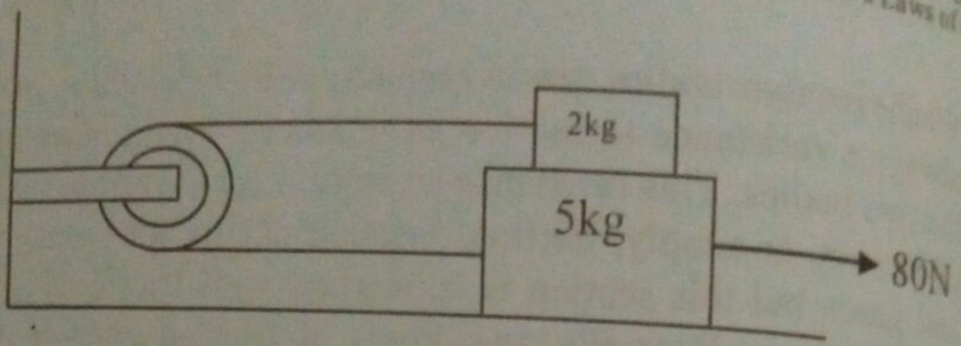


Figure 4.5: SAQ 4.5

**4.6.** A block of mass 20kg is at rest on a rough horizontal ground. The coefficient of friction between the block and the ground is 0.3. A horizontal force $F$ is applied steadily to the block according to the law: $F = t^2 - t + 4$, $F$ is in newtons and $t$ is the time in seconds. Calculate the time it takes the block to start moving. What is the speed of the block when $t = 4.0$s.

**4.7.** A constant force acts on a 2.0kg object and reduces its velocity from 7.0m/s to 3.0m/s in a time of 3.0s. What is the magnitude of the force?

**4.8.** A 8000kg engine pulls a 40 000kg train along a level track and gives it an acceleration 1.20m/s$^2$. What acceleration would the engine give to a 16 000kg train?

**4.9.** A 300g mass hangs at the end of a string. A second string hangs from the bottom of that mass and supports a 900g mass. (a) Find the tension in each string when the masses are accelerating upward at 0.700m/s$^2$. (b) Find the tension in each string when the masses are accelerating downward at 0.700m/s$^2$.

**4.10.** A horizontal force of 200N is required to cause a 15kg block to slide up a 20° incline with an acceleration of 25cm/s$^2$. Find (a) the friction force on the block and (b) the coefficient of friction.

**4.11.** A lawn tennis ball of mass $m$ and speed $v$ strikes a wall perpendicularly and rebounds with undiminished speed. If the time of collision is $t$, what is the average force exerted by the ball on the wall?

A. $mv/t$ B. $mv/2t$ C. $2mv/t$ D. $4mv/t$

**4.12.** A block of mass 2kg is pulled along a smooth horizontal surface by a horizontal force $P$. What is the value of the normal force exerted on the block by the surface, and determine the force $P$ required to give the block horizontal velocity of $4ms^{-1}$ in 2s.
A. 20N; 10N B. 20N; 4N C. 10N; 4N D. 10N; 10N

**4.13.** A block of mass $m_1$ = 2kg on a smooth horizontal surface which is pulled by a string which is attached to another block $m_2$ = 1kg hanging over a pulley, as shown in Figure 4.6. The acceleration of the system and the tension in the string are given as:
A. $2.2ms^{-2}$; 60N
B. $3.3ms^{-2}$; 6N
C. $4ms^{-2}$ 6.5N
D. $3.3ms^{-2}$; 6.7N

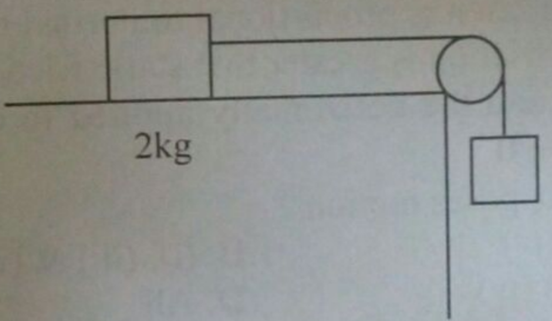


Figure 4.6: SAQ 4.13

**4.14.** Three forces act on a particle that moves with unchanging velocity **v** = (3m/s)**i** - (4m/s)**j**. Two of the forces are $\mathbf{F}_1$ = (2N)**i** + (3N)**j** +(-2N)**k** and $\mathbf{F}_2$ = (-5N)**i** + (8N)**j** +(-2N)**k**. What is the third force?
A. (3N)**i** +(-11N)**j** + (4N)**k**
B. (-11N)**i** + (3N)**j** +(4N)**k**
C. (4N)**i** + (3N)**j** +( -11N)**k**
D. (3N)**i** +(4N)**j** + (-11N)**k**

**4.15.** A car that weighs $1.3 \times 10^4$N was initially moving at a speed of 40km/h when the brakes were applied. The car was brought to a stop in 15m. Assuming that the force that stops the car is constant, find the magnitude of that force and the time required to bring the car to a stop.
A. $5.33 \times 10^3$N, 1.35s
B. $5.33 \times 10^3$N, 2.7s

C. $5.45 \times 10^3$N, 1.35s
D. $5.45 \times 10^3$N, 2.7s

**4.16.** A man pulls a crate across a frictionless floor by pulling rope tied to the crate. The man exerts a force of 450N on the which is inclined at 38° to the horizontal. Calculate the magnitude the acceleration of the crate if (i) its mass is 310kg and (ii) weight 310N.

A. $1.45 m/s^2$, $14.23 m/s^2$
B. $1.14 m/s^2$, $11.4 m/s^2$
C. $1.45 m/s^2$, $1.45 m/s^2$
D. $14.23 m/s^2$, $14.23 m/s^2$

**4.17.** Which of the following is/are true of friction?
(i) Limiting friction is proportional to normal reaction
(ii) Dynamic friction is greater that static friction
(iii) Friction and the horizontally applied force are not equal when acceleration $\neq 0$
(iv) Friction opposes motion

A. (i), (ii) & (iv)
B. (i), (iii) & (iv)
C. (i), (ii) and (iv)
D. All

**4.18.** The mass of a certain body varies as a function of time as $m = m_o e^{-0.5t}$. Find the force, in N, acting on the body if its velocity $v(t) = 2ti$ m/s. i is a unit vector in the x-direction.

A. $i(2-t)m_o e^{-0.5t}$
B. $2im_o e^{-0.5t}$
C. $-0.5im_o e^{-0.5t}$
D. $i(t-2)m_o e^{-0.5t}$

**4.19.** A tennis ball has a mass of 0.06kg. The impact with a wall changes its velocity from 25m/s towards the wall to 35m/s away from the wall. Determine the impulse the wall exerts on the tennis ball.

A. 2.16Ns
B. 21.6Ns
C. 3.6Ns
D. 0.36Ns

**4.20.** A man kicks a stationary ball of mass 200g, giving it a speed of 100cm/s. What impulse is imparted to the ball?

A. 0.2kgm/s
B. 20kgm/s
C. 2kgm/s
D. 200kgm/s

# CHAPTER FIVE

# CIRCULAR MOTION AND GRAVITATION

## 5.1 Circular Motion: Angular Speed

The direction of a particle's velocity changes when it moves along a curved path. This means that the particle must have a component of acceleration perpendicular to the path, even if the speed is constant. When a particle moves in a circle with constant speed, the motion is called uniform circular motion.

Let us suppose that a particle moves in a circle with a uniform speed $v$ round a fixed point $O$ as centre, as shown in Figure 5.1.

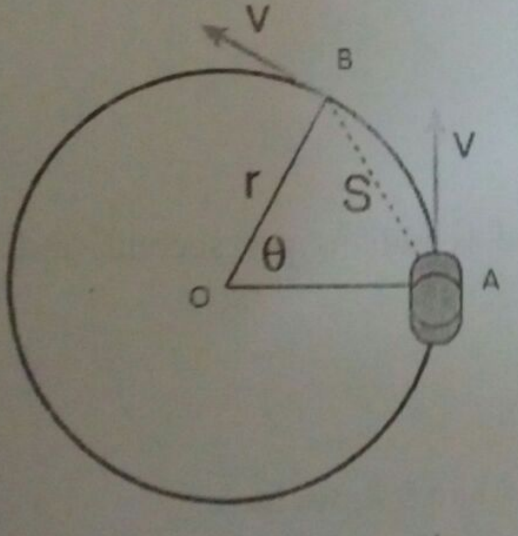


Figure 5.1: Circular motion

Consider the motion of the object in the time interval between $t_0 = 0$ and $t_f = t$. In this interval, the object rotates through an angle $\theta$ and traces out a circular arc of length $S$. The angle $\theta$ is measured anti-clockwise from the vertical line $OA$. If $\theta$ is in radian then we can write:

$$\theta = \frac{AB}{r} \qquad 5.1$$

where $AB = S$ is the length of the arc from $A$ to $B$ and $r$ is the radius of the circular path. Although the speed $v$ of the particle is constant, its velocity $\vec{v}$ is not because the direction of motion changes continuously as the particle moves round the circle. Therefore, the particle has acceleration. Let us now attempt to derive an expression for the acceleration of the motion. The acceleration here is different from the acceleration we encountered in Chapter 3. The speed $v$ is the rate of change of the arc length $AB$ with time. That is:

$$v = \frac{\Delta(r\cdot\theta)}{\Delta t} = \frac{r\Delta\theta}{\Delta t} \qquad 5.2$$

assuming the radius $r$ is constant.

The average angular speed of the motion, denoted by the symbol $\omega$, is the angular displacement divided by the total time taken to travel the distance $S$:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta - \theta_o}{t - t_o} \qquad 5.3$$

SI unit of angular speed is radians per second (rad/s). In calculus notation, we can have:

$$\omega = \frac{d\theta}{dt} \qquad 5.4$$

Substituting Equation 5.3 into 5.2, we have:

$$v = r\omega \qquad 5.5$$

Equation 5.5 gives the relationship between linear and angular speeds.

## 5.2 Angular Acceleration

The angular acceleration, $\alpha$, can be defined as the time rate of change of angular velocity:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_o}{t - t_o} \qquad 5.6a$$

So:

$$\omega = \omega_o + \alpha t \qquad 5.6b$$

where $t_o = 0$. The S.I. unit of angular acceleration is radians per second per second (rad s$^{-2}$). Equation 5.6 applies for constant angular acceleration only.

We can define angular displace $(\theta - \theta_o)$ as follows:

$$\theta - \theta_o = \left(\frac{\omega + \omega_o}{2}\right) \times t \qquad 5.7$$

Substituting Equation 5.6b into 5.7, we have:

$$\theta - \theta_o = \left(\frac{\omega_o + \alpha t + \omega_o}{2}\right) \times t$$

$$\theta - \theta_o = \omega_o t + \frac{1}{2}\alpha t^2 \qquad 5.8$$

Substituting for $t$ in Equation 5.6b using 5.7, we have:

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o) \qquad 5.9$$

Equations 5.6 to 5.9 are equations of motion for constant angular acceleration.

## 5.3 Centripetal Acceleration

Consider an object moving in a circular orbit of radius $r$ with uniform tangential speed $v$. Assume that the object possesses a velocity vector **v** whose magnitude is constant, but whose direction is continuously changing. This implies that the object must be *accelerating*, since (vector) acceleration is the rate of change of (vector) velocity and the (vector) velocity is indeed varying in time. The direction of the instantaneous acceleration at each point is always along a radius of the circle toward its centre. Because the speed is constant, the acceleration is always perpendicular to the instantaneous velocity. Figure 5.2 shows a particle moving with constant speed in a circular path of radius $r$ with centre $O$. The particle moves from $A$ to $B$ in a time interval $\Delta t$. The velocity of the particle changes from $\vec{v}_1$ *to* $\vec{v}_2$ as shown in Figure 5.2.

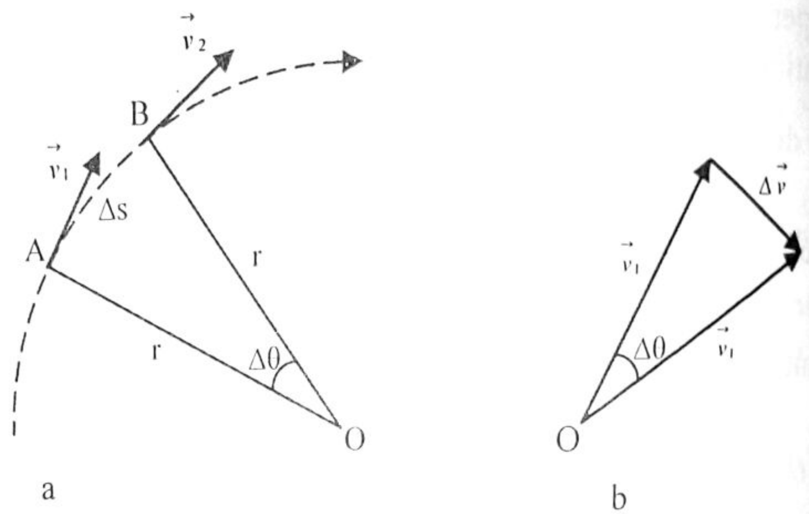


Figure 5.2: Centripetal Acceleration

The triangles in Figures 5.2a and 5.2b are similar since $\vec{v}_1$ is perpendicular to the line $OA$ and $\vec{v}_2$ is perpendicular to line $OB$. Hence, the ratios of corresponding sides of similar triangles are equal:

$$\frac{\left|\Delta \vec{v}\right|}{v_1} = \frac{\Delta s}{r} \quad or \quad \left|\Delta \vec{v}\right| = v_1 \frac{\Delta s}{r}$$

The magnitude $a_{av}$ of the average acceleration during the time interval $\Delta t$ is therefore:

$$a_{av} = \frac{\left|\Delta \vec{v}\right|}{\Delta t} = \frac{v_1}{r}\frac{\Delta s}{\Delta t}$$

The magnitude $a$ of the instantaneous acceleration $\vec{a}$ at point $A$ is the limit of the average acceleration as we take point $B$ closer and closer to point $A$, that is:

$$a = \lim_{\Delta t \to 0} \frac{v_1}{r}\frac{\Delta s}{\Delta t} = \frac{v_1}{r}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t} = \frac{v_1}{r} \times v_1 = \frac{v_1^2}{r}$$

The point $A$ can be any point on the path, so we can remove the subscript. Then:

$$a = \frac{v^2}{r} \qquad 5.10$$

Because the acceleration is always directed toward the centre of the circle, it is sometimes called centripetal acceleration, which means centre-seeking acceleration.

Using Equation 5.5, the centripetal acceleration equation can be written as:

$$a = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2 \qquad 5.11$$

The period $T$ of the motion is the time for one revolution (one complete trip around the circle). The particle travels a distance equal to the circumference $2\pi r$ of the circle in the time $T$, so its speed is:

$$v = \frac{2\pi r}{T} \qquad 5.12$$

When we substitute Equation 5.12 into 5.10, we obtain:

$$a = \frac{4\pi^2 r}{T^2} \qquad 5.13$$

From Newton's second law, the centripetal acceleration is:

$$F = ma = \frac{mv^2}{r} \qquad 5.14$$

where $m$ is the mass of the particle.

## 5.4 Conical Pendulum

Consider a small object of mass *m* tied to a string of length *L* fixed at point *O* and then whirled round in a horizontal circle of radius *r*, with the fixed point *O* directly above the centre of the circle, as shown in Figure 5.3. We assume that the circular speed of the object is constant and the string turns at a constant angle $\theta$ to the vertical. This arrangement is called a conical pendulum.

The tension *T* in the string *OA* has two components: $T\cos\theta$ and $T\sin\theta$. The horizontal component, $T\sin\theta$, of the tension *T* in the string provides the centripetal acceleration along the radius of the horizontal circular path. So that

$$T\sin\theta = \frac{mv^2}{r} \qquad 5.15$$

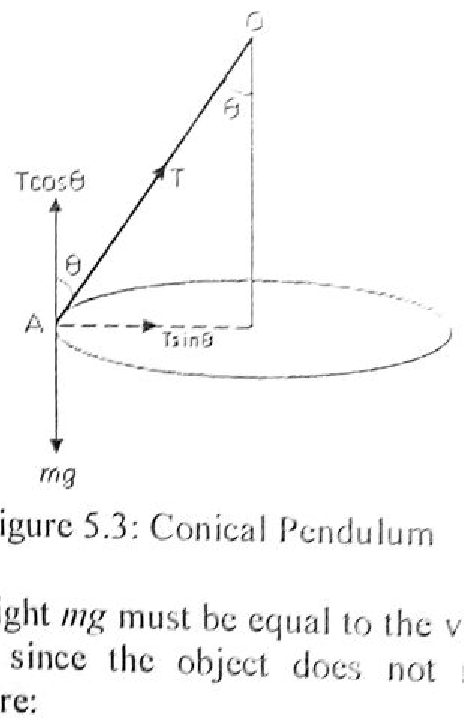


Figure 5.3: Conical Pendulum

The weight *mg* must be equal to the vertical component $T\cos\theta$ of the tension since the object does not move in a vertical direction. Therefore:

$$T\cos\theta = mg \qquad 5.16$$

Dividing Equation 5.15 by 5.16, we have:

$$\frac{T\sin\theta}{T\cos\theta} = \frac{mv^2}{r} \div mg$$

Then:

$$\tan\theta = \frac{v^2}{rg} \qquad 5.17$$

From Figure 5.3, we can see that $r = L\sin\theta$. Using Equation 5.13 we can write centripetal acceleration as:

$$a = \frac{4\pi^2 L\sin\theta}{T^2} \qquad 5.18$$

Substituting this into Equation 5.17 and using 5.10, we obtain:

$$\tan\theta = \frac{v^2}{rg} = \frac{a}{g} = \frac{4\pi^2 L\sin\theta}{gT^2} \qquad 5.19$$

which we can rewrite as:

$$T = 2\pi\sqrt{\frac{L\cos\theta}{g}} \qquad 5.20$$

## 5.5 Banking: The Motion of Vehicles on Curved Roads

Banking is a technique in road construction that involves tilting of the road surface at an angle $\theta$ above the horizontal, inward to the centre of the road, as in Figure 5.4. Suppose a car is moving round a curved part of a banked road or track.

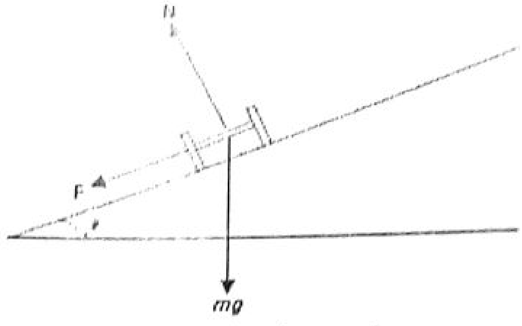


Figure 5.4: Banked road

The centripetal force is provided by the horizontal component of the frictional force and the horizontal component of the car's normal reaction, or:

$$f_k \cos\theta + N\sin\theta = \frac{mv^2}{r} \qquad 5.21$$

The maximum value of $f_k = \mu_k N$, hence Equation 5.21 becomes:

$$\mu_k N\cos\theta + N\sin\theta = \frac{mv^2}{r} \qquad 5.22$$

Since the car does not move in the vertical direction, we can write:

$$-\mu_k N\sin\theta + N\cos\theta = mg \qquad 5.23$$

Dividing Equation 5.22 by 5.23, we have:

$$\frac{\mu_k N\cos\theta + N\sin\theta}{-\mu_k N\sin\theta + N\cos\theta} = \frac{mv^2}{r} \div mg$$

or $$v = \sqrt{\frac{rg(\mu_k\cos\theta + \sin\theta)}{\cos\theta - \mu_k\sin\theta}} \qquad 5.24$$

In a special case of horizontal road, for which $\theta = 0$, we have:

$$v = \sqrt{\frac{rg(\mu_k\cos 0 + \sin 0)}{\cos 0 - \mu_k\sin 0}} = \sqrt{\mu_k rg} \qquad 5.25$$

Equations 5.21 and 5.23 can be solved for $f_k$. We multiply Equation 5.21 by $\cos\theta$ and Equation 5.23 by $\sin\theta$, and subtract the resulting equations to eliminate $N$. Then, we have:

$$f_k = \frac{mv^2}{r}\cos\theta - mg\sin\theta \qquad 5.26$$

The best, or optimum, angle of banking is the angle $\theta = \theta_o$ for which $f_k = 0$, as this produces a centripetal force which is due entirely to the horizontal component of the car's normal reaction. The wear on the car and its tyres, due to friction, is eliminated as $f_k = 0$. Then, from Equation 5.26, the optimum angle of banking $\theta_o$ is given by:

$$\tan\theta_o = \frac{v^2}{rg} \qquad 5.27$$

## 5.6 Newton's Law of Universal Gravitation

In 1687, Newton published the law of gravitation. Newton's law of universal gravitation states that every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. If the particles have masses $m_1$ and $m_2$ and are separated by a distance $r$, the law of gravitation can be stated as follows:

$$F_g \alpha \frac{m_1 m_2}{r^2}$$

or $$F_g = \frac{Gm_1 m_2}{r^2} \qquad 5.28$$

where $G$ is a fundamental physical constant called the gravitational constant. $G$ can be expressed inN.m$^2$kg$^{-2}$ and careful measurement shows that $G = 6.67 \times 10^{-11}$N.m$^2$kg$^{-2}$. Because 1N = 1kg.m/s$^2$, the units of $G$ can also be expressed (in fundamental SI units) as m$^3$/(kg.s$^2$).

In vector form, the law of gravitation can be written as:

$$\vec{F}_{12} = \frac{Gm_1 m_2}{r^2}\hat{r}_{12} \qquad 5.29a$$

According to Newton's third law of motion, the force exerted on particle 1 by particle 2, designated $\vec{F}_{12}$, is: equal in magnitude to $\vec{F}_{21}$ and in opposite direction. That is, $\vec{F}_{12} = -\vec{F}_{21}$.

**Gravitational field strength** (g) at a point is the gravitational force per unit mass at that point. It is a vector and its S.I. unit is Nkg$^{-1}$.

By definition:

$$g = \frac{F_g}{m}. \quad 5.29b$$

Substitute Equation 5.28, we have:

$$g = \frac{GMm}{r^2} \div m = \frac{GM}{r^2} \quad 5.29c$$

where $M$ is the mass of the object creating the gravitational field

## Determining the value of $G$

To determine the value of the gravitational constant $G$, we use gravitational torsion balance. The gravitational torsion bal reprises one of the great experiments in the history of physics. measurement of the gravitational constant, as performed by H Cavendish in 1798.

The Gravitational Torsion Balance (Figure 5.5) consists of two gram masses (small mass, $m_1$) suspended from a very thin, ver quartz fibre and two 1.5 kilogram masses (large mass, $m_2$), with arrangement positioned like an inverted T. The Gravitational Tor Balance is oriented so the force of gravity between the small b and the earth is negated (the pendulum is nearly perfectly alig vertically and horizontally). The large masses are brought near smaller masses and the gravitational force between the large small masses is measured by observing the twist of the tor ribbon. This attractive gravitational force twists the T throug small angle. To measure this angle, we shine a beam of light c mirror fastened to the T. The reflected beam strikes a scale, an the T twists, the reflected beam moves along the scale. A calibrating the Cavendish balance, we can measure gravitati forces and thus determine $G$.

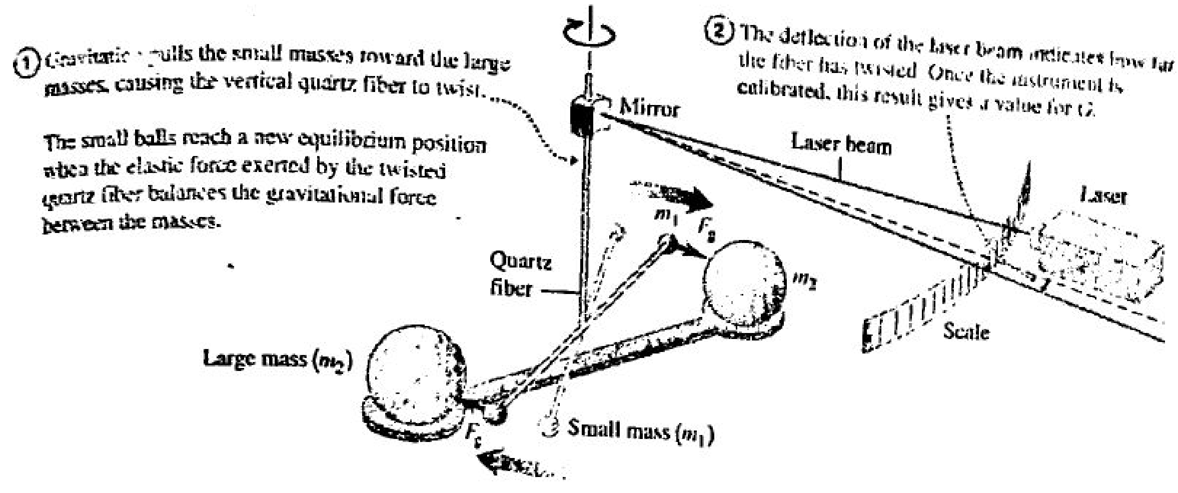


Figure 5.5: The principle of the Cavendish balance

## 5.7 Free-Fall Acceleration

The weight of a body is the total gravitational force exerted on the body by all other bodies in the universe. We can neglect all other gravitational forces when the body is near the surface of the Earth and consider the weight as just the Earth's gravitational attraction.

If the Earth is considered as a spherically symmetric body with radius $R_e$ and mass $M_e$, the weight $W$ of a small body of mass $m$ at the Earth's surface is:

$$W = F_g = \frac{GM_e m}{R_e^2}$$

$$mg = \frac{GM_e m}{R_e^2} \qquad \text{(since } W = mg\text{)}$$

$$g = \frac{GM_e}{R_e^2} \qquad 5.30$$

Equation 5.30 is the expression for the free-fall acceleration near the Earth's surface. This acceleration is independent of the mass $m$ of the body and this can also be shown in Equation 5.30.

The gravitational force obeys an inverse-square law at points outside the Earth, that is, $g \propto 1/r^2$, where $r$ is the distance to the centre of the

Earth. Inside the Earth, the value of $g$ is not inversely proportional to the square of the distance from the centre but varies linearly with the distance from the centre (line OP), as shown in Figure 5.6. Note that $g_s$ is the value on the surface of the Earth.

Let us now consider an object of mass $m$ located at a distance $h$ above the Earth's surface or a distance $r$ from the Earth's centre, where $r = R_e + h$.

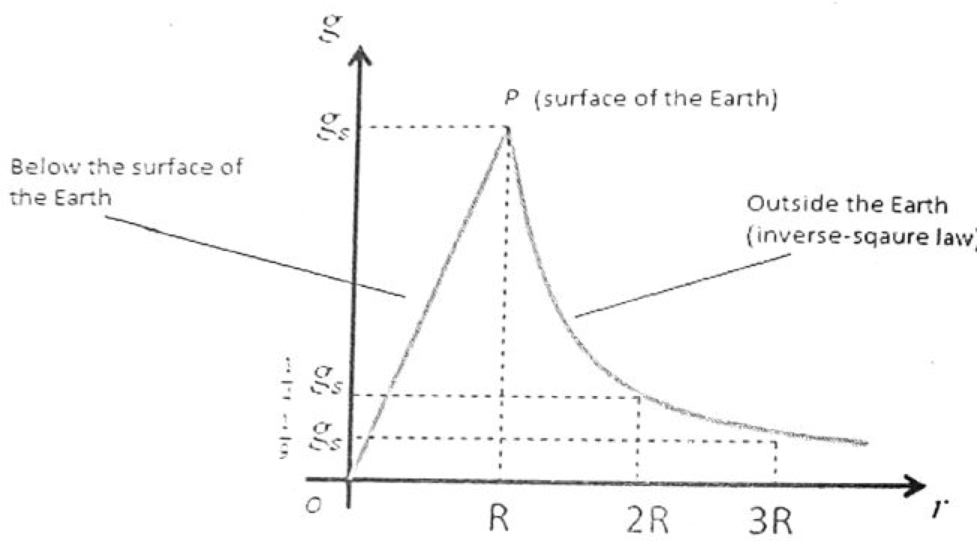


Figure 5.6: Variation of $g$ with $r$ (R is the radius of the earth)

The magnitude of the gravitational force acting on this object is:

$$F_g = mg = \frac{GM_e m}{r^2} = \frac{GM_e m}{(R_e + h)^2}$$

which gives:

$$g = \frac{GM_e}{(R_e + h)^2} \qquad 5.31$$

Thus, it follows that $g$ decreases with increasing altitude.

## 5.8 Gravitational Potential Energy

Consider a particle of mass $m$ displaced between two points $P$ and $Q$ above the Earth's surface. The change in the gravitational potential energy associated with the displacement is defined as the negative of the work done by the gravitational force during that displacement:

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)dr \qquad 5.32$$

The gravitational force can be expressed as (from Equation 5.28):

$$F(r) = -\frac{GM_e m}{r^2} \qquad 5.33$$

where the negative sign indicates that the force is attractive.

Hence Equation 5.32 becomes:

$$U_f - U_i = GM_e m \int_{r_i}^{r_f} \frac{1}{r^2} dr = GM_e m \left[ -\frac{1}{r} \right]_{r_i}^{r_f}$$

$$U_f - U_i = -GM_e m \left( \frac{1}{r_f} - \frac{1}{r_i} \right). \qquad 5.34$$

Taking the point $r_i = \infty$ (i.e $U_i = 0$) as the reference point, we obtain the important result:

$$U = -\frac{GM_e m}{r} \qquad 5.35$$

This expression (Equation 5.35) is valid provided that $r \geq R_e$. The result is not valid for particles inside the Earth, where $r < R_e$. If the mass $m$ is at an altitude $h$ above the Earth's surface, then:

$$U = -\frac{GM_e m}{R_e + h} \qquad 5.36$$

Equation 5.36 can be applied for any two particles. The gravitational potential energy for any pair of particles of masses $m_1$ and $m_2$ separated by a distance *r* is·

$$U = -\frac{Gm_1 m_2}{r} \qquad 5.37$$

This expression shows that the gravitational potential energy for any pair of particles varies as $1/r$, whereas the force between them varies as $1/r^2$.

If three or more particles are brought close to one another, the total gravitational potential energy is the sum over all the particles. Consider for example three particles of masses $m_1$, $m_2$ and $m_3$ in a system. The total gravitational potential energy is:

$$U_{total} = U_{12} + U_{13} + U_{23}$$

$$U_{total} = -G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_2 m_3}{r_{23}}\right) \qquad 5.38$$

## 5.9 Satellite Motion

Consider a satellite of mass *m* moving with speed *v* in an orbit around the Earth of mass $M_e$. The centripetal force required to keep the satellite in orbit is supplied by the gravitational attraction between the satellite and the Earth, therefore:

$$F = \frac{mv^2}{r} = \frac{GM_e m}{r^2}$$

Then:

$$v = \sqrt{\frac{GM_e}{r}} \qquad 5.39$$

where $r = R_e + h$.

The total mechanical energy of the two-body system is the sum of the kinetic energy of the satellite and the potential energy of the system:

$$E = K + U = \frac{1}{2}mv^2 - \frac{GM_e m}{r} \qquad 5.40$$

Substituting Equation 5.39 for (5.40), we have:

$$E = \frac{GM_e m}{2r} - \frac{GM_e m}{r} = -\frac{GM_e m}{2r} \qquad 5.41$$

This result (Equation 5.41) shows that the total mechanical energy is negative. Note that the kinetic energy is positive and equal to one-half the absolute value of the potential energy.

## 5.10 Velocity of Escape

Suppose an object of mass $m$ is projected from the Earth's surface at point $P$ with an initial speed $v_{esc}$ so that it just escapes from the gravitational influence of the Earth. Let $v_{esc}$ be the escape speed which is the minimum speed the object must have at the Earth's surface in order to escape from the influence of the Earth's gravitational attraction. From definition:

work done $W = m$ ×potential difference between infinity and the point $P$

$$W = m \times \frac{GM}{R_e} \qquad 5.42$$

We have the kinetic energy of object as:

$$K = \frac{1}{2}mv_{esc}^2 = m \times \frac{GM}{R_e}$$

$$\therefore v_{esc} = \sqrt{\frac{2GM}{R_e}} \qquad 5.43$$

From Equation 5.30, $gR_e^2 = GM_e$, substitute into Equation 5.4... have:

$$v_{esc} = \sqrt{2gR_e} \qquad 5.44$$

If $g = 9.8m/s^2$ and $R_e = 6.4 \times 10^6 m/s$ then:

$$v_{esc} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} ms^{-1} = 11.2 \times 10^3 ms^{-1} = 11.2k$$

With an initial velocity of about 11 $kms^{-1}$, an object or a rocket ... completely escape from the gravitational attraction of the Earth.

## 5.11 Kepler's Laws

Kepler (1571–1630) discovered three empirical laws that accura... describe the motions of the planets. Kepler's laws state that:

1. Each planet moves in an elliptical orbit, with the Sun at ... focus of the ellipse.
2. The line joining the Sun and the other planets sweeps ... equal areas in equal times.
3. The squares of the periods of revolution of the planets ... proportional to the cubes of their mean distances from ... Sun; that is, $T^2 \alpha\ r^3$.

Newton discovered that Kepler's laws can be derived; they ... consequences of Newton's laws of motion and the law of gravitati... Let us see how Kepler's third law arises for instance.

Consider a planet of mass $m_p$ with a circular orbit around the Sun ... mass $M_s$. Centripetal force is supplied by the force of gravity, that is:

$$\frac{m_p v^2}{r} = \frac{GM_s m_p}{r^2}$$

Then:

$$v = \sqrt{\frac{GM_s}{r}}$$

where $r$ is the average distance of the planet from the Sun.

But:

$$v = \frac{circumference}{period} = \frac{2\pi r}{T}$$

So:

$$\frac{2\pi r}{T} = \sqrt{\frac{GM_s}{r}}$$

Squaring both sides and solving for $T^2$ gives:

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right) r^3 \qquad 5.45$$

or:

$$T^2 = Kr^3 \qquad 5.46$$

## 5.12 Mass and Density of Earth

Assuming that the Earth is spherical and of radius $R_e$, the force of attraction of the Earth on a mass $m$ on the Earth surface is:

$$F = mg = \frac{GM_e m}{R_e^2}$$

$$\therefore M_e = \frac{gR_e^2}{G} \qquad 5.47$$

The volume of the Earth is $4\pi R_e^3/3$, since the Earth is regarded as a sphere. Therefore, the mean density $\rho$ of the Earth is given by:

$$\rho = \frac{M_e}{V} = \frac{gR_e^2}{4\pi R_e^3 G/3} = \frac{3g}{4\pi R_e G} \qquad 5.48$$

## Activity 5 Circular Motion

**5.1.** A satellite of mass *m* circles the Earth a distance $R$ from the centre of the Earth. If the radius of the Earth is $6.4 \times 10^6$m, calculate the height above the Earth's surface of the parking orbit and the velocity of the satellite in orbit. Take g = 9.8m/s$^2$.

A. 19600km, 2.4km/s
B. 6400km, 14.12km/s
C. 36000km, 3.1km/s
D. 956.78km, 14.12km/s

**Solution**

Period $T$ = 24 hrs = 24 x 3600s = 86400s (since the satellite is in parking orbit).

$$T^2 = \frac{4\pi^2 R^3}{gR_e^2}$$

$$R^3 = \frac{gR_e^2 T^2}{4\pi^2}$$

$$R^3 = \frac{9.8(6.4 \times 10^6)^2 (86400)^2}{4(3.142)^2} = 7.588 \times 10^{22}$$

$$R = \sqrt[3]{7.588 \times 10^{22}}\, m = 4.23 \times 10^7\, m$$

$$R = R_e + h$$

From which:

$$h = R - R_e = (4.23 \times 10^7 - 6.4 \times 10^6)\text{m} = 3.59 \times 10^7\text{ m} = 36000\text{km}$$

$$\frac{mv^2}{R} = \frac{GM_e m}{R^2}$$

$$v^2 = \frac{GM_e}{R} = \frac{gR_e^2}{R} \text{ (Using } GM_e = gR_e^2\text{)}$$

$$v = \sqrt{\frac{9.8 \times (6.4 \times 10^6)^2}{4.23 \times 10^7}} m/s = 3.08 \times 10^3 m/s = 3.1 km/s$$

The correct option is C.

**5.2.** Which of the following equations are expressions of Kepler's third law:

(i) $T^2 = \frac{4\pi^2 R^3}{gr^2}$ (ii) $\frac{Gm}{r^2} = \frac{4\pi^2 r}{T^2}$ (iii) $m\omega^2 r = \frac{GmM}{r^2}$

(iv) $mg = \frac{gr^2}{G}$

A. (i) & (ii) B. (i) & (iii) C. (i) & (iii) D. (ii) & (iv)

**Solution**

The correct option is A.

**5.3.** The law of universal gravitation can be expressed as:

A. $F = ma$ B. $F = \frac{m_1 m_2}{r}$ C. $F = \frac{Gm_1 m_2}{r^2}$ D. $F = kma$

**Solution**

The correct option is C.

**5.4.** The mass of the moon is about 1/81 that of the Earth and its radius is one-fourth that of the Earth. What is the acceleration due to gravity on the surface of the moon?

A. $\frac{GM_e}{R_e^2}$ B. $\frac{16GM_e}{81R_e^2}$ C. $\frac{4GM_e}{81R_e^2}$ D. $\frac{GM_e}{4R_e^2}$

**Solution**

$$M_m = \frac{1}{81} M_e; \; R_m = \frac{1}{4} R_e$$

$$g_m = \frac{GM_m}{R_m^2} = \frac{G \times \frac{1}{81}M_e}{\left(\frac{1}{4}R_e\right)^2} = \frac{16}{81}\frac{GM_e}{R_e^2}$$

The correct option is B.

**5.5.** Two objects of masses $m_1$ and $m_2$ are placed side by side at distance $r$ apart. What is the magnitude of the force of attraction on each other?

A. $F = ma$ B. $m_1 = km_2a$ C. $F = \frac{m_1m_2}{r}$ D. $F = \frac{Gm_1m_2}{r^2}$

**Solution**

The correct option is D.

**5.6.** A 3kg particle, resting on a smooth table and attached to a fixed point on the table by a rope 1.2 m long, is making 300 revmin$^{-1}$. Find the tension in the rope.

A. 3600N B. 360N C. 3600N D. 5600N

**Solution**

$$\omega = 300 rev/\min = \frac{300 \times 2\pi}{60} rad/s = 10\pi\ rad/s$$

Tension = centripetal force

$$T = \frac{mv^2}{R} = \frac{m(\omega R)^2}{R}$$

$$T = m\omega^2 R = 3(10\pi)^2 \times 1.2N = 3553.979N = 3600N \quad (2s.f)$$

The correct option is C.

**5.7.** A proposed communication satellite would revolve round the Earth in a circular orbit in the equatorial plane at a height of 35880km above the Earth's surface. Find the period of revolution of the satellite. $M_e = 5.98 \times 10^{24}$ kg, $R_e = 6370$ km,

$G = 6.6 \times 10^{-11} Nm^2kg^{-2}$

A. 23 hrs　　B. 22 hrs　　C. 24 hrs　　D. 25 hrs

**Solution**

$R = R_e + h = (35880 + 6370)$ km $= 42250$ km $= 4.2250 \times 10^7$ m

$$T^2 = \frac{4\pi^2 R^3}{gR_e^2}$$

$$T^2 = \frac{4 \times 3.142^2 \times (4.2250 \times 10^7)^3}{g \times (6.370 \times 10^6)^2} = 7.489235 \times 10^9$$

$T = 86.5403663 \times 10^3$ s $= 24.03$ hrs $\approx 24$ hrs

The correct option is C.

**5.8.** Calculate the period of a conical pendulum of length 16m, given that the string turns at a constant angle 60° to the vertical.

