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- The Most Comprehensive SSCE/JME Revision Text Available

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SSCE & JME

Mathematics

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Preface

This book has been written to serve as a companion to candidates preparing for the Secondary School Certificate Examination (SSCE) and the Joint Matriculation Examination (JME) in *Mathematics*. It consists of twenty-one chapters which cover the entire syllabuses of both the WASSCE and the JME. Strategies for taking the examinations are provided in the *Introduction*, while each chapter comprises:

- **Revision Notes**, highlighting basic concepts, definitions, formulae and problem-solving procedures.
- **Worked Examples** after each section, illustrating the applications of concepts discussed under the section.
- **Drills**, which are short questions designed to sharpen the candidate's skills in mathematical computations.
- **Answers to Drills**.
- **SSCE/JME-Type Objective Questions**, which are only slightly modified versions of actual past questions of the SSCE and the JME. These cover practically every type of question which has appeared in the SSCE (from 1988 to date) and the UME (from 1978 to date).
- **Answer Key** to the SSCE/JME-Type Questions.

Three Model Matriculation Examination papers are included at the end of the book which have been designed to be of the same standard as the JME papers.

The layout of the book has been carefully planned to ensure that the candidate derives maximum benefit from it even without the aid of a teacher. It is thus a comprehensive self-help study pack which guarantees an excellent performance in the examinations.

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Introduction

This revision book has been written with the sole purpose of aiding you, the candidate, in preparing for the West African Senior School Certificate and Joint Matriculation Examinations (WASSCE & JME) in Mathematics. It covers all the topics listed in the syllabuses for both of these examinations. The broad areas covered include:

- Numbers and Numeration
- Algebraic Processes
- Mensuration
- Plane Geometry
- Trigonometry
- Statistics and Probability, and
- Miscellaneous Topics (JME only).

Structure of the Examinations

The WASSCE consists of two papers:

Paper 1: a 1½ hr objective paper containing 50 multiple choice questions for a total of 50 marks.

Paper 2: a 2½ hr paper consisting of two parts, I and II. Part I contains five element questions which are compulsory for a total of 40 marks. Part II contains seven questions from which candidates are required to answer any five for a total of 60 marks.

The JME is a ¾ hr-objective paper containing 50 multiple choice questions for a total of 100 marks.

How to Use This Book

This book contains 21 chapters. Each topic covers revision notes with worked examples and drills, which are short questions designed to test the candidate's grasp of the concepts discussed. This is followed by multiple choice questions which are slightly adapted versions of actual past questions of the SSCE and the JME. To derive the maximum benefit from the book, you should adopt the following study procedure:

1. Review the concepts covered in each chapter and attempt the drill questions, for which answers are provided at the end of the notes section.
2. On completing the chapter review, attempt the *SSCE/JME Type Objective Questions*, allowing yourself roughly one minute to a question. If the chapter contains 50 questions, for instance, this should take you approximately 50 minutes to complete. If you have not finished solving all the problems at the end of the allotted time, do not stop, but go ahead and complete all the questions while noting how much extra time you have spent. This is an indication of how much faster you have to work under real examination conditions.
3. Score your answers with the *Answer Key* provided.
4. Where your answer is wrong, check under the *Solutions to Objective Questions* for the correct solution. Even where you obtain the correct answer, but only after much time-wasting effort, you should still check the solution for a possibly shorter (easier) approach.

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5. When you have completed all the chapters, and as close to the real examination as possible, attempt the three *Model Matriculation Examinations* at the end of the book. Score yourself on each paper and note your areas of weakness.

6. Return to the topics with which you have problems in (5) above, and where necessary, consult your textbook for further elaboration.

Strategies for Taking the Examinations

In addition to problem-solving strategies which you should have developed from your interactions with your mathematics teacher, the following hints should enhance your performance in the examinations:

1. **Answer the easy questions first.** In an objective paper, all questions carry equal marks even though the level of difficulty varies from question to question. It is thus a good strategy to first work quickly through the *whole* paper, answering the easy questions while saving the apparently difficult questions for later. Quite often, a question which appears difficult at first try may prove quite easy on the second run.

2. **Read the questions carefully.** It is important to make sure that you are answering the right question. If you do not read the question carefully, you may mistake an intermediate answer for the desired answer.

   Example: The angles of a triangle are given as $(x + 10)^\circ$, $(2x - 30)^\circ$ and $(3x - 10)^\circ$. The smallest angle of the triangle is

   A. 35° **B.** 40° **C.** 45° **D.** 48°

   Knowing that the angles of a triangle add up to 180°, we have $(x + 10) + (2x - 30) + (3x - 10) = 180$, or $x = 35^\circ$. Without further reflection, a candidate may pick option (A) corresponding to the calculated value of x, which has been deliberately included as a trap for the unwary candidate. To obtain the correct answer, the actual values of the angles must be worked out as $x + 10 = 45^\circ$, $2x - 30 = 40^\circ$, and $3x - 10 = 95^\circ$. Hence the smallest angle is 40°, option (B).

3. **Understand the meanings of terms used in questions.** Terms such as *integers, rational numbers, real numbers, odd* and *even* numbers, occur frequently in questions on numbers and numeration. It is important to be familiar with these terms and close attention must be paid to whether the question stipulates that such a number is positive or negative.

   Example: If x, y, and z are odd, negative integers, which of the following is necessarily an even, positive integer?

   A. xyz **B.** xy^2 **C.** $x(y + z)$ **D.** $\frac{x^2y}{z}$

   Although this question can be solved by trying out selected odd, negative integers, option (A) can be eliminated as the product of three negative numbers must give a negative number, and option (B) because y^2 must be positive and xy^2 negative. Option (D), though a positive number, will not necessarily yield an integer. The correct option is (C).