A. 4.2 s　　B. 3.6 s　　C. 8.62 s　　D. 5.62 s

**Solution**

$$T = 2\pi\sqrt{\frac{L\cos\theta}{g}} = 2 \times 3.142\sqrt{\frac{16\cos 60}{10}}s = 5.62s$$

The correct option is D.

**5.9.** The mass of the moon is about 1/81 that of the Earth and the distance from the centre of the Earth to that of the moon is about $4.0 \times 10^5$km. At what point between the moon and the Earth will the resultant gravitational force on a spacecraft become zero?

A. $3.6 \times 10^5$km from the centre of the Earth
B. $3.6 \times 10^5$km from the centre of the moon
C. $4.0 \times 10^5$km from the centre of the Earth
D. $4.0 \times 10^5$km from the centre of the moon

**Solution**

Let the point of zero gravitational force be $x$km from the centre of the Earth and a distance $(4 \times 10^5 - x)$ km from the moon. If the mass of the spacecraft is $m$, then:

$$\frac{GM_e m}{x^2} = \frac{GM_m m}{(4 \times 10^5 - x)^2}$$

Using $M_m = \frac{1}{81} M_e$ and rearranging, we have:

$$\frac{M_e}{M_m} = \frac{81}{1} = \frac{x^2}{(4 \times 10^5 - x)^2}$$

Taking the square root of both side:

$$9 = \frac{x}{4 \times 10^5 - x}$$

$$x = 36 \times 10^5 - 9x$$

$$x = 3.6 \times 10^5 km$$

Hence, the spacecraft will experience zero gravitational force at a distance of $3.6 \times 10^5$ km from the centre of the Earth.

The correct option is A.

**5.10.** A circular wheel moving at constant speed makes one complete rotation in 45s. Two objects are on the wheel, one at 3.0m from the centre of the ride and the other farther out, 6.0m from the centre. Calculate (a) the angular speed and (b) the tangential speed of each object?

A. (a) 0.14rad/s, 0.14 rad/s (b) 0.42m/s, 0.42m/s
B. (a) 0.10rad/s, 0.14 rad/s (b) 0.32m/s, 0.42m/s
C. (a) 0.14rad/s, 0.14 rad/s (b) 0.42m/s, 0.84m/s
D. (a) 0.14rad/s, 0.14 rad/s (b) 0.32m/s, 0.32m/s

**Solution**

(a) Both objects rotate at the same angular speed. All points on the wheel travel through $2\pi$ rad in the time it takes to make one rotation.

The angular speed $\omega = \frac{\theta}{t} = \frac{2\pi \, rad}{45s} = 0.14 rad/s$

(b) The tangential speed is different at different locations on the wheel. Thus:

$v_1 = r_1\omega = 3.0 \times 0.14 m/s = 0.42 m/s$

$v_2 = r_2\omega = 6.0 \times 0.14 m/s = 0.84 m/s$

The correct option is C.

**5.11.** A spacecraft is in circular orbit about the Earth at an altitude $h$ of 400km. Calculate the orbital speed and the centripetal acceleration if the spacecraft makes one revolution every 85 min.

A. $8.4 \times 10^3$ m/s, $9.8$ m/s$^2$
B. $4.8 \times 10^3$ m/s, $10.0$ m/s$^2$
C. $8.4 \times 10^3$ m/s, $10.4$ m/s$^2$
D. $4.8 \times 10^3$ m/s, $10.4$ m/s$^2$

**Solution**

$h = 400\text{km} = 4.0 \times 10^5 \text{m}$, $t = T = 85\text{min} = 5.1 \times 10^3$ s

The radius of the circular orbit is:

$$r = R_e + h = \left(6.4 \times 10^6 + 4.0 \times 10^5\right)m = 6.8 \times 10^6 \, m$$

Tangential speed:

$$v = \frac{2\pi r}{T} = \frac{2\pi \times 6.8 \times 10^6 \, m}{5.1 \times 10^3 \, s} = 8.4 \times 10^3 \, m/s$$

Then the centripetal acceleration is:

$$a = \frac{v^2}{r} = \frac{\left(8.4 \times 10^3\right)^2}{6.8 \times 10^6} m/s^2 = 10.4 m/s^2$$

The correct option is C.

**5.12.** Suppose that two masses, $m_1$ = 2.5kg and $m_2$ = 3.5kg respectively, are connected by light strings and are in uniform circular motion on a horizontal frictionless surface, where $r_1$ = 1.0m and $r_2$ = 1.3m, as shown in Figure 5.7 below.

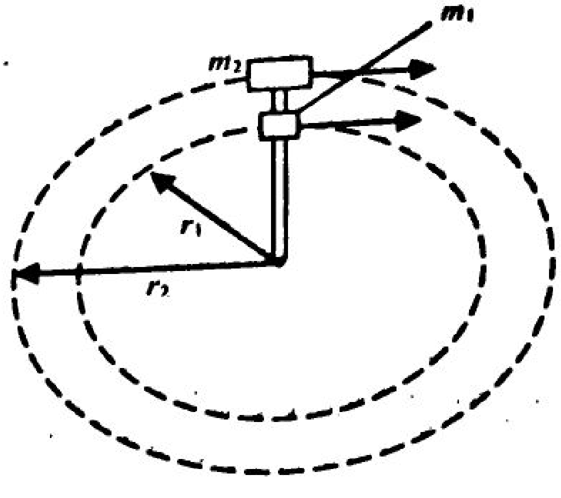


Figure 5.7: Activity 5.12

The forces acting on the masses are $T_2$ = 2.9N and $T_1$ = 4.5N. Find (a) the centripetal accelerations and (b) the magnitudes of the tangential velocities of the masses.

(A) $a_1 = 0.64 m/s^2$, $a_2 = 0.83 m/s^2$, $v_1 = 0.80 m/s$, $v_2 = 1.0 m/s$
(B) $a_1 = 0.60 m/s^2$, $a_2 = 0.63 m/s^2$, $v_1 = 0.70 m/s$, $v_2 = 1.0 m/s$
(C) $a_1 = 0.04\ m/s^2$, $a_2 = 0.083 m/s^2$, $v_1 = 0.080 m/s$, $v_2 = 0.10 m/s$
(D) $a_1 = 0.48\ m/s^2$, $a_2 = 0.89\ m/s^2$, $v_1 = 0.84 m/s$, $v_2 = 6.0 m/s$

**Solution**

The centripetal force for $m_2$ is provided by the tension in the string, since $T_2$ is the only force acting on $m_2$ toward the centre of its circular path. Thus:

$$T_2 = m_2 a_2$$

And:

$$a_2 = \frac{T_2}{m_2} = \frac{2.9N}{3.5kg} = 0.83 m/s^2$$

$$a_2 = \frac{v_2^2}{r}$$

or $$v_2 = \sqrt{a_2 r_2} = \sqrt{0.83 \times 1.3} m/s = 1.0 m/s.$$

However, there are two forces acting on $m_1$: the tensions $T_1$ and $T$ the strings. Also, by Newton's second law, in order to hav centripetal acceleration there must be a net force, which is given the difference in the two tensions. Conventionally, the tension in string always acts away from the mass. Hence, $T_1$ and $T_2$ are actin opposite directions. Thus we have:

$$F_R = T_1 + (-T_2) = m_1 a_1 = \frac{m_1 v_1^2}{r_1}$$

Then:

$$a_1 = \frac{T_1 - T_2}{m_1} = \frac{(4.5 + (-2.9))N}{2.5kg} = 0.64 m/s^2$$

And:

$$v_1 = \sqrt{a_1 r_1} = \sqrt{0.64 \times 1.0} m/s = 0.80 m/s$$

The correct option is A.

**5.13.** A compact disc accelerates uniformly from rest to an angu speed of 400rpm in 2.50s. Calculate the angular acceleration.
A. 16.72rad/s$^2$  B. 6.72rad/s$^2$  C. 7.2rad/s$^2$  D. 0.0rad/s$^2$

**Solution**

$$\omega_o = 0,\ \omega = 400rpm = \frac{400 \times 2\pi}{60} rad/s = 41.8 rad/s,\ t = 2.50 s$$

$$\alpha = \frac{\omega - \omega_o}{t} = \frac{41.8 rad/s - 0}{2.50 s} = 16.72 rad/s^2$$

**5.14.** A microwave oven has a rotating plate revolving at 900rpm which slows down uniformly to 300rpm while making 50 revolutions. Find (a) the angular acceleration and (b) the time required to turn through these 50 revolutions.

A. $-4.5\pi rad/s^2$, 5.0s
B. $4.5\pi rad/s^2$, 5.0s
C. $-4.0\pi rad/s^2$, 5.0s
D. $4.0\pi rad/s^2$, 5.0s

**Solution**

$\omega_0 = 900$ rev/min $= 30.0\pi$ rad/s, $\omega = 300$ rev/min $= 10.0\pi$ rad/s,
$\theta = 50$ rev $= 2\pi \times 50$ rad $= 100\pi$ rad

(a) From $\omega^2 = \omega_o^2 + 2\alpha\theta$, we have:

$$\alpha = \frac{\omega^2 - \omega_o^2}{2\theta} = \frac{(10.0\pi)^2 - (30.0\pi)^2}{2(100\pi)} = -4.0\pi rad/s^2$$

(b) Using:

$$\theta = \left(\frac{\omega + \omega_o}{2}\right)t = \left(\frac{30.0\pi + 10\pi}{2}\right) \times t$$

$$100\pi = 20.0\pi \times t$$

$$t = 5.0s$$

The correct option is C.

**5.15.** Find the angular speed of any point on one of the blade of a fan turning at a rate of 800rpm. Find the tangential speed of the tip of the fan blade if the distance from the centre to the tip is 15.0cm.

A. 5027.2 rad/s, 31.42m/s
B. 31.42 rad/s, 4.71m/s
C. 800 rad/s, 42.9m/s
D. 10.5 rad/s, 4.71m/s

**Solution**

$$f = 300\frac{rev}{\min} = 5rev/s$$

$$\omega = 2\pi f = 2 \times 3.142 \times 5rad/s = 31.42rad/s$$

$$v = \omega r = 31.42 \times 0.15m/s = 4.713m/s$$

The correct option is B.

**5.16.** Which of the following represents Kepler's law?

A. Each planet moves in an elliptical orbit with the Earth at one focus of the ellipse.

B. The line joining the Sun and the planet sweeps out unequal areas in equal times.

C. The squares of the periods of revolution of the planets are proportional to the cubes of their mean distances from the Sun, that is, $T^2 \alpha\ r^3$.

D. The squares of the periods of revolution of the planets are inversely proportional to the cubes of their mean distances from the Sun, that is, $T^2 \alpha\ 1/r^3$.

**Solution**

The correct option is C.

**5.17.** A wheel of radius 20cm is uniformly speeded up from rest to a speed of 300rpm in a time of 10s. Find the constant angular acceleration of the wheel and the tangential acceleration of a point on the wheel.

A. 16.72 rad/s$^2$, 3.344m/s

B. 6.72 rad/s$^2$, 3.044m/s

C. 16.72 rad/s$^2$, 3.344m/s$^2$

D. 16.72 rad/s, 3.344m/s

**Solution**

$r = 20\text{cm} = 0.2$ m, $\omega_o = 0$,

$$\omega = 300rpm = \frac{300 \times 2\pi}{60} rad/s = 31.4rad/s$$

*t* 10s

Then:

$$a = \alpha \times r = 16.72 \times 0.2 m/s^2 = 3.344 m/s^2$$

The correct option is C.

**5.18.** A pulley of 5.0cm radius, on a motor, slows down with constant angular acceleration of $10\pi$ rad/s$^2$ to 20 rev/s in 2.0s. Calculate the initial angular speed of the pulley and the number of revolutions it makes in this time.

A. 16.72 rad/s, 3.344 rad
B. 6.72 rad/s, 3.044 rad
C. 16.72 rad/s, 3.344 rad
D. 188.48 rad/s, 314.16 rad

**Solution**

$r = 5.0$, $\alpha = -10\pi$ rad/s$^2$ (negative since the object slows down)
$\omega = 20$ rev/s $= 2\pi \times 20$ rad/s $= 125.68$ rad/s, $t = 2.0$ s

$$\omega = \omega_o + \alpha t$$

$$125.68 = \omega_o + (-10\pi) \times 2.0$$

$$\omega_o = 188.48 rad/s$$

$$\theta = \left(\frac{\omega + \omega_o}{2}\right)t = \left(\frac{125.68 + 188.48}{2}\right) \times 2.0 rad = 314.16 rad$$

The correct option is D.

**5.19.** A car starts from rest and accelerates uniformly to a speed of 20m/s in a time of 4.0s. Find the angular acceleration of its 15cm radius wheels and the number of rotations one wheel makes in this time.

A. 33.3 rad/s$^2$, 266.4 rad
B. 33.3 rad/s$^2$, 44.1 rad
C. 16.72 rad/s$^2$, 44.1 rad
D. 16.72 rad/s$^2$, 266.4 rad

**Solution**

$v_0 = 0$, $v = 20\text{m/s}$, $t = 4.0$ s, $r = 15\text{cm} = 0.15$ m

$$a = \frac{v - v_o}{t} = \frac{20 - 0}{4.0} m/s^2 = 5m/s^2$$

Then:

$$\alpha = \frac{a}{r} = \frac{5}{0.15} rad/s^2 = 33.3 rad/s^2$$

Using:

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2 = \left(0 + \frac{1}{2} \times 33.3 \times 4^2\right) rad = 266.4 rad$$

The correct option is A.

**5.20.** A road of radius 20m is to be banked so that a car may make a turn at a maximum speed of 15m/s. What must be the banking angle?

A. 48.9° B. 46.2° C. 90.0° D. 8.9°

**Solution**

$$\tan\theta = \frac{v^2}{rg} = \frac{15^2}{20 \times 9.8} = 1.148$$

$$\theta = \tan^{-1}(1.148) = 48.9^o$$

The correct option is A.

## Summary of Chapter 5

In chapter 5, you have learned that:

1. When a particle moves in a circle with constant speed, the motion is called uniform circular motion.
2. The angular acceleration, $\alpha$, can be defined as the time rate of change of angular velocity: $\omega = \omega_o + \alpha t$.

3. The centripetal acceleration can be written as $a = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$.

4. Banking is a technique in road construction that involves tilting of the road surface at an angle $\theta$ above the horizontal, inward to the centre of the road.

5. Newton's law of universal gravitation states that every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

6. Kepler's laws state that:

    a. Each planet moves in an elliptical orbit with the Sun at one focus of the ellipse.
    b. The line joining the Sun and the planet sweeps out equal areas in equal times.
    c. The squares of the periods of revolution of the planets are proportional to the cubes of their mean distances from the Sun, that is, $T^2 \alpha\ r^3$.

## Self-Assessment Questions (SAQs) for Chapter 5

**5.1.** A satellite orbits the moon at a height of 20 000m. Find its speed and the time it takes for one orbit. (Take the mass on the moon as $7.34 \times 10^{22}$kg and radius of the moon as $1.738 \times 10^6$m.)

**5.2.** A 150g object is tied to the end of a cord and whirled in a horizontal circle of radius 1.5m. Assume that the angular speed is 3.0rev/s and that the cord is horizontal. Determine the acceleration of the object and the tension in the cord.

**5.3.** Calculate a value for the mass of the Earth, given that the acceleration of free fall at the Earth's surface is $9.81\text{ms}^{-2}$, and the radius of the Earth is 6400km.

**5.4.** A wheel revolving at 6.00 rev/s has an angular acceleration of 4.00 rad/s$^2$. Find the number of turns the wheel must make to reach 26.0 rev/s, and the time required.

**5.5.** A satellite orbits the Earth in a circle of radius 6570km. Find the speed of the satellite and the time taken to complete one revolution. Assume the Earth's mass is 6.0 x 10$^{24}$kg.

**5.6.** Calculate the speed and period of a 1000kg satellite orbiting at 7000km above the Earth's surface. How much work is done to place the satellite in orbit? Assume the Earth's mass is 6.0 x 10$^{24}$kg and the radius of the Earth is 6400km.

**5.7.** A curved road is part of a circle of radius 150m. The coefficient of kinetic friction between the road surface and the tyres of a car is 0.3. If the road is banked at 12°, calculate the speed with which the car can travel on it without skidding.

**5.8.** Three masses are position as shown in Figure 5.8. What is their total gravitational potential energy?

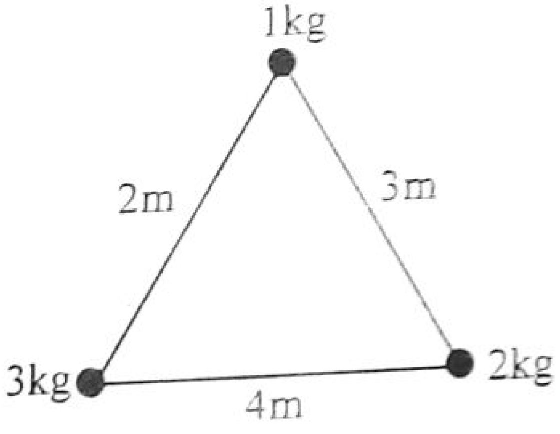


Figure 5.8: SAQ 5.8

**5.9.** A 1500kg car moving on a flat road negotiates a curve whose radius is 35m. If the coefficient of friction between the tyres and pavement is 0.50, find the maximum speed the car can have in order to make the turn successfully.

**5.10.** A car moving at 5.0m/s tries to round a corner in a circular arc of 8.0m radius. The roadway is flat. What must be the coefficient of friction between the wheels and roadway if the car is not to skid?

# CHAPTER SIX WORK, ENERGY AND POWER

## 6.1 Work Done by a Constant Force

Suppose a force $F$ acts on an object; if the object is free, it will move through a distance $\Delta x$. If, in moving this distance, the body experiences the same force of magnitude $F$ in the same direction, then $F$ is said to be a uniform or constant force, since $F$ is independent of time and the position of the particle. The work done by a constant force in moving an object is equal to the product of the magnitudes of the displacement and the component of the force parallel to the displacement.

Suppose that a constant force $F$ moves an object through a distance $\Delta x$ in the same direction as the force (Figure 6.1). The amount of work done by the force is given by:

$$W = F\Delta x \qquad 6.1$$

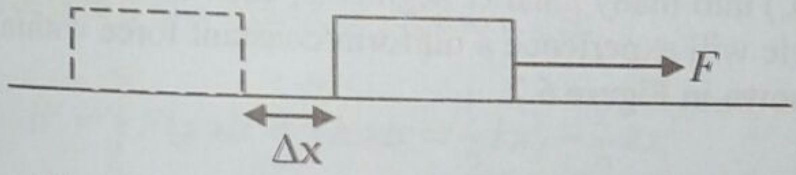


Figure 6.1: Work done by constant force

However, if the force acts at an angle $\theta$ to the object's displacement, then $F_{II} = F\cos\theta$ is the component of the force parallel to the displacement $r$. In this case, the amount of work done is given as:

$$W = F_{II} r = (F\cos\theta)r = Fr\cos\theta \qquad 6.2$$

where $F$ and $r$ are the magnitudes of $\vec{F}$ *and* $\vec{r}$ respectively. If $\vec{F}$ *and* $\vec{r}$ are in the same direction, $\theta$ is zero, then Equation 6.2

reproduces Equation 6.1. If $\theta$ is 90°, then $W = Fr\cos 90° =$
Thus, no work is done by a force if the point on which it acts is displaced in a direction which is perpendicular to the direction of the force.

Work involves moving an object through a distance. If a force is applied to an object and there is no motion (no displacement), then no work is done.

From the definition of scalar product and Equation 6.2, we also have that:

$$W = \vec{F} \cdot \vec{r} \qquad 6.3$$

In S.I. units, the unit of work is the newton-metre, with the special name joule (J).

## 6.2 Work Done by a Variable Force

We now consider the work done by a force that varies in magnitude, direction or both. Suppose a particle moves along the $x$-axis from point $x_i$ to $x_f$. To calculate the work done, we divide the interval of motion $(x_f - x_i)$ into many smaller segments, each of length $\Delta x$, such that the particle will experience a uniform/constant force within each segment as shown in Figure 6.2.

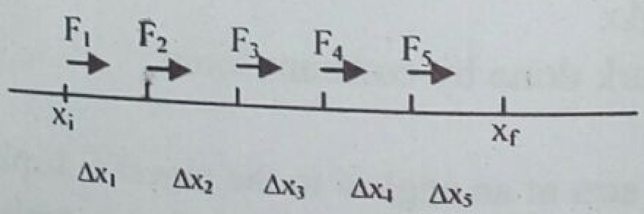


Figure 6.2: Work done by a variable

The work done by the force in the total displacement from $x_i$ to $x_f$ is approximately:

$$W = F_1\Delta x_1 + F_2\Delta x_2 + F_3\Delta x_3 + F_4\Delta x_4 + \dots \qquad 6.4a$$

Or:

$$W = \sum_{i=1}^{i=N} F_i \Delta x_i \qquad 6.4b$$

In the limit that the number of segments becomes very large and the width of each becomes very small, this sum becomes the integral of $F(x)$ from $x_i$ to $x_f$:

$$W = \int_{x_i}^{x_f} F(x)dx \qquad 6.5$$

Equation 6.5 shows that the work done by a variable force $F(x)$, in moving an object from $x_i$ to $x_f$ is the area under the graph of $F(x)$ against $x$, bounded by vertical lines at $x_i$ to $x_f$. An example of a variable force doing work is in the stretching of a spring. The work done in this case as the elongation goes from zero to a maximum value $X$ can be shown to be:

$$W = \int_0^X F(x)dx = \int_0^X kxdx = \frac{1}{2}kX^2 \qquad 6.6$$

Equation 6.6 assumes that the spring was originally unstretched. However, if the string is already stretched to a distance $x_i$, the work done to take it to a greater elongation $x_f$ is:

$$W = \int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f} kxdx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 \qquad 6.7$$

## 6.3 Energy

Energy is mostly defined as the capacity to do work. This is because in many circumstances in which work is done, the agent that is exerting the force must expend something called energy. For example, work is done by the restoring force in a stretched spring in moving an attached mass $m$ to and fro about a fixed point. In this example, energy was earlier expended by whatever agent or arrangement had, in the first place, stretched the spring. In general, the more energy the agent has at its disposal, the more work it is able to do. However, the expenditure of energy does not in itself guarantee that work is done. For example, if a force $F$ acts on a particle and fails to move the particle in its direction, the work done is zero although energy may have been expended by the agent which provided the force. Basically, energy is something that objects have whereas work is something that is done on objects.

## 6.3.1 Forms of Energy

The main forms of energy are:

- Heat
- Chemical
- Electromagnetic
- Nuclear
- Electrical
- Mechanical

The internal motion of the atoms is called heat energy because moving particles produce heat. Heat energy can be produced by friction and it causes changes in temperature and phase of any form of matter.

Chemical energy is the energy required to bond atoms together. This energy is released when the bonds are broken. Fuel and food are forms of stored chemical energy.

Light is a form of electromagnetic energy. Each component colour of light presents a different amount of electromagnetic energy. Electromagnetic energy is also carried by X-rays, radio waves and laser light.

The nucleus of an atom is the source of nuclear energy. During nuclear fission (splitting of nucleus), nuclear energy is released in the form of heat energy and light energy. Nuclear energy is also released during nuclear fusion (joining of nuclei). The sun's energy is produced from a nuclear fusion reaction in which hydrogen nuclei fuse to form helium nuclei. Nuclear energy is the most concentrated form of energy.

Mechanical energy is the energy an object acquires when work is done on it.

### 6.3.2 Energy Conversion

Energy can neither be created nor destroyed but it can be changed from one form to another. These changes in the form of energy are called **energy conversions**. All forms of energy can be converted into other forms. For instance, the sun's energy through solar cells can be converted directly into electricity (electrical energy); green plants convert the sun's energy (electromagnetic) into starches and sugars (chemical energy) and electromagnetic energy is converted to mechanical energy in an electric motor. Other examples of energy conversion are:

- Conversion of chemical energy into electromagnetic energy in a battery;
- Conversion of mechanical energy of a waterfall to electrical energy in a generator;
- In an automobile engine, fuel is burned to convert chemical energy into heat energy. The heat energy is then changed into mechanical energy;
- Conversion of potential energy into kinetic energy and vice-versa.

## 6.4 Kinetic Energy: the Work-Energy Theorem

**Kinetic energy** is a form of energy that is closely associated with work. Kinetic energy is defined as the energy a body possesses by virtue of its motion. The kinetic energy of a particle of mass $m$ travelling with speed $v$ is given as:

$$K = \frac{1}{2}mv^2 \qquad 6.8$$

Let us now derive an expression relating work done with kinetic energy. Consider a particle of mass $m$ acted upon by a force $F$. The force causes the object to accelerate and from Equation 3.15 we have:

$$a = \frac{v^2 - v_o^2}{2x}$$

When we multiply this equation by $m$, we find:

$$F = ma = m\frac{v^2 - v_o^2}{2x}$$

Using this expression in the equation for work, we get:

$$W = Fx = \left(m\frac{v^2 - v_o^2}{2x}\right)x = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 \qquad 6.9$$

Equation 6.9 is called the **work-energy theorem**, which states that for an object which moves under the influence of a given force from one point to another the work done by the force on the object is the change in the kinetic energy of the object.

## 6.5 Conservative Forces: Potential Energy

A force is said to be conservative if the work done by or against it in moving an object is independent of the object's path. The implication of this statement is that the work done by a conservative force depends only on the initial and final positions of an object. Examples of conservative forces include the force due to gravity and the spring force. If an object falls from rest through a vertical height $h$, the maximum amount of mechanical work done irrespective of the path taken is given by:

$$W = mgh = mg(y_2 - y_1) \qquad 6.10$$

where the object has travelled from $y_1$ to its final position $y_2$.

We can define the gravitational potential energy function $U(y)$ of an object of mass $m$, at the vertical position $y$ as:

$$U(y) = -mgy \qquad 6.11$$

The maximum amount of work which an object of mass $m$ can do, if falling under the Earth's gravity, from a vertical position $y_2$ to a lower position $y_1$, is known as its potential energy relative to the Earth's surface:

$$W = U(y_1) - U(y_2) = mg(y_2 - y_1) \qquad 6.12$$

This expression is similar to the work-energy theorem. The S.I. unit of energy is joule (J).

For non-conservative or dissipative force, any work done against it in moving an object from one position to another depends on the details of the path between the initial and final positions and not just on the end points. It is not possible to define a potential energy function for a non-conservative force. Examples of non-conservative forces are the frictional force between two solid surfaces and the drag force for motion through a fluid.

## 6.6 The Law of Conservation of Mechanical Energy

The law of conservation of mechanical energy is a consequence of the work-energy theorem (Equations 6.9 and 6.12). This law is valid only for motion under a conservative force. Consider an object moving from an initial position $x_1$ to another position $x_2$, under a conservative force $F(x)$. The work done by $F(x)$ according to the work-energy theorem is:

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

where $v_1$ is the speed at position $x_1$ and $v_2$ is the speed at position $x_2$.

The work done according to Equation 6.12 is:

$$W = U_1 - U_2$$

These expressions refers to the same value of work $W$, so:

$$U_1 - U_2 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Or:

$$U_1 + \frac{1}{2}mv_1^2 = U_2 + \frac{1}{2}mv_2^2 \qquad 6.13$$

This is the mathematical statement of the **law of conservation of mechanical energy**, which states that in a conservative system, the total mechanical energy is conserved.

## 6.7 Power

**Power** is the rate at which work is done. Like work and energy, power is a scalar quantity. When a quantity of work $\Delta W$ is done during a time interval $\Delta t$, the average power, $P_{av}$, is defined to be:

$$P_{av} = \frac{\Delta W}{\Delta t} \qquad 6.14$$

We can rewrite Equation 6.14 as:

$$P_{av} = \frac{\vec{F}.\vec{r}}{t} = \frac{Fr\cos\theta}{t} = Fv\cos\theta \qquad 6.15$$

where $v$ is the magnitude of the instantaneous velocity.

We can also express Equation 6.15 in terms of the scalar product:

$$P_{av} = \vec{F}.\vec{v} \qquad 6.16$$

The instantaneous power $P$ is defined as:

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \qquad 6.17$$

The S.I. unit of power is the watt (W). One watt equals 1 joule per second, that is $1\text{W} = 1\text{J/s}$. The kilowatt ($1\text{kW} = 10^3\text{W}$), the megawatt ($1\text{MW} = 10^6\text{W}$) and horsepower ($1\text{hp} = 746\text{W} = 0.746\text{kW}$) are also commonly used as unit of power.

The **efficiency** ($\varepsilon$) is defined as:

$$\varepsilon = \frac{\textit{power output}}{\textit{power input}} \times 100\% \qquad 6.18$$

We can also write efficiency in terms of work done as:

$$\varepsilon = \frac{work\ output}{work\ input} \times 100\% \qquad 6.19$$

## Activity 6 Work, Energy and Power

6.1. A water skier is pulled by a tow rope behind a boat. He skies off to the side and the rope makes an angle of 60° with his direction of motion. The tension in the rope is 120N. How much work is done on the skier by the rope during a displacement of 500m?

A. $5.64 \times 10^4$J  B. $3.00 \times 10^4$J
C. $2.05 \times 10^4$J  D. $2.18 \times 10^4$J

**Solution**

Work done:
W = force x displacement = 120 cos 60° x 500J = $3.00 \times 10^4$ J
The correct option is B.

6.2. The spring of a spring gun has a force constant of 500N/m. It is compressed 0.05m and a ball of mass 0.01kg is placed in the barrel against the compressed spring. Compute the speed with which the ball leaves the gun when released.

A. 11.2m/s  B. 125m/s  C. 12.5m/s  D. 112m/s

**Solution**

Energy stored in spring = Kinetic energy of ball

$$\frac{1}{2} ke^2 = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{ke^2}{m}} = \sqrt{\frac{k}{m}}\, e = \sqrt{\frac{500}{0.01}} \times 0.05 m/s$$

$$v = \sqrt{50000} \times 0.05 m/s = 11.18 m/s = 11.2 m/s$$

The correct option is A.

6.3. An object of mass 5.4kg is falling from the top of a building. If the velocity of the object at height 20.0m above the sea level is

10.0m/s, how far is the top of the building from the sea level? Assume $g = 9.8m/s^2$.

A. 30.0m B. 23.4m C. 25.1m D. 30.2m

**Solution**

Using the law of conservation of mechanical energy, sum of potential and kinetic energies at the top of the building = sum of potential and kinetic energies at any other point. The object's initial speed is zero.

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

$$\frac{1}{2} \times 0 + 9.8 \times H = \frac{1}{2} \times 10^2 + 9.8 \times 20$$

$$9.8H = 50 + 196$$

$$H = 25.1m$$

The correct option is C.

**6.4.** A force of 12.0N which is applied at 50° to the vertical pulls a load of mass 1.5kg along a horizontal surface through a distance 10.0m. If the frictional force between the two surfaces is 1.2N, calculate the effective work done on the load.

A. 120.0J B. 91.93J C. 108.0J D. 79.93J

**Solution**

Since the force is applied 50° to the vertical it implies that it is 40° to the horizontal.

Effective force:
$F_R$ = component of the 12.0N force along the horizontal - frictional force
$F_R = (12\cos 40° - 1.2)N = 7.9925N$

Work done:
$W = F_R \times d = 7.9925 \times 10J = 79.93J$

The correct option is D.

6.5. A spring is stretched by a force F through a distance of 10.000cm. What is the work done if the stiffness of the spring is 0.08N/m?

A. $8.0 \times 10^{-4}$ J  B. $4.0 \times 10^{-4}$ J
C. $8.0 \times 10^{-3}$ J  D. $4.0 \times 10^{-3}$ J

**Solution**

$e = 10.00\text{cm} = 0.1$ m
Work done $= \frac{1}{2}ke^2 = \frac{1}{2} \times 0.08 \times 0.1^2\text{J} = 4 \times 10^{-4}$J

The correct option is B.

6.6. A block is pushed 2m along a fixed horizontal surface by a horizontal force of 2N. The opposing force of friction is 0.4N. How much work is done by the frictional force?

A. 0.2J  B. 0.4J  C. 0.8J  D. 10.0J

**Solution**

Work done by friction force = friction force x distance moved

$= 0.4 \times 2\text{J} = 0.8\text{J}$

The correct option is C.

6.7. What is the work done by the 2N force in Q6.6?

A. 3.2J  B. 4J  C. 4.2J  D. 4.6J

**Solution**

Work done by the 2N force = force x distance moved

$= 2 \times 2\text{Nm} = 4\text{J}$

The correct option is B.

6.8. How much work is done when a bucket of mass 2kg with 20kg of water in it is pulled up from the bottom of a well 10m deep?

A. 901.6J  B. 1121J  C. 2200J  D. 2156J

**Solution**

Work done $W$ = potential energy = $mgh = (2 + 20) \times 10 \times 10\text{J} = 2200\text{J}$

The correct option is C.

**6.9.** A force of 20N is needed to hold a spring extended 5.0cm from its equilibrium position. How much work is done in extending the spring?

A. 400J B. 0.5J C. 400N D. 0.5N

**Solution**

Work done, W = Energy stored in the spring

$$W = \frac{1}{2}Fe = \frac{1}{2} \times 20 \times 0.05 J = 0.5 J$$

The correct option is B.

**6.10.** A constant force 20N moves a body of mass 2kg with constant speed of 0.2m/s. Calculate the power expended.

**Solution**

$$P_{av} = Fv = 20 \times 0.2W = 4W$$

**6.11.** A 4kg ball moving with a velocity of $10ms^{-1}$ collides with a 16kg ball moving with a velocity of $4ms^{-1}$ in the opposite direction. Calculate the velocity of the balls if they coalesce on impact and the loss of energy resulting from the impact.

A. $5.2ms^{-1}$; 58J B. $1.2ms^{-1}$; 314J

C. $5.2ms^{-1}$; 314J D. $1.2ms^{-1}$; 58J.

**Solution**

$m_1$ = 4kg, $m_2$ = 16kg, $u_1$ = 10m/s, $u_2$ = -4m/s, $v_1 = v_2 = v$

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$4 \times 10 + 16 \times (-4) = (4 + 16)v$$

$$40 - 64 = 20v$$

$$20v = -24$$

$$v = -1.2ms^{-1}$$

Loss of energy, $\Delta E$ = final total K.E - initial total K.E.

$$\Delta E = \frac{1}{2}(m_1 + m_2)v^2 - \frac{1}{2}\left(m_1u_1^2 + m_2u_2^2\right)$$

$$\Delta E = \frac{1}{2}(4+16)(-1.2)^2 - \frac{1}{2}\left(4\times10^2 + 16\times(-4)^2\right) = -314J$$

The correct option is B.

**6.12.** An object of mass 100kg is released from rest and falls through a distance of 10 m. Calculate the work done by gravity and the change in P.E.

A. $-10 \times 10^3$ J; $2 \times 10^3$ J
B. $10 \times 10^3$ J; $10^3$ J
C. $10 \times 10^2$ J; 10 J
D. $10 \times 10^2$ J; $-10 \times 10^{-3}$ J

**Solution**

Work done by gravity = $mgh$ = 100 x 10 x 10J = $10 \times 10^3$ J
Change in P.E. = $mg(y_2 - y_1)$ = 100 x 10 (0-10)J = $-10 \times 10^3$ J

The correct option is D.

**6.13.** A 10kg mass is to be raised from the bottom to the top of an incline 5m long and 3m off the ground at the top. How much work must be done by a force parallel to the incline pushing the mass up at a constant speed?

A. 294J
B. 194J
C. 260J
D. 59J

**Solution**

Work done = $mgh$ = 10 x 10 x 3J = 294J

The correct option is A.

**6.14.** A force of 150N is applied to drag a box of mass 12kg along a horizontal floor through a distance of 10m. If the friction between the box and the floor is 25N and the direction of the applied force makes angle 35° with the horizontal, calculate the effective work done on the box.

A. 1500J
B. 1228.73J
C. 978.73J
D. 1250J

**Solution**

Resultant force $F_R = (150\cos 35° - 25)N = 97.873N$
Work done $= F_R \times d = 97.873 \times 10J = 978.73J$

The correct option is C.

**6.15.** A force of 6N acts horizontally on a stationary mass of 2kg for 4s. The kinetic energy gained by the mass is:
A. 12J B. 24J C. 48J D. 144J

**Solution**

From Newton's second law of motion:

$F_R = ma$
$6 = 2 \times a$
$a = 3m/s$

Using, $v = u + at$

$v = (0 + 3 \times 4)m/s = 12m/s$

K.E gained, $K.E. = \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times 12^2 J = 144J$

The correct option is D.

**6.16.** A body is released from rest and allowed to fall freely from a given height under gravity. The kinetic energy at its halfway point is:
A. a little above half of its initial energy
B. a little below half of its initial energy
C. a half of its initial energy
D. a little above its initial energy

**Solution**

The correct option is C.

**6.17.** An egg falls from a nest at a height of 4m. What speed will it have when it is 1m from the ground? Neglect air resistance and take $g = 9.8m/s^2$.
A. 7.1m/s B. 4.43m/s C. 58.8m/s D. 7.67m/s

**Solution**

Total energy at initial point = Total energy at any point

Therefore:

$$mgh_1 = mgh_2 + \frac{1}{2}mv^2$$

$$m \times 9.8 \times 4 = m \times 9.8 \times 1 + \frac{1}{2} \times m \times v^2$$

$$v^2 = 58.8$$

$$v = 7.67m/s$$

The correct option is D.

6.18. A car of mass 1200kg accelerates from rest to a speed of 25m/s in a time of 8.0 s. What is the average power produced?

A. 7.1kW B. 4.43kW C. 47kW D. 7.67kW

**Solution**

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2} \times 1200 \times (25^2 - 0)J = 3.75 \times 10^5 J$$

$$P = \frac{work}{time} = \frac{3.75 \times 10^5}{8.0}W = 46.9 \times 10^3 W = 47kW$$

The correct option is C.

6.19. A 4kg block is pushed a distance 5m along a level floor at constant speed by a 20N along a horizontal path. The coefficient of friction between the block and the floor is 0.25. How much work is done?

A. 392J B. 0J C. 584.4J D. 58J

**Solution**

Resultant force, $F_R = (20 - 0.25 \times 8 \times 10)N = 0$

Work done, $W = F_R \times d = 0 \times 5J = 0J$

The correct option is B.

**6.20.** The force generated from the engine of a car is found to be 300N. Calculate the power developed when the car moves with a constant speed of 5m/s.

A. 7.1W B. 4.43W C. 1500W D. 7.67W

**Solution**

$P =$ force x velocity $= 300 \times 5W = 1500W$

The correct option is C.

## Summary of Chapter 6

In chapter 6, you have learned that:

1. The work done by a constant force in moving an object is equal to the product of the magnitudes of the displacement and the component of the force parallel to the displacement.
2. From the definition of scalar product and work done by a force is $W = \vec{F}.\vec{r}$. In S.I. units, the unit of work is the newton-metre, with the special name joule (J).
3. The work done by the force in the total displacement from $x_i$ to $x_f$ is $W = \int_{x_i}^{x_f} F(x)dx$.
4. Energy is defined as the capacity to do work.
5. Kinetic energy is defined as the energy a body posses by virtue of its motion. The kinetic energy of a particle of mass $m$ travelling with speed $v$ is given as: $K = \frac{1}{2}mv^2$.
6. The work-energy theorem states that for an object which moves under the influence of a given force, from one point to another, the work done by the force on the object is the change in the kinetic energy of the object: $W = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2$.

7. A force is said to be conservative if the work done by or against it in moving an object is independent of the object's path. The implication of this statement is that the work done by a conservative force depends only on the initial and final positions of an object. Examples of conservative forces include the force due to gravity and the spring force.

8. If an object falls from rest, through a vertical height $h$, the maximum amount of mechanical work done, irrespective of the path taken, is given by $W = mgh = mg(y_2 - y_1)$, where the object has travelled from $y_1$ to its final position $y_2$.