4. **Remember basic mathematical facts** such as the trigonometric ratios of standard angles (0°, 30°, 45°, 60° and 90°), laws of logarithms, etc. A recall of such facts is necessary to enable you answer certain questions correctly.

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Example: If $0° < x < 360°$, the values of x which satisfy the equation $\cos^2 x - 1 = 0$ are

A. $0°, 90°$ B. $0°, 180°$ C. $90°, 270°$ D. $0°, 270°$

The equation is solved as $\cos^2 x = 1$, or $\cos x = \pm 1$. You should know that the angle whose cosine is 1 is $0°$, while $\cos^{-1}(-1) = 180°$, $\cos 90° = \cos 270° = 0$. Hence (B) is the correct option. A list of mathematical facts with which you should be familiar is provided in the Appendix.

5. **Remember elementary mathematical identities** such as

$a^2 - b^2 = (a - b)(a + b)$, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, $\sin x = \cos(90° - x)$, $\sin^2 x + \cos^2 x = 1$, etc.

At least a few problems will require a knowledge of these identities.

Example: The value of $15.82^2 - 14.18^2$ is

A. 2.69 B. 16.40 C. 30.00 D. 49.20

Although this question can be answered correctly by actually expanding 15.82^2 and 14.18^2 and taking their difference, valuable time is saved once it is recognised that
$15.82^2 - 14.18^2 = (15.82 - 14.18)(15.82 + 14.18) = 1.64 \times 30 = 49.20$, option (D).

The identities and formulas which you should know are given in the Appendix.

6. **Set up an equation where necessary.** Word problems involving more than one variable are often more easily solved by setting up an algebraic equation in which the variables are represented by symbols.

Example: The length of a rectangular lawn is three times its breadth. If the area of the lawn is 48 m^2, its perimeter is

A. 12 m B. 24 m C. 30 m D. 32 m

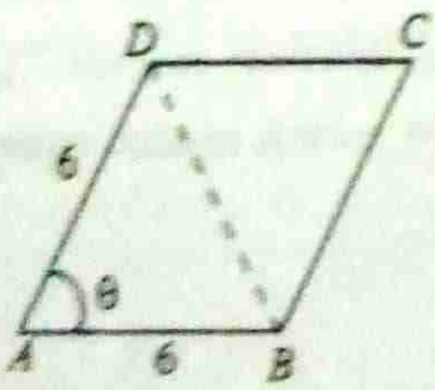
Let l, b represent the length and breadth, respectively.
Then $l = 3b$ and $lb = 48$
Thus $3b^2 = 48$, $b = 4$ m and $l = 12$ m
Perimeter $= 2(l + b) = 2(12 + 4) = 32$ cm, option (D).

7. **Sketch diagrams where necessary.** Problems on geometry and trigonometry are more easily solved with a diagram in view. If the diagram is not provided with the question, it is often helpful to sketch your own diagram.

Example: A rhombus of side 6 cm has an area of 18 cm^2. The sine of one of the angles of the rhombus is

A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. $\frac{1}{\sqrt{2}}$ D. $\frac{2}{3}$

The first step is to sketch a diagram showing the given data. The required information is $\sin\theta$. Noting that the diagonal BD bisects the rhombus into two equal areas, area of ΔABD is



9 cm^2. But this is also equal to $\frac{1}{2}(6)(6)\sin\theta$. Hence $18\sin\theta = 9$, or $\sin\theta = \frac{1}{2}$, option (C).

Diagrams are also useful when tackling problems on loci and construction (Paper 2 SSCE) for which a sketch provides a rough guide regarding the order of the steps to be taken in making the construction.

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8. **Simplify or expand algebraic expressions** through factorization and the use of algebraic identities.

Example: The reciprocal of $\frac{x+y}{\frac{x}{y}-\frac{y}{x}}$ is

A. $\frac{1}{y}-\frac{1}{x}$ **B.** $\frac{xy}{x-y}$ **C.** $\frac{x+y}{y}$ **D.** $\frac{x-y}{x}$

The reciprocal required is $\frac{\frac{x}{y}-\frac{y}{x}}{x+y}$.

Simplify the numerator to give $\frac{x^2-y^2}{xy}=\frac{(x-y)(x+y)}{xy}$

The expression becomes $\frac{(x-y)(x+y)}{xy(x+y)}=\frac{1}{y}-\frac{1}{x}$, option (A).

9. **Work backwards from the answers.** Many problems on algebraic processes do not require complete solutions of the equations provided. In this type of problem, you can obtain the solution by working backwards from the options provided.

Example: One of the roots of the equation $x^3-x^2-10x-8=0$ is

A. 2 **B.** 1 **C.** 0 **D.** –2

It is not necessary to attempt to solve the given cubic equation as the correct option can be ascertained by substituting each of the options into the equation:

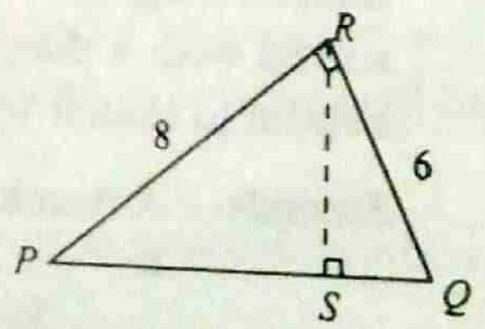
For $x=2$: $(2)^3-(2)^2-10(2)-8\neq 0$
For $x=1$: $(1)^3-(1)^2-10(1)-8\neq 0$
For $x=0$: $(0)^3-(0)^2-10(0)-8\neq 0$
For $x=-2$: $(-2)^3-(-2)^2-10(-2)-8=0$

Hence the correct option is (D).

10. **Make an educated guess by elimination.** You can improve your scores on the apparently difficult questions by making intelligent guesses through the elimination of "unreasonable" options.

Example: In the figure shown, PQR is a right-angled triangle with $P\hat{R}Q = 90^\circ$, $PR = 8$ and $QR = 6$. The length of the altitude RS is

A. 4.0 **B.** 4.8
C. 6.0 **D.** 7.5



If you do not know exactly how to approach this problem, you can at least eliminate some options once you recognise that the hypotenuse is the longest side in any right-angled triangle. Thus in ΔRSQ RS must be less than QR, which immediately rules out options (C) and (D). If you have to guess between options (A) and (B), your chance of guessing correctly is 50 percent, up from 25 percent for a completely random guess.

[The correct approach to solving the problem is to use Pythagoras theorem to find PQ in ΔPQR : $PQ^2 = 8^2 + 6^2$, or $PQ = 10$. Area of $\Delta PQR = \frac{1}{2}(8)(6) = 24$ which is also equal to $\frac{1}{2}(PQ)(RS) = 5RS$. Hence $5RS = 24$, or $RS = 4.8$. option (B)].

11. **Good Luck!**

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1 Numbers and Basic Arithmetic Operations

1.1 Definitions

- An **integer** is a whole number which could be positive (1, 2, 3 ,....), negative (–1, –2, –3, ...) or zero (0).
- An **even** number is an integer which is divisible by 2, e. g. –6, –4, 2, 6, 10 ...
- An **odd number** is an integer which is not divisible by 2, e. g. –3, –1, 3, 5, 9, ...
- A **prime number** is divisible only by 1 and by itself, e. g. 2, 3, 5, 7, 11, 17, 19, 23 ...
- A **rational number** is one which is expressed as a quotient of two integers, e. g. $^{7}/_{3}$, $^{1}/_{4}$, $^{2}/_{5}$, $^{8}/_{9}$,...
- The number Z is a **factor** of a number A if the division of A by Z results in an integer, e. g. 5 is a factor of 30 since 30 ÷ 5 is 6. Other factors of 30 are 2, 3, 6, 10 and 15.
- The **prime factors** of a number are the factors which are also prime numbers. Thus the prime factors of 30 are 2, 3, and 5.
- The **least common multiple** (LCM) of two or more numbers is the smallest multiple of the numbers, e.g. the multiples of 2 and 3 are 6, 12, 18, 24 ..., and the LCM of 2 and 3 is 6.
- The **highest common factor** (HCF) of two or more numbers is the largest factor which is common to the numbers, e.g. the numbers 16 and 40 can be written in terms of their factors as
  16 = 2 x 2 x 2 x 2, and
  40 = 2 x 2 x 2 x 5
  The HCF of both numbers is 2 x 2 x 2, or 8.

DRILL 1

(a) List the prime numbers between 20 and 50
(b) How many prime numbers are also even ?
(c) List the prime factors of
  (i) 330 (ii) 504 (iii) 1040
(d) Find the LCM of (i) 3, 5 and 6
  (ii) 4, 7 and 8
(e) Find the HCF of (i) 24, 36 and 72
  (ii) 28, 154 and 210.

1.2 Number Bases

- A number in base b is composed of integers from 0 to b– 1, i.e.
  Base 10..... (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
  Base 7 ..... (0, 1, 2, 3, 4, 5, 6)
  Base 5..... (0, 1, 2, 3, 4)
  Base 2 (binary)..... (0, 1)
- The number 4132 in base 10 means
  $4(10^3) + 1(10^2) + 3(10) + 2$
  and the same number in base b means
  $4(b^3) + 1(b^2) + 3(b) + 2$
- The number 572.43 in base 10 means
  $5(10^2) + 7(10) + 2 + 4(10^{-1}) + 3(10^{-2})$,
  while the same number in base b means
  $5(b^2) + 7(b) + 2 + 4(b^{-1}) + 3(b^{-2})$.

EXAMPLE 1-1

Convert the following numbers to base 10:

(a) 1100101_2 (b) 3154_7
(c) 538_9 (d) 1101.111_2

Solutions

(a) $1100101_2 = 1(2^6) + 1(2^5) + 0(2^4) + 0(2^3) + 1(2^2) + 1(2) + 1$
$= 64 + 32 + 0 + 0 + 4 + 0 + 1 = 101_{10}$

(b) $3154_7 = 3(7^3) + 1(7^2) + 5(7) + 4$
$= 1029 + 49 + 35 + 4 = 1117_{10}$

(c) $538_9 = 5(9^2) + 3(9) + 8 = 405 + 27 + 8$
$= 440_{10}$

(d) $1101.111_2 = 1(2^3) + 1(2^2) + 0(2) + 1 + 1(2^{-1}) + 1(2^{-2}) + 1(2^{-3})$
$= 8 + 4 + 0 + 1 + 0.5 + 0.25 + 0.125$
$= 13.875_{10}$

DRILL 2

Convert the following numbers to base 10:

(a) 101101_2 (b) 403_5 (c) 726_8
(b) 101.101_2 (e) 31.24_5

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1.3 Conversion from Base 10 to Another Base

- To convert a number from base 10 another base:
  (i) Divide the number repeatedly by the new base until the dividend is zero.
  (ii) Write the remainder after each division.
  (iii) Obtain the new number by writing the remainder in reverse order.