9. For non-conservative or dissipative force, any work done against it in moving an object from one position to another depends on the details of the path between the initial and final positions, and not just on the end points. Examples of non-conservative forces are the frictional force between two solid surfaces, and the drag force for motion through a fluid.

10. The law of conservation of mechanical energy states that in a conservative system, the total mechanical energy is conserved. That is: $U_1 + \frac{1}{2}mv_1^2 = U_2 + \frac{1}{2}mv_2^2$.

11. Power is the rate at which work is done. When a quantity of work $\Delta W$ is done during a time interval $\Delta t$, the average power $P_{av}$ is defined to be $P_{av} = \frac{\Delta W}{\Delta t}$. The S.I. unit of power is the watt (W).

12. The efficiency ($\varepsilon$) is defined as $\varepsilon = \frac{power\ output}{power\ input} \times 100\%$.

## Self-Assessment Questions (SAQs) for Chapter 6

**6.1.** A 3.0kg object is lifted 3.5m. How much work is done against the Earth's gravity?

**6.2.** Calculate the work done in moving a 1300kg car from rest to a speed of 20m/s in a distance of 80m.

**6.3.** A 200kg drum is pushed up an incline plane 7.0 m long to a platform 1.5m above the starting point. Calculate the work done by the pushing force if a friction force of 150N opposes the motion.

**6.4.** A water pump takes water through a distance of 12m. Water is discharged at the surface at a rate of 2.0kg/s, and the discharge speed of each drop is 3.0m/s. Find the minimum power of the pump.

**6.5.** An object of mass 10kg, starting from rest, is pushed a distance 15m along a frictionless surface by a force of 300N. (a) Calculate how much work is done by the force. (b) Using the work-energy theorem, calculate the final speed and the final kinetic energy of the object.

**6.6.** An object of mass 12kg, initially at rest, is lifted from the ground to a final height of 10m by a uniform force, so that it arrives at its final height with a speed of 5m/s. (a) Calculate the force applied to the object. (b) How much work is done in lifting the object? (c) How much of this work is done against gravity?

**6.7.** A 1200kg car is moving down a 30° hill. The driver applies the brakes at a time that the car's speed is 12m/s. What constant force $F$ must result if the car is to stop after travelling 100m?

**6.8.** An object $A$ of mass 15kg is moving with a velocity of 6m/s. Calculate the kinetic energy and its momentum.

**6.9.** A ball of mass 0.1kg is thrown vertically upwards with an initial speed of 20m/s. Calculate (a) the time taken to return to the thrower, (b) the maximum height reached, (c) the kinetic and potential energies of the ball half-way up.

**6.10.** An object of mass 5kg is held at a height of 1 metre above the ground for 15 seconds. The work done within this period is:

A. 30J B. 20J C. 50J D. 0J

# CHAPTER SEVEN

# MECHANICS OF RIGID BODIES

## 7.1 Introduction

Our study of mechanics has been restricted to the analysis of the dynamics of point masses or particles (i.e., objects whose spatial extent is either negligible or plays no role in their motion). In this chapter, we will study the mechanics of rigid bodies. A rigid body is a system of particles, such that the distances between the particles are fixed, and do not change throughout the motion. Examples of rigid bodies include a stone, a football and a bullet. Unlike the case of a point mass which can only exhibit translational motion, the motion of a rigid body could be translational and rotational motion. In describing rotational motion, we will define the following terms: **moment**, **torque** and **angular momentum**.

## 7.2 Moment of a vector

The moment of a vector **A** about a point $O$ is a vector whose magnitude is the product of the magnitude of **A** and the perpendicular distance of $O$ from the direction of **A**. The moment of the vector **A** in Figure 7.1 is defined as:

Moment = vector **A** x perpendicular distance from axis  7.1

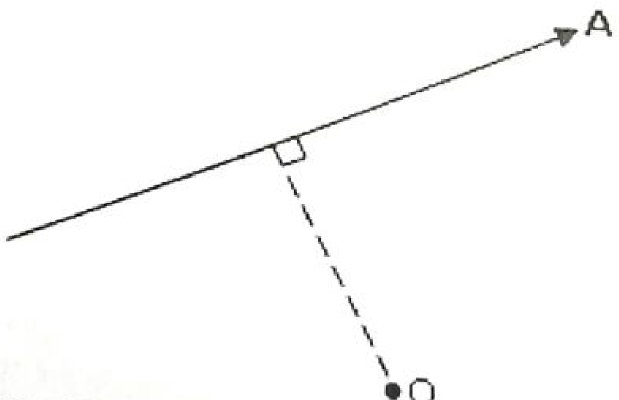


Figure 7.1: Moment of vector **A** about O

If the vector is a force, its moment is called a torque. Thus, torque is the moment of a force. The moment, about the origin, of a force **F** which passes through a point with position vector **r**, is given by the vector product:

$$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta \qquad 7.2$$

where $\theta$ is the angle between the line $r$ and the force vector **F**. The S.I. unit of torque is newton-meter (N.m).

The moment, about the origin, of linear momentum **p** which passes through a point with position vector **r**, is given by:

$$\vec{L} = \vec{r} \times \vec{p} \qquad 7.3$$

$L$ is called the orbital angular momentum, or simply, the angular momentum.

## 7.3 Mechanical Equilibrium

A rigid body is said to be in translational equilibrium if the resultant force acting on it is equal to zero, $\sum \vec{F} = 0$, that is, the body remains either at rest or in motion with a constant velocity. This statement is known as the condition for translational equilibrium. If the sum of torques acting on an object about any point is zero, $\sum \vec{\tau} = 0$, then the object is in rotational equilibrium. In other words, sum of clockwise moment about a point (pivot) is equal to sum of anticlockwise moment about the same point. In this case the object is rotationally at rest or rotates with a constant angular velocity. For example, if the system in Figure 7.2 is in equilibrium, we have:

$$W_1 d_1 = W_2 d_2$$

A body is said to be in mechanical equilibrium when the conditions for both translational and rotational equilibrium are satisfied. That is $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$. These are usually referred to as the condition for equilibrium.

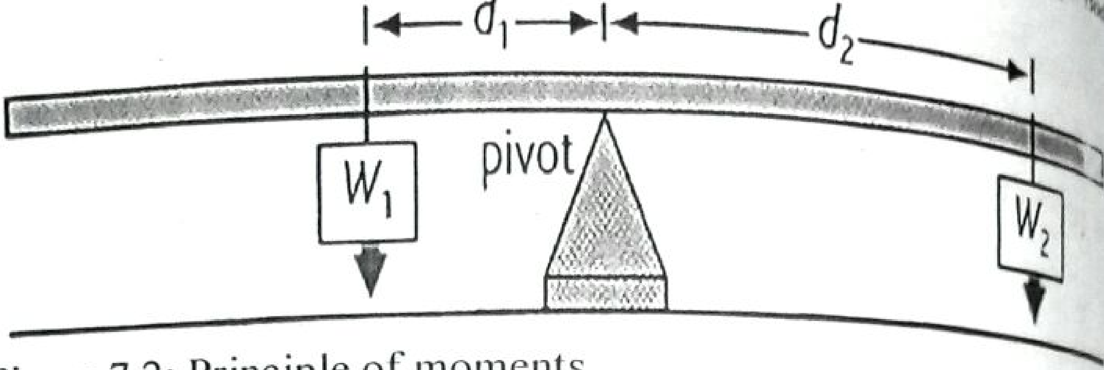


Figure 7.2: Principle of moments

All static equilibrium problems are solved the same way:

i. Find all external forces
ii. Choose a pivot
iii. Find all external torques i.e. the moments of the all the forces about the pivot
iv. Set net force to zero, i.e. sum of upward forces equal to sum of downward forces
v. Set net torque to zero i.e. sum of all clockwise moments equal to sum of anticlockwise moments
vi. Solve for unknown quantities

It is generally simpler to choose the pivot at the point of application of the force for which you have the least information.

## 7.4 Rotation of Rigid Bodies

In chapter five we considered the motion of an object in a circular path. We defined the angular speed/velocity and angular acceleration which are applicable for rotational motion of rigid bodies. For rotational motion about a fixed axis when the angular acceleration is constant, we have derived equations for angular velocity and angular position (Section 5.2) using exactly the same procedure that we used for straight-line motion in Section 3.2. The equations are as follows:

$$\omega = \omega_o + \alpha t \qquad 7.4$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2 \qquad 7.5$$

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o) \qquad 7.6$$

$$\theta-\theta_o=\frac{1}{2}(\omega_o+\omega)t \qquad 7.7$$

## 7.5 Centre of Mass and Centre of Gravity

Consider a system of particles of masses $m_1$, $m_2$, $m_3$,..., located at position ($x_1$, $y_1$, $z_1$), ($x_2$, $y_2$, $z_2$), ($x_3$, $y_3$, $z_3$),..., respectively (Figure 7.3). We define the centre of mass of the system as the point that has coordinates ($x_{cm}$, $y_{cm}$, $z_{cm}$) given by:

$$x_{cm}=\frac{m_1x_1+m_2x_2+m_3x_3+\ldots}{m_1+m_2+m_3+\ldots}=\frac{\sum_i m_ix_i}{\sum_i m_i} \qquad 7.8a$$

$$y_{cm}=\frac{m_1y_1+m_2y_2+m_3y_3+\ldots}{m_1+m_2+m_3+\ldots}=\frac{\sum_i m_iy_i}{\sum_i m_i} \qquad 7.8b$$

$$z_{cm}=\frac{m_1z_1+m_2z_2+m_3z_3+\ldots}{m_1+m_2+m_3+\ldots}=\frac{\sum_i m_iz_i}{\sum_i m_i} \qquad 7.8c$$

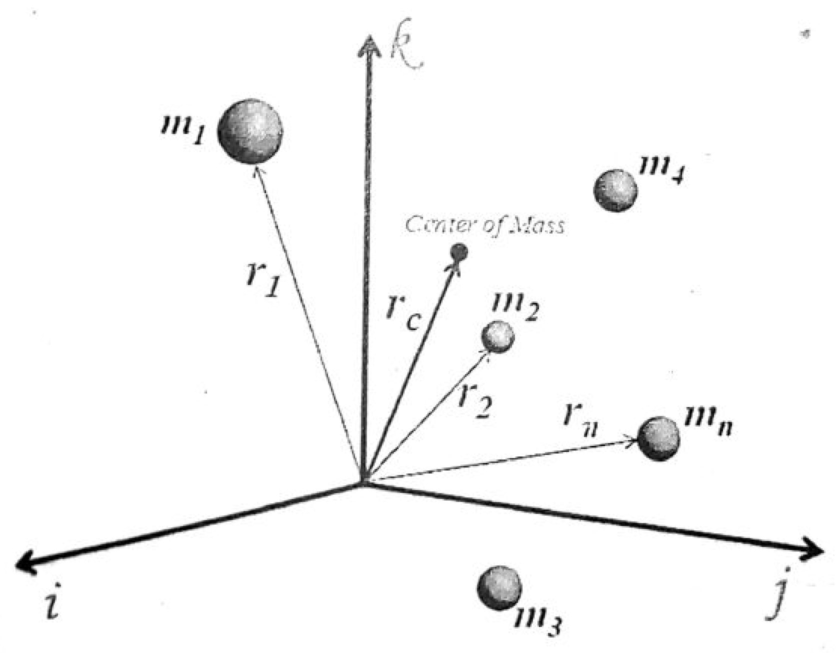


Figure 7.3: Centre of mass

The position vector $\vec{r}_{cm}$ of the centre of mass can be expressed in terms of the position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots$ of the particles as:

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} \qquad 7.9$$

The centre of gravity is the average location of the weight of an object. It is a geometric property of any object. The motion of any object can be completely described through space in terms of the translation of the centre of gravity of the object from one place to another and the rotation of the object about its centre of gravity if it is free to rotate.

For a general shaped object, there is a simple mechanical way to determine the centre of gravity:

1. If you just balance the object using a string or an edge, the point at which the object is balanced is the centre of gravity.
2. Another, more complicated way is a two-step method shown in Figure 7.4. In Step 1, you hang the object from any point and you drop a weighted string from the same point. Draw a line on the object along the string. For Step 2, repeat the procedure from another point on the object. You now have two lines drawn on the object which intersect. The centre of gravity is the point where the lines intersect. This procedure works well for irregularly shaped objects that are hard to balance.

If the mass of the object is not uniformly distributed, we must use calculus to determine centre of gravity.

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$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} \qquad 7.9$$

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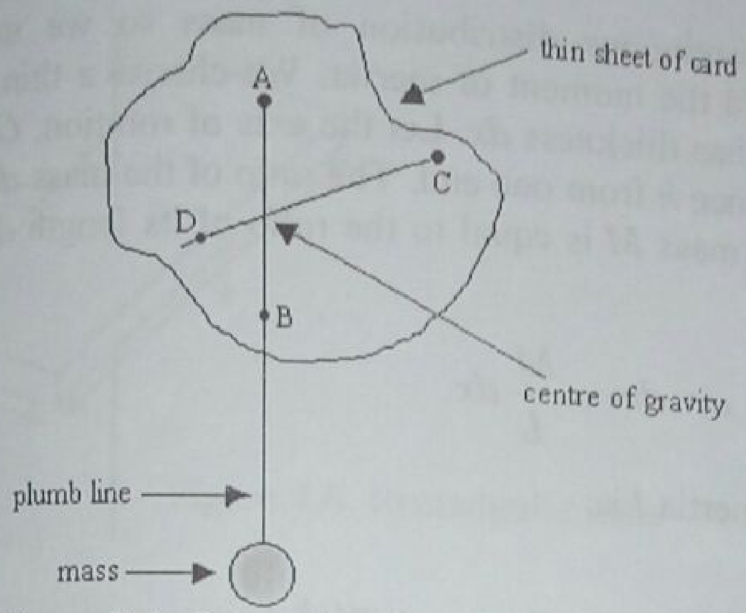



Figure 7.4: Determination of centre of gravity

## 7.6 Moment of Inertia

Consider a rigid body made up of a large number of particles, with masses $m_1$, $m_2$,... at perpendicular distances $r_1$, $r_2$,..., respectively, from the axis of rotation. The moment of inertia $I$, of the rigid body is defined as:

$$I = m_1 r_1^2 + m_2 r_2^2 + ... = \sum_i m_i r_i^2 \qquad 7.10$$

We now examine in some detail how moment of inertia, about a given axis $OX$, is computed for some uniform bodies.

Consider a uniform bar of length $L$ and mass $M$, about an axis through the bar's centre and which is normal to its length as shown in Figure 7.5.

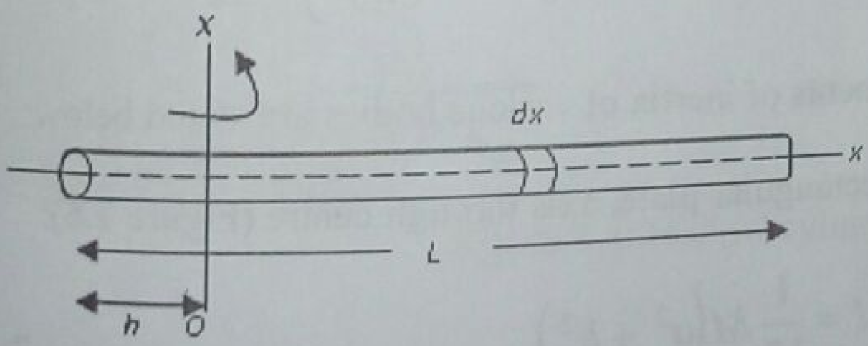



Figure 7.5: Slender rod

The rod is a continuous distribution of mass so we must use integration to find the moment of inertia. We choose a thin slice of mass $dm$, which has thickness $dx$. Let the axis of rotation, $OX$, be at an arbitrary distance $h$ from one end. The ratio of the mass $dm$ of the slice to the total mass $M$ is equal to the ratio of its length $dx$ to the total length $L$:

$$\frac{dm}{M} = \frac{dx}{L} \quad or \quad dm = \frac{M}{L}dx$$

The moment of inertia $I$ is:

$$I = \int dmx^2 = \int_{-h}^{L-h} \frac{M}{L}x^2dx = \left[\frac{M}{L}\left(\frac{x^3}{3}\right)\right]_{-h}^{L-h} = \frac{1}{3}M\left(L^2 - 3Lh + 3h^2\right)$$

If the axis is at the left end of the rod, i.e $h = 0$, then:

$$I = \frac{1}{3}M\left(L^2 - 3L\times 0 + 3\times 0\right) = \frac{1}{3}ML^2 \qquad 7.11$$

If the axis is at the right end of the rod $h = L$, then:

$$I = \frac{1}{3}M\left(L^2 - 3L\times L + 3\times L^2\right) = \frac{1}{3}ML^2$$

If the axis passes through the centre $h = L/2$, then:

$$I = \frac{1}{3}M\left(L^2 - 3L\times\frac{L}{2} + 3\times\left(\frac{L}{2}\right)^2\right) = \frac{1}{12}ML^2 \qquad 7.12$$

The moments of inertia of various bodies are stated below:

1. Rectangular plate, axis through centre (Figure 7.6):

$$I = \frac{1}{12}M\left(a^2 + b^2\right) \qquad 7.13$$

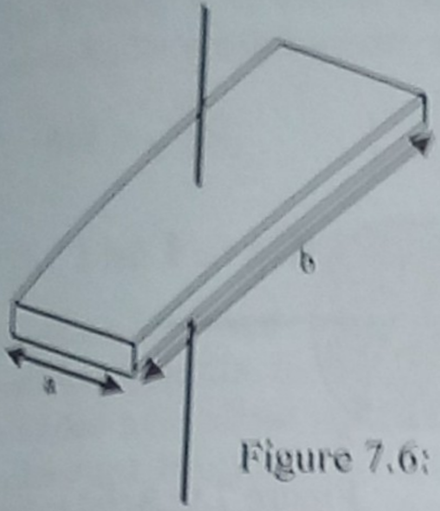



Figure 7.6: Rectangular plate

2. Thin rectangular plate, axis along edge (Figure 7.7):

$$I = \frac{1}{12} Ma^2 \qquad 7.14$$

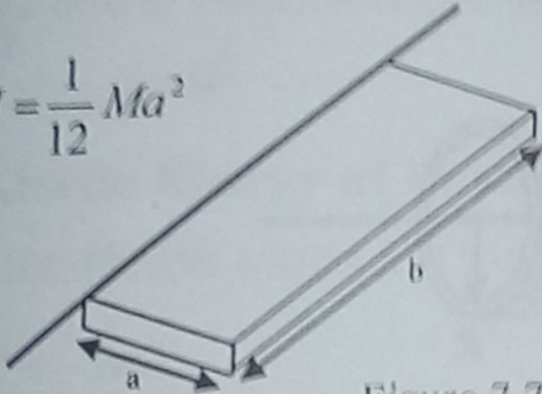



Figure 7.7: Thin rectangular plate

3. Hollow cylinder (Figure 7.8):

$$I = \frac{1}{2} M\left(R_1^2 + R_2^2\right) \qquad 7.15$$


Figure 7.8: Hollow cylinder

4. *Solid cylinder (Figure 7.9):*

$$I = \frac{1}{2}MR^2 \qquad 7.16$$

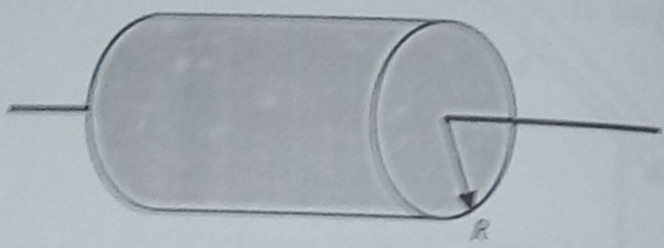


Figure 7.9: Solid cylinder

5. *Thin-walled hollow cylinder (Figure 7.10):*

$$I = MR^2 \qquad 7.17$$

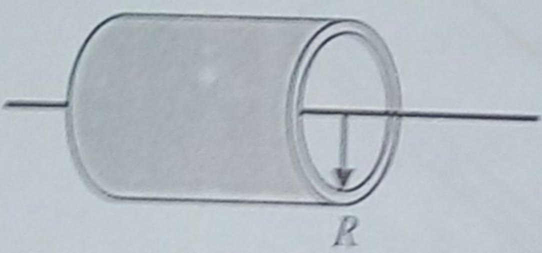


Figure 7.10: Hollow cylinder

6. *Solid sphere (Figure 7.11):*

$$I = \frac{2}{5}MR^2 \qquad 7.18$$

Figure 7.11: Solid sphere

All the results in Equations 7.11 to 7.18 can be written in the form:

$$I = Mk^2 \qquad 7.19$$

The number $k$ is called the radius of gyration of the object. For example, for a solid sphere about a diametrical axis:

$$k = \sqrt{\frac{2}{5}}R \qquad 7.20$$

## 7.7 The Parallel Axis Theorem

Equations 7.11 and 7.12 show that a body can have more than one moment of inertia. In fact, it has an infinite number because there are unlimited axes about which it might rotate. The parallel axis theorem explains the relationship between the moment of inertia $I_{cm}$ of a body of mass $M$ about an axis through its centre of mass and the moment of inertia $I$ about any other axis parallel to the original one, at a distance $d$ from it. The parallel axis theorem states that:

$$I = I_{cm} + Md^2 \qquad 7.21$$

## 7.8 Kinetic Energy of Rotation

The rotational kinetic energy $K$ of a rigid body is:

$$K = \frac{1}{2}I\omega^2 \qquad 7.22$$

where $I$ is the moment of inertia of the rigid body.

The motion of a rigid body can always be divided into translation of the centre of mass and rotation about the centre of mass. Hence, the kinetic energy of a rigid body has both translational and rotational parts, that is:

$$K = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}Mv_{cm}^2 \qquad 7.23$$

where the subscript *cm* refers to the centre of mass.

Consider a uniform cylinder rolling down an inclined plane without slipping, as shown in Figure 7.12.

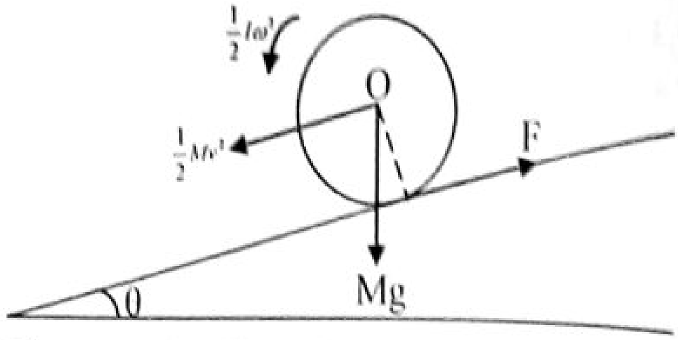



Figure 7.12: Rolling without slipping

The total kinetic energy of the cylinder, according to Equation 7.23, is:

$$K = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} M v_{cm}^2$$

Since the cylinder does not slip, then $v = r\omega$. Therefore:

$$K = \frac{1}{2} I_{cm}\left(\frac{v_{cm}}{r}\right)^2 + \frac{1}{2} M v_{cm}^2 = \frac{1}{2} v_{cm}^2\left(\frac{I}{r^2} + M\right) \qquad 7.24$$

Suppose the cylinder rolls without slipping from rest through a distance $x$ along the plane. The conservation of mechanical energy stipulates that the loss of potential energy = gain in kinetic energy, i.e.:

$$Mgx\sin\theta = \frac{1}{2} v_{cm}^2\left(\frac{I}{r^2} + M\right)$$

Rearranging, we have:

$$v_{cm}^2 = \frac{2Mgx\sin\theta}{M + \left(I/r^2\right)}$$

If $a$ is the acceleration down the plane, we have:

$$v^2 = v_o^2 + 2ax = 2ax = \frac{2Mgx\sin\theta}{M + \left(I/r^2\right)} \qquad \text{since } v_0 = 0$$

Thus:

$$a = \frac{Mg\sin\theta}{M + (I/r^2)} \qquad 7.25$$

Substituting Equation 7.16 (since the moving body is a solid cylinder) into Equation 7.25, we have:

$$a = \frac{2g\sin\theta}{3} \qquad 7.26$$

## 7.9 Work and Power in Rotational Motion

Newton's second law for rotational motion states that the total external torque $\tau$ is equal to the product of the moment of inertia $I$ and the angular acceleration $\alpha$ :

$$\tau = I\alpha \qquad 7.27$$

The angular acceleration $\alpha$ of a rotating body is defined as:

$$\alpha = \frac{d\omega(t)}{dt} \qquad 7.28$$

The total work done by the torque during an angular displacement from $\theta_1$ to $\theta_2$ is defined as:

$$W = \int_{\theta_1}^{\theta_2} \tau\, d\theta \qquad 7.29$$

If the torque remains constant while the angle changes by a finite amount, then:

$$W = \int_{\theta_1}^{\theta_2} \tau\, d\theta = \tau(\theta_2 - \theta_1) \qquad 7.30$$

...k on a rotating rigid body, the change in the ... of the rigid body equals the work done by

$$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt} = \vec{v} \times m\vec{v} + \vec{r} \times m\vec{a}$$

$$\frac{d\vec{L}}{dt} = 0 + \vec{r} \times m\vec{a} = \vec{r} \times \vec{F} = \tau \qquad 7.36a$$

Hence:

$$d\vec{L} = \tau dt \qquad 7.36b$$

If torque $\tau$ acting on the rigid body is constant, then:

$$\int_{L_1}^{L_2} d\vec{L} = \int_{0}^{t} \tau dt$$

$$\vec{L}_2 - \vec{L}_1 = \tau \times t$$

Hence:

$$\tau \times t = I\omega_2 - I\omega_1 \qquad 7.37$$

Equation 7.37 shows that the rate of change of angular momentum of a particle equals the torque of the net force acting on it.

We can use Equation 7.36 to infer the principle of conservation of angular momentum. If the external torque is equal to zero then:

$$\frac{d\vec{L}}{dt} = 0 \text{ and therefore } \vec{L} \text{ is a constant.}$$

The principle of conservation of angular momentum states that when the net external torque acting on a system is zero, the total angular momentum of the system is constant. Using $L = I\omega$, the conservation of angular momentum equation is:

$$I_1\omega_1 = I_2\omega_2 \qquad 7.38$$

# Activity 7 Rigid bodies

**7.1.** A flywheel of radius 20cm and mass 15kg is mounted on a horizontal axle. Calculate its moment of inertia about the axis passing through its centre of mass.

A. $10kgm^2$ B. $2.0kgm^2$ C. $3kgm^2$ D. $0.3kgm^2$

**Solution**

$$I = \frac{MR^2}{2} = \frac{1}{2} \times 15 \times 0.2^2\, kgm^2 = 0.3kgm^2$$

The correct option is D.

**7.2.** A grind stone in the form of a solid cylinder has a radius of 0.5 m and a mass of 50kg. What torque will bring it from rest to an angular velocity of 300 rev/min in 10s?

A. 187.5Nm B. 6.3Nm C. 19.6Nm D. 3.1Nm

**Solution**

For solid cylinder, moment of inertia, $I = \frac{1}{2} MR^2$ = ½ x 50 x $0.5^2$ $kgm^2 = 6.26kgm^2$

$$\omega = 300 rev/\min = \frac{300 \times 2\pi}{60} rad/s = 10\pi\, rad/s$$

$$\omega = \omega_o + \alpha t$$

$$10\pi = 0 + \alpha \times 10$$

$$\alpha = \pi\ rad/s^2$$

Torque, $\tau = I\alpha = 6.25 \times \pi Nm = 19.6Nm$

The correct option is C.

**7.3.** A solid cylinder of mass 20kg rolls without slipping down a 30° slope. Find the acceleration and the frictional force needed to prevent slipping. Take $g = 9.8m/s^2$.

A. $3.27m/s^2$, 32.6N

B. $7.07m/s^2$, 70.7N

Moment of inertia of hollow cylinder:

$$I = \frac{1}{2}M(R_1^2 - R_2^2)$$

Moment of inertia of thin cylindrical shell, $I = MR^2$. Since they have the same moment of inertia:

$$MR^2 = \frac{1}{2}(R_1^2 - R_2^2)$$

$$R^2 = \frac{1}{2}(10^2 + 20^2) = \frac{1}{2}(100 + 400) = \frac{500}{2} = 250$$

$R = 15.8cm$

The correct option is D.

76. When the net external torque acting on a system is zero, the total angular momentum of the system is constant. This statement is the principle conservation of:

A. angular momentum
B. angular acceleration
C. linear momentum
D. energy

Solution

The correct option is A.

77. A system consists of three particles, masses $m_1 = 2kg$, $m_2 = 1kg$, $m_3 = 3kg$ with instantaneous positions and velocities as follows:

$\vec{r}_1 = 2\hat{i} + \hat{j} + \hat{k}$ $\quad\quad$ $\vec{v}_1 = -\hat{i}$

$\vec{r}_2 = \hat{i} + \hat{k}$ $\quad\quad$ $\vec{v}_2 = \hat{i} + 2\hat{j}$

$\vec{r}_3 = \hat{k}$ $\quad\quad$ $\vec{v}_3 = \hat{i} + \hat{j} + \hat{k}$

Calculate the angular momentum of the system about the origin.

Soln: B

$$L=\sum_{i=1}^{3}\left(\vec{r}_i\times m_i\vec{v}_i\right)$$

$$L=(2i+j-k)\times(-2i)+(i-k)\times(i+2j)+(k)\times(3i+3j-3k)$$

$$L=\begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ -2 & 0 & 0 \end{vmatrix}+\begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 1 & 2 & 0 \end{vmatrix}+\begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 3 & 3 & -3 \end{vmatrix}$$

$$L=-i+4j+4k$$

28. A solid sphere of mass 20kg rolls without slipping down a 30° slope. Calculate the acceleration and the frictional force needed to prevent slipping.

A. 10m/s², 40N
B. 7.07m/s², 1.40N
C. 4.87m/s², 17.17N
D. 3.50m/s², 28N

Solution

For a rolling object acceleration:

$$a=\frac{Mg\sin\theta}{\left(M+\frac{I}{R^2}\right)}$$

For a solid sphere:

$$I=\frac{2}{5}MR^2$$

$$\Rightarrow\frac{I}{R^2}=\frac{2}{5}M$$

$$a=\frac{Mg\sin\theta}{M+\frac{2}{5}M}=\frac{Mg\sin\theta}{7M/5}=\frac{5}{7}g\sin\theta$$

$$a = \frac{5}{7} \times 9.8 \sin 30^{\circ}\ m/s^2 = 3.50 m/s^2$$

Frictional force

$F = mg\sin\theta - ma = (20 \times 9.8\ \sin 30 - 20 \times 3.5)N = (98 - 70)N = 28N$

The correct option is D.

**7.9.** Find the moment of inertia of a rod 4cm in diameter and 2m long, of mass 8kg about an axis perpendicular to the rod and passing through one end.

A. 5.6kgm$^2$ B. 10.5kgm$^2$ C. 10.67kgm$^2$ D. 11.0kgm$^2$

**Solution**

$$I = \frac{1}{3}ML^2 = \frac{1}{3} \times 8 \times 2^2 kgm^2 = \frac{32}{3} kgm^2 = 10.67 kgm^2$$

The correct option is C.

**7.10.** A wheel 1.0m in diameter is rotating about a fixed axis with an inertia angular velocity of 2revs$^{-1}$. The angular acceleration is 3rads$^{-2}$. Compute the angular velocity after 6s.

A. 20 revs$^{-1}$ B. 18 revs$^{-1}$ C. 17.5 revs$^{-1}$ D. 19.5 revs$^{-1}$

**Solution**

$\omega_t = \omega_o + \alpha t = (2 + 3 \times 6)$ rev/s $= 20$ rev/s

The correct option is A.

**7.11.** A flywheel requires 3s to rotate through 234radians. Its angular velocity at the end of this time is 108rads$^{-1}$. Find the constant angular acceleration.

A. 20 rads$^{-2}$ B. 10 rads$^{-2}$ C. 5 rads$^{-2}$ D. 2 rads$^{-2}$

**Solution**

$\theta - \theta_o = 234$rad, $t = 3$s, $\omega_t = 108$rad/s

$$\theta - \theta_o = \left(\frac{\omega + \omega_i}{2}\right)t$$

$$234 = \left(\frac{\omega_o + 108}{2}\right) \times 3$$

$$\omega_o = 48 rad/s$$

$\theta - \theta_o = \omega_o t + ½ \alpha t^2$
$234 = 48 \times 3 + ½ \alpha \times 3^2$
$90 = 9/2\ \alpha$
$\alpha = 180/9\ rad/s^2 = 20\ rad/s^2$

The correct option is A.

7.12. Find the moment of inertia of a solid sphere 2m in diameter and of mass 5kg about an axis passing through the centre.
A. $6.0kgm^2$ B. $10.5kgm^2$ C. $10.67kgm^2$ D. $1.0kgm^2$

**Solution**

Moment of inertia passing through the centre:

$$I = \frac{2}{5}MR^2 = \frac{2}{5} \times 5 \times 1^2 kgm^2 = 2\ kgm^2$$

The correct option is C.

7.13. A heavy flywheel of moment of inertia $0.3kgm^2$ is mounted on a horizontal axle of radius 0.01m and negligible mass compared with the flywheel. Neglecting friction, find the angular acceleration if a force of 40N is applied tangentially to the axle and the angular velocity of the flywheel after 10 seconds from rest.
A. $1.3\ rad/s^2$, 13 rad/s
B. $1.3\ rad/s^2$, 1.3 rad/s
C. $13r\ ad/s^2$, 13 rad/s
D. $13\ rad/s^2$, 0.13 rad/s

**Solution**

Torque, $\tau = 40 \times 0.01Nm$

angular acceleration:

$$\alpha = \frac{\tau}{I} = \frac{0.4}{0.3} rads^{-1} = 1.3 rads^{-1}$$

After 10 s, angular velocity :

$\omega = \alpha t = 1.3 \times 10$ rad/s = 13 rad/s

The correct option is A.

**7.14.** The flywheel of a stationary engine has a moment of inertia of $60 kgm^2$. What is the kinetic energy if its angular acceleration is $2 rads^{-2}$?

A. 10J B. 20J C. 30J D. 15J E. 60J

**Solution**

K.E = ½ $I\alpha$ = ½ x 60 x 2J = 60J

The correct option is E.

**7.15** A heavy flywheel rotating on its axis is slowing down because of friction on its bearing. At the end of the first 90 seconds, its angular velocity is 0.5 times its initial angular velocity. Find its angular velocity at the end of the second minute.

A. $0.90\omega_o$ B. $0.55\omega_o$ C. $0.33\omega_o$ D. $0.80\omega_o$

**Solution**

After 90s $\omega_t = 0.5\omega_o$

$\omega_t = \omega_o + \alpha t$

$0.5\omega_o = \omega_o + \alpha \times 90$

$\alpha = (-1/180)\omega_o rad/s^2$

$\omega_t = \omega_o + \alpha t = \omega_o + (-1/180)\omega_o \times 120 = \omega_o - 2\omega_o/3 = \omega_o/3 = 0.33\omega_o$

The correct option is C.

**7.16.** A circular plate of radius 0.5m starts from rest with constant angular acceleration. 20s later its angular velocity has increased to $150 rads^{-1}$. Calculate the angle through which the plate has turned.

A. 500 rad B. 50 rad C. 1500 rad D. 5 rad

**Solution**

$\omega = \omega_0 + \alpha t$
$150 - 0 = \alpha \times 20$
$\alpha = 7.5 rad/s^2$

$\theta - \theta_0 = \omega_0 t + ½ \alpha t^2 = 0 \times 20 + ½ \times 7.5 \times 20^2$ rad = 1500rad

The correct option is C.

117. A heavy flywheel of mass 15kg and radius 20cm is mounted on a horizontal axle of radius 1cm. If a force of 40N is applied tangentially to the axle, find the angular velocity of the fly wheel after 10 sec. ($I_{wheel} = Mr^2/2$.)
A. 13rad s$^{-1}$  B. 1.3rad s$^{-1}$  C. 130rad s$^{-1}$  D. 0.13rad s$^{-1}$

**Solution**

$I_{wheel} = Mr^2/2 = 15 \times (20 \times 10^{-2})^2/2 kgm^2 = 0.3kgm^2$
Torque, $\tau = FR = 40 \times 0.01Nm = 0.4Nm$
Torque, $\tau = I\alpha$
$0.4 = 0.3 \times \alpha$
$\alpha = 1.3rad/s^2$
$\omega = \alpha t = 1.3 \times 10 rad/s = 13rad/s$

The correct option is A.

118. A circular disc of mass 20kg and radius 15cm is mounted on a horizontal cylindrical axle of radius 1.5cm. Calculate the K.E. of the disc at the end of the 12s if a force of 20N is applied tangentially to the axle.
A. 29J  B. 16J  C. 20J  D. 24J

**Solution**

Moment of Inertia, $I = ½Ma^2 = ½ \times 20 \times 0.15kgm^2 = 0.225kgm^2$
Torque due to 20N tangential to axle = 20 x 0.015Nm = 0.3Nm
Angular acceleration, $\alpha$ = Torque/$I$ = (0.3/0.225) rad/s$^2$

Therefore after 12 seconds:
Angular velocity, $\omega = \alpha t$ = (0.3/0.225) x 12 rad/s = 16rad/s

K.E of disc after 12 seconds:

K.E. = $\frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.225 \times 16^2$ J = 28.8J ≈ 29J

The correct option is A.

**7.19.** A sphere rolls down a plane inclined at 30° to the horizontal. Calculate the velocity and acceleration of the sphere after it has moved 5m from rest along the plane ($I_{sphere} = 2MR^2/5$).

A. 3.6ms$^{-1}$; 4.0ms$^{-2}$
B. 6.0ms$^{-1}$; 3.6ms$^{-2}$
C. 6.0ms$^{-1}$; 44.6ms$^{-2}$
D. 4.0ms$^{-1}$; 3.6ms$^{-2}$

**Solution**

$$I = \frac{2MR^2}{5}$$

$$\frac{I}{R^2} = \frac{2M}{5}$$

Velocity of an object rolling down an inclined plane without slipping is:

$$v^2 = \frac{2Mgs\sin\theta}{\left[M + \left(\frac{I}{R^2}\right)\right]}$$

$$v^2 = \frac{2Mgs\sin\theta}{\left[M + \left(\frac{2M}{5}\right)\right]}$$

$$v^2 = \frac{2Mgs\sin\theta}{\frac{7M}{5}} = \frac{10gs\sin\theta}{7} = \frac{10 \times 10 \times 5 \times \sin 30}{7} = 35.714$$

$$v = 5.976m/s \approx 6.0m/s$$

$$a = \frac{Mg\sin\theta}{M + \frac{I}{R^2}}$$

$$a = \frac{Mg\sin\theta}{M + \frac{2M}{5}} = \frac{5g\sin\theta}{7}$$

$$a = \frac{5\times 10\times \sin 30}{7} m/s^2 = 3.57 m/s^2 \approx 3.6 m/s^2$$

The correct option is B.

7.20. A disc of mass 0.5kg and radius 10cm is rotating freely about axis $O$ through its centre at 30 $rev.min^{-1}$. If a 50g mass is now dropped gently unto the disc at a distance of 8cm from the centre $O$, what is the new r.p.m of the disc?

A. 35 r.p.m B. 20 r.p.m C. 27 r.p.m D. 10 r.p.m

**Solution**

Moment of inertia of the disc, $I_1 = \frac{M_1 R_1^2}{2}$

$$I_1 = \frac{0.5\times\left(10\times 10^{-2}\right)^2}{2} kgm^2 = 2.5\times 10^{-3} kgm^2$$

Moment of inertia of the mass:

$$I_2 = M_2 R_2^2 = 50\times 10^{-3}\times\left(8\times 10^{-2}\right)^2 kgm^2 = 3.2\times 10^{-4} kgm^2$$

The disc slows down to another speed corresponding to an angular momentum about $O$ of disc plus mass:

$= I_1\omega_2 + I_2\omega_2 = 2.5 \times 10^{-3}\omega_2 + 3.2 \times 10^{-4}\omega_2 = 2.82 \times 10^{-3}\omega_2$

From the conservation of angular momentum for the disc and mass about $O$:

$2.5 \times 10^{-3} \times 30 = 2.82 \times 10^{-3}\omega_2$

$\omega_2 = 26.5957$ rev/min = 27rpm

The correct option is C.