EXAMPLE 1-2

Convert: (a) 375_{10} to base 6
(b) 73_{10} to base 2

Solutions

(a)

6	357 rem
6	59.... 3
6	9.....5
6	1.....3
	0.....1

Hence $375_{10} = 1353_6$

(b)

2	73 rem
2	36....1
2	18....0
2	9.....0
2	4.....1
2	2.....0
2	1.....0
	0.....1

$\therefore 73_{10} = 1001001_2$

DRILL 3

Convert the following numbers:
(a) 89_{10} to base 2 (b) 37_{10} to base 2
(b) 598_{10} to base 4 (d) 7421_{10} to base 6

1.4 Conversion from One Base to Another

- To convert from one base (other than 10) to a new base, first convert the number to base 10, then to the required base.

EXAMPLE 1-3

Convert 101101_2 to base 6

Solution

$101101_2 = 1(2^5) + 0(2^4) + 1(2^3) + 1(2^2) + 0(2) + 1$
$= 45_{10}$

Now convert 45_{10} to base 6:

6	45
6	7 ...rem 3
6	1 ...rem 1
	0 ...rem 1

Hence $101101_2 = 113_6$

1.5 Determining an Unknown Base

- The unknown base is determined by writing out the number in explicit form and solving the resulting algebraic equation.

EXAMPLE 1-4

Determine b if (i) $305_b = 152_{10}$
(ii) $101_b = 122_4$

Solutions

(i) $305_b = 3(b^2) + 0(b) + 5 = 3b^2 + 5$
$3b^2 + 5 = 152$, $3b^2 = 147$
$b^2 = 49$ and $b = 7$
Hence $305_7 = 152_{10}$

(ii) $101_b = 1(b^2) + 0\,(b) + 1 = b^2 + 1$
$122_4 = 1(4^2) + 2(4) + 2 = 26$
Hence $b^2 + 1 = 26$, $b^2 = 25$ and $b = 5$
$\Rightarrow 101_5 = 122_4$

DRILL 4

Determine the value of the base x if
(i) $1000_x = 27_{10}$ (ii) $21_x = 9_{10}$

1.6 Addition and Subtraction

- Addition and subtraction of numbers in bases (b) other than 10 are performed following the same rules as for base 10, although it must be constantly borne in mind that only digits from 0 to b - 1 are allowed, e.g.
  For base 2: $1 + 1 = 10$, $1 + 1 + 1 = 11$,
  $1 + 1 + 1 + 1 = 100$, etc.
  For base 4: $2 + 2 = 10$, $3 + 3 = 12$, $3 + 3 + 2 = 20$, etc.

DRILL 5

Copy and complete the following addition table in base two:

+	1	10	11	100	101
1				101	
10		100			
11			110		
100	101				
101				1001	

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DRILL 6

Copy and complete the following addition table in base **six**:

+	2	3	4	5
2	4			
3			11	
4				
5		12		

EXAMPLE 1-5

Simplify (i) $110010_2 + 10111_2$
(ii) $110101_2 - 11011_2$
(iii) $1323_4 + 3102_4 + 2133_4$
(iv) $6325_8 - 3717_8$

Solutions

(i) 110010 (base 2)
+ 10111
1001001

(ii) 110101 (base 2)
− 11011
11010

(iii) 1323 (base 4)
+ 3102
+ 2133
13230

(iv) 6325 (base 8)
− 3717
2406

1.7 Multiplication and Division

- Multiplication and division are performed in exactly the same way as for base 10.

EXAMPLE 1-6

Perform the following operations:
(a) $11011_2 \times 101_2$ (b) $11001101_2 \div 101_2$
(c) $341_5 \times 32_5$ (d) $10025_7 \div 26_7$

Solutions

(a)
11011
x 101
11011..... 11011 x 1
00000..... 11011 x 0
11011 ..... 11011 x 1
10000111 ..... add up

(b)

```
        101001
     ----------
101 | 11001101
    -101          .....101 x 1
     010
    -000          .....101 x 0
      101
     -101         .....101 x 1
      001
    -000          .....101 x 0
     010
    -000          .....101 x 0
      101
     -101         .....101 x 1
      000
```

Hence 11001101 ÷ 101 = **101001**

(c)
341
x 32
1232 ..... 341 x 2 (base 5)
2123 ..... 341 x 3
23012 ..... add up

(d)

```
        232
     ----------
26  | 10025
    -55           ..... 26 x 2 (base 7)
     122
    -114          ..... 26 x 3
      55
     -55          ..... 26 x 2
      00
```

Hence 10025 ÷ 26 = **232**

DRILL 7

Copy and complete the following multiplication table in base **two**:

x	10	11	100	101
10				
11				
100				
101				

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1.8 Fractions

- **Proper fractions** are of the form $^1/_2$, $^3/_4$, $^7/_8$,..., in which the numerator is less than the denominator.
- **Improper fractions** are of the form $1^1/_2$, $2^1/_6$, $3^5/_8$, which consist of integers and proper fractions.
- When a simplification involving various arithmetical operations is to be performed on fractions, the order in which the operation should be carried out is easily remembered using the term "**BODMAS**", where the symbols stand for **B**racket, **O**f, **D**ivision, **M**ultiplication, **A**ddition and **S**ubtraction, respectively.