7.21. What is the tension in the supporting cable of a 14m, 11,000kg drawbridge?

**Solution**

The forces to be considered include: force at pivot, tension in cable and weight of the bridge.

Pivot: A sensible choice is the hinge since we do not know the exact direction of the hinge force, nor do we care about it. The force diagram is shown in Figure 7.13b.

Torques:

Due to the weight, $\tau_g = (L/2)\,mg\sin\theta_1$ = clockwise moment

Due to the tension, $\tau_T = LT\sin\theta_2$ = anticlockwise moment

Sum of clockwise moment = sum of anticlockwise moment

$$T = \frac{mg\sin\theta_1}{2\sin\theta_2} = \frac{11000\times10\times\sin 120}{2\times\sin 165}N = 1.84\times10^5\,N$$

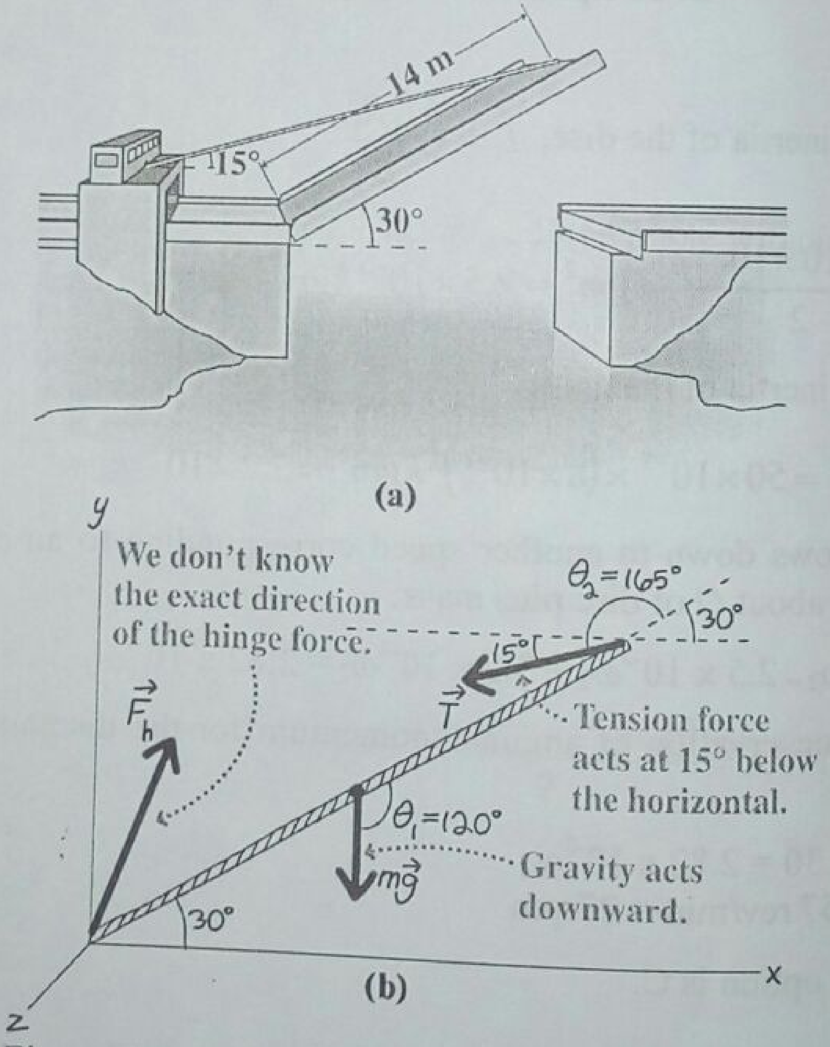



Figure 7.13: A drawbridge

7.22. At what minimum angle can the ladder lean without slipping (Figure 7.14a)?

**Solution**

The wall is frictionless and there is friction between the floor and the ladder. The force diagram is shown in Figure 7.14b.

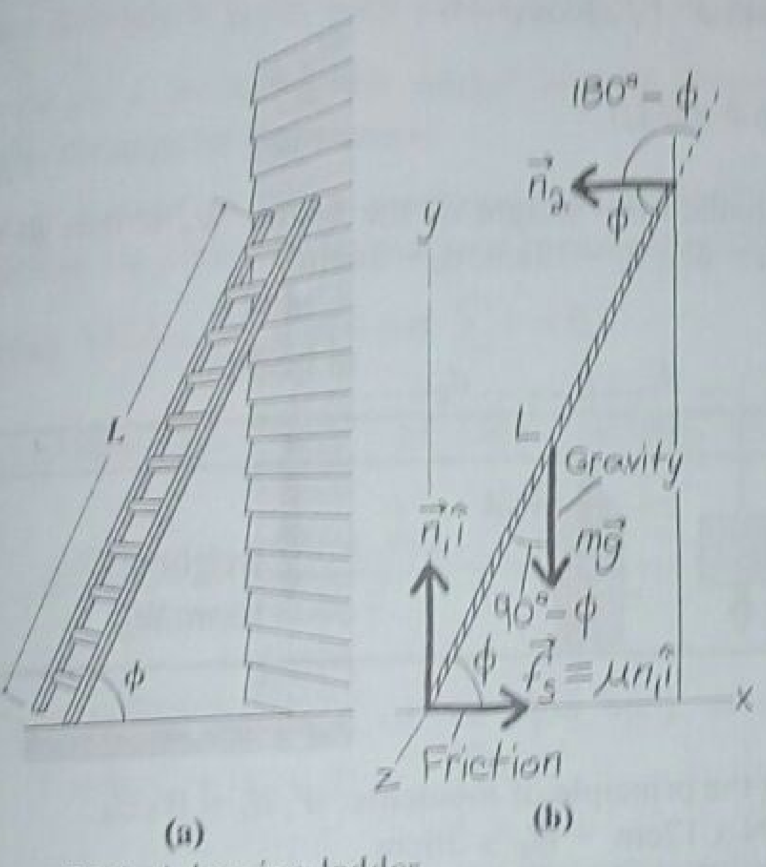



Figure 7.14: A leaning ladder

Forces:

1. Normal force at bottom of ladder
2. Friction force at bottom of ladder
3. Ladder's weight
4. Normal force at top of ladder

Pivot:

Choose bottom of ladder

Torques:

1. Due to ladder's weight
2. Due to normal force at top of ladder

Solve:

Force, $x$: $\mu n_1 - n_2 = 0$
Force, $y$: $n_1 - mg = 0$
Torque: $Ln_2 \sin\phi - (L/2)mg\cos\phi = 0$

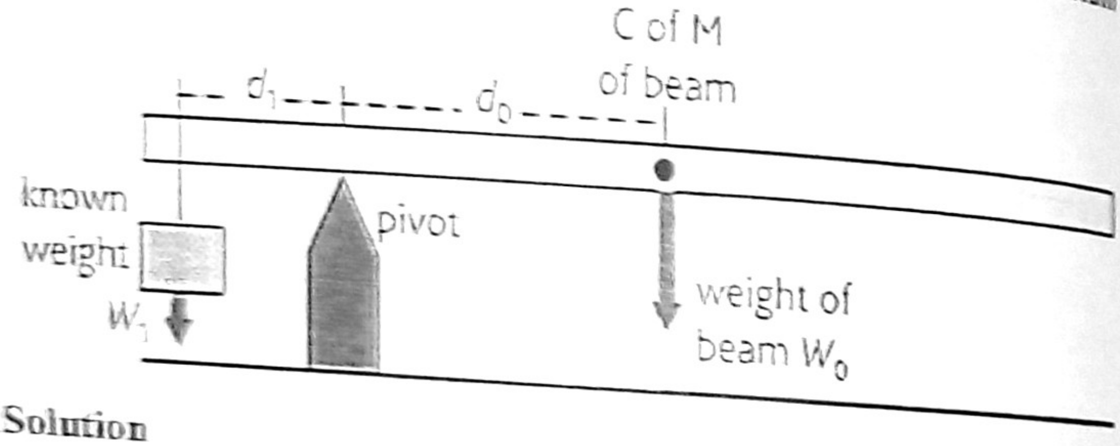
From the force equations we get $n_2 = \mu mg$.
Therefore:

$$\mu \sin\phi - (1/2)\cos\phi = 0$$

and so:

$$\tan\phi = 1/2\mu$$

**7.23.** Calculate the weight of the beam, $W_o$ if it is in equilibrium when: $W_1 = 6N$; $d_1 = 12cm$; $d_0 = 36cm$.




**Solution**

Applying the principle of moments: $W_1 d_1 = W_0 d_0$
Hence, 6N x 12cm = $W_0$ x 36cm
Therefore, $W_0$ = (72 / 36)N
$W_0$ = 2N

## Summary of Chapter 7

In chapter 7, you have learned that:

1. The moment of a vector **A** about a point $O$ is a vector whose magnitude is the product of the magnitude of **A** and the perpendicular distance of $O$ from the direction of **A**.
   Moment = vector **A** x perpendicular distance from axis.

2. The moment, about the origin, of a force **F** which passes through a point with position vector **r**, is given by the vector product: $\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta$ where $\theta$ is the angle between the line $r$ and the force vector **F**.

3. The moment, about the origin, of linear momentum **p** which passes through a point with position vector **r**, is given by: $\vec{L} = \vec{r} \times \vec{p}$. $L$ is called the orbital angular momentum, or simply, the angular momentum.

4. A body is said to be in mechanical equilibrium when the conditions for both translational and rotational equilibrium are satisfied. That is, $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$.

5. The centre of mass of the system is defined as $\vec{r}_{cm} = \dfrac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \dfrac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$.

6. The moment of inertia $I$, of the rigid body, is defined as $I = m_1 r_1^2 + m_2 r_2^2 + \ldots = \sum_i m_i r_i^2$.

7. The parallel axis theorem explains the relationship between the moment of inertia $I_{cm}$ of a body of mass $M$ about an axis through its centre of mass and the moment of inertia $I$ about any other axis parallel to the original one, at a distance $d$ from it.

8. The rotational kinetic energy $K$ of a rigid body is $K = \frac{1}{2} I\omega^2$, where $I$ is the moment of inertia of the rigid body.

9. Newton's second law for rotational motion states that the total external torque $\tau$ is equal to the product of the moment of inertia $I$ and the angular acceleration $\alpha$ : $\tau = I\alpha$ .

10. The total work done by the torque during an angular displacement from $\theta_1$ to $\theta_2$ is defined as: $W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$ .

11. The power associated with work done by a torque acting on a rotating body is $P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$ .

12. The principle of conservation of angular momentum states that when the net external torque acting on a system is zero, the total angular momentum of the system is constant. Using $L = I\omega$, the conservation of angular momentum equation is $I_1\omega_1 = I_2\omega_2$ .

## Self-Assessment Questions (SAQs) for Chapter 7

7.1. An astronaut is being tested in a centrifuge. The centrifuge has a radius of 10m and, in starting, rotates according to $\theta = 0.30t^2$, where $t$ is in seconds and $\theta$ is in radians. When $t = 5.0$s, what are the magnitude of the astronaut's angular velocity and linear velocity.

7.2. Calculate the rotational inertial of a wheel that has a kinetic energy of 24,400J when rotating at 602 rev/min.

A. 24.50kgm$^2$ B. 6.14kgm$^2$ C. 3.07kgm$^2$ D. 12.3kgm$^2$

7.3. A solid cylinder of mass 800kg and radius 6.0cm rolls without slipping down an inclined plane which makes an angle of 30° with the horizontal. Calculate the linear speed of the cylinder after rolling a distance of 1.5m. Assume that the moment of inertia of the cylinder about an axis through its middle, perpendicular to its plane is ½$MR^2$, where $M$ is its mass and $R$ its radius.

7.4. A helicopter has a propeller of mass of 70kg and radius of gyration of 0.75m. Find its moment of inertia. Calculate the torque needed to produce an angular acceleration of 4 rev/$s^2$.

7.5. A 0.5kg uniform sphere of 7.0cm radius spins at 30 rev/s on an axis through its centre. Find its (a) K.E, (b) angular momentum, and (c) radius of gyration.

7.6. A force of 200N acts tangentially on the rim of a wheel 25cm in radius. Calculate the torque.

7.7. A 20kg solid disk rolls on a horizontal surface at the rate of 4.0m/s. Calculate its total kinetic energy.

7.8. A disc of moment of inertia 10kg$m^2$ about its centre rotates steadily about the centre with an angular velocity of 20rad/s. Calculate (i) its rotational energy, (ii) its angular momentum about the centre, (iii) the number of revolutions per second of the disc.

7.9. A constant torque of 200Nm turns a wheel about its centre. The moment of inertia about this axis is 100kg$m^2$. Find (i) the angular velocity gained in 4s, (ii) the kinetic energy gained after 20revs.

7.10. A solid spherical ball of radius 0.12 m and mass 1.5kg rolls without slipping, moving in a straight line on a horizontal surface. If the velocity of the ball is 0.2m/s, calculate its kinetic energy. (b) If, instead, the ball slides without rolling, at the same speed, calculate its kinetic energy.

# CHAPTER EIGHT MECHANICAL PROPERTIES OF SOLIDS

## 8.1 Introduction

External forces can be applied to a solid to produce a change in the shape or volume of the solid. This change in shape or volume is known as deformation. In some cases when the deforming forces are removed, solids return to their original shapes and volumes. **Elasticity** is the ability of a solid to return to its original shape and volume when the deforming force has been removed. Such kinds of solids are called elastic solids and the deformation is said to be an **elastic deformation**. This type of deformation is reversible. Once the forces are no longer applied, the object returns to its original shape.

**Plastic deformation** is a process in which enough force is placed on metal or plastic to cause the object to change its size or shape in a way that is not reversible. In other words, the changes are permanent; even when the force is removed, the material will not go back to its original shape. Sometimes referred to simply as **plasticity**, this type of deformation can be conducted under controlled circumstances as well as unintentionally. Both the deformation of plastic and the deformation of metals involve changes to the makeup of the material itself. For example, metals that undergo this process of plastic deformation experience a condition known as **dislocation**. As force is exerted on the metal, the material reaches a point known as the yield strength. When this point is achieved, the pattern of the molecules that make up the metal begin to shift. The end result is that the molecules realign in a pattern that is shaped by the exterior force placed on the object.

Another type of deformation is **fracture**. This type of deformation is also irreversible. A break occurs after the material has reached the end of the elastic, and then plastic, deformation ranges. At this point forces accumulate until they are sufficient to cause a fracture. All materials will eventually fracture, if sufficient forces are applied.

## 8.2 Hooke's Law

When a deforming force $F$ is applied to a wire of initial length $L_o$, this produces a change in length $\Delta L$ of the wire. In an experiment in which the applied force is increased and the corresponding increase in length (extension, $e = \Delta L$) are recorded - if the extension, $e$, is plotted against the force $F$ in the wire, a graph as shown in Figure 8.1 is obtained. The elastic region $OA$ shows that the extension $e$ is proportional to the force $F$ and the wire returned to its original length when the applied force is removed. $A$ is the proportional limit of the wire. The force at $B$ is called the elastic limit that is the limit of elasticity.

Hooke's law states that the extension is proportional to the force in a wire if the proportional limit is not exceeded:

$$F = k(\Delta L) \qquad 8.1$$

where $k$ is a constant of proportionality.

Beyond the elastic limit, the wire is no longer elastic and it has a permanent extension $OP$ when the force is removed at $E$. The extension increases rapidly along the curve $AEC$ as the force on the wire is further increased and at $D$ the wire breaks.

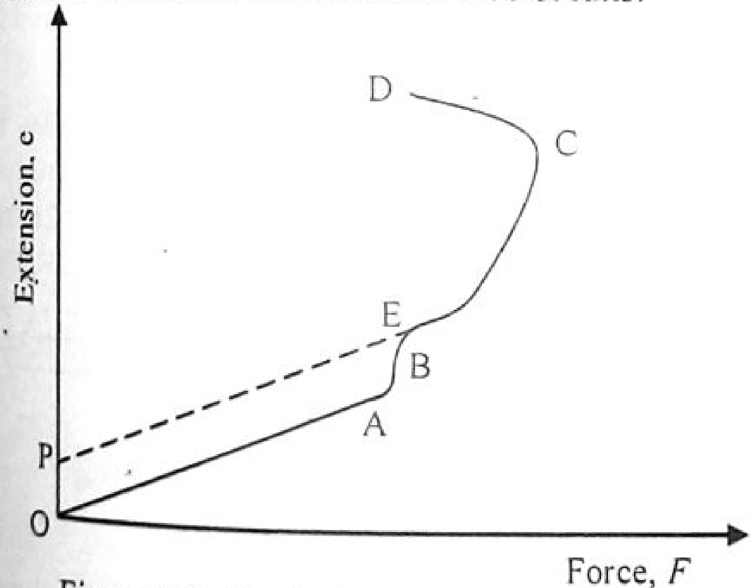



Figure 8.1: Hooke's Law

## 8.3 Stress and Strain

When a force $F$ is applied to the end of a wire of cross-sectional area $A$ and original length $L_o$, this produces what is known as a stress on the wire. Stress is defined as the force per unit area:

$$stress = \frac{F}{A} \qquad 8.2$$

S.I. unit of stress is newton per square meter (N/m$^2$).

Strain is defined as the extension per unit length:

$$strain = \frac{\Delta L}{L_o} = \frac{L - L_o}{L_o} \qquad 8.3$$

where it is assumed that the applied force stretches the wire to a length $L = L_o + \Delta L$. Strain is a unitless quantity.

Hooke's law also states that the strain is proportional to the stress if the proportional limit is not exceeded:

$$strain \propto stress$$

Rewriting this as an equation, Hooke's law states that

$$stress = Y \times strain \qquad 8.4$$

where $Y$, the constant of proportionality, is known as the modulus of elasticity (or Young's modulus) for the deformation.

If the applied force is perpendicular to the surface of the wire, then $F$ can either be inward, as in Figure 8.2a, to give what is known as a compressional stress, or $F$ can be outward, as in Figure 8.2b, to give a tensile stress.

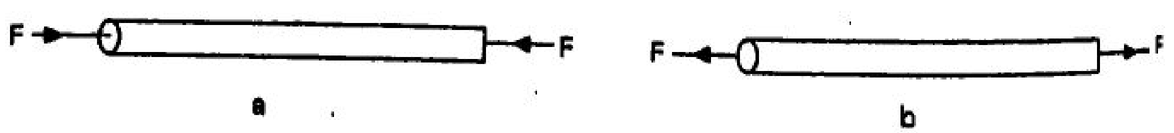



Figure 8.2: (a) Compressional stress, and (b) tensile stress

When applied to a compressional or a tensile stress and the corresponding strain, Hooke's law gives:

$$\frac{F}{A} = Y \times \frac{\Delta L}{L_o}$$

which implies:

$$F = \frac{YA}{L_o} \Delta L \qquad 8.5$$

Comparing Equations 8.1 and 8.5, we have:

$$k = \frac{YA}{L_o} \qquad 8.6$$

If the applied force $F$ is parallel to the surface (Figure 8.3) we have what is known as a shear stress producing a shear strain which is defined as the angle $\theta$, measured in radian.

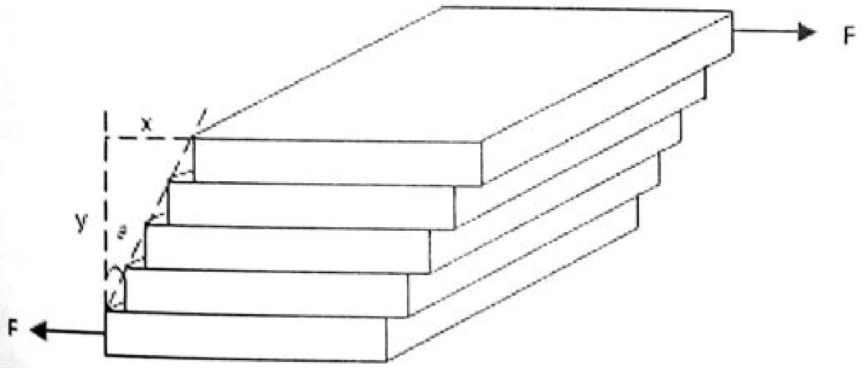



Figure 8.3: Shear strain

From the diagram above, we can also define shear strain as the ratio of the displacement $x$ to the transverse dimension $y$:

$$Shear\ strain = \theta \approx \tan\theta = \frac{x}{y} \qquad 8.7$$

Hooke's law, applied to a shear strain $\theta$ and the corresponding shear stress, gives

$$\frac{F}{A} = n\theta \qquad 8.8$$

where the constant of proportionality $n$ is known as the shear modulus of the substance or modulus of rigidity.

The third kind of stress-strain situation is called bulk. A solid object can be compressed by applying the same compressional stress to all its faces, as shown in Figure 8.4.

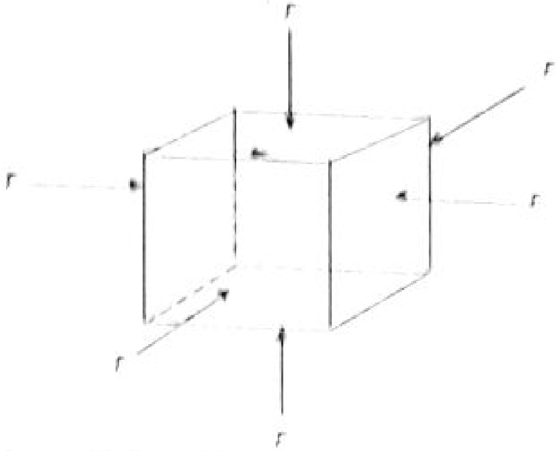



Figure 8.4: Bulk strain

Bulk (volume) stress, which is the same as change in pressure $\Delta p$, is defined as:

$$\Delta p = \frac{F}{A} \qquad 8.9$$

A bulk stress produces a reduction $\Delta V$ in the volume of the object, to give what is defined as the bulk strain:

$$Bulk\ \ strain = \frac{\Delta V}{V_o} \qquad 8.10$$

Applying Hooke's law to the bulk strain, we have:

$$\Delta p = -B\frac{\Delta V}{V_o} \qquad 8.11$$

where the constant of proportionality $B$ is known as the bulk modulus of the object or substance. The S.I. unit of $B$ is pascal (Pa) or $N/m^2$.

From Equation 8.11, we have:

$$B = -V_o \frac{\Delta p}{\Delta V} \quad 8.12$$

We include a minus sign in Equations 8.11 and 8.12 because an increase of pressure always causes a decrease in volume.

The reciprocal of the bulk modulus is called the compressibility and is denoted by $C$. From Equation 8.12:

$$C = \frac{1}{B} = -\frac{1}{V_o}\frac{\Delta V}{\Delta p} \quad 8.13$$

The unit of compressibility is $Pa^{-1}$ or $atm^{-1}$.

## 8.4 Energy Stored in a Tensile or Compressional Strain

When a solid object is stretched or compressed, energy is stored in it. This energy is released when the applied force is removed and provided the deformation is elastic.

Consider a solid object of cross-sectional area $A$ and initial length $L_o$, with one end initially on the origin of the $x$-axis (Figure 8.5). If a force $F(x)$ is applied causing deformation $x$, then from Equation 8.5, we have:

$$F(x) = \frac{YAx}{L_o} \quad 8.14$$

The work done against the restoring force in stretching the object by an additional small length $dx$ is given as:

$$W = \int_0^x dW = \int_0^x F(x)dx = \int_0^x \frac{YAx}{L_o}dx$$

$$W = \frac{YAx^2}{2L_o} \qquad 8.15$$

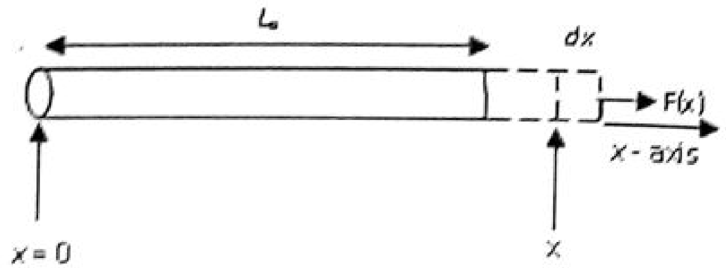



Figure 8.5: Energy stored

$W$ is stored in the solid as potential energy of deformation $E$. Given that the volume of the object $V$ is $AL_o$, the energy stored per unit volume is:

$$\frac{E}{V} = \frac{YAx^2}{2L_o} \times \frac{1}{AL_o} = \frac{Yx^2}{2L_o^2}$$

$$\frac{E}{V} = \frac{1}{2} \times \frac{Yx}{L_o} \times \frac{x}{L_o} = \frac{1}{2} \times (stress) \times (strain) \qquad 8.16$$

We see that for a tensile strain, the energy stored per unit volume of the object is $\frac{1}{2}(stress)(strain)$. We can also show that the energy stored in a shear strain for a cube of sides of length $L$ is given as:

$$E = \frac{1}{2} L^3 n \theta_o^2 \qquad 8.17$$

The volume of the cube $V$ is $L^3$. Then, the energy stored per unit volume is:

$$\frac{E}{V} = \frac{1}{2} n \theta_o^2 \qquad 8.18$$

Rewriting Equation 8.18, using Equation 8.8:

$$\frac{E}{V}=\frac{1}{2}\left(\frac{F}{A}\right)(\theta)=\frac{1}{2}\times(stress)\times(strain) \qquad 8.19$$

The energy stored in a bulk strain in compressing a spherical solid by a volume $V$ is (where $V_o$ is the initial volume):

$$E=\frac{1}{2}B\frac{V^2}{V_o} \qquad 8.20$$

The energy stored per unit volume is:

$$\frac{E}{V_o}=\frac{1}{2}B\frac{V^2}{V_o^2} \qquad 8.21$$

Rearranging the right hand side, we have:

$$\frac{E}{V_o}=\frac{1}{2}\left(\frac{BV}{V_o}\right)\left(\frac{V}{V_o}\right) \qquad 8.22$$

## Activity 8 Elasticity

8.1. A wire is subjected to a tensile stress of $2.5 \times 10^9 Nm^{-2}$, calculate the Young's modulus if the length increases by 5% of its original length.

A. $1.25 \times 10^{10} Nm^{-2}$
B. $5.0 \times 10^{10} Nm^{-2}$
C. $2.5 \times 10^{10} Nm^{-2}$
D. $12.5 \times 10^{10} Nm^{-2}$

**Solution**

Tensile strain $=\frac{\Delta L}{L_o}=\frac{5\%\ of\ L_o}{L_o}=\frac{0.05L_o}{L_o}=0.05$

Young's Modulus,

$$Y = \frac{tensile\ stress}{tensile\ strain} = \frac{2.5\times10^{9}}{0.05} N/m^2 = 5\times10^{10}\ N/m^2$$

The correct option is B.

8.2. Determine the Poisson's ratio of a solid material of Young's modulus $5.5 \times 10^{10}\ Nm^{-2}$ and shear modulus $2.5 \times 10^{10}\ Nm^{-2}$.

A. 0.10 B. 0.20 C. 0.15 D. 0.12

**Solution**

$Y = 2n\ (1 + \sigma)$, where $Y$ = Young's modulus, $n$ = Shear modulus, $\sigma$ = Poisson's ratio

$$5.5 \times 10^{10} = 2 \times 2.5 \times 10^{10}\ (1 + \sigma)$$

$$1 + \sigma = \frac{5.5 \times 10^{10}}{5.0 \times 10^{10}}$$

$$\sigma = 0.10$$

The correct option is A.

8.3. A spherical object has its pressure increased by $1.205 \times 10^{5}\ Nm^{-2}$ and the volume reduced to 2/3 of the original volume. Calculate the compressibility if the original volume is $1.2 \times 10^{-3}\ m^3$.

A. $-0.1670 \times 10^{-8} pa^{-1}$
B. $-0.3600 \times 10^{-8} pa^{-1}$
C. $-0.2766 \times 10^{-8} pa^{-1}$
D. $-0.1666 \times 10^{-8} pa^{-1}$

**Solution**

$$\Delta V = V_o - V = V_o - {}^2/_3 V_o = (1.2 \times 10^{-3} - {}^2/_3 \times 1.2 \times 10^{-3})\ m$$

$$= 1.2 \times 10^{-3}\ (1 - {}^2/_3)\ m = 4 \times 10^{-4} m^3$$

$$P = \frac{-1}{C}\frac{\Delta V}{V}$$

$$1.205 \times 10^{8} = -\frac{4\times10^{-4}}{C\times1.2\times10^{-3}}$$

$C = -2.766 \times 10^{-9}pa^{-1} = -0.2766 \times 10^{-8}pa^{-1}$

The correct option is C.

8.4. A wire of length 3m and diameter 1mm suspended vertically with one attached to a rigid ceiling is stretched when a block is attached to the other end. If the Young's modulus is $20 \times 10^{10}$ pascal, calculate the force constant on the wire.

A. $5.2 \times 10^{4}$N/m
B. $6.4 \times 10^{5}$N/m
C. $1.2 \times 10^{10}$N/m
D. $3.2 \times 10^{5}$ N/m

**Solution**

Area $A = \frac{\pi d^2}{4} = \frac{3.142\times(1\times10^{-3})^2}{4} m^2 = 0.0000007855m^2$

Young's modulus $Y = \frac{Stress}{Strain} = \frac{F}{A}\times\frac{L_o}{e} = \frac{KL_o}{A}$

$K = \frac{YA}{l_o} = \frac{20\times10^{10}\times7.855\times10^{-7}}{3} N/m = 5.2\times10^{4} N/m$

The correct option is A.

8.5. A wire of radius 0.2mm is extended by 0.1% of its length when it supports a load of 1kg. Calculate the Young's modulus for the material of the wire.

A. $7.8 \times 10^{10}$N/m$^2$
B. $7.8 \times 10^{4}$N/m$^2$
C. $5.4 \times 10^{10}$N/m$^2$
D. $5.4 \times 10^{4}$N/m$^2$

**Solution**

$r = 0.2mm = 0.2 \times 10^{-3}\ m$, $e = 0.1\%$ of $L = (0.1/100)L = 0.001L$,
$m = 1kg$

$F = mg = 1 \times 9.8N = 9.8N$

Young's modulus:

$$Y = \frac{Stress}{Strain} = \frac{F}{A} \div \frac{e}{L_0} = \frac{F}{A} \times \frac{L_0}{e} = \frac{9.8}{\pi r^2} \times \frac{L_0}{0.001L}$$

$$Y = \frac{9.8}{3.14 \times (0.2 \times 10^{-3})^2} \times \frac{1}{0.001} N/m^2 = 7.8 \times 10^{10} N/m^2$$

The correct option is A.

8.6. The rubber cord of a catapult has a cross sectional area $1.0mm^2$ and total unstretched length 10cm. It is stretched to 12cm and then released to project a missile of mass 5.0g. Calculate the velocity of projection taking Young's modulus for the rubber as $5.0 \times 10^8 N/m^2$.

A. 10m/s B. 5m/s C. 20m/s D. 40m/s

**Solution**

$A = 1.0mm^2 = 1.0 \times 10^{-6}\ m^2$, $L_0 = 10cm = 10 \times 10^{-2}\ m$,
$L_1 = 12cm = 12 \times 10^{-2}\ m$,
$e = L_1 - L_0 = (12 - 10)cm = 2 \times 10^{-2}\ m$, $m = 5\ g = 5 \times 10^{-3}$ kg

$Y = 5.0 \times 10^8 N/m^2$

Energy stored in wire = Kinetic energy

$$\frac{1}{2}\frac{YA}{L}e^2 = \frac{1}{2}mv^2$$

$$\frac{1}{2} \times \frac{5.0 \times 10^8 \times 1.0 \times 10^{-6}}{10 \times 10^{-2}} \times (2 \times 10^{-2})^2 = \frac{1}{2} \times 5 \times 10^{-3} v^2$$

$$1 = 2.5 \times 10^{-3} v^2$$

$$v^2 = 400$$

$$v = 20m/s$$

The correct option is C.

8.7. The rubber cord of a catapult is pulled back until its original length has doubled. Assuming that the cross section is 2mm square and that Young's modulus for rubber is $10^7$ N/m$^2$, calculate the tension in the cord.

A. 20N B. 40N C. 80N D. 10N

**Solution**

$L_1 = 2L_o$, $e = 2L_o - L_o = L_o$, $A = 2\text{mm}^2 = 2 \times 10^{-6}\,\text{m}^2$, $Y = 10^7\text{N/m}^2$

$$F = \frac{YAe}{L} = \frac{10^7 \times 2 \times 10^{-6} \times L_o}{L_o} N = 20N$$

The correct option is A.

8.8. When a spiral spring is stretched by a weight attached to it, the strain is:

A. tensile B. shear C. bulk D. elastic

**Solution**

The correct option is A.

8.9. A spherical object has its pressure increased by $1.205 \times 10^8\text{Nm}^{-2}$ and the volume reduced to $^5/_6$ of the original volume. Calculate the Bulk modulus. If the original volume is $1.2 \times 10^{-3}\,\text{m}^3$, what is the radius of the object under this pressure?

A. $7.23 \times 10^8\text{Nm}^{-2}$, 6.590cm
B. $1.44 \times 10^8\text{Nm}^{-2}$, 6.204cm
C. $7.23 \times 10^8\text{Nm}^{-2}$, 6.204cm
D. $1.44 \times 10^8\text{Nm}^{-2}$, 6.590cm

**Solution**

$P = 1.205 \times 10^8\,\text{Nm}^{-2}$, $V_1 = (5/6)V_o$, $V_o = 1.2 \times 10^{-3}\,\text{m}^3$

$$P = -\frac{B\Delta V}{V_o}$$

$$1.289\times10^{9} = -B\left(\frac{\frac{5}{6}V_o - V_o}{V_o}\right)$$

$$1.289\times10^{9} = -B\left(-\frac{1}{6}\right)$$

$$B = 7.73\times10^{9}\ N/m^{2}$$

New volume of sphere $V_1 = \frac{4\pi r_1^3}{3} = \frac{5}{6}V_o = 1\times10^{-3}m^3$

$$r_1 = \sqrt[3]{\frac{3\times1\times10^{-3}}{4\times3.142}}\,m = 0.06203m = 6.203cm$$

The correct option is C.

8.10. A wire is subjected to a tensile stress of $4.5\times10^{8}Nm^{-2}$. Calculate the Young's modulus if the length increases by 6% of its original length.
A. $6.79\times10^{9}Nm^{-2}$
B. $3.7\times10^{9}Nm^{-2}$
C. $6.23\times10^{9}Nm^{-2}$
D. $7.5\times10^{9}Nm^{-2}$

**Solution**

Tensile stress $= 4.5\times10^{8}N/m$

New length $L = L_o + \frac{6}{100}L_o$

$$\text{Tensile strain} = \frac{\Delta L}{L_o} = \frac{\left(\frac{6}{100}L_o\right)}{L_o} = \frac{3}{50}$$

Young Modulus

$$Y = \frac{stress}{strain} = \frac{4.5\times10^{8}}{3/50}N/m^{2} = 7.5\times10^{9}N/m^{2}$$

$$1.205\times10^{8} = -B\left(\frac{\frac{5}{6}V_o - V_o}{V_o}\right)$$

$$1.205\times10^{8} = -B\left(-\frac{1}{6}\right)$$

$$B = 7.23\times10^{8}\, N/m^{2}$$

New volume of sphere $V_1 = \frac{4\pi r_1^3}{3} = \frac{5}{6}V_o = 1\times10^{-3}\, m^3$

$$r_1 = \sqrt[3]{\frac{3\times1\times10^{-3}}{4\times3.142}}m = 0.06203m = 6.203\,cm$$

The correct option is C.

**8.10.** A wire is subjected to a tensile stress of $4.5 \times 10^{9}Nm^{-2}$. Calculate the Young's modulus if the length increases by 6% of its original length.

A. $4.79 \times 10^{-9}Nm^{-2}$
B. $2.7 \times 10^{-8}Nm^{-2}$
C. $4.23 \times 10^{-2}Nm^{-2}$
D. $7.5 \times 10^{10}Nm^{-2}$

**Solution**

Tensile stress = $4.5 \times 10^{9}$ N/m

New length $L = L_o + \frac{6}{100}L_o$

Tensile strain $= \frac{\Delta L}{L_o} = \frac{\left(\frac{6}{100}L_o\right)}{L_o} = \frac{3}{50}$

Young Modulus

$$Y = \frac{stress}{strain} = \frac{4.5\times10^{9}}{3/50}N/m^{2} = 7.5\times10^{10}N/m^{2}$$

The correct option is D.

**8.11.** A catapult is made of two identical strips of rubber, each of unstretched length of 0.3m and area of cross-section $1.7 \times 10^{-4} m^2$. Young's modulus for the rubber is $5 \times 10^6 Nm^{-2}$. If each of the catapult rubber is stretched by 0.2m to fire a stone of mass 250g vertically upward and all the elastic energy stored in the rubber is converted to kinetic energy of the motion of the stone, calculate the maximum height to which the stone rises.

A. 46.26m  B. 5.628m  C. 23.14m  D. 2.314m

**Solution**

Energy stored $E = \frac{1}{2} Fe = \frac{1}{2} \frac{YA}{L} e^2$

Since there are two identical strips of rubbers, total energy stored:

$$E = 2 \times \frac{1}{2} \frac{YA}{L} e^2 = \frac{YA}{L} e^2 =$$

$$E = \frac{5.0 \times 10^6 \times 1.7 \times 10^{-4} \times (0.2)^2}{0.3} = 1.13 \times 10^2 J$$

Energy stored = Potential energy at maximum height

$$1.13 \times 10^2 = mgh$$

$$h = \frac{1.13 \times 10^2}{0.25 \times 9.8} m = 46.258m$$

The correct option is A.

**8.12.** A solid cube of sides 3.5cm made of material of shear modulus $6.53 \times 10^8 Nm^{-2}$ is sheared through 0.06 rad by a force of 48000N applied parallel to one of its face, while the opposite face is clamped in position. Calculate the elastic energy stored in the deformed cube.

A. 100.8J  B. 50.4J  C. 25.5J  D. 12.6J

**Solution**

Energy stored

$$W = \frac{1}{2} L^3 n \theta_0^2 = \frac{1}{2} \times (3.5\times10^{-2})^3 \times 6.53\times10^8 \times (0.06)^2 J = 50.4J$$

The correct option is B.

**8.13.** Determine the Poisson's ratio of solid material of Young's modulus 7.5 x $10^{10}Nm^{-2}$ and shear modulus 2.52 x $10^{10}Nm^{-2}$.

A. 4.14 B. 0.15 C. 0.49 D. 2.98

**Solution**

$Y = 2n(1 + \sigma)$

$7.5 \times 10^{10} = 2 \times 2.52 \times 10^{10}(1 + \sigma)$

$1.4880952 = 1 + \sigma$

$\sigma = 0.488 = 0.49$

The correct option is C.

**8.14.** A cylindrical copper bar of length 2.5m and radius of cross-section 2.25cm compressed by a force of 1.62 x $10^6$ N. If the Young's modulus and Poisson's ratio for copper are 12.98 x $10^{10}Nm^{-2}$ and 0.34 respectively. Calculate the decrease in length and the increase in diameter.

A. 0.0196m, 0.012cm
B. 0.0503m, 0.012cm
C. 0.0196cm, 0.0308cm
D. 0.0503m, 0.0308cm

**Solution**

$$A = \pi r^2 = 3.142\times (2.25 \times10^{-2})^2 m^2 = 15.9 \times 10^{-4} m^2$$

Young's Modulus:

$$Y = \frac{FL}{Ae}$$

$$e = \frac{FL}{YA} = \frac{1.62\times10^6 \times 2.5}{12.98\times10^{10}\times15.9\times10^{-4}} m = 0.019623804m = \Delta L$$

Position's ratio $\sigma$ is defined:

$$\frac{\Delta w}{w} = -\frac{\sigma \Delta L}{L}$$

Diameter of the rope,

$w = (2)\,(\text{radius}) = 2 \times 2.25 \times 10^{-2}\,m = 4.5 \times 10^{-2}\,m$

$$\frac{\Delta w}{4.5\times10^{-2}} = -\frac{0.34 \times 0.019623804}{2.5}$$

$$\Delta w = 0.0001201m$$

Therefore, the change in diameter of the bar is 0.012cm

The correct option is A.

8.15. In an experiment to measure Young's modulus, a load of 500kg hanging from a steel wire of length 3m and cross section $0.20cm^2$ was found to stretch the wire 0.4cm above its no-load length. What is the Young's Modulus for the steel of which the wire is composed?

A. $2.45 \times 10^8$ Pa  B. $2.54 \times 10^8$ Pa
C. $1.84 \times 10^{11}$ Pa  D. $18.4 \times 10^{11}$ Pa

**Solution**

$m = 500kg$, $F = 500 \times 9.8N = 4900N$, $A = 0.20 \times 10^{-4}\,m^2$,
$e = 0.4 \times 10^{-2}\,m$, $L_o = 3\,m$

Young modulus,

$$Y = \frac{F}{A} \times \frac{L_o}{e}$$

$$Y = \frac{4900}{0.2\times10^{-4}} \times \frac{3}{0.4\times10^{-2}}\,Pa = \frac{14700}{8\times10^{-8}}\,Pa = 1.84 \times 10^{11}\,Pa$$

The correct option is C.

8.16. Find the maximum load in kg which may be placed on a steel wire of diameter 0.10cm, if the permitted strain must not exceed 0.001. Young's modulus for steel $= 2.0 \times 10^{11}Nm^{-2}$.

A. 157kg B. 15.7kg C. 1.57kg D. 10kg

**Solution**

Young's modulus:

$$Y = \frac{Stress}{Strain}$$

$$2.0 \times 10^{11} = \frac{Stress}{0.001}$$

Stress = $2.0 \times 10^{8}$N/m

Area = $\pi d^2/4 = (3.142 \times (0.10 \times 10^{-2})^2/4)$ m$^2$ = $7.855 \times 10^{-7}$ m$^2$

Stress = $F/A$
$2.0 \times 10^{8} = F/7.855 \times 10^{-7}$
$F = 157.1$N

Therefore, load, $L$ = 15.7kg

The correct option is B.

**8.17.** A wire of length 5m of uniform circular cross section of radius 1mm is stretched by 1.5mm when subjected to a tension of 100N. Calculate the strain energy per unit volume.
A. $6.77 \times 10^{3}$Jm$^{-3}$
B. $4.77 \times 10^{2}$Jm$^{-3}$
C. $6.77 \times 10^{2}$Jm$^{-3}$
D. $4.77 \times 10^{3}$Jm$^{-3}$

**Solution**

Strain energy, $W = ½Fe = ½ \times 100 \times 0.0015$J = 0.075J

Volume of wire, $V$ = length x area = $5 \times \pi \times 0.001^2$ m$^3$

Energy per unit volume :

$$u = \frac{Energy}{Volume}$$

$$u = \frac{0.075}{5 \times \pi \times 0.001^2} Jm^{-3} = 4.77 \times 10^{3}\ Jm^{-3}$$

The correct option is D.

118. A wire of [illegible] cross-sectional area [illegible] is stretched [illegible]. Calculate the Young's modulus of the wire [illegible] its original length.

A. [illegible] B. 250N/m² C. [illegible] D. [illegible]

Solution

Extension $e = \frac{3}{2}L_0 - L_0 = \frac{1}{2}L_0$

Strain $= \frac{e}{L_0} = \frac{1}{2}$

Stress $= \frac{Force}{Area} = \frac{2.5}{0.02} N/m^2 = 125 N/m^2$

Young's modulus, $Y = \frac{stress}{strain} = \frac{125}{1/2} N/m^2 = 250 N/m^2$

The correct option is B.

119. A wire is subjected to a tensile stress of $5 \times 10^8 Nm^{-2}$ producing a strain of 0.5 on the wire. Calculate the Young's modulus of the wire.

A. $1.2 \times 10^9 Nm^{-2}$
B. $5.0 \times 10^9 Nm^{-2}$
C. $2.0 \times 10^9 Nm^{-2}$
D. $1.0 \times 10^9 Nm^{-2}$

Solution

Young's modulus $Y = \frac{stress}{strain} = \frac{5 \times 10^8}{0.5} N/m^2 = 1.0 \times 10^9 N/m^2$

The correct option is D.

120. The shear and Young modulus of an isotropic material are $0.2 \times 10^{11} Nm^{-2}$ and $0.5 \times 10^{11} Nm^{-2}$ respectively. Calculate the bulk modulus of the material.

A. $0.42 \times 10^{11} Nm^{-2}$
B. $0.33 \times 10^{11} Nm^{-2}$

C. $0.25 \times 10^{11}\,Nm^{-2}$
D. $0.12 \times 10^{11}\,Nm^{-2}$

**Solution**

For an isotropic material $Y = 2n(1 + \sigma)$

$$0.5 \times 10^{11} = 2 \times 0.2 \times 10^{11}(1 + \sigma)$$
$$\sigma = 0.25$$

Also:

$$Y = 3B(1 - 2\sigma)$$
$$0.5 \times 10^{11} = 3B(1 - 2 \times 0.25)$$
$$B = 0.33 \times 10^{11}\,Nm^{-2}$$

The correct option is B.

## Summary of Chapter 8

In chapter 8, you have learned that:

1. Elasticity is the ability of a solid to return to its original shape and volume when the deforming force has been removed. Such kinds of solids are called elastic solids and the deformation is said to be an elastic deformation.
2. Hooke's law states that the extension is proportional to the force in a wire if the proportional limit is not exceeded. $F = k(\Delta L)$, where $k$ is a constant of proportionality.
3. Stress is defined as the force per unit area; $stress = \dfrac{F}{A}$.
4. Strain is defined as the extension per unit length; $strain = \dfrac{\Delta L}{L_o} = \dfrac{L - L_o}{L_o}$.
5. Hooke's law also states that the strain is proportional to the stress if the proportional limit is not exceeded.

6. Hooke's law also states that $stress = Y \times strain$ where $Y$, the constant of proportionality, is known as the modulus of elasticity (or Young's modulus) for the deformation.

7. The work done against the restoring force in stretching the object by an additional small length $dx$ is given as $W = \int dW = \int F(x)dx = \int_0^x \frac{YAx}{L_o} dx = \frac{YAx^2}{2L_o}$.

8. The energy stored per unit volume is $\frac{E}{V} = \frac{1}{2} \times (stress) \times (strain)$.

## Self-Assessment Questions (SAQs) for Chapter 8

8.1. When a force of 5.0N is applied to a spiral spring, it was stretched by 4mm. What will be the extension if the force is increased 30%?

8.2. When a piece of wire is stretched by a weight attached to it, the stress experience is:

A. Tensile stress B. Shear stress
C. Bulk stress D. Normal stress

8.3. A solid cylindrical steel column is 4.0m long and 9.0cm in diameter. What will be its decrease in length when carrying a load of 80 000kg? Young's modulus, $Y = 1.9 \times 10^{11}$ Pa.

8.4. Calculate the volume contraction when 100mL of water is subjected to a pressure of 1.5MPa, given that the bulk modulus of water is 2.1 GPa.

8.5. A 2.0kg object is attached to the end of a 5.0m long and of $0.0058cm^2$ cross-sectional area and stretches the wire elastically. The object undergoes vertical SHM when pulled down a little and released. Find the period of its vibration, given that the Young's modulus of the wire is 200 GPa. [illegible]

9.6. A wire 4.0m long and 0.3mm² in cross-section area [illegible] 1.0mm when a mass of 2.25kg is hung from its lower end [illegible] the Young's modulus for the wire.

9.7. A copper wire of length 3m and area of cross-section [illegible] $4 \times 10^{-6} m^2$ is stretched so that it is under [illegible] $1.52 \times 10^{-3}$. For this deformation, calculate (a) the [illegible] extension, and (c) the strain, if Young's modulus for the copper [illegible] $12.00 \times 10^{10} N/m^2$.

9.8. A rubber rope used to pull water from a well [illegible] under a load of 150N, which is twice the weight of [illegible] water. The original length and radius of the rope are [illegible] respectively. (a) Find Young's modulus for the rubber. (b) [illegible] Poisson's ratio for this rubber is 0.46, calculate the change in the rope's diameter under this load.

9.9. A certain solid object made of material of bulk modulus $4.6 \times 10^{10} N/m^2$ has volume $0.05 m^3$ when it is placed under a pressure of one atmosphere. If the solid is now placed under a hydrostatic pressure of 50 atmospheres, calculate its volume.

9.10. A wire of length 2m and area of cross-section $6.5 \times 10^{-6} m^2$ is stretched 2.4mm by a force of 1200N. If the deformation is elastic, calculate (a) the strain, (b) the stress, (c) Young's modulus for the wire, and (d) the energy stored in the wire in this deformation.

CHAPTER NINE

# MECHANICAL PROPERTIES OF FLUIDS

## 9.1 Introduction

A fluid is a group of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container. Both liquids and gases are fluids. They have the ability to flow and to assume the shape of any containing vessel. In this chapter, we consider fluid statics: mechanics of fluid at rest; and fluid dynamics: mechanics of fluids in motion. For a fluid, we would like to know the values of mechanical quantities such as density, pressure, compressibility, bulk modulus, Young's modulus and viscosity.

## 9.2 Density

The density $\rho$ of a substance is defined as its mass per unit volume:

$$\rho = \frac{m}{V} \qquad 9.1$$

The S.I. unit of density is the kilogram per cubic metre ($kg/m^3$). The cgs unit, the gram per cubic centimetre ($g/cm^3$), is also widely used: $1\ g/cm^3 = 1000 kg/m^3$.

The relative density $RD$ of a substance is the ratio of its density to the density of water:

$$RD = \frac{density\ of\ substance}{density\ of\ water} \qquad 9.2a$$

$RD$ has no unit.

It can easily be seen that the relative density of a body can be defined alternatively as:

$$RD = \frac{mass\ of\ a\ body}{mass\ of\ equal\ volume\ of\ wat} \qquad 9.2b$$

RD of water is obviously 1.

A hydrometer is an instrument for determining the relative density of a liquid, base on the law of floatation. It usually consists of a long graduated thin stem A, a large bulb B to provide buoyancy and a small bulb C with some lead shots to enable the hydrometer to float upright. It should be noted that the readings of the graduations increase downwards and the graduations are not equally apart.

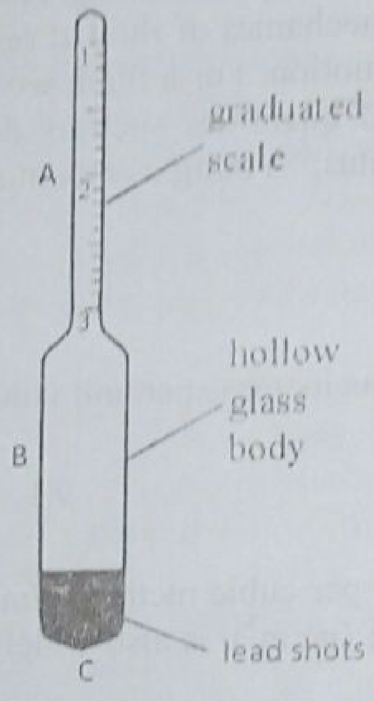



Figure 9.1: Hydrometer

## 9.3 Pressure in a Fluid

When a fluid is at rest, it exerts a force perpendicular to any surface in contact with it. This force is due to molecules in motion colliding with their surroundings. We define the pressure $P$ at any point as the normal force per unit area:

$$P = \frac{F_\perp}{A} \qquad 9.3$$

where $F_\perp$ is the net normal force on one side of the surface. The S.I. unit of pressure is the pascal, where 1 pascal (Pa) = 1N/m$^2$.

Now let us derive a general relationship between the pressure $P$ at any point in a fluid at rest and the depth $h$, as shown in Figure 9.2. We assume that the density $\rho$ of the fluid is constant: this means that the fluid is incompressible. The force exerted by the outside liquid on the bottom face of the cylinder is $PA$ and the force exerted on the top face of the cylinder is $P_oA$, where $P_o$ is the atmospheric pressure. If the mass of the liquid is $M = \rho V = \rho Ah$, then the weight of the liquid in the cylinder is $Mg = \rho Ahg$. Assuming the cylinder is in equilibrium, the net force acting on it must be zero:

$$\sum F_y = PA - P_oA - Mg = 0$$

Or:

$$PA - P_oA - \rho Ahg = 0$$

$$PA - P_oA = \rho Ahg$$

$$P - P_o = \rho gh$$

$$P = P_o + \rho gh \qquad 9.4$$

where $P_o$ is the pressure applied to the liquid surface.

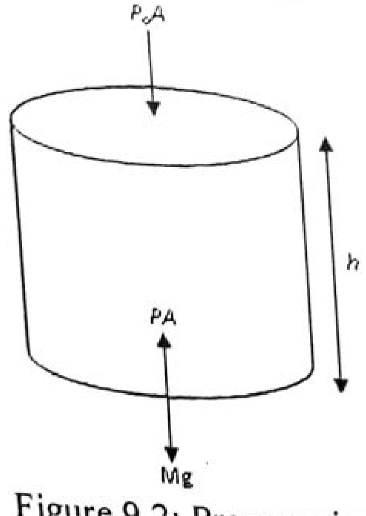



Figure 9.2: Pressure in liquid

$$P = \frac{F_{\perp}}{A} \qquad 9.3$$

where $F_{\perp}$ is the net normal force on one side of the surface. The S.I. unit of pressure is the pascal, where 1 pascal (Pa) = 1N/m$^2$.

Now let us derive a general relationship between the pressure $P$ at any point in a fluid at rest and the depth $h$, as shown in Figure 9.2. We assume that the density $\rho$ of the fluid is constant: this means that the fluid is incompressible. The force exerted by the outside liquid on the bottom face of the cylinder is $PA$ and the force exerted on the top face of the cylinder is $P_oA$, where $P_o$ is the atmospheric pressure. If the mass of the liquid is $M = \rho V = \rho Ah$, then the weight of the liquid in the cylinder is $Mg = \rho Ahg$. Assuming the cylinder is in equilibrium, the net force acting on it must be zero:

$$\sum F_y = PA - P_oA - Mg = 0$$

Or:

$$PA - P_oA - \rho Ahg = 0$$

$$PA - P_oA = \rho Ahg$$

$$P - P_o = \rho gh$$

$$P = P_o + \rho gh \qquad 9.4$$

where $P_o$ is the pressure applied to the liquid surface.

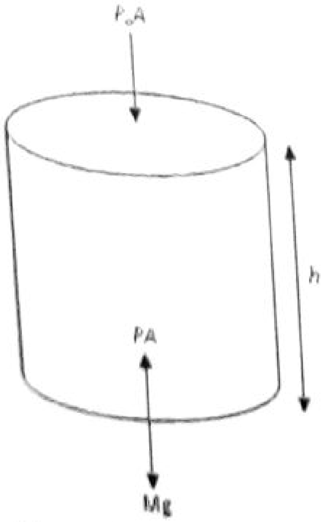



Figure 9.2: Pressure in liquid

Equation 9.4 shows that pressure $P$ at depth $h$ is greater than the pressure $P_0$ at the surface by an amount $\rho gh$. The average atmospheric pressure at sea level is used as a unit and is called an atmosphere (atm):

$$1 \text{ atm} = 1.01325 \times 10^5 \text{ Pa} = 1.01325 \times 10^5 \text{N/m}^2$$

There are two types of pressure gauge that we will consider in this chapter: Open-tube manometer and Mercury barometer (Figure 9.3). The open-tube manometer contains a liquid of density $\rho$, often mercury or water. One end of the tube is connected to the container where the pressure $p$ is to be measured, and the other end is open to the atmosphere at pressure $P_0 = P_{atm}$. The pressure at the bottom of the tube due to fluids at both sides are shown in the figure as $P + \rho gy_1$ and $P_{atm} + \rho gy_2$. These pressures are measured at the same level, so they must be equal:

$$p + \rho gy_1 = P_{atm} + \rho gy_2$$
$$p - P_{atm} = \rho g(y_2 - y) = \rho gh$$

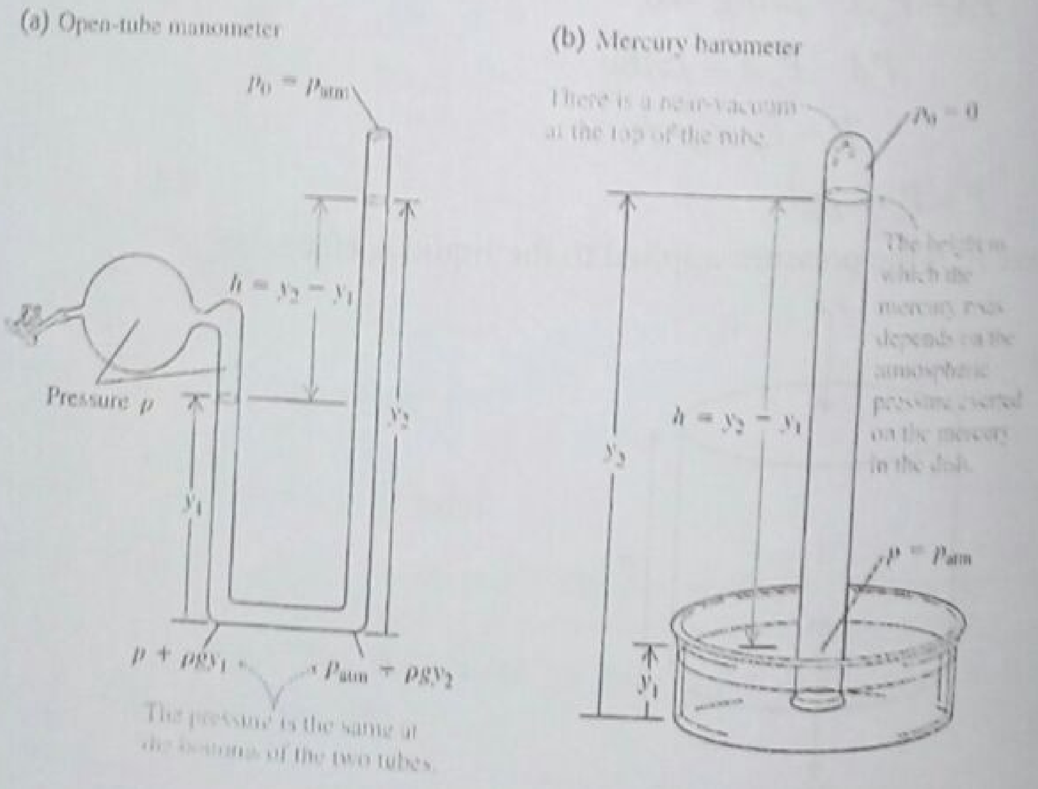



Figure 9.3: Pressure gauges

The excess pressure above the atmospheric pressure is usually called gauge pressure, and the total pressure is called absolute pressure. The mercury barometer consists of a long glass tube, closed at one end that has been filled with mercury and then inverted in a dish of mercury (Figure 9.3b). From Equation 9.4:

$$P_a = P = 0 + \rho g(y_2 - y_1) = \rho g h.$$

This implies that the mercury barometer reads the atmospheric pressure $p_{atm}$ directly from the height of the mercury column.

## 9.4 Pascal's Principle

When the pressure $p_o$ at the top surface of a fluid is increased using a piston that fits tightly inside the container to push down on the fluid surface, the pressure $p$ at any depth inside the fluid increases by exactly the same amount. The transmission of pressure in fluids was studied in 1653 by the French scientist Blaise Pascal and the result is called Pascal's Principle. Pascal's principle states that pressure applied to an enclosed fluid is transmitted undiminished to every point in the fluid and to the walls of the container.

We can use the hydraulic lift shown in Figure 9.4 to illustrate Pascal's principle. A piston with small cross-sectional area $A_1$ exerts a force $F_1$ on the surface of a liquid. The applied pressure $P_1 = F_1/A_1$ is transmitted through the liquid inside the connecting pipe to a larger piston of area $A_2$. The pressure $p_1$ is the same in both cylinders, so:

$$P_1 = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

or

$$F_2 = \frac{A_2}{A_1} F_1 \qquad 9.5$$

Sin[illegible]arger than $A_1$ the force $F_2$ will be larger than $F_1$.

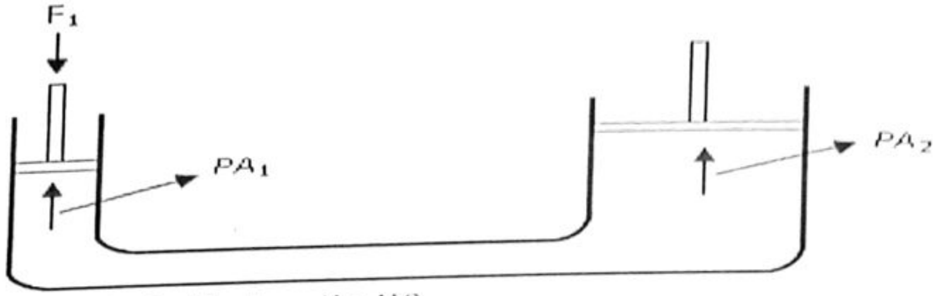



Figure 9.4: Hydraulic lift

## 9.5 Buoyancy and Archimedes' Principle

Archimedes' principle explains the phenomenon of buoyancy. A body floats because it is buoyant, or is buoyed up. Archimedes' principle states that when a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.

This upward force is called the upthrust or buoyant force $F_b$. Hence:

$$F_b = m_f g = \rho_f V_f g \qquad 9.6$$

where $m_f$, $\rho_f$ and $V_f$ are the mass, density and volume of the fluid displaced by immersed body.

Let us consider a solid uniform object totally immersed in a fluid (Figure 9.5). The weight of the object in air is:

$$W_1 = m_1 g = \rho_1 V_1 g \qquad 9.7$$

If the object is completely immersed, the volume of the fluid displaced is equal to the volume of the object. Equation 9.6 divided by Equation 9.7 gives:

$$\frac{F_b}{W_1} = \frac{\rho_f V_f g}{\rho_1 V_1 g}$$

Or:

$$F_b = W_1\left(\frac{\rho_f}{\rho_1}\right) \quad 9.8$$

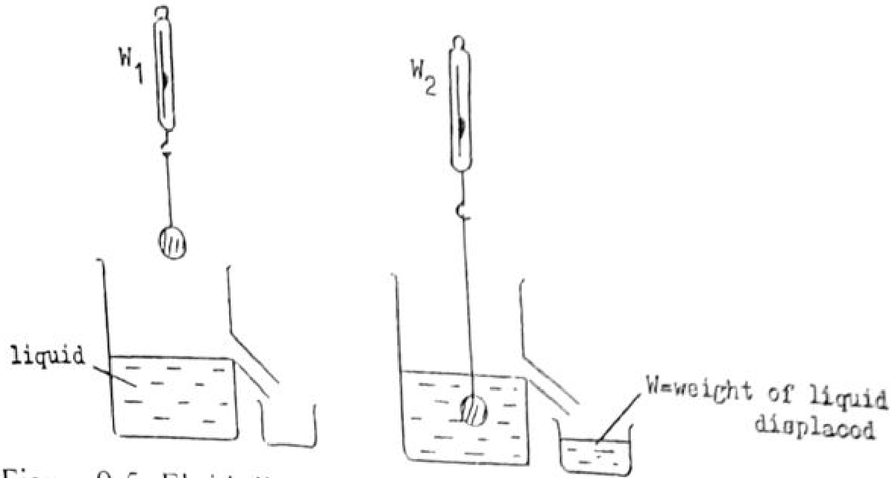



Figure 9.5: Fluid displaced when solid are immersed

When an object is lowered into a liquid, it is found to lose weight. It is because the liquid exerts an upthrust on the object. It can be shown that:

upthrust = apparent lost in weight of the object $= W_1 - W_2$.

Experimentally:

$$W_1 - W_2 = W = \text{weight of liquid displaced}$$

Hence:

upthrust acting on a body = weight of liquid displaced by the body

**Principle of floatation** states that a floating body displaces its own weight of fluid in which it floats, i.e. weight of floating body is equal to weight of liquid displaced by the body. Actually, principle of floatation is a special case of Archimedes' Principle. A body with average density $\rho$ can float in a liquid with density $\rho'$ if $\rho < \rho'$. A body (density $\rho$) is said to be just floating in a liquid (density $\rho'$) if $\rho = \rho'$.

## 9.6 Fluid Dynamics and Bernoulli's Equation

When a fluid of density $\rho$ and viscosity $\eta$ flows from one place to another, the flow may be laminar or turbulent, depending on the value of **Reynold's Number** $R$ for the flow. $R$ for the flow of a given fluid at an average speed $v$ through a pipe of radius $r$ is defined as:

$$R = \frac{2r\rho v}{\eta} \quad 9.9$$

When $R$ is less than 2000, the flow is laminar; and when $R$ is greater than 3000, the flow is turbulent. For values of $R$ between 2000 and 3000, the flow fluctuates unpredictably between laminar and turbulent.

In fluid dynamics, an ideal fluid is considered to have the characteristics. In such a fluid, flow is:

1. Steady: Steady flow means that all the particles of a fluid have the same velocity as they pass a given point.
2. Irrotational: Irrotational flow means that a fluid element (a small volume of the fluid) has no net angular velocity, which eliminates the possibility of whirlpools and eddy currents (the flow is non-turbulent).
3. Non-viscous: non-viscous flow means that viscosity is neglected. Viscosity refers to a fluid's internal friction or resistance to flow. No energy is lost in a truly flowing non-viscous fluid and there would be no frictional drag between the fluid and walls containing it. When a liquid flows through a pipe, the speed is lesser near the walls because of frictional drag and is greater toward the centre of the pipe.
4. Incompressible: Incompressible flow means that the fluid's density is constant.

Let us consider a flow in which the fluid is bounded by lines as shown in Figure 9.6. We define the volume rate of discharge of flowing fluid at a place where the area of cross-section is $A$ as:

$$\text{Volume rate of discharge} = vA \quad (\text{m}^3/\text{s}) \quad 9.10$$

where $v$ is the velocity of the fluid at the place.

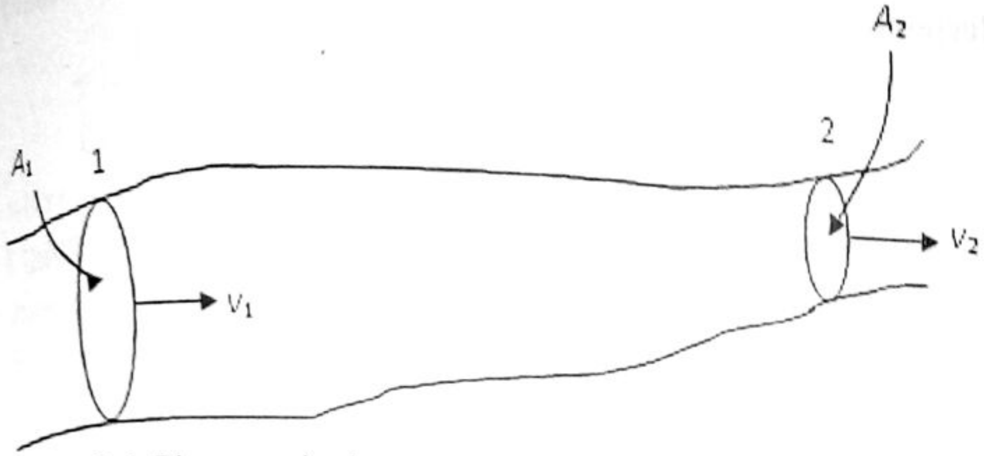



Figure 9.6: Flow continuity

The amount of mass per second passing through point 1 is

$$M_1 = \rho_1 v_1 A_1$$

where $A_1$, $\rho_1$ and $v_1$ are the area, density and velocity of the fluid at point 1.

Similarly, the mass discharged per second where the area of cross-section is $A_2$ and the density is $\rho_2$ is:

$$M_2 = \rho_2 v_2 A_2$$

If the fluid is not allowed to leave the tube through the sides, we have:

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2 \qquad 9.11$$

Equation 9.11 is known as the equation of continuity for fluid flow, and is obeyed in all cases of fluid flow in which there are no sources or sinks of fluid.

For incompressible fluid, $\rho_1 = \rho_2$ and the continuity equation becomes:

$$v_1 A_1 = v_2 A_2 \qquad 9.12$$

According to the continuity equation, the fluid's speed of flow can vary along the paths of the fluid. Daniel Bernoulli in 1738 derived an important expression that relates the pressure, flow, speed and height for flow of an ideal, incompressible fluid. This relationship is called **Bernoulli's equation**:

$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant} \quad 9.13$$

where $p$ is the pressure at any given point within the flowing fluid, $v$ is the speed of flow at that point, and $h$ is height of the same point from a fixed reference level.

## 9.7 Surface Tension; Viscosity and Poiseulle's Law

Despite the fact that molecules are electrically neutral, there is often some slight asymmetry of charge that gives rise to van der Waals forces of attraction. The molecules of a liquid hence exert small attractive forces on each other. The net force on a molecule completely surrounded by other molecules within the liquid is zero. However, for molecules at the surface of the liquid, the attractive force from above the surface is small and negligible and this result in a net forces due to the surrounding molecules experienced by molecules of the surface layer. This inward pull on the surface layer molecules causes the surface of the liquid to contract and to resist being stretched or broken and results in what is called surface tension.

Surface tension is the reason that the surface of a liquid appears to be covered by a skin and this explains why the surface of a liquid can support the weight of a small insect or a pin. It can be shown experimentally that increasing the temperature of a liquid and adding soap can decrease the surface tension of the liquid. Decreasing the surface tension of liquid implies increasing the surface area of the liquid. It is found that the amount of work $dW$ required to increase the area of a liquid is proportional to the increase in area:

$$dW = \gamma dA \quad 9.14$$

where the constant of proportionality $\gamma$ is known as the surface tension of the liquid boundary. The unit of surface tension is $Jm^{-2}$

Alternatively, we can define **surface tension** as:

$$\gamma = \frac{T}{L} \qquad 9.15$$

where $T$ is the surface tension force and $L$ is length of the surface. Equation 9.15 also gives alternative units for $\gamma$ as N/m.

## 9.8 Angle of Contact and Capillarity

When a small quantity of liquid comes in contact with a plane solid body, we have one of the situations shown in Figure 9.7. The line $AB$ is a tangent to the liquid surface at $A$. The angle of contact of the liquid with the surface is the angle $\theta$ which $AB$ makes with the solid surface. The size of angle $\theta$ depends on three factors:

1. how clean the surface is;
2. the material with which the surface is made; and
3. the type of liquid.

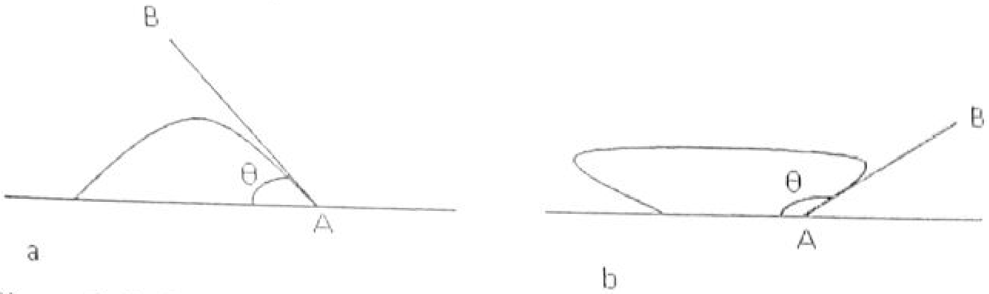



Figure 9.7: Contact angles

In Figure 9.7a, the angle of contact is less than 90° and the liquid tends to spread out over the surface. Then, we say that the liquid wets the surface. If, however, the angle of contact is greater than 90°, the liquid gathers in small drops (Figure 9.7b).

It is observed that if a capillary tube is dipped in water, the level of water inside the tube rises above the level outside (Figure 9.8a) and if the tube is dipped into mercury, the level of the mercury inside the capillary tube falls below its level outside (Figure 9.8b). This phenomenon is known as capillarity. The rising or falling of the liquid in the capillary tube depends only on whether the liquid wets or does not wet the tube. If $F$ is the surface tension force, the vertical component of $F$ is balanced by the weight of the liquid column of height $h$, at equilibrium:

$$F\cos\theta = Mg = \rho\pi r^2 hg \qquad 9.16$$

where the volume the liquid is $\pi r^2 h$.

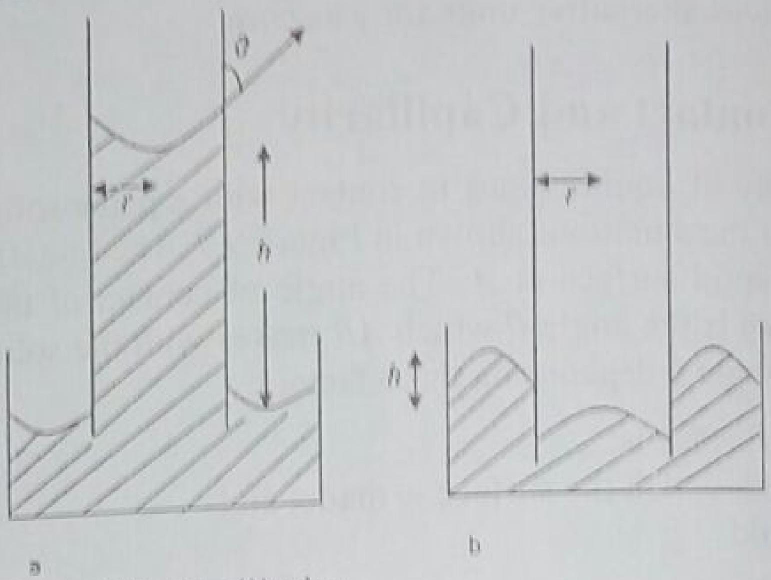



Figure 9.8: Capillarity

From Equation 9.15, the surface tension force is,

$$F = \gamma L = \gamma(2\pi r) \qquad 9.17$$

where $r$ is the radius of cross-section of the capillary tube and $L = 2\pi r$ is the circumference of the circle of contact along which the surface tension force acts.

Substituting for $F$ from Equation 9.17 into 9.16:

$$\gamma(2\pi r)\cos\theta = \rho\pi r^2 hg$$

$$h = \frac{2\gamma\cos\theta}{\rho g r} \qquad 9.18$$

## 9.9 Pressure in a Bubble

Consider a spherical thin film of liquid which encloses a gas, such as is the case with a soap bubble (Figure 9.9). The thin film of liquid has two surfaces $S_1$ and $S_2$, one inside the bubble and the other outside. The enclosed gas is under excess pressure $p$, acting upwards as a result of the surface tension forces developed over the surface of the bubble. Let us imagine that the excess pressure $p$ acts to increase the bubble's volume by $dV$. Then, the work done by $p$ is:

$$dW = pdV \tag{9.19}$$

From the definition of the surface tension in Equation 9.14, we have:

$$dW = 2\gamma dA \tag{9.20}$$

where $dA$ is the increase in the surface area of the bubble. The factor 2 is added because a soap bubble has two surfaces. Combining Equations 9.19 and 9.20,

$$pdV = 2\gamma dA \tag{9.21}$$

The soap bubble is a sphere and the area is $A = 4\pi r^2$,

and the volume is $V = \frac{4}{3}\pi r^3$

Differentiating, $dA = 8\pi r dr$, and $dV = 4\pi r^2 dr$. With these expressions, Equation 9.21 becomes $4\pi r^2 p dr = 16\pi\gamma r dr$,

or:

$$p = \frac{4\gamma}{r} \tag{9.22}$$

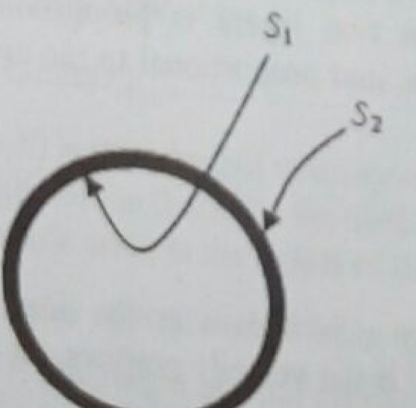



Figure 9.9: Bubble with two surfaces

A bubble can also be a spherical volume of gas inside a liquid. In this case $dW = \gamma dA$ since this type of bubble has only one surface. The excess pressure can be shown to be:

$$p = \frac{2\gamma}{r} \qquad 9.23$$

## 9.10 Viscosity and Poiseuille's Law

Viscosity is described as the internal resistance to flow in fluids. It is considered to be the friction between the molecules of a fluid. Figure 9.10 shows a fluid flowing in the positive $x$-direction. The two layers $X$ and $Y$ in the fluid are a distance $dy$ apart.

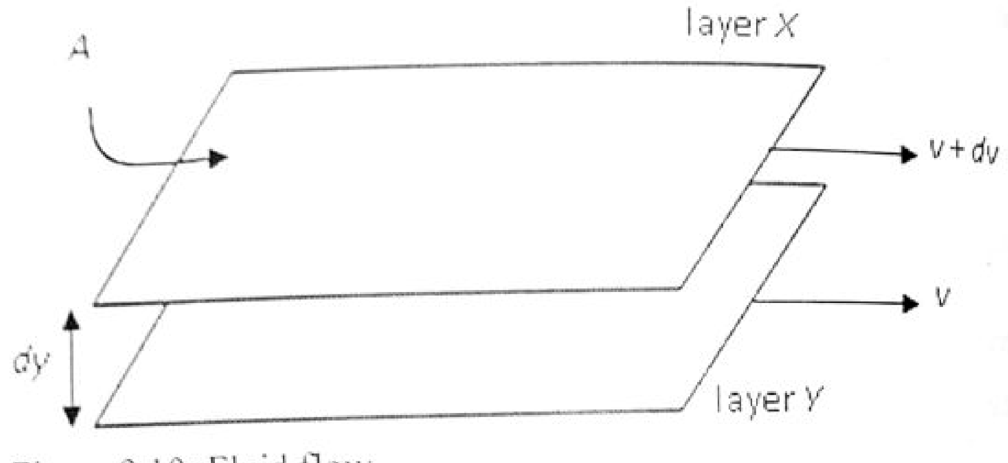



Figure 9.10: Fluid flow

The velocity gradient between any two layers in the fluid produces a force known as viscous force to bring the layers to the same velocity. The viscous force $F$ between two layers is proportional to the velocity gradient between them, and proportional to the area $A$ with which the layers overlap:

$$F = -\eta A \frac{dv}{dy} \qquad 9.24$$

The constant of proportionality $\eta$ is known as the coefficient of viscosity of the fluid and $dv/dy$ is the velocity gradient. In S.I., $\eta$ is measured in $Nsm^{-2}$.

Jean Poiseuille in 1842 studied flow in pipes and tubes, assuming constant viscosity and steady or laminar flow, and derived the following expression known as Poiseuille's law: The flow rate volume per unit time, is:

$$Q = \frac{V}{t} = \frac{\pi r^4 p}{8\eta L} \qquad 9.25$$

where $r$ is the radius of the pipe and $L$ is its length.

## 9.11 Stokes' Law

Irishman George Stokes used viscosity and the equations of fluid flow to predict the drag force on a sphere moving through a fluid. Stokes' law applies to objects moving at low enough speeds that the flow of fluid is streamlined or laminar.
Stokes' law states that the viscous force $F$ with which a fluid of viscosity $\eta$ opposes the motion of a spherical object through it is given as:

$$F = 6\pi\eta rv$$

where $r$ is the radius of the object and $v$ is the speed of travel through the fluid.