EXAMPLE 1–7

Arrange in increasing order:

$\frac{3}{4}$, $\frac{1}{2}$, $\frac{7}{10}$, $\frac{3}{5}$, $\frac{13}{20}$

Solution

Place the fractions on a common denominator (LCM) to give

$\frac{15}{20}$, $\frac{10}{20}$, $\frac{14}{20}$, $\frac{12}{20}$, $\frac{13}{20}$

The order required is thus

$\frac{1}{2}$, $\frac{3}{5}$, $\frac{13}{20}$, $\frac{7}{10}$, $\frac{3}{4}$

DRILL 8

Arrange the following in decreasing order:

$\frac{11}{15}$, $\frac{1}{3}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{7}{9}$

EXAMPLE 1–8

Simplify:

(a) $1\frac{3}{4}-3\frac{1}{3}\div 1\frac{2}{3}+2\frac{1}{2}$

(b) $\left[2\frac{3}{5}+\frac{2}{3}\right]\times\frac{5}{7}-1\frac{2}{3}$

(c) $3\frac{1}{3}-\frac{5}{16}\div\frac{1}{4}\,of\,\frac{5}{8}$

Solutions

(a) "Division" is evaluated first to give

$$1\frac{3}{4}-\frac{10}{3}\times\frac{3}{5}+2\frac{1}{2}=\frac{7}{4}-2+\frac{5}{2}$$

$$=\frac{7-8+10}{4}=\frac{9}{4}=2\frac{1}{4}$$

(b) Evaluating 'Bracket" first gives

$$\left[\frac{13}{5}+\frac{2}{3}\right]\times\frac{5}{7}-\frac{5}{3}=\frac{49}{15}\times\frac{5}{7}-\frac{5}{3}$$

"Multiplication" next gives

$$\frac{7}{3}-\frac{5}{3}=\frac{2}{3}$$

(c) Evaluate "Of" to give

$$\frac{10}{3}-\frac{5}{16}\div\frac{1}{4}\times\frac{5}{8}=\frac{10}{3}-\frac{5}{16}\div\frac{5}{32}$$

"Division" next gives

$$\frac{10}{3}-\frac{5}{16}\times\frac{32}{5}=\frac{10}{3}-2=1\frac{1}{3}$$

DRILL 9

Simplify (a) $3-8\div 2+3\times 5$

(b) $10+{}^1/_3$ of $(18-6)\div 2$

(c) $\frac{1}{3}\times 2\frac{1}{4}\div 1\frac{1}{2}-\frac{3}{4}+\frac{7}{8}$

1.9 Decimal Notation and Significant Figures

- A **decimal** is a fraction of 10 or of multiples of 10, **e.g.** 0.3 = 3/10, 0.07 = 7/100, 0.319 = 319/1000.
- A number containing a decimal is often given to a specified number of **decimal places**. This is the number of digits after the decimal point.
- To correct a given number to n decimal places:
  (i) If the $(n+1)^{th}$ digit after decimal is less than 5, drop the $(n+1)^{th}$ and higher digits.
  (ii) If the $(n+1)^{th}$ digit is equal to or greater than 5, increase the n^{th} digit by 1 and drop the $(n+1)^{th}$ and higher digits.
- The number of **significant figures** is the number of digits, counting from the left and including numbers before and after the decimal point.
- The procedure for correcting a number to a specified number of significant figures is similar to that described above for decimal places.
- Where a given number containing a decimal starts with zero, the first non–zero digit is the first significant figure.

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EXAMPLE 1–9

Write the number
(a) 7.0384 correct to 3, 2 and 1 decimal place(s)
(b) 0.00582 correct to 4, 3 and 2 decimal places.

Solutions
(a) 7.0384 is
7.038 to 3 decimal places (4^{th} digit after dec. is less than 5).
7.04 to 2 decimal places (3^{rd} digit is greater than 5).
7.0 to 1 decimal place (2^{nd} digit is less than 5).

(b) 0.00582 is
0.0058 to 4 decimal places (5^{th} digit is less than 5).
0.006 to 3 decimal places (4^{th} digit is greater than 5).
0.01 to 2 decimal places (3^{rd} digit is greater than 5).

EXAMPLE 1–10

Correct
(a) 326.847 to 4, 3 and 1 significant figure(s)
(b) 0.008751 to 3, 2 and 1 significant figure(s)
(c) 3.0064 to 4, 3 and 2 significant figures.

Solutions
(a) 326.847 is
326.8 to 4 sig. figures (5^{th} digit is less than 5)
327 to 3 sig. figures (4^{th} digit is greater than 5)
300 to 1 sig. figure (2^{nd} digit is less than 5)

(b) 0.008751 is
0.00875 to 3 significant figures (the first two zeros are not counted)
0.0088 to 2 significant figures
0.009 to 1 significant figure

(c) 3.0064 is
3.006 to 4 significant figures
3.01 to 3 significant figures
3.0 to 2 significant figures

DRILL 10

Correct 82.0519 to
(a) 3 decimal places
(b) 1 decimal place
(c) 4 significant figures
(d) 2 significant figures
(e) 1 significant figure

1.10 Decimal Division and Multiplication

- To evaluate a fraction in which the denominator contains a decimal, multiply the numerator and denominator by 10 or multiples of 10 such that the denominator becomes a whole number.

EXAMPLE 1–11

Simplify

(a) $\frac{132}{0.11}$ (b) $\frac{15.25}{0.005}$ (c) $\frac{0.091}{0.7}$

Solutions

(a) $\frac{132}{0.11} = \frac{132}{0.11} \times \frac{100}{100} = \frac{13200}{11} = 1200$

(b) $\frac{15.25}{0.005} = \frac{15.25}{0.005} \times \frac{1000}{1000} = \frac{15250}{5} = 3050$

(c) $\frac{0.091}{0.7} = \frac{0.091}{0.7} \times \frac{10}{10} = \frac{0.91}{7} = \mathbf{0.13}$

- To multiply two numbers containing decimals:
  (i) Multiply the numbers while ignoring the decimal points.
  (ii) Insert the decimal point such that the number of decimal places (counting from the right) is equal to the sum of the decimal places in the original numbers.