Consider a spherical object falling through a fluid. The object reaches a terminal velocity $v$ very quickly because of friction with the air in the chamber. The drag force acting on the drop can then be calculated using Stokes' law:

$$F = 6\pi\eta\, rv$$

where $v = l/t$ is the terminal velocity of the falling object, $\eta$ is the viscosity of the fluid, $l$ is the distance travelled by the object, $t$ is the time taken and $r$ is the radius of the object.

For a perfectly spherical object, the apparent weight $W$ is:

$W$ = the weight of fluid displaced by the object
$W$ = true weight – upthrust

$$W = \frac{4\pi r^3}{3}(\rho - \rho_f)g$$

9.26

where $\rho$ and $\rho_f$ are densities of object and density of fluid respectively.

At terminal velocity the object is not accelerating, therefore the total force acting on it must be zero and the two forces $F$ and $W$ must cancel one another out (that is, $F = W$). This implies:

$$\frac{4\pi r^3}{3}(\rho - \rho_f)g = 6\pi\eta rv$$

$$\eta = \frac{2gr^2(\rho - \sigma)}{9v}$$

9.27

Stokes' law is often used in experiments to determine the viscosity $\eta$ of a fluid.

## Activity 9 Fluids

**9.1.** The piston of a hydraulic automobile lift is 30cm in diameter. What pressure, in mm of mercury, is required to lift a car of mass 1500kg? ($1.015 \times 10^5$Pa = 760mmHg).

A. 1208mmHg
B. 760mmHg
C. 50mmHg
D. 1556.9mmHg

**Solution**

Diameter $d = 30\text{cm} = 30 \times 10^{-2}$ m,

Force = $mg = 1500 \times 9.8\text{N} = 14700\text{N}$

Pressure $P = \frac{14700}{0.070695} N/m^2 = 207935.5 N/m^2$

$$P = \frac{207935.5}{1.015 \times 10^5} \times 760 mmHg = 1556.9 mmHg$$

The correct option is D.

**9.2.** If water is flowing through a pipe of diameter 2m, with a velocity of 20m/s, calculate the value of Reynold's Number. Assume the density and coefficient of viscosity of water are $10^3$kgm$^{-3}$ and $10^{-4}$Nsm$^{-2}$ respectively.

A. $2.0 \times 10^8$
B. $3.5 \times 10^8$
C. $4.0 \times 10^8$
D. $5.0 \times 10^8$

**Solution**

Reynolds's Number, $R = \frac{2r\rho v}{\eta} = \frac{2 \times 1 \times 10^3 \times 20}{10^{-4}} = 4 \times 10^8$

The correct option is C.

**9.3.** A small metal sphere with diameter 10mm is dropped into glycerin. The viscous force on the object was 2.5N. If the terminal velocity of the object is 6m/s, calculate the coefficient of viscosity "$\eta$" of the fluid.

A. 3.01Nsm$^{-2}$ B. 5.60Nsm$^{-2}$ C. 4.42Nsm$^{-2}$ D. 2.32Nsm$^{-2}$

**Solution**

By Stoke's law, the viscous force is:

$F = 6\pi\eta rv$

$2.5 = 6 \times 3.142 \times \eta \times 5 \times 10^{-3} \times 6$

$$\eta = \frac{2.5}{6 \times 3.142 \times 5 \times 10^{-3} \times 6} Nsm^{-2} = \frac{2.5}{0.56556} Nsm^{-2} = 4.42\, Nsm^{-2}$$

The correct option is C.

**9.4.** Water moving with a speed of 5.0m/s through a pipe with cross-sectional area of 4.0cm$^2$ gradually descends 10m as the pipe increases in area to 8.0cm$^2$. Calculate the speed of flow and pressure at the lower level if the pressure at the upper level is $1.50 \times 10^5$ Pa.

A. 3.0m/s, $1.2 \times 10^5$ Pa
B. 2.0m/s, $1.8 \times 10^5$ Pa
C. 2.5m/s, $2.6 \times 10^5$ Pa
D 4.1m/s, $3.0 \times 10^5$ Pa

**Solution**

$v_1A_1 = v_2A_2$

$5 \times 4 = v_2 \times 8$

$v_2 = 20/8 m/s = 2.5 m/s$

Using the Bernoulli's equation

$$\rho_1 + \frac{1}{2}\rho v_1 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^3\right) + \rho g\left(h_1 - h_2\right)$$

$$P_2 = 1.5\times 10^5 + \frac{1}{2}\times 1000\left(5^2 - 2.5^2\right) + 1000\times 9.8\times 10$$

$$P_2 = 257375 N/m^2 = 2.6\times 10^5 Pa$$

The correct option is C.

9.5. Castor oil at 20°C has a coefficient of viscosity 2.42[illegible] and a density of $940 kgm^{-3}$. Calculate the terminal velocity of [illegible] radius 2.0mm falling under gravity in the oil, taking the density of steel as $7{,}800 kgm^{-3}$.

A. 0.025m/s B. 0.315m/s C. 0.971m/s D. [illegible]

**Solution**

$T = 20°C = 293$ K, $\eta = 2.42 Nsm^{-2}$, $\rho = 7800 kgm^{-3}$,

$r = 2.0 mm = 2.0 \times 10^{-3}$ m, $\sigma = 940 kgm^{-3}$

$$\frac{4}{3}\pi r^3 g(\rho - \sigma) = 6\pi\eta r v$$

$$\frac{4}{3}r^2 g(\rho - \sigma) = 6\eta v$$

$$v = \frac{4r^2 g(\rho - \sigma)}{3\times 6\eta} = \frac{4\times(2.0\times 10^{-3})^2\times 9.8}{18\times 2.42}(7800 - 940) m/s = [illegible]$$

The correct option is A.

9.6. A capillary tube is immersed in water of surface [illegible] $7 \times 10^{-2} N/m$ and rises 6.2cm. By what depth will [illegible] depressed if the same capillary is immersed in it? [illegible]

of mercury = 0.54N/m angle of contact between mercury and glass = 140°; density of mercury = 13600kgm$^{-3}$).
A. 2.7cm B. 5.4cm C. 6.2cm D. 4.3cm

**Solution**

Using:

$$h = \frac{2\gamma\cos\theta}{\rho g r}$$

$$\Rightarrow r = \frac{2\gamma_1 \cos\theta_1}{h_1\rho_1 g_1} = \frac{2\gamma_2 \cos\theta_2}{h_2\rho_2 g_2}$$

$$\frac{2\times7\times10^{-2}\cos 0}{6.2\times10^{-2}\times1000\times9.8} = \frac{2\times0.54\cos 140}{h_2\times13600\times9.8}$$

$$0.000230414 = \frac{6.21\times10^{-6}}{h}$$

$$h = 0.0269m = 2.7cm$$

The correct option is A.

**9.7.** A soap bubble has a diameter of 4mm. Calculate the pressure inside it if the atmospheric pressure is $10^5$N/m (Surface tension of soap solution = 2.8 x $10^{-2}$N/m).
A. 5.0026 x $10^5$ N/m
B. 5.0026 x $10^2$ N/m
C. 1.00056 x $10^5$ N/m
D. 1.00056 x $10^2$ N/m

**Solution**

Excess pressure $P = 4\gamma/r$

$$P = \frac{4\times2.8\times10^{-2}}{2\times10^{-3}} N/m^2 = 56N/m^2$$

Pressure inside $P_{in}$ = excess pressure + atm pressure = 56 + $10^5$N/m$^2$
= 1.00056 x $10^5$N/m$^2$

The correct option is C.

**9.8.** A fluid of density $1760kgm^{-3}$ and viscosity $0.084Nsm^{-2}$ through a pipe of radius 2.5cm with an average speed of 2 Calculate the Reynolds's Number $R$ and decide whether the f turbulent on laminar.

A. 1926, turbulent
B. 1926, laminar
C. 2619, unpredictable
D. 2619, turbulent

**Solution**

$$R = \frac{2r\rho v}{\eta} = \frac{2 \times 2.5 \times 10^{-2} \times 1760 \times 2.5}{0.084} = 2619$$

For values of R between 2000 and 3000, the flow fluctu unpredictably between laminar and turbulent.

The correct option is C.

**9.9.** A large reservoir tank is filled with water to height of 16 from the tap located near its base. If the cross-sectional area of tap is $1.26 \times 10^{-3}m^2$, calculate the velocity of flow from the tap a the volume of water leaving the tap per second.

A. $32.34 ms^{-1}$, $4.075 \times 10^{-2} m^3 s^{-1}$
B. $5.687 ms^{-1}$, $7.165 \times 10^{-3} m^3 s^{-1}$
C. $32.34 ms^{-1}$, $7.165 \times 10^{-3} m^3 s^{-1}$
D. $5.687 ms^{-1}$, $4.075 \times 10^{-2} m^3 s^{-1}$

**Solution**

$V = (2gh)^{½} = (2 \times 9.8 \times 1.65)^{½} m/s = 5.687m/s$

Volume rate of discharge

$= vA = 5.687 \times 1.26 \times 10^{-3} m^3s^{-1} = 7.165 \times 10^{-3}m^3s^{-1}$

The correct option is B.

**9.10.** A flat square surface plate of sides 0.25m is placed flat on a liquid of surface tension $0.0586Nm^{-1}$. Assuming that the angle of contact is zero, calculate the downward force on the plate due to surface tension.

A. 0.01465N
B. 0.02930N
C. 0.03215N
D. 0.0586N

**Solution**

Length in contact $L = 4l = 4 \times 0.25$ m = 1.0 m

Surface tension $\gamma = F/L$

F = 0.0586 x 1N = 0.0586N

The correct option is D.

**9.11.** A capillary tube of radius of cross section 0.42mm is dipped vertically inside a liquid of surface tension coefficient $0.085Nm^{-1}$. The angle of contact of the liquid with the wall of capillary tube is 32° and the density of the liquid is $1260kgm^{-3}$. Calculate the height through which the liquid rise or fall.

A. 2.78cm
B. 27.35cm
C. 3.28cm
D. 32.8cm

**Solution**

$$h = \frac{2\gamma \cos\theta}{\rho g r}$$

$$h = \frac{2 \times 0.085 \cos 32}{1260 \times 9.8 \times 0.42 \times 10^{-3}} m = 0.02779m = 2.78cm$$

The correct option is A.

**9.12.** The size of contact angle $\theta$ of any liquid with the wall of container depends on all of the following except

A. how clean the surface is
B. the substance of which the container is made
C. area in contact
D. the type of the liquid

**Solution**

The correct option is C.

**9.13.** A spherical ball of radius 0.65cm is dropped in a fluid of density $1260kgm^{-3}$. If the viscosity of the liquid is $1.495Nsm^{-2}$ and the terminal velocity is 2.25m/s. Calculate the upthrust on the falling ball.

A. 14204.48N
B. 0.0150N
C. 2.185N
D. 0.0142N

**Solution**

Upthrust

$$U = \rho_L Vg = 1260 \times \frac{4}{3}\pi r^3 \times 9.8N$$

$$U = 1260 \times \frac{4}{3} \times 3.142 \times \left(6.5 \times 10^{-3}\right)^3 \times 9.8N = 0.0142N$$

The correct option is D.

**9.14.** Compute the atmospheric pressure on a day when the height of a barometer is 76.0cm. ($\rho$ = 13.6 x $10^3$ $kgm^{-3}$)

A. 1.013 x $10^5$ Pa
B. 133.28 x $10^3$ Pa
C. 1.012 x $10^2$ Pa
D. 10.12 x $10^3$ Pa

**Solution**

$P = \rho gh$ = 13.6 x $10^3$ x 9.8 x 0.76 Pa = 101292.8 Pa = 1.013 x $10^5$pa

The correct option is C.

**9.15.** Castor oil at 20°C has a coefficient of viscosity 2.42 $Nsm^{-2}$ and a density 940$kgm^{-3}$. Calculate the terminal velocity of a steel ball of radius 2.0mm falling under gravity ($\rho_{steeel}$ = 7800$kgm^{-3}$).

A. 0.25 $ms^{-1}$
B. 2.5 $ms^{-1}$
C. 0.0025 $ms^{-1}$
D. 0.025 $ms^{-1}$

**Solution**

Terminal velocity:

$$v = \frac{2gr^2(\rho - \sigma)}{9\eta}$$

$$v = \frac{2 \times 9.8 \times 0.002^2(7800 - 940)}{9 \times 2.42} m/s = 0.025m/s$$

The correct option is D.

**9.16.** Assuming an isothermal atmosphere, the variation of pressure $p$ with altitude $h$ is given as $dp/dh = -\rho g$ where $\rho$ is the air density. Given that $pV = mRT$ and $p_o$ is the pressure at the Earth's surface, the atmospheric pressure at any height may be written as:

A. $p = p_o\exp(-gh/RT)$
B. $p = p_o\exp(-gR/hT)$
C. $p = p_o\exp(-ghT/R)$
D. $p = p_o\exp(RT/gh)$

**Solution**

$pV = mRT$
$p = mRT/V = \rho RT$
$\rho = p/RT$ (i)
$dp/dh = -\rho g$ (ii)

Subtitute Eqn. (i) into eqn (ii)

$dp/dh = -pg/RT$
$dp/p = -gdh/RT$

Integrating, we have,

$\ln(p/p_o) = -gh/RT$
$p/p_o = \exp(-gh/RT)$
$p = p_o\exp(-gh/RT)$

The correct option is A.

**9.17.** A capillary tube of 0.4mm diameter is placed vertically inside a liquid of density 800kgm$^{-3}$ surface tension $5 \times 10^{-2}$ Nm$^{-1}$ and angle of contact 30°. Calculate the height to which the liquid rises in the capillary tube.

A. 4.4cm B. 5.4mm C. 5.5cm D. 4.4mm

**Solution**

$$h = \frac{2\gamma\cos\theta}{\rho g r}$$

$$h = \frac{2 \times 5 \times 10 \times \cos 30}{800 \times 9.8 \times 0.2 \times 10^{-3}} m = 0.0555m = 5.55cm$$

The correct option is C.

**9.18.** A rectangular plate of dimensions 6cm by 4cm and thickness 2mm is placed with its largest face flat on the surface of water. Calculate the force due to surface tension on the plate and the downward force if the plate is placed vertically and its longest side just touches the water. (S.T. of water = $7 \times 10^{-2}$ $Nm^{-1}$).

A. $1.0 \times 10^{-2}$ N; $8.6 \times 10^{-3}$ N
B. $1.4 \times 10^{-2}$ N; $9 \times 10^{-3}$ N
C. $8.6 \times 10^{-2}$ N; $1.4 \times 10^{-8}$ N
D. $1.4 \times 10^{-2}$ N; $8.68 \times 10^{-3}$ N

Solution (i)

For the largest face, $L = 2(l + b) = 2(6 + 4)$cm = 20cm = 0.2 m

$F = \gamma L = 7 \times 10^{-2} \times 0.2\text{N} = 1.4 \times 10^{-2}\text{N}$

Similarly:

$L = 2(0.06 + 0.002)$ m = 0.124 m

$F = \gamma L = 0.07 \times 0.124\text{N} = 8.68 \times 10^{-3}\text{N}$

The correct option is D.

**9.19.** Water at 20°C flows through a horizontal pipe of radius 1.0cm. If the flow velocity at the centre is 0.2cm/s, find the pressure drop along a 4m section of the pipe due to viscosity $1.00 \times 10^{-3}$ $Nsm^{-2}$.

A. 0.32$Nm^{-2}$ B. 0.61$Nm^{-2}$
C. 0.55$Nm^{-2}$ D. 0.72$Nm^{-2}$

**Solution**

Velocity:

$$v = \frac{pR^2}{4\eta L}$$

$$\therefore p = \frac{4\eta L v}{R^2}$$

$$p=\frac{4\times1\times10^{-3}\times4\times0.002}{\left(1\times10^{-2}\right)^{2}}Nm^{-2}=0.32Nm^{-2}$$

The correct option is A.

**9.20.** Calculate the flow rate of water in Q9.19 above.

A. $4.5\times10^{-7}\,m^3/s$ B. $3.0\times10^{-7}\,m^3/s$

C. $5.2\times10^{-7}\,m^3/s$ D. $2.7\times10^{-7}\,m^3/s$

**Solution**

Assuming laminar flow, we apply Poiseuille's law which is:

$$\eta=\frac{\pi T\,pR^4}{8VL}$$

Rearranging this, we have:

$$\frac{V}{T}=\frac{p\pi R^4}{8\eta L}=flow\ rate$$

$$\frac{V}{T}=\frac{0.32\times3.142\times\left(1\times10^{-2}\right)^4}{8\times1.0\times10^{-3}\times4}m^3s^{-1}=3.142\times10^{-7}m^3s^{-1}$$

The correct option is B.

**9.21.** Find the mass of air inside a room measuring 10m × 8m × 3m, if the density of air is $1.28kgm^{-3}$.

**Solution**

Mass $m=\rho V=1.28\times10\times8\times3kg=307.2kg$

**9.22.** A container of volume $0.05m^3$ is full of ice. When the ice melts into water, how many kg of water should be added to fill it up? (density of ice = $900kgm^{-3}$; density of water = $1000kgm^{-3}$)

**Solution**

Mass of ice, $m_{ice}=\rho_{ice}\times0.05kg=45kg$

Mass of water, $m_{water} = \rho_{water} \times 0.05 = 50kg$
Mass of water to be added = (50 – 45)kg = 5kg

**9.23.** If $1m^3$ of water is mixed with $3m^3$ of a liquid to form a mixture of density $850kgm^{-3}$, find he density of the liquid, assuming that there is no contraction of volume. (Density of water = $1000kgm^{-3}$)

**Solution**

$$\rho = \frac{total\ mass}{total\ volume} = \frac{m_w + m_l}{V_w + V_l} = \frac{\rho_w V_w + \rho_l V_l}{1+3} = 850$$

$$\rho_l = 800kgm^{-3}$$

**9.24.** An alloy consists of 60% aluminium and 40% tin by weight. (Density of aluminium = $2700kgm^{-3}$; density of tin = $7300kgm^{-3}$.) (a) What is the density of the alloy? (b) What is the mass of tin in $0.02m^3$ of the alloy?

**Solution**

$$\text{(a) } \rho = \frac{m}{V_a + V_t} = \frac{m}{\dfrac{0.6m}{\rho_a} + \dfrac{0.4m}{\rho_t}} = 3610kgm^{-3}$$

$$\rho = \frac{m}{V} \Rightarrow 3610kgm^{-3} = \frac{m}{0.02} = 72.2kg$$

$$m_t = 0.4m = 0.4 \times 72.2kg = 28.9kg$$

**9.25.** A log of wood of volume of $0.1m^3$ and density of $700kgm^{-3}$ floats in water of density $1000kgm^{-3}$. Calculate (a) the weight of the log, (b) the mass of water displaced by the log (c) the volume of the log immersed in water.

**Solution**

(a) Mass of log, $m_{block} = \rho_{block} \times V_{block} = 70kg$. Hence, weight of the log = 70 x 10N = 700N

(b) Principle of floatation states that a floating body displaces its own

weight of fluid in which it floats, i.e. weight of floating body = weight of liquid displaced by the body.
The mass of water displaced by the log = mass of floating body = 70kg

(c) the volume of the log immersed in water
= volume of water displaced

$$= \frac{mass\ of\ water\ displaced}{density\ of\ water} = \frac{70}{1000} m^3 = 0.07\, m^3$$

## Summary of Chapter 9

In chapter 9, you have learned that:

1. The density $\rho$ of a substance is defined as its mass per unit volume: $\rho = \frac{m}{V}$. The relative density $RD$ of a substance is the ratio of its density to the density of water: $RD = \frac{density\ of\ substance}{density\ of\ water}$.

2. The pressure $p$ at any point is defined as the normal force per unit area: $p = \frac{F_\perp}{A}$.

3. The pressure $p$ at any point in a fluid at rest and at depth $h$ is $p = p_o + \rho gh$, where $p_o$ is the pressure applied to the liquid surface.

4. Pascal's principle states that pressure applied to an enclosed fluid is transmitted undiminished to every point in the fluid and to the walls of the container.

5. Archimedes' principle states that when a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.

6. When a fluid of density $\rho$ and viscosity $\eta$ flows from one place to another, the flow may be laminar or turbulent, depending on the value of *Reynolds's Number R* for the flow. $R$ for the flow of a given fluid at an average speed $v$ through a pipe of radius $r$ is defined as $R = \frac{2r\rho v}{\eta}$.

7. When $R$ is less than 2000, the flow is laminar; and when $R$ is greater than 3000, the flow is turbulent. For values of $R$ between 2000 and 3000, the flow fluctuates unpredictably between laminar and turbulent.

8. The equation of continuity for fluid flow is $\rho_1 v_1 A_1 = \rho_2 v_2 A_2$.

9. The Bernoulli's equation: $p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$, where $p$ is the pressure at any given point within the flowing fluid, $v$ is the speed of flow at that point, and $h$ is height of the same point from a fixed reference level.

10. Surface tension is define as $\gamma = \frac{T}{L}$, where $T$ is the surface tension force and $L$ is length of the surface.

11. The viscous force $F$ between two layers is proportional to the velocity gradient between them and proportional to the area $A$ with which the layers overlap: $F = -\eta A \frac{dv}{dy}$. The constant of proportionality $\eta$ is known as the coefficient of viscosity of the fluid and $dv/dy$ is the velocity gradient. In S.I., $\eta$ is measured in $Nsm^{-2}$.

12. The Poiseuille's law states that the flow rate, volume per unit time, is $Q = \frac{V}{t} = \frac{\pi r^4 p}{8\eta L}$, where $r$ is the radius of the pipe and $L$ is its length.

## Self-Assessment Questions (SAQs) for Chapter 9

**9.1.** Which of the following is correct? In fluids, pressure:

A. decreases with height, increases with depth and independent of area.

B. increases with height, increases with depth and depend on area

C. decreases with height, decreases with depth and depend on area

D. increases with height, decreases with depth and independent of area

**9.2.** Two reliable devices routinely employed to measure the pressure of a confined fluid are:

A. Hydrometer and manometer

B. Thermometer and Mercury-barometer

C. Manometer and Mercury-barometer

D. Barometer and Spectrometer

**9.3.** Calculate the excess pressure inside a soap bubble of diameter 5mm, if the surface tension $\gamma = 25 \times 10^{-3}\text{Nm}^{-1}$.

**9.4.** A block of brass of mass 1.5kg and density of $4.0 \times 10^3$ kg/m$^3$ was suspended from a string. Find the tension in the string if the block is completely immersed in water. Density of water = 1000kg/m$^3$

**9.5.** Water enters a pipe of 4.0cm inlet diameter at a velocity of 5m/s and pressure $2.5 \times 10^5$ N/m$^2$. The outlet of the pipe 2cm in diameter is 6m above the inlet. Find the pressure at the outlet.

**9.6.** A circular hole 4cm in diameter is cut in the side of a large cylinder filled with water 10 below the water level. Find the volume discharged per unit time.

**9.7.** A steel ball 1mm in radius falls with zero initial velocity in a tank of glycerine with viscosity $8.3 \times 10^{-1}$ Ns/m$^2$. What is the terminal velocity of the ball? Density of glycerine = 1.32 gcm$^{-3}$ and density of steel = 8.5 gcm$^{-3}$.

**9.8.** Methanol at 20°C flows with a speed of 40cm/s through a horizontal pipe of uniform radius 1.5mm. What is the Reynold's Number and the nature of flow if viscosity of methanol at that temperature is $0.584 \times 10^{-3}$ Ns/m$^2$ and its density is 806kg/m$^3$?

**9.9.** Calculate the gauge pressure in a large fire hose if the nozzle is to shoot water straight upward to a height of 20.0m. (density of water = 1.0 gcm$^{-3}$)

**9.10.** How much water will flow in 60.0s through 200mm of capillary tube of 1.50mm inner diameter if the pressure differential across the tube is 5.00cm of mercury? The viscosity of water is $8.01 \times 10^{-4}$ kg/m.s and density for mercury is 13 600kg/m$^3$.

# CHAPTER TEN

# THERMAL PHYSICS

## 10.1 Introduction

Thermal physics, naturally, begins with the definitions of temperature and heat. The **temperature** of a body is a measure of its hotness or coldness. A measure of temperature is obtained by using a thermometer. The thermometers use some measurable properties of substances which are sensitive to changes in temperature. For example, the constant-volume gas thermometer uses the pressure change with temperature of a gas at constant volume. The mercury-in-glass thermometer depends on the change in volume of mercury with temperature and the resistance thermometer uses the change of electrical resistance of a pure metal with temperature.

**Heat** is the net energy transferred from one object to another because of a temperature difference. The total mechanical energy of all molecules of a system or body is referred to as the internal energy of the system or body.

## 10.2 Temperature Scales and Thermometers

There are basically three different types of temperature scales. These are Celsius temperature scale, Fahrenheit temperature scale and thermodynamic temperature scale. All these temperature scales depend on the properties of a particular substance. Each of these scales has two fixed points where the temperature is always the same and easily reproducible and reliable. On the Celsius scale, water freezes at 0°C and boils at 100°C, while on Fahrenheit scale, water freezes at 32°F and boils at 212°F. This is shown in Figure 10.1. In the thermodynamic temperature scale, the fixed point is the triple point of water. It is the temperature at which ice, water and water-vapour coexist in equilibrium and is defined as 273.16 K; denoted by $T_{tr}$.

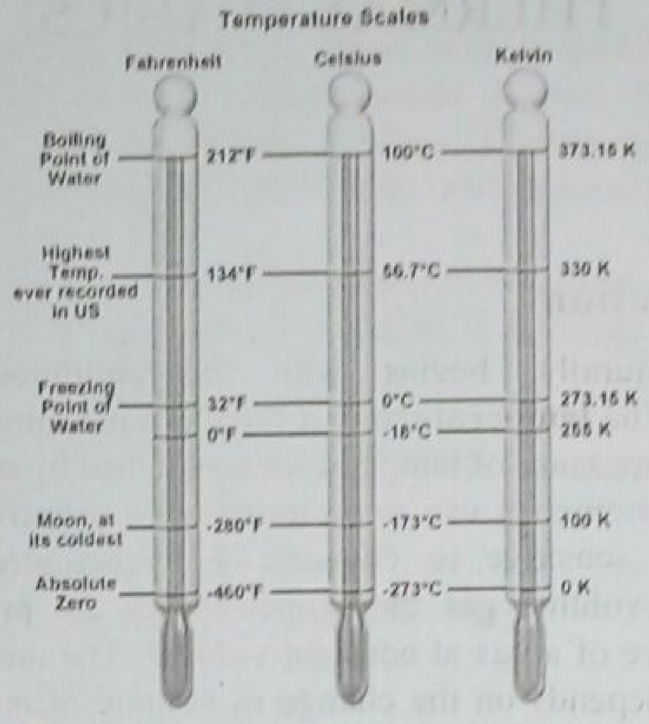



Figure 10.1: Temperature scales

The thermodynamic (or absolute) temperature scale is chosen as standard temperature scale adopted for scientific measurement and it is based on the second law of thermodynamics. Thermodynamic temperature is denoted by the symbol $T$ and is measured in kelvin, symbol K. The conversions between the three scales are:

$$T_F = \frac{9}{5}T_C + 32 \qquad 10.1a$$

$$T_C = \frac{5}{9}(T_F - 32) \qquad 10.1b$$

$$T = 273.15 + T_C \qquad 10.2$$

where $T_F$ and $T_C$ are Fahrenheit and Celsius temperatures respectively.

To establish any of these scales, the following are important:

i. some physical properties of a substance which increases continuously with increasing temperature but is constant at constant temperature;

ii. fixed temperatures which can be accurately reproduced in the laboratory.

Thermometers are the instruments designed to measure temperature. There are many types of thermometers. Their operation always depends on some physical properties of matter that change with temperature. These properties are known as **thermometric properties**. The following are examples of thermometric properties:

- most solids and liquids expand when they are heated;
- electrical resistance change when heated;
- in a gas, pressure and volume change when the gas is heated; and
- radiation from the surface of a body depends on the surface temperature.

A liquid-in-glass thermometer is a temperature-measuring instrument consisting of a thin-walled glass bulb, which serves as the reservoir for the thermometric substance. The glass bulb is attached to a glass stem (the capillary tube through which the meniscus of the liquid moves with a change in temperature). The portion of the bulb-stem space that is not occupied with the thermometer liquid is usually filled with a dry inert gas under sufficient pressure to prevent separation of the thermometer liquid.

A scale is provided to indicate the height to which the liquid column rises in the stem and this reading is made to indicate closely the temperature of the bulb. A contraction chamber (an enlargement of the capillary) is often provided below the main capillary to avoid the need for a long length of capillary or to prevent contraction of the liquid column into the bulb.

The operation of a liquid-in-glass thermometer thus depends on the coefficient of expansion of the liquid being greater than that of the containing glass bulb. Any increase in the bulb temperature causes the liquid to expand and rise in the stem, with the difference in volume between the bulb and the stem serving to magnify the change in volume of the liquid.

Figure 10.2 shows the constant-volume gas thermometer, which is an example of liquid-in-glass thermometer. The chamber at the left is the sensing bulb. The gas in the bulb changes volume when it experiences a temperature change. That causes the levels of mercury in the U-tube manometer to shift, and $h$ gives the measure of the gas pressure and therefore of the temperature. Lifting the movable mercury column on the right shifts the other two mercury levels and allows the volume of gas to be kept constant.

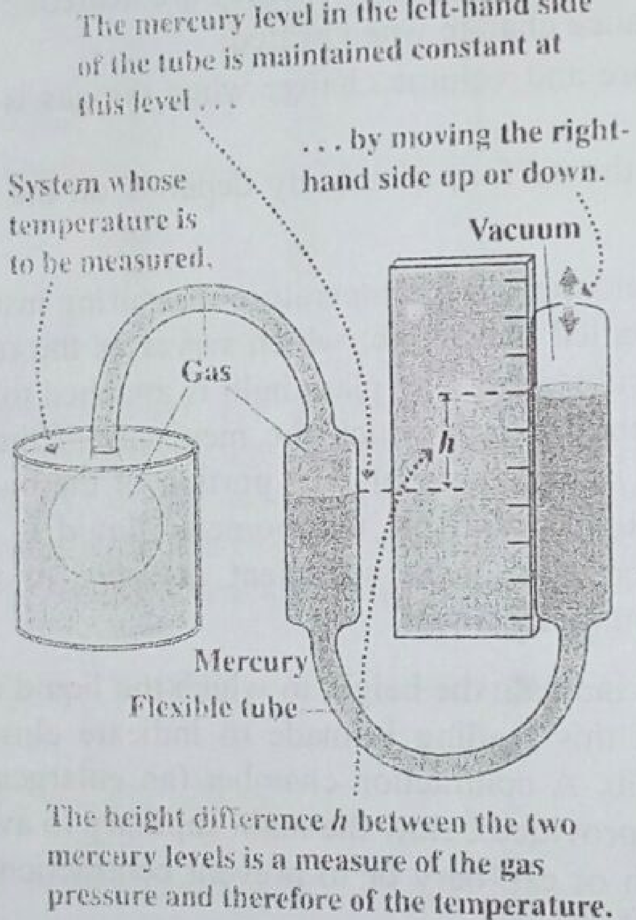



Figure 10.2: Constant-volume gas thermometer

The absolute or Kelvin temperature $T$ at any point is defined, using a constant-volume gas thermometer for an ideal gas, as:

$$T = \frac{P}{P_{tr}} \times 273.16K \qquad 10.3$$

where $P_{tr}$ is the pressure of the gas in the thermometer at the triple point temperature of water, and $P$ is the pressure in the thermometer when it is at the point where $T$ is being determined.

Similarly, using the platinum resistance thermometer, if $R_{tr}$ is the resistance at the triple point of water, 273.16 K, and the resistance $R$ is measured at an unknown temperature $T$ on the thermodynamic scale, then, by definition:

$$T = \frac{R}{R_{tr}} \times 273.16K \qquad 10.4$$

Suppose a constant-volume gas thermometer is calibrated at only the ice and steam points. Then, since there are 100°C between these two points, any other temperature $\theta$ on the Celsius scale is given by:

$$\theta = \frac{P_\theta - P_o}{P_{100} - P_o} \times 100^o C \qquad 10.5$$

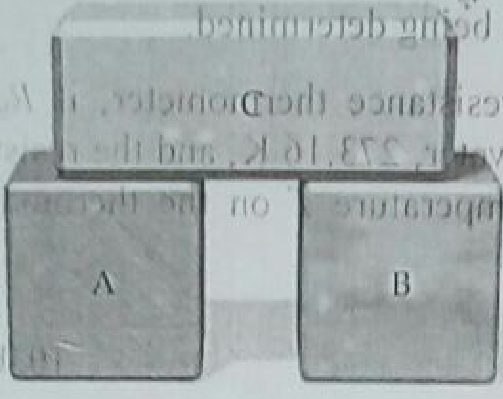
where $P_\theta$ is the pressure at $\theta$°C, $P_{100}$ is the pressure at the steam point and $P_o$ is the pressure at the ice point.

## 10.3 Zeroth Law of Thermodynamics

Two bodies are said to be in thermal contact when heat is transferred between them, whether or not they are touching. When there is no longer a net heat transfer between objects in thermal contact, they tend to come to the same temperature and are said to be in **thermal equilibrium**. This implies that two systems are in thermal equilibrium if and only if they have the same temperature. If systems A and B are each in thermal equilibrium with a third body C, then A and B are also in thermal equilibrium with each other (Figure 10.3). This is called the **zeroth law of thermodynamics**.

Two objects are defined to have the same temperature if they are in thermal equilibrium with each other. Temperature may be defined as the property of a system that determines whether it is in thermal equilibrium with other system. Temperature is one of the seven basic physical quantities by which all other physical quantities are defined.

It is an intensive property, as pressure or density. Length and mass are extensive.



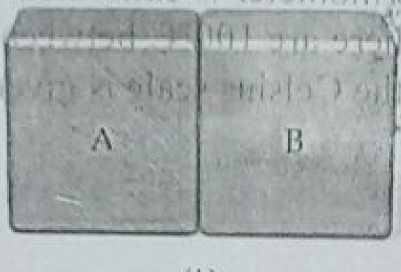




Figure 10.3: Thermal equilibrium

## 10.4 Thermal Expansion

When the temperature of most materials is increased they expand. This expansion in the length, area or volume of the material is called thermal expansion.

### 10.4.1 Linear expansion

Suppose that a rod of a particular material has length $L_0$ at some initial temperature $T_o$. If the rod is subjected to heat such that its length changes to $L$ when the temperature changes to $T$, then the coefficient of linear expansion $\alpha$ which describes the thermal expansion properties of a particular material is defined as:

$$\alpha = \frac{\Delta L}{L_o \Delta T} = \frac{L - L_o}{L_o (T - T_o)} \qquad 10.6$$

Rearranging Equation 10.6, we have:

$$L = L_o(1 + \alpha\Delta T) \qquad 10.7$$

The unit of $\alpha$ is $K^{-1}$ or $(°C)^{-1}$.

### 10.4.2 Area Expansion

Similarly, the coefficient of area expansion $\beta$ which describes the thermal expansion properties of a particular material is defined as:

$$\beta = \frac{\Delta A}{A_o \Delta T} = \frac{A - A_o}{A_o(T - T_o)} \qquad 10.8$$

The new area is given as:

$$A = A_o(1 + \beta\Delta T), \qquad 10.9$$

where $\beta = 2\alpha$. The unit of $\beta$ is $K^{-1}$ or $(°C)^{-1}$.

### 10.4.3 Volume Expansion

Experiments show that increase in volume $\Delta V$ for both solid and liquid materials is approximately proportional to both the temperature change $\Delta T$ and the initial volume $V_o$:

$$\gamma = \frac{\Delta V}{V_o \Delta T} = \frac{V - V_o}{V_o(T - T_o)} \qquad 10.10$$

The new volume is given as:

$$V = V_o(1 + \gamma\Delta T), \qquad 10.11$$

where $\gamma = 3\alpha$ . The unit of $\gamma$ is $K^{-1}$ or $(C°)^{-1}$.

## 10.5 Thermal Stress

When a metal rod of length $L_o$ is heated and clamped rigidly to prevent expansion or contraction, tensile or compressive stresses called thermal stresses develop in the metal. The thermal stress is given as:

$$\frac{F}{A} = Y\frac{\Delta L}{L_o} = Y \times \frac{\alpha L_o \Delta T}{L_o} = Y\alpha\Delta T$$

$$\therefore F = YA\alpha\Delta T \qquad 10.12$$

where $F$ is the force; $A$ the surface area; $Y$ the Young's modulus; $\Delta T$ the change in temperature and $\alpha$ the coefficient of linear expansion.

## 10.6 Quantity of Heat

When heat is applied to an object, the heat energy may increase the random molecular motion and thereby increase the temperature of the object. The quantity of heat ($Q$) required to change the temperature of an object is proportional to the mass ($m$) of the object and to the change in the temperature ($\Delta T$). That is:

$$Q \,\alpha\, m\Delta T$$

Or:

$$Q = mc\Delta T \qquad 10.13$$

where $c$ is the specific heat capacity with unit J/(kg.K).

The **specific heat capacity** of a substance is the amount of heat required to raise the temperature of a unit mass of the substance through one degree change in temperature.

For an infinitesimal temperature change $dT$ and corresponding quantity of heat $dQ$:

$$dQ = mcdT$$

and:

$$c = \frac{1}{m}\frac{dQ}{dT} \qquad 10.14$$

The relations 10.13 and 10.14 does not apply if a phase change is encountered because the heat added or removed during a phase change does not change the temperature.

The molar heat capacity (or molar specific heat) denoted by $C$ is define as:

$C$ = molar mass of substance x specific heat capacity

$$C = Mc \qquad 10.15$$

Heat capacity can also be defined as the quantity of heat required to change the temperature of a substance by one degree.

The total mass $m$ of object is equal to the mass per mole $M$ multiplied by the number of moles $N$:

$$m = nM \qquad 10.16$$

Using Equations 10.15 and 10.16, Equations 10.13 and 10.14 becomes;

$$Q = nC\Delta T \qquad 10.17$$

and;

$$C = \frac{1}{n}\frac{dQ}{dT} \qquad 10.18$$

The unit of $C$ is J/mol.K.

## 10.7 Latent Heat

Matter normally exists in one of three phases or states: the solid state, the liquid state or the gaseous state. **Phase change** or phase transition is a transition from one state to another. For any given pressure a phase transition takes place at a definite temperature, usually accompanied by absorption or emission of heat. For example, when ice at 0°C absorbs heat slowly at normal atmospheric pressure, its temperature does not change but it melts to form liquid water. The absorbed heat (per unit mass) is called the heat of fusion.

**The specific latent heat of fusion** of a solid is the heat required to change the unit mass of it, at its melting point, into liquid at the same temperature. The quantity of heat required to melt a mass $m$ of an object that has specific latent heat of fusion $L_f$ is:

$$Q = mL_f \qquad 10.19$$

The unit of $L$ is J/kg.

The **specific latent heat of vaporisation** of a liquid is the heat required to change the unit mass of it, at its boiling point, into vapour at the same temperature. The quantity of heat required to vaporise a liquid of mass $m$ that has specific latent heat of vaporisation $L_v$ is:

$$Q = mL_v \tag{10.20}$$

## 10.8 Heat Transfer

The three mechanisms or modes of heat transfer are conduction, convection and radiation.

### 10.8.1 Conduction

The **conduction** of heat may be defined as the transfer of heat from one part of a substance to another in which the average position of the intervening material remains constant. Experiment shows that the quantity of heat $Q$ flowing through a small part $XY$, in Figure 10.4, of a lagged bar of uniform cross-section of area $A$ in the steady state, in time $t$, is given as:

$$Q = ktA \times \text{temperature gradient} \tag{10.21}$$

where $k$ is a constant of the material called its **thermal conductivity** and is measured in $Wm^{-1}K^{-1}$.

The temperature gradient $g$, is defined as the ratio:

$$g = \frac{\textit{temperature difference}}{\textit{length } XY} = \frac{\theta_2 - \theta_1}{L} \tag{10.22}$$

Combining Equations 10.21 and 10.22, we have:

$$\frac{Q}{t} = kA\frac{\theta_2 - \theta_1}{L} \tag{10.23}$$

If the temperature varies in a non-uniform state, Equation 10.23 may be written generally as:

$$\frac{dQ}{dt} = -kA\frac{d\theta}{dL} \tag{10.24}$$

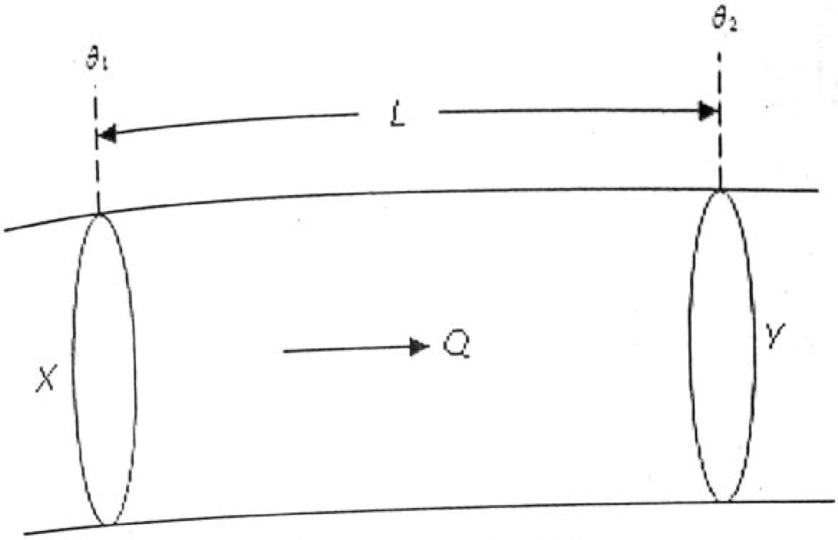



Figure 10.4: Heat flow in steady state

The minus indicates that the temperature $\theta$ along a bar diminishes as $L$ increases, so that $d\theta/dL$ has a negative value and $dQ/dt$ is then positive.

### 10.8.2 Convection

**Convection** is the transfer of heat by mass motion of a fluid from one region of space to another. There are two types of convection: **natural and forced convection**.

Natural convection cycles occur when cold water, when being heated, rises with the heat and cold water replaces the rising warm water. Such cycles occur naturally and are important in atmospheric processes. During the day, natural convection cycles give rise to sea breezes in which the ground heats up more quickly than do sea. This is made possible because the water has a greater specific heat than the land and because convection currents disperse the absorbed heat through the great volume of water. The air in contact with the warm ground expands, becoming less dense than the surrounding cooler air. As a result, the warm air from the land rises and the cooler air from the sea descends to take the place of the warm air and a thermal convection cycle is set up which transfers heat away from the land (Figure 10.5). The process is usually called **sea breeze.**

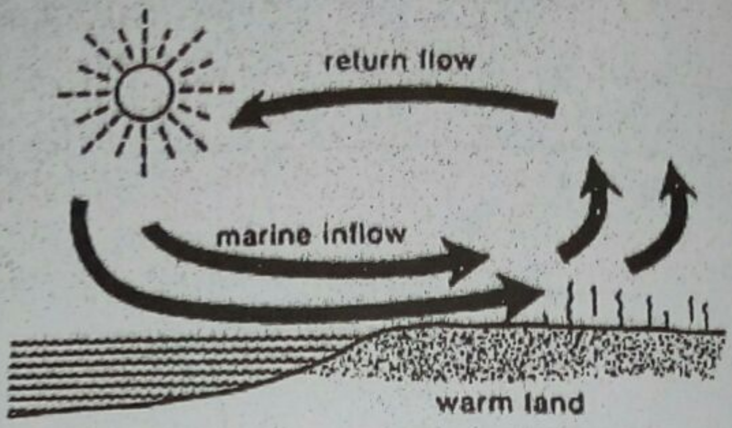



Figure 10.5: Sea breeze

At night, the ground loses its heat more quickly than the water, and the water surface is warmer than the land. As a result, the warm air from the sea rises and a cooler air from the land descends (Figure 10.6). This is called **land breeze.**

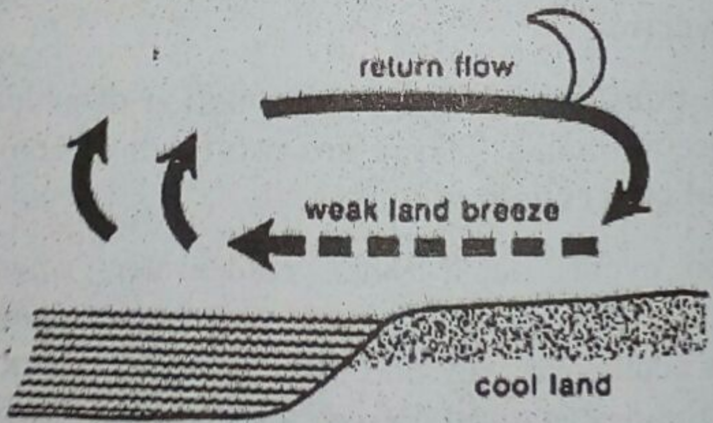



Figure 10.6: Land breeze

### 10.8.3 Radiation

**Radiation** is the transfer of heat by electromagnetic waves suc[h as] visible light, infrared and ultraviolet radiation. Unlike conduc[tion] and convection, radiation needs no transfer medium. Hea[t is] transferred to the Earth from the Sun through space by radiation.

The rate of energy radiation from a surface is found to depend o[n the] following:

i. surface area $A$
ii. temperature $T$
iii. nature of the surface; this dependence is described by a quantity $e$ called **emissivity**.

**Stefan-Boltzmann law** for a black body radiation can be expressed as:

$$\dot{H} = \frac{dQ}{dt} = \sigma A e T^4 \qquad 10.25$$

where $H$ is the power radiated in watts (W), $\sigma$ is the Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8}$ W/(m$^2$.K$^4$).

When an object is in thermal equilibrium with its surroundings, the temperature $T$ of the body is equal to the temperature $T_s$ of the surroundings; hence the rate of radiation to the surrounding is equal to the rate at which the body absorbs heat. For this to be true, the rate of absorption must be given as $H = \sigma A e T_s^4$. Then, the net rate of energy loss or gain per unit time (power) is given by:

$$H_{net} = \sigma A e\left(T^4 - T_s^4\right) \qquad 10.26$$

In Equation 10.26 a positive value of $H_{net}$ means a net heat flow out of the body.

## 10.9 Gas Laws

The gas laws explain the relationship between the variables that describe the behaviour of a given mass of gas. The variables are pressure, volume and temperature. Such a relation is called an **equation of state**. Let us consider some of these equations.

**Boyle's law** states that the volume of a fixed mass of gas at constant temperature is inversely proportional to the pressure:

$$p \alpha \frac{1}{V}$$

which implies:

$$pV = \text{constant}$$

or:

$$p_1V_1 = p_2V_2 \qquad 10.27$$

Figure 10.7 shows four different graphs that depict Boyle's law.

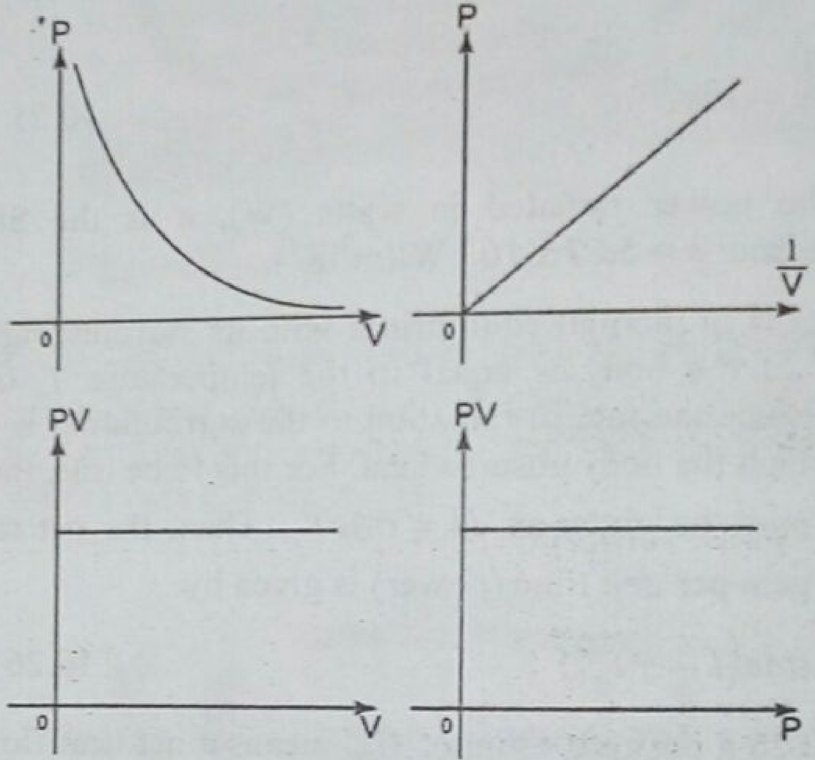



Figure 10.7: Graphs showing Boyle's law

**Charles's law** states that the volume of a fixed mass of gas at constant pressure is directly proportional to the absolute temperature:

$$V \alpha T$$

which implies:

$$\frac{V}{T} = \text{constant}$$

or:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad 10.28$$

Charles found that when the pressure is not too high and is kept constant, the volume of a gas increases with temperature at a linear rate, as shown in Figure 10.8. The graph is essentially a straight line and if projected to lower temperatures , as shown by the dashed line , it crosses the axis at about − 273°C. The implication of this is that if a gas could be cooled to −273°C, it would have zero volume, and at lower temperatures a negative volume, which makes no sense. Hence, the scientist argued that − 273°C is the lowest temperature possible. This temperature is called the **absolute zero** of temperature. Its value has been determined to be − 273.15°C.

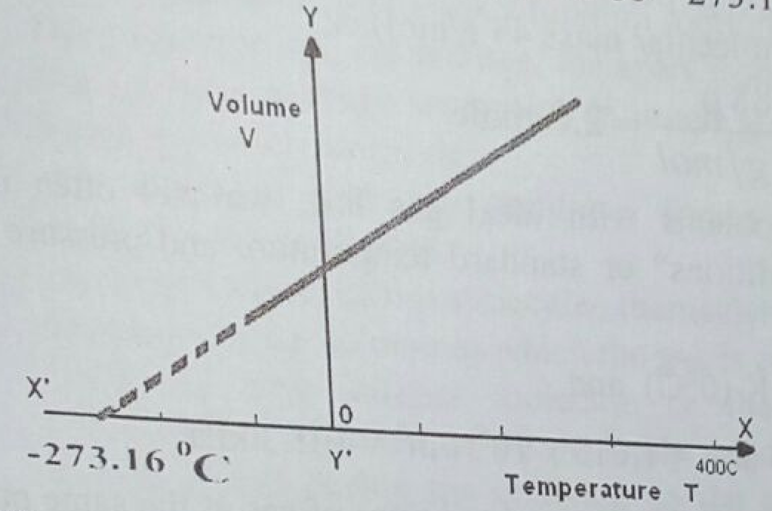



Figure 10.8: Graph showing Charles's law

Combining Equations 10.27 and 10.28, we have:

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \qquad 10.29$$

This relationship (Equation 10.29) is the ideal gas law and can also be written as:

$$\frac{pV}{T} = nR$$

or:

$$pV = nRT \qquad 10.30$$

where $n$ is the number of moles (mol) of the gas and $R$ is called the universal gas constant: $R = 8.31\text{J/(mol.K)}$.

One mole is defined as the amount of substance that contains as many atoms or molecules as there are in precisely 12 grams of carbon 12. It is the quantity of substance whose mass in grams is numerically equal to the molecular mass of the substance. For example, the molecular mass of hydrogen gas ($H_2$) is 2.0u (since each molecule contains two atoms of hydrogen and each atom has an atomic mass of 1.0u). Thus 1 mole of $H_2$ has a mass of 2.0g. The number of moles $n$ in a given sample of a pure substance is equal to the mass of the sample in grams divided by the molecular mass specified as grams per mole. For example, the number of moles in 88g of $CO_2$ (molecular mass 44 g/mol) is:

$$n = \frac{88\,g}{44\,g/mol} = 2.0 \text{ mole.}$$

In solving problems with ideal gas law, we will often refer to "standard conditions" or standard temperature and pressure (STP), which means:

$$T = 273 \text{ K } (0^{\circ}\text{C}) \text{ and}$$

$$P = 1.00 \text{ atm} = 1.013 \times 10^5 \text{N/m}^2 = 101.3 \text{ kPa}$$

Avogadro discovered that equal volumes of gas at the same pressure and temperature contain equal numbers of molecules. This is usually called **Avogadro's hypothesis**. The number of molecules in one mole of any pure substance is known as **Avogadro's number,** $N_A$ ($N_A = 6.02 \times 10^{23}$). Since the total number of molecules $n$ in a gas is equal to the number per mole times the number of moles ($N = nN_A$). The ideal gas law, Equation 10.30, can be written as:

$$pV = nRT = \frac{N}{N_A} RT,$$

Or:

$$pV = NkT \qquad 10.31$$

where $N$ is the number of molecules in the sample of gas and $k = R/N_A$ is the Boltzmann's constant: $k = 1.38 \times 10^{-23}$ J/K.

## 10.10 Kinetic Theory of Gases

So far we have dealt with macroscopic variables of gases: pressure, volume and temperature. Now, we want to describe all these variables on a microscopic level using some assumptions about the molecules in a gas.

The assumptions, which represent the basic postulates of the kinetic theory for an ideal gas, are:

1. Gases consist of large numbers of molecules $n$ each of mass $m$, that are in continuous random motion.
2. The molecules are, on average, far apart from one another. That is, their average separation is much greater than the diameter of each molecule.
3. The attraction and the repulsive forces between the molecules are negligible.
4. The total volume of the molecules themselves is negligible compared to the volume in which the gas is contained.
5. Collisions with another molecule or the wall of the container are assumed to be perfectly elastic. No kinetic energy is lost during the collision so the average kinetic energy per molecule does not change with time.
6. The time spent during a collision is negligible, compared to the time during which the molecules move independently.
7. The average kinetic energy of the molecules is proportional to the absolute temperature.

The pressure exerted on the wall of a gas container is due to the constant bombardment of molecules. If the volume is reduced, the molecules are closer together and many more molecules will be striking a given area of the wall per second. Hence we expect the pressure to increase, in agreement with Boyle's law.

Matter is treated as a collection of molecules. We can apply Newton's laws of motion in a statistical manner to a collection of particles to provide a reasonable description of the thermodynamic processes and derive an expression for the pressure a gas exerts on its container based on kinetic theory. Consider molecules inside a square container (at rest) whose ends have area $A$ and whose length is $d$, as shown in Figure 10.9.

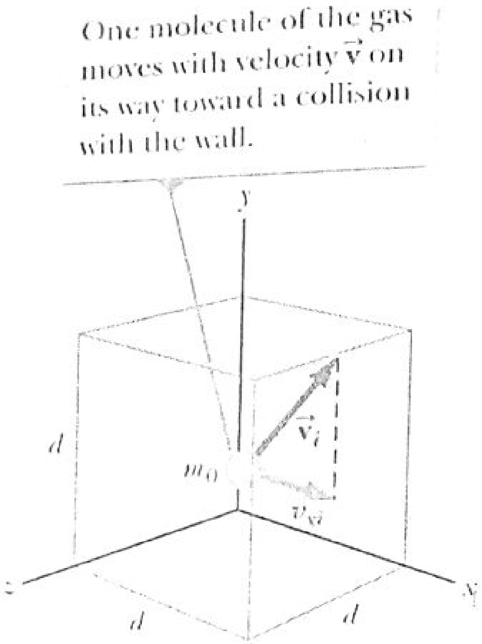



Figure 10.9: Molecules of a gas moving about in a container

Let us consider a molecule of mass $m_0$ moving with velocity $\vec{v}$ on its way toward a collision with the wall. This molecule exerts a force on the wall, and according to Newton's third law of motion, the wall exerts an equal and opposite force back on the molecule. The magnitude of this force on the molecule is equal to the molecule's time rate of change of momentum $F = dp/dt$ (Newton's second law of motion). If we assume that the collision is elastic, the change in the molecule's momentum is:

$$\Delta p = m_o v_x - (-m_o v_x) = 2m_o v_x.$$

This molecule will make many collisions with the wall of the container and each collision is separated by a time $\Delta t$. This time interval $\Delta t$ is the time it takes the molecule to travel across the container and back again, a distance equal to $2d$. Hence:

$$\Delta t = \frac{2d}{v_x}.$$

The $\Delta t$ is very small, so the number of collisions per second is large. This implies that the average force will be equal to the momentum change during one collision divided by the time between collisions:

$$F = \frac{\Delta p}{\Delta t} = \frac{2m_o v_x}{2d/v_x} = \frac{m_o v_x^2}{d}$$

This force is due to one molecule. To calculate the force due to all the molecules in the container, we have to add the contributions of each molecule. Thus the net force on the wall is:

$$F = \frac{m}{d}\left(v_{x1}^2 + v_{x2}^2 + \cdots + v_{xN}^2\right), \qquad 10.32$$

where $v_{x1}$, $v_{x2}$, ... $v_{xN}$ represent the $x$ components of the velocity of each molecules.

The average value of the square of the $x$ component of velocity is:

$$\overline{v_x^2} = \frac{v_{x1}^2 + v_{x2}^2 + \cdots + v_{xN}^2}{N} \qquad 10.33$$

Using Equation 10.33, we can write Equation 10.32 as:

$$F = \frac{m}{d} N\overline{v_x^2}. \qquad 10.34$$

According to Pythagoras theorem, the square of any vector is equal to the sum of the squares of its components, hence:

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}.$$

The $x$, $y$, $z$ axes are called the molecules' degree of freedom: they are in three directions such that the motion of the molecule along any one is independent of its motion along the others.

In kinetic theory, we have assumed that the velocities of the molecules in our gas to be random, there is therefore no preference to one direction or another. Hence:

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}.$$

Combing this relation with the one above, we get:

$$\overline{v^2} = 3\overline{v_x^2}.$$

We substitute this into Equation 10.34:

$$F = \frac{m}{d} N \frac{\overline{v^2}}{3}.$$

The pressure on the wall is then:

$$P = \frac{F}{A} = \frac{1}{3} \frac{Nm\overline{v^2}}{Ad}$$

or:

$$P = \frac{1}{3} \frac{Nm\overline{v^2}}{V}, \qquad 10.35a$$

where $V = Ad$ is the volume of the container.

We can rewrite Equation 10.35a as:

$$PV = \frac{2}{3} N \frac{m\overline{v^2}}{2}. \qquad 10.35b$$

The quantity $\frac{m\overline{v^2}}{2}$ is the average kinetic energy $\overline{K}$ of the molecules in the gas. If we compare Equation 10.35b with Equation 10.31, we see that:

$$\frac{2}{3}\left(\frac{m\overline{v^2}}{2}\right) = kT,$$

or:

$$\overline{K} = \frac{1}{2} m\overline{v^2} = \frac{3}{2} \qquad 10.36$$

The square root of $\overline{v^2}$ is called the **root-mean-square speed**, $v_{rms}$:

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}, \qquad 10.37a$$

or:

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3RT}{M}}. \qquad 10.37b$$

where $M$ = molar mass = $mN_A$.

Equation 10.36 shows that the average translational kinetic energy of molecules in random motion in an ideal gas is directly proportional to the absolute temperature of the gas. The higher the temperature, the faster the molecules are moving on the average. We can conclude that pressure and temperature relate directly to molecular motion in a sample of gas.

Therefore the average kinetic energy of a monoatomic molecule, in each degree of freedom, is:

$$\frac{1}{2}m\overline{v_x^2} = \frac{1}{2}m\overline{v_y^2} = \frac{1}{2}m\overline{v_z^2} = \frac{1}{2}kT.$$

So the molecule has kinetic energy $\frac{1}{2}kT$ per degree of freedom. A monoatomic gas has one atom per molecule. If the gas molecules contain more than one atom, then the rotational and vibrational energy of the molecules must also be taken into account. We have seen that the internal energy of an ideal gas depends only on temperature and the number of moles of gas. This internal energy is shared between the translations and rotations of its molecules. The average kinetic energy of a molecule, in each degree of freedom, rotational as well as translational, is $\frac{1}{2}kT$. This statement is called the [illegible] **equipartition of energy**. The statement is valid at roo[illegible]re and above but it fails when the gas is near liquefaction, at very low temperatures. At ordinary temperature, we have:

Average kinetic energy of monoatomic molecule $= \frac{3}{2}kT$ (trans.);

Average kinetic energy of diatomic molecule

$$= \frac{3}{2}kT\,(\text{trans.}) + \frac{2}{2}kT\,(\text{rot.}) = \frac{5}{2}kT;$$

Average kinetic energy of polyatomic molecule

$$= \frac{3}{2}kT\,(\text{trans.}) + \frac{3}{2}kT\,(\text{rot.}) = \frac{6}{2}kT.$$

## 10.11 First Law of Thermodynamics

The internal energy of an ideal gas is the sum of the kinetic energy of thermal motion of its molecules and its magnitude depends on the temperature of the gas and on the number of atoms in its molecule. We denote the internal energy with the symbol $U$ and changes in internal energy denoted by $\Delta U$.

Hence:

$$U = N_A\left(\frac{1}{2}m\overline{v^2}\right) = \frac{3}{2}N_A kT \qquad 10.38a$$

(since there are $N_A$ molecules mole where $N_A$ is the Avogadro constant)

or:

$$U = \frac{3}{2}nRT, \qquad 10.38b$$

for an ideal monoatomic gas.

The internal energy of real gases depends to some extent on pressure and volume, although the internal energy depends mainly on temperature.

When we add a quantity of heat $Q$ to a system and the system does work $W$ by expanding against its surroundings, the total change in internal energy is:

$$U_f - U_i = Q - W$$

or:

$$Q = \Delta U + W \qquad 10.39$$

Equation 10.39 is known as the mathematical formulation of the first law of thermodynamics.

The implication of Equation 10.39 is that in general, when heat $Q$ is added to a system, some of the heat energy changes the internal energy of the system by an amount $\Delta U$; the remainder leaves the system again as the system does work $W$ against its surroundings. $\Delta U$ can be positive, negative or zero for different processes since $W$ and $Q$ may be positive, negative, or zero.

For a process involving only infinitesimal changes, the first law of thermodynamics becomes:

$$dQ = dU + dW \qquad 10.40$$

If the work $dW$ is given by:

$$dW = PdV \qquad 10.41$$

then the first law can be stated as:

$$dQ = dU + PdV \qquad 10.42$$

## 10.12 Molar Specific Heats for Gases

The molar specific heats, $C_V$ and $C_P$, are defined as the heat required to raise 1 mol of the gas by 1°C at constant volume and at constant pressure respectively. Hence the heat $Q$ needed to raise the temperature of $n$ moles of gas by $\Delta T$ is:

$$Q_V = nC_V\Delta T \qquad \text{[volume constant]} \qquad 10.43a$$

$$= nC_P\Delta T \qquad \text{[pressure constant]} \qquad 10.43b$$

From the definition of molar specific heat that:

$$C_V = Mc_V \qquad \text{[volume constant]} \qquad 10.44a$$

$$C_P = Mc_P \qquad \text{[pressure constant]} \qquad 10.44b$$

where $M$ is the molecular mass of the gas ($M = m/n$ in grams/mol). Consider an ideal gas slowly heated through two different processes; first at constant volume, and then at constant pressure. The temperature is made to increase by the same amount in both processes. In the process done at constant volume, no work is done since $\Delta V = 0$ (Equation 10.41). Hence the heat added all goes into increasing the internal energy of the gas. You can see this by considering Equation 10.42:

$$Q_V = dU. \qquad 10.45$$

The heat added at constant pressure will increase internal energy and also be used to do work $W = P\Delta V$. Thus, more heat must be added in this process at constant pressure than in the first process at constant volume. At constant pressure, we have:

$$Q_P = dU + PdV \qquad 10.46$$

Since the change in temperature is the same, $dU$ is same in the two processes. Combining the two equations above:

$$Q_P - Q_V = PdV. \qquad 10.47$$

Substitute Equations 10.30 and 10.43 into the Equation 10.47:

$$nC_P\Delta T - nC_V\Delta T = P\left(\frac{nR\Delta T}{P}\right)$$

$$C_P - C_V = R. \qquad 10.48$$

For an ideal monoatomic gas, Equation 10.45 becomes:

$$\Delta U = \frac{3}{2}nR\Delta T = nC_V\Delta T \qquad 10.49a$$

or:

$$C_V = \frac{3}{2}R. \qquad 10.49b$$

and using Equation 10.48:

$$C_P = \frac{5}{2}R. \qquad 10.49c$$

## 10.13 Thermodynamics Processes

A process in which there are changes in the state of a thermodynamic system is called thermodynamic process. In this section we describe four kinds of thermodynamic processes that occur often in practical situations. The PV diagram for all the processes is shown in Figure 10.10.

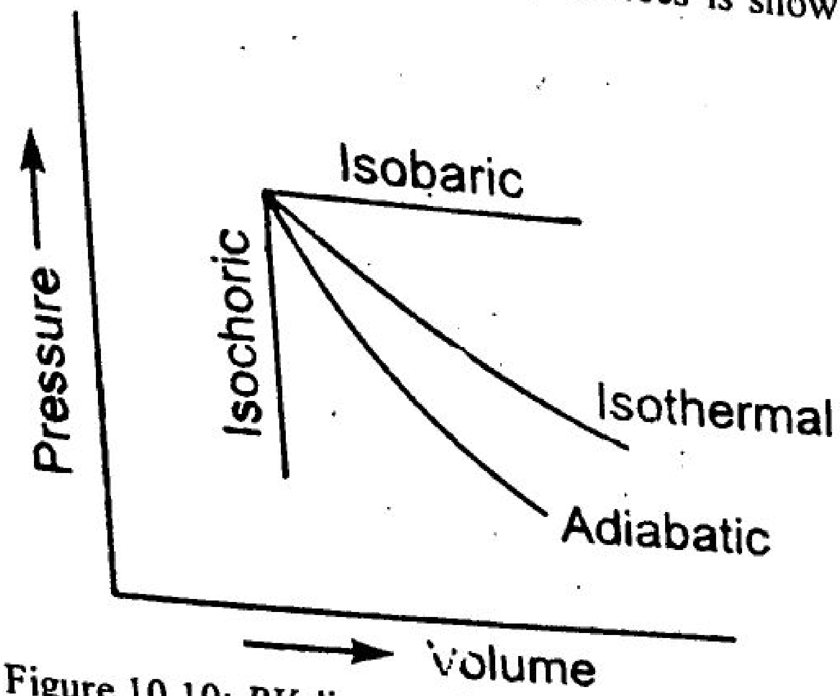



Figure 10.10: *PV* diagram for thermodynamic processes

### 10.13.1 Adiabatic Processes

An adiabatic process is defined as one with no heat transfer into or out of a system: $Q = 0$. For every adiabatic process Equation 10.39 becomes:

$$U_f - U_i = \Delta U = -W \qquad 10.50$$

When a system expands adiabatically, $W$ is positive, that is, the system does work on its surroundings, so $\Delta U$ is negative and the internal energy decreases. When a system is compressed adiabatically, $W$ is negative, that is, work is done on the system by its surroundings, and $U$ increases.

For an ideal gas changing from conditions ($P_1$, $V_1$, $T_1$) to ($P_2$, $V_2$, $T_2$) in an adiabatic process:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} \text{ and } T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \quad 10.51$$

where $\gamma = C_P/C_V$ = ratio of heat capacities.

The work done by an ideal gas during an adiabatic expansion can be expressed in terms of the initial and final values of temperature, or in terms of the initial and final values of pressure and volume:

$$W = nC_V(T_1 - T_2) \quad 10.52a$$

or:

$$W = \frac{1}{\gamma - 1}(P_1V_1 - P_2V_2) \quad 10.52b$$

## 10.13.2 Isobaric Processes

An isobaric process is one in which the pressure is constant. If the volume changes from $V_1$ to $V_2$ as the pressure $P$ remains constant, the work done by the system is:

$$W = p(V_2 - V_1) \quad 10.53$$

$\Delta U$, $Q$, and $W$ are not equal to zero in an isobaric process.

## 10.13.3 Isochoric Processes

An isochoric process is one in which the volume is constant. When the volume of a thermodynamic system is constant, it does no work on its surroundings. Then $W = 0$ and:

$$U_f - U_i = \Delta U = Q \quad 10.54$$

### 10.13.4 Isothermal Processes

An isothermal process is a constant-temperature process. For an ideal gas, $\Delta U = 0$ in an isothermal change and so the first law becomes:

$$Q = W \tag{10.55a}$$

In an expansion from $V_1$ to $V_2$, the work done is:

$$W = \int dW = \int_{V_1}^{V_2} PdV$$

From Equation 10.30, $P = \frac{nRT}{V}$

So:

$$W = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \ln\left(\frac{V_2}{V_1}\right) \tag{10.55b}$$

## 10.14 Second Law of Thermodynamics

The second law of thermodynamics has several forms:

i. It is impossible for any self-acting machine working in a cyclical process unaided by an external agency to make heat pass from one body to another at a higher temperature – Clausius' statement of the second law.

ii. It is impossible by means of inanimate material agency to derive mechanical effect from any portion of matter by cooling it below the temperature of the coldest body of its surroundings – Kelvin's statement of the second law.

iii. It is impossible to construct an engine which, working in a complete cycle, will produce no effect other than doing mechanical work and the cooling of a heat reservoir – Planck statement of the second law.

In summary [illegible] statements indicate that processes occur in a certain di[illegible] in just any direction. Physical processes in nature can spontaneously proceed toward equilibrium. For example, water flows down a waterfall, gases expand from a high pressure to a low pressure and heat flows from a high temperature to a low temperature. The reverse of these processes will not occur spontaneously. In other words, the processes will not occur on their own. A spontaneous process can be reversed, but it will not reverse itself spontaneously. To reverse a spontaneous process some external energy must be expended. We know by experience that heat flows spontaneously from a high temperature to a low temperature. But heat flowing from a low temperature to a higher temperature with no expenditure of energy to cause the process to take place would violate the second law of thermodynamics but will not violate the first law of thermodynamics. The first law is concerned with the conversion of energy from one form to another. Heat and work are not completely interchangeable forms of energy. This is shown in Joule's experiments. His experiment showed that energy in the form of heat could not be completely converted into work; however, work energy can be completely converted into heat energy. It is the second law of thermodynamics that controls the direction processes may take and how much heat is converted into work. A process will not occur unless it satisfies both the first and the second laws of thermodynamics.

You will encounter the following terms as we discuss second law of thermodynamics in details:

### Heat (thermal) reservoir

A heat reservoir is a sufficiently large system in stable equilibrium to and from which finite amounts of heat can be transferred without any change in its temperature. A high temperature heat reservoir from which heat is transferred is sometimes called a **heat source**. A low temperature heat reservoir to which heat is transferred is sometimes called a **heat sink**.

### Work reservoir

A **work reservoir** is a sufficiently large system in stable equilibrium to and from which finite amounts of work can be transferred adiabatically without any change in its pressure.

### Thermodynamic cycle

A system has completed a **thermodynamic cycle** when the system undergoes a series of processes and then returns to its original state so that the properties of the system at the end of the cycle are the same as at its beginning.

### Heat Engine

A **heat engine** is a thermodynamic system operating in a thermodynamic cycle to which net heat is transferred and from which net work is delivered. The system, or working fluid, undergoes a series of processes that constitute the heat engine cycle.

### Reversible Process

A reversible process is a quasi-equilibrium, or quasi-static, process with a more restrictive requirement. A reversible process is one that is carried out infinitely slowly, so that the process can be considered as a series of equilibrium states, and the whole process could be done in reverse with no change in magnitude of the work done or heat exchanged.

## 10.14.1 Thermal Efficiency

The thermal efficiency $e$ of a heat engine is defined as the ratio of the net work output $W$ (the desired result) to the heat input, $Q_H$, (the costs to obtain the desired result) at the high temperature:

$$e = \frac{W}{Q_H}. \qquad 10.56a$$

Since energy is conserved, the heat input $Q_H$ must equal the work done plus the heat that flows out at the low temperature $Q_L$ (Figure 10.11):

$$Q_H = W + Q_L.$$

Equation 10.56a becomes:

$$e = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}. \qquad 10.56b$$

The thermal efficiency is always less than 1 or less than 100 percent.

Cyclic devices such as heat engines, refrigerators and heat pumps often operate between a high-temperature reservoir at temperature $T_H$ and a low-temperature reservoir at temperature $T_L$.

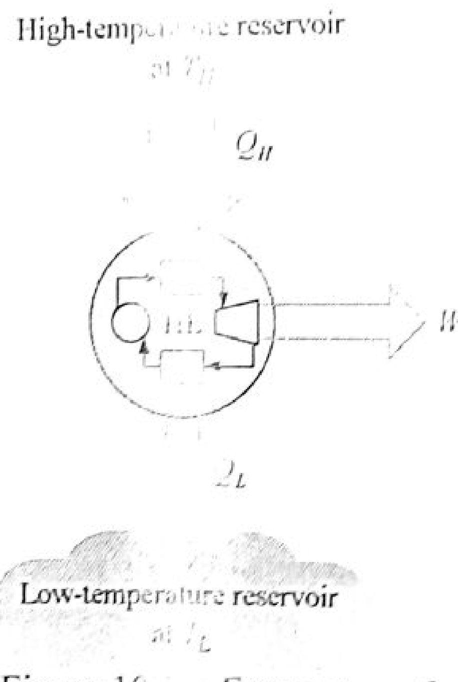



Figure 10. Energy transfers for heat engine

### 10.14.2 Carnot Cycle

Nicolas Sadi Carnot (1769-1832) was among the first to study the principles of the second law of thermodynamics. He introduced the concept of cyclic operation and devised a reversible cycle that is composed of four reversible processes, two isothermal and two adiabatic. The processes are:

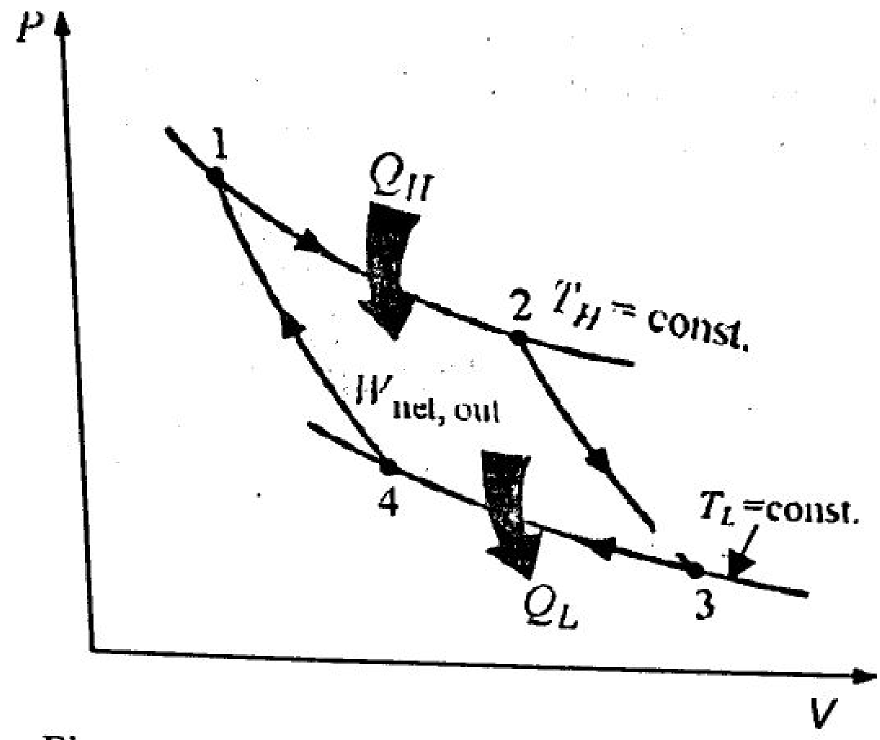



Figure 10.12: The Carnot cycle

Process 1 to 2: Reversible isothermal heat addition at high temperature, $T_H > T_L$, to the working fluid (an ideal gas) in a piston-cylinder device that does some boundary work.

Process 2 to 3: Reversible adiabatic expansion during which the system does work as the working fluid temperature decreases from $T_H$ to $T_L$.

Process 3 to 4: The system is brought in contact with a heat reservoir at $T_L < T_H$ and a reversible isothermal heat exchange takes place while work of compression is done on the system.

Process 4 to 1: A reversible adiabatic compression process increases the working fluid temperature from $T_L$ to $T_H$. The working fluid is hence returned back to its original state.

The net work done in one cycle by a Carnot engine (or any other type of engine using a reversible cycle) is equal to the area enclosed by the curve representing the cycle on the PV diagram, the curve 1234 in Figure 10.12.

We can show that the efficiency of a Carnot engine using an ideal gas depends only on the temperatures of the heat reservoirs, $T_H$ and $T_L$. The work done by the gas in the isothermal process 1 to 2 is:

$$W_{1\to 2} = nRT_H \ln\frac{V_2}{V_1},$$

where $n$ is the number of moles of the ideal gas.

For isothermal process the temperature is constant, hence the change in internal energy is zero. The first law of thermodynamics shows that the heat added to the gas equals the work done by the gas:

$$Q_H = nRT_H \ln\frac{V_2}{V_1}.$$

Similarly, the heat lost by the gas in the isothermal process 3 to 4 is:

$$Q_L = nRT_L \ln\frac{V_3}{V_4}.$$

The path 2 to 3 and 4 to 1 are adiabatic, so we have:

$$P_2V_2^\gamma = P_3V_3^\gamma \text{ and } P_4V_4^\gamma = P_1V_1^\gamma.$$

Also, from the gas law:

$$\frac{P_2V_2}{T_H} = \frac{P_3V_3}{T_L} \text{ and } \frac{P_4V_4}{T_L} = \frac{P_1V_1}{T_H}.$$

Dividing these last equations with the corresponding set of equations on the line above, we obtain:

$$T_HV_2^{\gamma-1} = T_LV_3^{\gamma-1} \text{ and } T_LV_4^{\gamma-1} = T_HV_1^{\gamma-1}.$$

Now we divide the equation on the left by the one on the right:

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

We can show that the efficiency of a Carnot engine using an ideal gas depends only on the temperatures of the heat reservoirs, $T_H$ and $T_L$. The work done by the gas in the isothermal process 1 to 2 is:

$$W_{1\to 2} = nRT_H \ln\frac{V_2}{V_1},$$

where $n$ is the number of moles of the ideal gas.

For isothermal process the temperature is constant, hence the change in internal energy is zero. The first law of thermodynamics shows that the heat added to the gas equals the work done by the gas:

$$Q_H = nRT_H \ln\frac{V_2}{V_1}.$$

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The path 2 to 3 and 4 to 1 are adiabatic, so we have:

$$P_2V_2^\gamma = P_3V_3^\gamma \text{ and } P_4V_4^\gamma = P_1V_1^\gamma.$$

Also, from the gas law:

$$\frac{P_2V_2}{T_H} = \frac{P_3V_3}{T_L} \text{ and } \frac{P_4V_4}{T_L} = \frac{P_1V_1}{T_H}.$$

Dividing these last equations with the corresponding set of equations on the line above, we obtain:

$$T_HV_2^{\gamma-1} = T_LV_3^{\gamma-1} \text{ and } T_LV_4^{\gamma-1} = T_HV_1^{\gamma-1}.$$

Now we divide the equation on the left by the one on the right:

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

or:

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}.$$

Substitute this result in our equations for $Q_H$ and $Q_L$ above, we obtain

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} \qquad \text{[Carnot cycle]} \qquad 10.57$$

Hence the efficiency of a reversible Carnot engine can now be written as:

$$e = 1 - \frac{T_L}{T_H}. \qquad 10.58$$

Thus the efficiency of a Carnot engine depends only on the temperature $T_L$ and $T_H$.