EXAMPLE 1–12

Simplify:
(a) 0.71×0.03 (b) 4.6×3.17
(c) $\frac{7.50 \times 0.3}{0.015 \times 2.5}$ (d) $\frac{(0.3)^3 \times 12}{(0.9)^2}$

Solutions
(a) $71 \times 3 = 213$, and the total number of decimal places is 4
$\therefore 0.71 \times 0.03 = 0.0213$

(b) $46 \times 317 = 14582$. The number of decimal places is 3
$\therefore 4.6 \times 3.17 = 14.582$

(c) Multiply numerator and denominator by 10000 to give

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$$\frac{75\times 3\times 100}{15\times 25} = 60$$

(d) This can be written as

$$\frac{0.3 \text{ x } 0.3 \text{ x } 0.3 \text{ x } 12}{0.9 \text{ x } 0.9}$$

Multiply numerator and denominator by 1000 to give

$$\frac{3 \text{ x } 3 \text{ x } 3 \text{ x } 12}{9 \text{ x } 9 \text{ x } 10} = \frac{4}{10} = 0.4$$

DRILL 11

Evaluate the following

(a) 0.02 x 0.47 (b) 380 x 0.0014
(c) $1.728 \div 0.32$ (d) $0.00325 \div (0.05)^2$

1.11 Percentage Errors

- If a = actual value of a quantity being measured and m = measured value, then,
- Error = $|a - m|$, i.e. the absolute value of $a - m$.
- The absolute value of a number, $|x|$ is the positive value of x i.e. $|7| = 7$, $|-7| = 7$, $|0.25| = 0.25$, $|-0.25| = 0.25$,etc.
- Percentage error $= \frac{\text{error}}{\text{actual value}} \text{ x } 100\% = \frac{|a-m|}{a} \text{ x } 100\%$

EXAMPLE 1–13

Determine the percentage error for

(a) a 50 cm – bar which is measured as 49.5 cm
(b) a 20 kg – weight which is measured as 20.25 kg, and
(a) a time interval of 5.0 s which is measured as 5.2 s.

Solutions

(a) Error = $|50 - 49.5| = 0.5$ cm

Percent error $= \frac{0.5}{50} \text{ x } 100 = 1.0\%$

(b) Error = $|20 - 20.25| = 0.25$ kg

Percent error $= \frac{0.25}{20} \text{ x } 100 = 1.25\%$

(c) Error = $|5.0 - 5.2| = 0.2$ s

Percent error $= \frac{0.2}{5.0} \text{ x } 100 = 4.0\%$

DRILL 12

Find the values of a, b, and c in the following table

Actual Value	Measured Value	Percent Error
25 kg	24.5 kg	a
b	15.0 cm	5.0
50.0 s	c	4.0

ANSWERS TO DRILLS

1. (a) 23, 29, 31, 37, 41, 43, 47
   (b) one – and that is the number 2
   (c) (i) 2, 5 and 11 (ii) 2, 3 and 7 (iii) 2, 5 and 13
   (d) (i) 30 (ii) 56
   (e) (i) 12 (ii) 14

2. (a) 45 (b) 103 (c) 470
   (d) 5.625 (e) 16.56

3. (a) 1011001 (b) 100101
   (c) 21112 (d) 54205

4. (i) 3 (ii) 4

5.

+	1	10	11	100	101
1	10	11	100	101	110
10	11	100	101	110	111
11	100	101	110	111	1000
100	101	110	111	1000	1001
101	110	111	1000	1001	1010

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6.

+	2	3	4	5
2	4	5	10	11
3	5	10	11	12
4	10	11	12	13
5	11	12	13	14

7.

x	10	11	100	101
10	100	110	1000	1010
11	110	1001	1100	1111
100	1000	1100	10000	10100
101	1010	1111	10100	11001

8. In descending order the fractions are

$$\frac{5}{6}, \frac{7}{9}, \frac{4}{5}, \frac{11}{15}, \frac{1}{3}$$

9. (a) 14 (b) 12 (c) $^5/_8$

10. (a) 82.052 (b) 82.1
(c) 82.05 (d) 82
(e) 80

11. (a) 0.0094 (b) 0.532
(c) 5.4 (d) 1.3

12. (a) 2.0
(b) 14.29 kg or 15.79 kg
(c) 48.0 s or 52.0 s

SSCE/JME–TYPE OBJECTIVE QUESTIONS

1. Convert 69_{10} to a number in base two
A. 1001101 **B.** 1010001
C. 1000101 **D.** 100101

2. The number 110101_2 converted to base ten is equal to
A. 4 **B.** 23 **C.** 49 **D.** 53

3. The number 101.011_2 converted to base ten is equal to
A. 2.200 **B.** 3.125 **C.** 4.250 **D.** 5.375

4. The value of $(111_2)^2$, expressed in base two is
A. 110001 **B.** 111111
C. 100011 **D.** 110011

5. The number 83_{10}, converted to base seven is
A. 641_7 **B.** 146_7 **C.** 61_7 **D.** 16_7

6. The number 635_8, converted to base five is
A. 413_5 **B.** 3213_5 **C.** 3123_5 **D.** 4132_5

7. The value of $5410_6 - 2535_6$ is
A. 2431_6 **B.** 3241_6 **C.** 3141_6 **D.** 2531_6

8. Evaluate $11010_2 - 11011_2 + 10011_2$
A. 1001_2 **B.** 11001_2 **C.** 11000_2 **D.** 10010_2

9. Evaluate $313_4 - 131_4 + 333_4$
A. 1312_4 **B.** 1121_4 **C.** 211_4 **D.** 1311_4

10. If $101010_2 = x_{10} + 1111_2$, the value of x is
A. 32 **B.** 27 **C.** 19 **D.** 13

11. If $(x_{\text{base }2})^3 = 11011_2$, the value of x is
A. 11 **B.** 10 **C.** 100 **D.** 101

12. Find y if $(y_{\text{ base }3})^2 = 301_4$
A. 11 **B.** 12 **C.** 21 **D.** 22

13. If $(1Y14)_5 = 184_{10}$, the value of Y is
A. 0 **B.** 2 **C.** 3 **D.** 4

14. If $32_x = 20_{\text{ten}}$, the value of x is
A. 4 **B.** 5 **C.** 6 **D.** 7

15. If $45_y = 100001_2$, the value of y is
A. 7 **B.** 6 **C.** 8 **D.** 9

16. If $301_{\text{base }a} = 193_{10}$, the value of a is
A. 5 **B.** 6 **C.** 7 **D.** 8

17. If $101_2 \times P_2 = 11110_2$, the value of P is

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A. 11_2 B. 10_2 C. 101_2 D. 110_2

18. Solve for y in the following equation given in base two: $11(y + 101) = 1000y$
A. 10 B. 11 C. 110 D. 111

19. Solve for x if $(5x)_7 = 37_{10}$
A. 2 B. 3 C. 4 D. 6

20. Solve for x if $404_{\text{five}} = 150_x$
A. 6 B. 7 C. 8 D. 9

21. A housewife bought five yams at W34 per yam and three oranges at W5 each where W is the currency reckoned in base six. The total amount spent by the housewife is
A. $W225_6$ B. $W252_6$ C. $W315_6$ D. $W352_6$

22. If $641_x + 156_x = 807_x$, the number base x is
A. 8 B. 9 C. 10 D. 11

23. A trader bought 132 oranges at C30 each, and sold them at C42 each, where C is the currency. If all reckonings are in base four, the profit is
A. $C231_4$ B. $C2210_4$ C. $C2310_4$ D. $C3210_4$

24. If $14_{\text{twelve}} = y_{\text{eight}}$, the value of y is
A. 20 B. 16 C. 15 D. 14

25. The prime factors of 1260 are
D. 2,5,9 B. 2,7,9 C. 2,3,5,7 D. 2,3,7,9.

26. The sum of the prime numbers between 1 and 15 is
A. 42 B. 41 C. 40 D. 39

27. The lowest common multiple of the prime numbers between 1 and 10 is
A. 210 B. 105 C. 70 D. 35

28. The sum of the prime numbers between 40 and 50, divided by the sum of the prime numbers between 1 and 6 is
A. 18.0 B. 13.1 C. 9.0 D. 8.4

29. Divide the L.C.M. of 36, 54 and 90 by their HCF
A 15 B. 20 C. 30 D. 45

30. The smallest number by which 180 can be multiplied to obtain a perfect square is
A. 2 B. 3 C. 4 D. 5

31. The smallest length of a rod which can be cut into exactly equal sections, each of either 30 cm or 36 cm in length is
A. 90 cm B. 180 cm C. 360 cm D. 540 cm

32. Which of the following fractions is less than one quarter ?
A. $\frac{14}{55}$ B. $\frac{3}{11}$ C. $\frac{21}{80}$ D. $\frac{17}{70}$

33. Arrange the following numbers in ascending order of magnitude $^5/_6$, $^{11}/_{13}$, 0.837.
A. $^5/_6$, 0.837, $^{11}/_{13}$ B. $^5/_6$, $^{11}/_{13}$, 0.837
C. $^{11}/_{13}$, 0.837, $^5/_6$ D. 0.837, $^5/_6$, $^{11}/_{13}$

34. Simplify: $3^1/_2 \div 2^1/_3 - {}^3/_4$
A. $^3/_4$ B. $1^1/_2$ C. $2^4/_{19}$ D. $5^1/_{12}$

35. Simplify $4^1/_3 - (2^1/_2 - 1^3/_5)$
A. $^7/_{30}$ B. $2^1/_2$ C. $3^1/_{10}$ D. $3^{13}/_{30}$

36. Evaluate $^1/_4 \div [^2/_3(2^7/_8 - 1 + {}^3/_4)]$
A. $^1/_{14}$ B. $^1/_7$ C. $^3/_7$ D. $1^3/_{14}$

37. Simplify $\dfrac{2^1/_4}{4 \div {}^1/_8 \text{ of } 2^2/_3}$
A. $2^3/_4$ B. $1^1/_2$ C. $^3/_{16}$ D. $^1/_8$

38. Simplify $1^1/_2 - 1 \div {}^4/_5 + {}^1/_4$
A. 0 B. $^1/_2$ C. 1 D. $1^1/_4$

39. Simplify $\dfrac{(^3/_4 - {}^2/_5) - {}^1/_3 \text{ of } {}^3/_{10}}{2 - (1/1^3/_7)}$
A. $^5/_{26}$ B $^1/_2$ C. $-{}^1/_5$ D. $^1/_4$

40. Simplify $^1/_4 + \dfrac{1}{1 - \dfrac{1}{1 - {}^1/_4}}$
A. $^1/_2$ B. $-{}^1/_2$ C. $2^3/_4$ D. $-2^3/_4$

41. The reciprocal of $\dfrac{^1/_4}{^1/_3 + {}^3/_4}$ is
A. $^3/_{13}$ B. $4^1/_3$ C. $^8/_9$ D. $1^1/_8$

42. Three girls shared a packet of sweets. The first received $^2/_5$ of the sweets, the second received $^5/_9$ of the remainder and the third got 8 sweets. How many sweets were in the packet ?
B. 20 B. 25 C. 30 D. 40

43. The number 327,036 was corrected to 327,000. Which of the following can exactly describe the degree of approximation used ?
I. to 3 significant figures
II. to 4 significant figures
III. to the nearest hundred
IV. to the nearest thousand
A. I and III only B. II and IV only
C. I, II and III only D. I, II, III and IV.