The second law of thermodynamics puts limits on the operation of cyclic devices as expressed by the Kelvin-Planck and Clausius statements. A heat engine cannot operate by exchanging heat with a single heat reservoir, and a refrigerator cannot operate without net work input from an external source.

The heat engine we described above is operating between two fixed temperature reservoirs at $T_H > T_L$. We can point out two conclusions about the thermal efficiency of reversible and irreversible heat engines:

- The efficiency of an irreversible heat engine is always less than the efficiency of a reversible one operating between the same two reservoirs.
- All reversible engines operating between the same two constant temperatures $T_H$ and $T_L$ have the same efficiency.

As the result of the above, Lord Kelvin in 1848 used energy as a thermodynamic property to define temperature and devised a temperature scale that is independent of the thermodynamic substance.

### 10.14.3 Refrigerator

The operating principle of refrigerators, air conditioners and heat pumps is just the reverse of a heat engine. A refrigerator is a device that operates on a thermodynamic cycle and extracts heat from a low-temperature medium. A heat pump is a thermodynamic system operating in a thermodynamic cycle that removes heat from a low-temperature body and delivers heat to a high-temperature body. To accomplish this energy transfer, the refrigerator receives external energy in the form of work or heat from the surroundings.

Figure 10.13 shows the schematic diagram of energy transfers for a refrigerator or air conditioner. By doing work $W$, heat is taken from a low-temperature region, $T_L$ (such as inside a refrigerator), and a greater amount of heat is exhausted at a high temperature, $T_H$ (the room). The work is usually done by an electric motor which compresses a fluid.

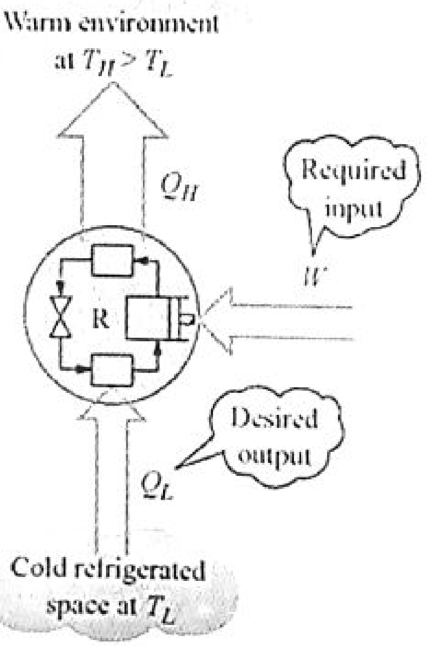


Figure 10.13: Energy transfers for refrigerator

The Carnot cycle may be reversed, in which it operates as a refrigerator. The refrigeration cycle operates in the counter-

clockwise direction as shown in Figure 10.14. Compare
refrigeration cycle with the Carnot cycle.

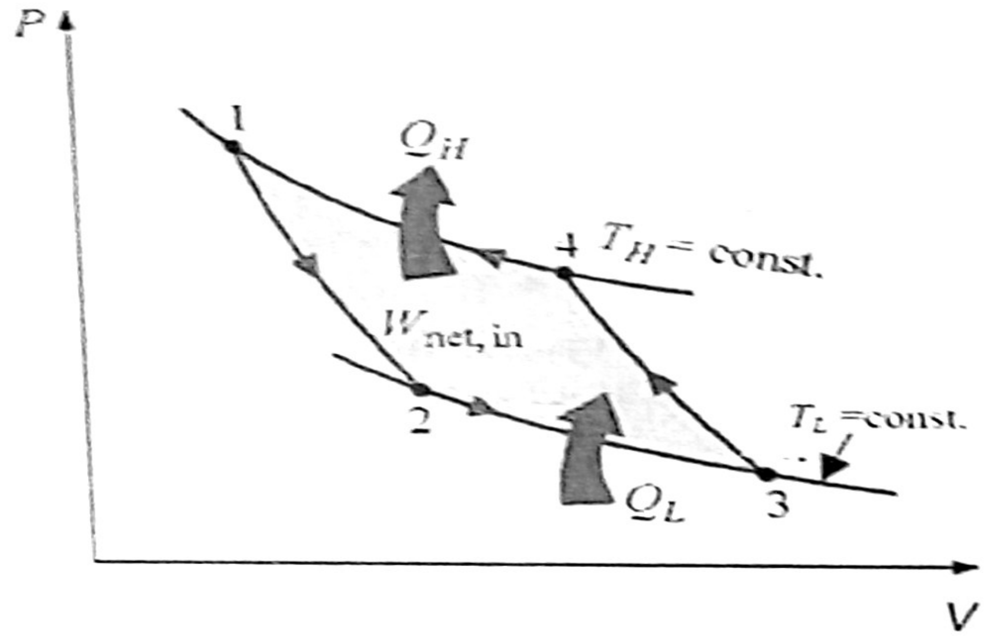


Figure 10.14: The Refrigerator cycle

The coefficient of performance (COP) of a refrigerator is defin
the heat $Q_L$ removed from the low-temperature area (insi
refrigerator) divided by the work $W$ done to remove the heat:

$$\text{COP} = \frac{Q_L}{W}. \quad 10.59a$$

This shows that the more heat $Q_L$ that can be removed fro
inside of the refrigerator for a given amount of work, the
efficient the refrigerator is. From first law of thermodynamics w
write:

$$Q_H = Q_L + W \qquad \text{(see Figure 10.13)}$$

$$\text{COP} = \frac{T_L}{T_H - T_L}. \qquad 10.59c$$

For the device acting like a heat pump, the primary function of the device is the transfer of heat to the high-temperature system. The coefficient of performance for a heat pump is:

$$\text{COP}_{\text{HP}} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} \qquad 10.60$$

The $\text{COP}_{\text{HP}}$ is necessarily greater than 1. Most heat pumps can be "turned around" and used as air conditioners in the summer.

## 10.15 Entropy

Entropy is defined as a measure of the order or disorder of a system. When we deal with entropy it is the change in entropy during a process that is important, not the absolute amount. The change in entropy $S$ of a system when an amount of heat $Q$ is added to it by a reversible process at constant temperature, is given by:

$$\Delta S = \frac{Q}{T}, \qquad 10.61$$

where $T$ is the kelvin temperature.

If the temperature is not constant, we define entropy $S$ by the relation:

$$dS = \frac{dQ}{T}. \qquad 10.62$$

For a reversible process, the change in entropy between two states $a$ and $b$ is given by:

$$\Delta S = S_b - S_a = \int_a^b dS = \int_a^b \frac{dQ}{T}. \qquad 10.63$$

## Activity 10 Temperature and Heat

**10.1.** A gas has a volume of 120 $m^3$ at STP. If the gas is compressed and adiabatically to a volume of 25$m^3$ the new temperature, pressure and the work done on the gas respectfully are:
(Hint: $C_p = 1.68$ kJkg$^{-1}$K$^{-1}$, $C_v = 1.20$ kJkg$^{-1}$K$^{-1}$
A. $T = 344$ K, $p = 2.3 \times 10^5$Nm$^{-2}$, $W = 3.7 \times 10^6$ J
B. $T = 427.4$ K, $p = 1.6 \times 10^5$ Nm$^{-2}$, $W = 3.7 \times 10^7$ J
C. $T = 527.4$ K, $p = 9.6 \times 10^6$ Nm$^{-2}$, $W = 1.7 \times 10^7$ J
D. $T = 511$ K, $p = 9.1 \times 10^5$ Nm$^{-2}$, $W = 2.7 \times 10^7$ J

**Solution**

At STP, $T_1 = 273$K, $P_1 = 1.013 \times 10^5$N/m$^2$

$$\gamma = \frac{C_p}{C_v} = \frac{1.68}{1.20} = 1.40$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$273 \times 120^{1.40-1} = T_2 \times 25^{1.40-1}$$

$$1852.828172 = T_2 \times 3.623898318$$

$$T_2 = 511.28 \text{ K}$$

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

$$1.013 \times 10^5 \times 120^{1.4} = p_2 \times 25^{1.4}$$

$$8.250 \times 10^7 = p_2 \times 90.59745796$$

$$p_2 = 9.106 \times 10^5 \text{N/m}^2 = 9.1 \times 10^5 \text{N/m}^2$$

$$W = \frac{1}{\gamma - 1}(p_1 V_1 - p_2 V_2)$$

$$W = \frac{1}{1.4-1}\left(1.013 \times 10^5 \times 120 - 9.1 \times 10^5 \times 25\right) J = 2.7 \times 10^7\, J$$

The correct option is D.

**10.2.** A quantity of a diatomic gas expands adiabatically from an initial pressure of 3 x $10^5$ Nm$^{-2}$ and volume of 3 x $10^{-3}$m$^3$ at temperature 20°C to thrice its original volume. The work done by the gas is (hint: for a diatomic gas $\gamma = 1.4$).

A. 791J    B. 801J    C. 810J    D. -810J

**Solution**

$$p_1V_1^\gamma = p_2V_2^\gamma$$

$$3\times10^5\left(3\times10^{-3}\right)^{1.4} = p_2\times\left(3\times3\times10^{-3}\right)^{1.4}$$

$$p_2 = 6.4439403 \text{ x } 10^4\text{N/m}^2$$

$$W = \frac{1}{\gamma-1}\left(p_1V_1 - p_2V_2\right)$$

$$W = \frac{1}{1.4-1}\left(3\times10^5\times3\times10^{-3} - 6.44\times10^4\times9\times10^{-3}\right)J = 800.1J$$

The correct option is B.

**10.3.** A volume of gas $V$ at a temperature $T_1$ and a pressure $p_1$ is enclosed in sphere. It is connected to another sphere of volume $V/2$ by a tube and stopcock. The second sphere is initially evacuated and the stopcock is closed. If the stopcock is opened the temperature of the gas in the second sphere becomes $T_2$. The first sphere is maintained at a temperature $T_1$. The final pressure $p_2$ within the apparatus is:

A. $p_2 = \dfrac{3p_1T_2}{2T_2+T_1}$    B. $p_2 = \dfrac{2p_1T_1}{2T_2+T_1}$

C. $p_2 = \dfrac{2T_2T_1}{2p_1T_2}$    D. $p_2 = \dfrac{2T_1+T_2}{2p_1T_1}$

**Solution**

The total number of moles of air in the two containers remains constant, although some air is transferred from the hot to the cold sphere.

From $n = \dfrac{pV}{RT}$,

Initially, total moles of air:

$$= \frac{pV}{RT} = \frac{p_1\left(V + \frac{V}{2}\right)}{RT_1} = \frac{3p_1V}{2RT_1}$$

and finally, total moles of air:

$$= \frac{p_2 \times V}{RT_1} + \frac{p_2 \times V/2}{RT_2}$$

where $p_2$ is the new pressure in both containers after heating.

Equating the two numbers of moles, which are unchanged by any transfer:

$$\frac{p_2 \times V}{RT_1} + \frac{p_2 \times V/2}{RT_2} = \frac{3p_1V}{2RT_1}$$

$$\frac{p_2}{T_1} + \frac{p_2}{2T_2} = \frac{3p_1}{2T_1}$$

$$\frac{2p_2 + p_2T_1}{2T_1T_2} = \frac{3p_1}{2T_1}$$

$$\frac{p_2(2T_2 + T_1)}{T_2} = 3p_1$$

$$p_2 = \frac{3p_1T_2}{2T_2 + T_1}$$

The correct option is A.

**10.4.** An ideal gas 27°C and a pressure of 760mm of mercury is compressed isothermally until its volume is doubled. It is then expanded reversibly and adiabatically to half its original volume. If the value of $\gamma$ for the gas is 1.4, calculate the final pressure and temperature of the gas.

A. $p_2 = 218$mmHg, $T_2 = 172$ K
B. $p_2 = 2646.5$mmHg, $T_2 = 325.5$ K

C. $p_2 = 380\text{mmHg},\ T_2 = 325.2$ K
D $p_2 = 1520\text{mmHg},\ T_2 = 300$ K

**Solution**

For isothermal process:

$$p_1V_1 = p_2V_2$$

$$760 \times V = p_2 \times 2V$$

$$p_2 = 380\text{mmHg}$$

$V_2 = 2V,\ V_3 = V/2$

$$p_1V_1^\gamma = p_2V_2^\gamma$$

$$380 \times (2V)^{1.4} = p_2 \times \left(\frac{V}{2}\right)^{1.4}$$

$$380 \times 2^{1.4} \times 2^{1.4} = p_2$$

$$p_2 = 2646.5\text{mmHg}$$

$$T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$$

$$300 \times (2V)^{1.4-1} = T_3 \times \left(\frac{V}{2}\right)^{1.4-1}$$

$$T_3 = 300 \times 2^{0.4} \times 2^{0.4}\ K = 522.3K$$

**10.5.** The root mean square velocity of hydrogen molecule is $1839\text{ms}^{-1}$ at STP. Calculate the density of hydrogen.

A. $0.09\text{kgm}^{-3}$ B. $0.9\text{kgm}^{-3}$
C. $0.009\text{kgm}^{-3}$ D. $11.2\text{kgm}^{-3}$

**Solution**

r.m.s velocity

$$V_{rms} = \sqrt{\frac{3p}{\rho}}$$

$$\rho = \frac{3p}{V_{rms}^2}$$

$$\rho = \frac{3\times 1.013\times 10^{5}}{1839^{2}}\, kgm^{-3}$$

$$\rho = 0.08986 kgm^{-3} = 0.09 kgm^{-3}$$

The correct option is A.

**10.6.** A lagged bar of length 60cm and cross-sectional area $2.0cm^2$ made of copper attains a steady state with one end at a temperature 120°C and the other at 30°C. Find the time rate of heat transfer (thermal conductivity for copper is 380 $Wm^{-1}K^{-1}$).

A. 14.1 W B. 1.1 W C. 11.4 W D. 3.84 W

**Solution**

$$\frac{dQ}{dt} = kA\left(\frac{\theta_2 - \theta_1}{L}\right)$$

$$\frac{dQ}{dt} = 380\times 2.0\times 10^{-4}\left(\frac{120-30}{60\times 10^{-2}}\right)W = 11.4W$$

The correct option is C.

**10.7.** At 25°C and a pressure of 75cm of mercury, the density of air is $1.26kgm^{-3}$. What is the density at the top of a hill when the pressure is 50cm of mercury and temperature is -43°C.

A. $1.09kgm^{-3}$ B. $10.9kgm^{-3}$ C. $1.43kgm^{-3}$ D. $1.38kgm^{-3}$

**Solution**

$$\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho} \qquad \text{(i)}$$

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \qquad \text{(ii)}$$

Substitute (i) into (ii):

$$\frac{p_1 m}{\rho_1 T_1} = \frac{p_2 m}{\rho_2 T_2}$$

$$\frac{50}{1.26\times 298}=\frac{50}{\rho_2\times 230}$$

$$\rho_2 = 1.088kgm^{-3} = 1.09kgm^{-3}$$

The correct option is A.

**10.8.** The density of an oxygen gas is $1.83kgm^{-3}$ at STP. If its specific heat capacity at constant volume is $Cv = 20.8Jmol^{-1}K^{-1}$, calculate the ratio of its specific heat capacity at constant pressure to that at constant volume.

A. 1.40 B. 1.67 C. 1.31 D. 1.33

**Solution**

The gas constant:

$$R=\frac{pV}{T}=\frac{pm}{T\rho}$$

$$R=\frac{1.013\times 10^5\times 32\times 10^{-3}}{273\times 1.83}=6.489$$

$$C_p = C_v + R$$

$$C_p = 20.8 + 6.489 = 27.289Jmol^{-1}$$

$$\gamma=\frac{C_p}{C_v}=\frac{27.289}{20.8}=1.31$$

The correct option is C.

**10.9.** Calculate the average kinetic energy of a gas molecule at room temperature (27°C). (Hint: $k_B = 1.38 \times 10^{23} JK^{-1}$).

A. 0.0353eV B. 0.386eV C. 0.013eV D. 0.0386 eV

**Solution**

Average kinetic energy:

$$= \frac{3kT}{2} = \frac{3 \times 1.38 \times 10^{-23} \times 300}{2} J$$

$$= 6.21 \times 10^{-21} J = \frac{6.21 \times 10^{-21}}{1.6 \times 19^{-19}} eV = 0.0388125 eV$$

The correct option is D.

**10.10.** Calculate the maximum efficiency of a steam engine if the temperature of the input steam is 185K and the temperature of the exhaust is 90K.

A. 40.7% B. 26.2% C. 51.4% D. 20.7%

**Solution**

$$\text{Efficiency } e = \frac{T_1 - T_2}{T_1} \times 100\%$$

$$e = \frac{185 - 90}{185} \times 100\% = 51.4\%$$

The correct option is C.

**10.11.** A 0.02kg ice cube was melted at 0°C by an application of heat which further increased the temperature of the resulted water to 40°C. Calculate the total heat supplied to the system within this temperature range (hint: specific latent heat of fusion of water is $3.34 \times 10^5 \text{ Jkg}^{-1}$, specific heat capacity of water is $4200 \text{Jkg}^{-1}\text{K}^{-1}$).

A. 20080J B. 19880J C. 10040J D. 3320J

**Solution**

Quantity of heat required to melt the ice:

$Q = mL = 0.02 \times 3.34 \times 10^5 \text{ J} = 6.680 \times 10^3 \text{ J}$

Quantity of heat required to increase temperature to 40°C,

$Q = mc(\theta_2 - \theta_1) = 0.02 \times 4200(40 - 0)\text{J} = 3.360 \times 10^3 \text{J}$

Total heat required $Q = (6.680 \times 10^3 + 3.360 \times 10^3) \text{ J} = 10040 \text{ J}$

The correct option is C.

**10.12.** 0.01kg of ice and 0.2kg of water at 20°C are in a container. Steam at 100°C is passed in until all the ice melted and the final temperature of mixture is 30°C. Calculate the mass of water that is now in the container. (hint: specific latent heat of vaporisation of water = $2.3 \times 10^6 Jkg^{-1}$, specific latent heat of fusion of water = $3.34 \times 10^5 Jkg^{-1}$, specific heat capacity of water = $4200 Jkg^{-1}K^{-1}$).

A. 0.228kg B. 0.182kg C. 0.192kg D. 0.3kg

**Solution**

Quantity of heat required to melt the ice and increase its temperature:

$Q = mL + mc\Delta\theta$

$Q = 0.01 \times 3.34 \times 10^5 J + 0.01 \times 4200 \times 30J = 4.6 \times 10^3 J$

Quantity of heat required to increase the temperature of water from 20°C to 30°C:

$Q = mc\Delta\theta = 0.2 \times 4200 \times (30 - 20)J = 8400J$

Heat lost by steam:

$Q = mL + mc\Delta\theta$

$Q = m \times 2.3 \times 10^6 + m \times 4200(100 - 30) = 2.594 \times 10^6 mJ$

Heat lost = heat gained

$4600 + 8400 = 2.594 \times 10^6 m$

$$m = \frac{13000}{2.594 \times 10^6} kg = 0.00501 kg$$

Total mass of water = (0.01 + 0.2 + 0.005)kg = 0.215kg = 0.220kg

**10.13.** A carnot engine has an efficiency of 22.0%. It operates between constant temperature reservoirs differing in temperature by 75°C. What are the temperatures of the two reservoirs?

A. 266 K and 341 K
B. 225 K and 300 K
C. 300 K and 375 K

D. 175 K and 250 K

**Solution**

Efficiency:

$$e = \frac{T_1 - T_2}{T_1} \times 100\%$$

$$22 = \frac{75}{T_1} \times 100\%$$

$$T_1 = 340.9 \text{ K} = 341 \text{ K}$$

$$T_1 - T_2 = 75$$
$$341 - T_2 = 75$$

$$T_2 = 265.9 \text{ K} = 266 \text{ K}$$

The correct option is A.

**10.14.** At what temperature is the Fahrenheit scale twice the centigrade scale?

A. -40°F B. 320°C C. 160°C D. 160°F

**Solution**

$$t_f = \left(\frac{9}{5}t + 32\right)$$

with $t_f = 2t$

$$2t = \left(\frac{9}{5}t + 32\right)$$

$$2t - \frac{9t}{5} = 32$$

$$t = 160^o C$$

The correct option is C.

**10.15.** The volume of liquid relative to glass in a certain type of liquid-in-glass thermometer at temperature of 273 K is given by:

$V_\theta = V_o\left(1 + a\theta + b\theta^2\right)$ where a = 2 x $10^{-3}$ and b = 3 x $10^{-6}$. The temperature on this thermometer when the temperature on the gas scale is 300 K will be:

A. 297.4 K B. 300 K C. 397.4 K D. 290.4 K

**Solution**

$$V_\theta = V_o\left(1 + a\theta + b\theta^2\right)$$

$$\theta = 300\text{K} = 27°\text{C}$$

$$\frac{X_\theta - X_o}{X_{100} - X_o} = \frac{\theta}{100}$$

$$\frac{V_o\left(1 + 2\times10^{-3}\times27 + 3\times10^{-6}\times27^2\right) - V_o}{V_o\left(1 + 2\times10^{-3}\times100 + 3\times10^{-6}\times100^2\right) - V_o} = \frac{\theta}{100}$$

$$\frac{0.054 + 0.002187}{0.2 - 0.03} = \frac{\theta}{100}$$

$$\theta = 24.43°\text{C} = 297.4\text{ K}$$

The correct option is A.

**10.16.** What temperature, at constant pressure, will the root mean square velocity of hydrogen will be thrice its value at STP?

A. 1092 K B. 2457 K
C. 819 K D. 1258 K

**Solution**

Let

$v_{rms} = \sqrt{\dfrac{3RT}{M}}$ be value at STP, then

$$\sqrt{\frac{3RT_1}{M}} = 3\sqrt{\frac{3RT}{M}}$$

Squaring both sides,

$$\frac{3RT}{M} = 9\times\frac{3RT}{M}$$

$\Rightarrow T_1 = 9T = 9 \times 273K = 2457K$

The correct option is B.

**10.17.** A certain platinum resistance thermometer has a resistance of 12.6 Ω when immersed in a triple point cell. When the thermometer is placed in surroundings where its resistance becomes 21.6 Ω, what is the temperature that will be shown on the thermometer? (Hint: triple – point of water is 273.16 K.)

A. 159.34 K B. 743.4 K
C. 468.27 K D. 2458.44 K

**Solution**

$$T = \frac{X_T}{X_{tr}} \times 273.16$$

$$T = \frac{21.6}{12.6} \times 273.16K = 468.27K$$

The correct option is C.

**10.18.** Pressure $p$, volume $V$ and temperature $T$ for a certain material are related by $p = \frac{AT - BT^2}{V}$, where $A$ and $B$ are constants. An expression for the work done by the material if the temperature changes from $T_1$ to $T_2$ with constant pressure is:

A. $(T_2 - T_1)[A - B(T_2 + T_1)]$
B. $(T_2 - T_1)[A + B(T_2 - T_1)]$
C. $(AT_2^2 - BT_1^2)$
D. $A(T_2^2 - T_1^2) - AB(T_2 - T_1)$

**Solution**

$$p = \frac{AT - BT^2}{V}$$

Rearranging, we have:

$$V_1 = \frac{AT_1 - BT_1^2}{p}, V_2 = \frac{AT_2 - BT_2^2}{p}$$

Work done = $p(V_2 - V_1)$

$$W = AT_2 - BT_2^2 - AT_1 + BT_1^2$$
$$W = A(T_2 - T_1) - B(T_2^2 - T_1^2)$$
$$W = A(T_2 - T_1) - B(T_2 - T_1)(T_2 + T_1)$$
$$W = (T_2 - T_1)[A - B(T_2 + T_1)]$$

The correct option is A.

**10.19.** The pressure recorded by a constant volume gas thermometer at a kelvin temperature $T$ is $5.80 \times 10^4 Nm^{-2}$. Calculate $T$ if the pressure at the triple point, 273.16 K, is $5.20 \times 10^4 Nm^{-2}$.

A. 304.7 K B. 312 K
C. 273 K D. 549 K

**Solution**

$$T = \frac{P_T}{P_{tr}} \times 273.16K$$

$$T = \frac{5.80 \times 10^4}{5.20 \times 10^4} \times 273.16K = 304.7K$$

The correct option is A.

**10.20.** A constant mass of gas maintained at constant pressure has a volume of $200.0 cm^3$ at the temperature of melting ice, $273.2 cm^3$ at the temperature of water boiling under standard pressure, and $525.1 cm^3$ at the normal boiling-point of sulphur. A platinum wire has resistance of 2.000, 2.778 and 5.280 Ω at the temperatures. Calculate the values of the boiling-point of sulphur given the two sets of observations, and comment on the results.

A. 444.1°C, 214.6°C
B. 444.1°C, 421.6°C

C. 434.1°C, 214.6°C
D. 434.1°C, 421.6°C

**Solution**

On the gas thermometer scale, the boiling-point of sulphur is given by:

$$T = \frac{V_\theta - V_o}{V_{100} - V_o} \times 100$$

$$T = \frac{525.1 - 200.0}{273.2 - 200.0} \times 100^o C = 444.1^o C$$

On the gas platinum resistance scale, the boiling-point of sulphur is given by:

$$\theta_P = \frac{R_\theta - R_o}{R_{100} - R_o} \times 100$$

$$\theta_P = \frac{5.280 - 2.000}{2.778 - 2.000} \times 100^o C = 421.6^o C$$

The correct option is B.

## Summary of Chapter 10

In chapter 10, you have learned that:

1. The temperature of a body is a measure of its hotness or coldness. A measure of temperature is obtained by using a thermometer.
2. Heat is the net energy transferred from one object to another because of a temperature difference. The total mechanical energy of all molecules of a system or body is referred to as the internal energy of the system or body.
3. There are basically three types of temperature scales. These are Celsius temperature scale, Fahrenheit temperature scale and thermodynamic temperature scale.

4. The conversions between the three scales are:

$$T_F = \frac{9}{5}T_C + 32^o \quad \text{and} \quad T = 273.15 + T_C,$$

where $T_F$ and $T_C$ are Fahrenheit and Celsius temperatures, respectively.

5. If systems A and B are each in thermal equilibrium with a third body C, then A and B are also in thermal equilibrium with each other. This is called the Zeroth law of thermodynamics.

6. The coefficient of linear expansion $\alpha$ is defined as:

$$\alpha = \frac{\Delta L}{L_o \Delta T} = \frac{L - L_o}{L_o(T - T_o)}$$

. The units of $\alpha$ are $K^{-1}$ or $(°C)^{-1}$.

7. The coefficient of area expansion $\beta$ which describes the thermal expansion properties of a particular material, is defined as:

$$\beta = \frac{\Delta A}{A_o \Delta T} = \frac{A - A_o}{A_o(T - T_o)}$$

.The units of $\beta$ are $K^{-1}$ or $(°C)^{-1}$.

8. Experiments show that increase in volume $\Delta V$ for both solid and liquid materials is approximately proportional to both the temperature change $\Delta T$ and the initial volume $V_o$:

$$\gamma = \frac{\Delta V}{V_o \Delta T} = \frac{V - V_o}{V_o(T - T_o)}.$$

9. The thermal stress is given as:

$$\frac{F}{A} = Y\frac{\Delta L}{L_o} = Y \times \frac{\alpha L_o \Delta T}{L_o} = Y\alpha\Delta T,$$

where $F$ is the force; $A$ the surface area; $Y$ the Young's modulus; $\Delta T$ is the change in temperature and $\alpha$ is the coefficient of linear expansion.

10. The quantity of heat ($Q$) required to change the temperature of an object is proportional to the mass ($m$) of the object and to

the change in the temperature ($\Delta T$). That is, $Q = mc\Delta T$, where $c$ is the specific heat capacity.

11. The *specific latent heat of fusion* of a solid is the heat required to change the unit mass of it, at its melting point, into liquid at the same temperature. The quantity of heat required to melt a mass $m$ of an object that has specific latent heat of fusion $L_f$ is: $Q = mL_f$.

12. The three mechanisms or modes of heat transfer are conduction, convection and radiation.

13. Experiment shows that the quantity of heat $Q$ flowing through a small part of a lagged bar of uniform cross-section of area $A$ in the steady state, in time $t$, is given as: $Q = ktA \times$ temperature gradient, where $k$ is a constant of the material called its thermal conductivity and is measured in $Wm^{-1}K^{-1}$.

14. Convection is the transfer of heat by mass motion of a fluid from one region of space to another. There are two types of convection: natural and forced convection.

15. The rate of energy radiation from a surface is found to depend on the following: surface area $A$, temperature, $T$ and nature of the surface; this dependence is described by a quantity $e$ called emissivity.

16. Stefan's law can be expressed as: $H = \frac{dQ}{dt} = \sigma AeT^4$, where $H$ is the power radiated in watts (W), $\sigma$ is the Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8}$ W/(m$^2$.K$^4$).

17. Boyle's law states that the volume of a fixed mass of gas at constant temperature is inversely proportional to the pressure: $pV =$ constant.

18. Charles's law states that the volume of a fixed mass of gas at constant pressure is directly proportional to the absolute temperature: $V \alpha T$.

19. For a process involving only infinitesimal changes, the first law of thermodynamics is:

$$dQ = dU + dW.$$

20. An adiabatic process is defined as one with no heat transfer into or out of a system; $Q = 0$. For an ideal gas changing from conditions ($p_1$, $V_1$, $T_1$) to ($p_2$, $V_2$, $T_2$) in an adiabatic process,

$p_1V_1^{\gamma} = p_2V_2^{\gamma}$, where ratio of capacities $\gamma = C_P/C_V$.

21. An isobaric process is one in which the pressure is constant. An isochoric process is one in which the volume is constant. An isothermal process is a constant-temperature process.

22. The second law of thermodynamics states that it is impossible for any self-acting machine working in a cyclical process unaided by an external agency to make heat pass from one body to another at a higher temperature – Clausius' statement of the second law.

## Self-Assessment Questions (SAQs) for Chapter 10

**10.1.** The result of an experiment shows that the resistance $R_0$ of a platinum wire at temperature $\theta$°C, measured on the scale, is given by $R_\theta = R_0(1 + a\theta + b\theta^2)$, where $a = 3.800 \times 10^{-3}$ and $b = -5.6 \times 10^{-7}$. What temperature will the platinum thermometer indicate when the temperature on the gas scale is 300°C?

**10.2.** A process which involves no exchange of heat is:
A. isothermal B. isobaric
C. isochoric D. adiabatic

**10.3.** The heat taken in when $n$ moles of a gas expands isothermally from $V_1$ and $V_2$ is:

A. $RT \ln\left(\frac{P_1}{P_2}\right)$ B. $nRT \ln\left(\frac{V_2}{V_1}\right)$

C. $nRT \ln(P_1 - P_2)$ D. $nRT \ln\left(\frac{V_1}{V_2}\right)$

**10.4.** Which of the following temperature reservoir relationships would have the highest efficiency for a carnot engine?

A. $T_{cold} = 0.90T_{hot}$ B. $T_{cold} = 0.25T_{hot}$

C. $T_{cold} = 0.5T_{hot}$ D. $T_{cold} = 0.15T_{hot}$

**10.5.** An ideal gas system expands from $1m^3$ to $3.0m^3$ at S.T.P. If the system absorbs $5.0 \times 10^4 J$ of heat in the process, what is the change in the internal energy of the system?

**10.6.** In an experiment, the voltmeter reads 12V, the ammeter reads 4.0A and the block, mass 1.0kg, rises by 16°C in 300s. Calculate the specific heat capacity of the metal.

**10.7.** In a constant flow experiment, water flows at the rate of $0.1500kgmin^{-1}$ through a tube and is heated by a heater dissipating 25.2W. The inflow and outflow water temperatures are 15.2°C and 17.4°C respectively. When the rate of flow is increased to $0.2318kgmin^{-1}$ and the rate of heating to 37.8W, the inflow and outflow temperature are unaltered. Find the specific heat capacity of water.

**10.8.** With respect to **Q10.7**, calculate the rate of loss of heat from the tube.

**10.9.** A metal of mass 0.2kg at 100°C is dropped into 0.08kg of water at 15°C contained in a calorimeter of mass 0.12kg and specific heat capacity $400Jkg^{-1}K^{-1}$. The final temperature reached is 35°C. Calculate the specific heat capacity of the metal. The specific heat capacity of water is $4200Jkg^{-1}K^{-1}$.

**10.10.** An electric kettle with a 2.0 kW heating element has a heat capacity of $400 JK^{-1}$. 1.0kg of water at 20°C is placed in the kettle. The kettle is switched on and it is found that 13 minutes later the mass of water in it is 0.5kg. Ignoring heat losses, calculate a value for the specific latent heat of vaporisation of water. (Specific heat capacity of water = $4.2 \times 10^3 Jkg^{-1}K^{-1}$.)

**10.11.** A faulty barometer tube has some air at the top above the mercury. When the length of the air column is 250mm, the reading of the mercury above the outside level is 750mm. When the length of the air column is decreased to 200mm, by depressing the barometer tube further into the mercury, the reading of the mercury above the outside level becomes 746mm. Calculate the atmospheric pressure.

**10.12.** Oxygen gas contained in a cylinder of volume $V$ of $1 \times 10^{-2}$ $m^3$ has a temperature $T$ of 300 K and a pressure $p_1$ of $2.5 \times 10^5$ Pa. After some of the oxygen is used at constant temperature, the pressure falls to $1.3 \times 10^5$ Pa. Calculate the mass of the oxygen gas used.

**10.13.** Two gases containers with volumes of $100cm^3$ and $1000cm^3$ respectively are connected by a tube of negligible volume, and contain air at a pressure of 1000mmHg. If the temperature of both vessels is originally 0°C, how much air will pass through the connecting tube when the temperature of the smaller is raised to 100°C? Give your answer in $cm^3$ measured at 0°C and 760mmHg.

**10.14.** The amount of heat required to raise the temperature of a 1kg body through 1°C is:

A. thermal energy
B. thermal capacity
C. heat lost
D. specific heat capacity

**10.15.** Calculate the quantity of heat conducted through 2 $m^2$ of a brick wall 10cm thick in 1 hour if the temperature on one side is 10°C and on the other side is 20°C. (Thermal conductivity of brick = $0.13 Wm^{-1}K^{-1}$)

# ANSWERS

## CHAPTER 1

1.1. $f = \frac{k}{l}\sqrt{\frac{T}{\mu}}$

1.2. 2, 1, 1
1.3. The expression is correct.
1.4. C
1.5. D
1.6. B
1.7. C
1.8. B
1.9. C
1.10. D
1.11. C
1.12. D
1.13. C
1.14. A
1.15. D
1.16. B
1.17. D
1.18. B
1.19. C
1.20. A
1.21. D
1.22. A
1.23. B

## CHAPTER 2

2.1. 50m/s at 36.9° south east
2.2. 100N at 110°
2.3. 14.4 units at -56.3°
2.4. C
2.5. 33.4
2.6. 100.1°
2.7. -21
2.8. B
2.9. D
2.10. A
2.11. B
2.12. B
2.13. D
2.14. A
2.15. A

## CHAPTER 3

3.1. 20 m
3.2. 17.3m/s
3.3. 20m/s
3.4. 6.5m/s, 21.5m/s
3.5. 15m/s, 35m/s
3.6. (a) 1.5m/s (b) 2 s (c) 20.1m/s, 85.7°
3.7. (i) 5 s (ii) 62.5 m (iii) 17.3m/s
3.8. 1.25m/s$^2$, 10m/s
3.9. (a) 3.33m/s$^2$ (b) 15 m
3.10. (a) 43.83 m, 16.15m/s ; 21.7° (b) 2.598 s (c) 77.9 m
3.11. $1.62 \times 10^{15}$m/s$^2$
3.12. 150m/s, -100m/s$^2$
3.13. -1.875m/s$^2$, 5.3 s
3.14. 180 m, 12 s
3.15. 261 m
3.16. 77.5m/s, 20.1 m
3.17. 34.6 m, 1.73 s
3.18. 5 m, 2 s
3.19. 225 m
3.20. 13.25 m

## CHAPTER 4

4.1. 1.35m/s, 21.4° above the $x$-axis
4.2. 0.75m/s
4.3. 200N
4.4. 2m/s, 4m/s
4.5. 4.51m/s$^2$
4.6. 8 s, 0
4.7. 2.7N
4.8. 2.40m/s$^2$
4.9. (a) 12.84N, 9.63N (b) 11.16N, 8.37N
4.10. (a) 132.89N (b) 0.63
4.11. C
4.12. B
4.13. D
4.14. A
4.15. B
4.16. B
4.17. B
4.18. A
4.19. C
4.20. A

## CHAPTER 5

5.1. 1668.8m/s, 6619.9 s
5.2. 533.1m/s$^2$, 80N
5.3. 6.02 x 10$^{24}$kg
5.4. 502.72 rev, 31.42 s
5.5. 7804.7m/s, 5289.9 s
5.6. 7729m/s, 5447.4 s, 3.47 x 10$^{10}$J
5.7. 28.7m/s
5.8. 2.45 x 10$^{-10}$J
5.9. 13.2m/s
5.10. 0.31

## CHAPTER 6

6.1. 105J
6.2. 2.6 x 10$^5$J
6.3. 4050J
6.4. 235.2 W
6.5. (a) 4500J (b) 30m/s, 4500J
6.6. (a) 135N (b) 1350J (c) 1200J
6.7. 6864N
6.8. 270J, 90kgm/s
6.9. (a) 4 s (b) 20 m (c) 10J, 10J
6.10. D

## CHAPTER 7

7.1. 3.0 rad/s, 30m/s
7.2. D
7.3. 3.16m/s
7.4. 39.4kgm$^2$, 990.4Nm
7.5. (a) 17.41J (b) 0.185kgm$^2$/s (c) 0.044m
7.6. 50Nm
7.7. 240J
7.8. (i) 2000J (ii) 200kgm$^2$/s (iii) 3.18 rev/s
7.9. (i) 8 rad/s, (ii) 25,136J
7.10. (a) 0.042J, (b) 0.03J

## CHAPTER 8

8.1. 7.2 x 10$^{-3}$ m
8.2. A
8.3. 2.65mm
8.4. 0.0714 mL

8.5. 0.047 s
8.6. $1.8 \times 10^{11} N/m^2$
8.7. (a) $3.8 \times 10^{9} N/m^2$ (b) 0.1464 (c) 0.029
8.8. (a) $6.238 \times 10^{6} Nm^{-2}$ (b) 1.02mm
8.9. $4.404 \times 10^{-7}\ m^3$
8.10. (a) $8.0 \times 10^{-4}$ (b) $2.0 \times 10^{8} N/m^2$
(c) $2.5 \times 10^{11} N/m^2$ (d) 0.156J

## CHAPTER 9

9.1. A
9.2. C
9.3. $40Nm^{-2}$
9.4. 11.25N
9.5. $2500N/m^2$
9.6. $0.0178\ m^3/s$
9.7. 0.0192m/s
9.8. 1656.16, laminar
9.9. $2.0 \times 10^{5}$ Pa
9.10. $3.16 \times 10^{-4}\ m^3$

## CHAPTER 10

10.1. 291°C
10.2. D
10.3. B
10.4. D
10.5. $1.526 \times 10^{5} J$
10.6. $900Jkg^{-1}K^{-1}$
10.7. $4200Jkg^{-1}K^{-1}$
10.8. 2.1 W
10.9. $590.8Jkg^{-1}K^{-1}$
10.10. $2.38 \times 10^{6} Jkg^{-1}$
10.11. 766mmHg
10.12. 0.0154kg
10.13. $33cm^3$
10.14. D
10.15. 93600J

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