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44. The population of a school is 1,376. Express this to three significant figures
A. 1,370 B. 1,376 C. 1,380 D. 138

45. The number 1.5714286, corrected to 5 decimal places, is
A. 1.57142 B. 1.57143
C. 1.57140 D. 1.57100

46. The value of $\sqrt{42{,}000}$, correct to three significant figures is
A. 64.8 B. 648 C. 20.5 D. 205

47. The number 0.0047049, correct to three significant figures is
A. 0.005 B. 0.0047 C. 0.00470 D. 0.004705

48. Correct each of the numbers 38.7152 and 0.05381 to two significant figures and find their product, correct to two significant figures.
A. 2.1 B. 2.0 C. 1.9 D. 0.21

49. Find correct to one decimal place: $0.31744 \div 0.0407$
A. 0.8 B. 7.8 C. 7.9 D. 8.0

50. The difference between 0.003286 correct to three significant figures and 0.003286 correct to four decimal places is
A. 0.0003 B. 0.000004
C. 0.00001 D. 0.00009

51 Evaluate $\frac{4512}{0.05}$ correct to three significant figures
A. 902 B. 9020 C. 90240 D. 90200

52. Evaluate $\frac{8.25 - 3.64}{3.47 - 3.01}$
correct to two significant figures.
A. 1.002 B. 10 C. 10.02 D. 100

53. A ball has diameter 20 cm. The volume of the ball, correct to two significant figures is
A. 3,500 cm^3 B. 4,000 cm^3
C. 4,200 cm^3 D. 5,000 cm^3

54. Evaluate correct to two decimal places
$50 + \frac{1}{50} + \frac{3}{500} + \frac{25}{5000}$.
A. 50.03 B. 50.02 C. 50.20 D. 50.30

55. Evaluate, without using tables,
$$\frac{0.35 \times 0.84}{0.0105}$$
A. 0.28 B. 2.8 C. 28.0 D. 280

56. Evaluate $\frac{0.48 \times 6.3 \times 0.56}{2.8 \times 1.2 \times 1.8}$
A. 0.028 B. 0.28 C. 0.56 D. 2.8

57. Reduce each number in the following expression to two significant figures and then evaluate:
$$\frac{0.04371 \times 2.44913}{0.047512}$$
A. 2.4 B. 2.2 C. 1.8 D. 1.6

58. The length of a wall which is 3.8 m long is measured as 4.0 m. The percentage error is
A $5^5/_{19}\%$ B. 6% C. $2^1/_2\%$ D. 5%

59. The radius of a circle is given as 10 cm subject to an error of 0.2 cm. the error in the area of the circle is
A. $^1/_4\%$ B. $^1/_{50}\%$ C. 2% D. 4%

60. A carpenter made a stool with a rectangular top of dimension 24 cm x 32 cm while the order stipulates a dimension of 25 cm x 30 cm. The percentage error in the area of the top is
A. 1.2 B. 2.3 C. 2.4 D. 4.8

61. The percentage error in the perimeter of the stool top in question 60 is
A. 1.50 B. 1.82 C. 1.79 D. 2.00

62. A student who was asked to correct 0.02539 to two significant figures gave its value to two decimal places. His percentage error is
A. 20 B. $16^2/_3$ C. $15^5/_{13}$ D. 0

63. The measured length of a rod of actual length 50 cm exceeded the actual length by 5 percent. The measured length of the rod is
A. 50.5 cm B. 50.25 cm
C. 51.25 cm D. 52.5 cm

64. A bar of length 30 cm was measured as 31.7 cm. The percentage error correct to 2 significant figures is
A. 5.0 B. 5.4 C. 5.7 D. 5.67

65. The diameter of a rod is measured as 3.70 cm to three significant figures. The maximum error in the measurement is
A. 0.5 cm B. 0.05 cm
C. 0.045 cm D. 0.005 cm

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66. A student measured the length of a certain bar as 20 cm, which exceeded the actual length by 10%, The actual length of the bar was

A. 18.2 cm **B.** 19.0 cm
C. 21.0 cm **D.** 22.2 cm

ANSWER KEY

1. C	12. C	23. C	34. A	45. B	56. B
2. D	13. B	24. A	35. D	46. D	57. B
3. D	14. C	25. C	36. B	47. C	58. A
4. A	15. A	26. B	37. C	48. A	59. D
5. B	16. D	27. A	38. B	49. B	60. C
6. C	17. D	28. B	39. A	50. C	61. B
7. A	18. B	29. C	40. D	51. D	62. A
8. D	19. A	30. D	41. B	52. B	63. D
9. B	20. C	31. B	42. C	53. C	64. C
10. B	21. C	32. D	43. D	54. A	65. D
11. A	22. B	33. A	44. C	55. C	66. A

SOLUTIONS TO OBJECTIVE QUESTIONS

1. (C)

2	69
2	34 rem 1
2	17 0
2	8 1
2	4 0
2	2 0
2	1 0
2	0 1

$\therefore 69_{10} = 1000101_2$

2. **(D)** $110101_2 = 2^5 + 2^4 + 0(2^3) + 2^2 + 0(2) + 1$
$= 32 + 16 + 4 + 1 = 53$

3. **(D)** $101.011_2 = 2^2 + 0(2) + 1 + 0(2^{-1}) + 2^{-2} + 2^{-3}$
$= 4 + 1 + 0.25 + 0.125 = 5.375$

4. **(A)**
$$\begin{array}{r} 111 \\ \times\ 111 \\ \hline 111 \\ 111 \\ \underline{111} \\ 110001 \end{array}$$

$\therefore (111_2)^2 = 111_2 \times 111_2 = 110001_2$

5. **(B)**

7	83
7	11 rem . 6
7	1 4
7	0 1

Hence $83_{10} = 146_7$

6. (C) $635_8 = 6(8^2) + 3(8) + 5 = 413_{10}$

5	413
5	82 rem .3
5	16 2
5	3 1
	0 3

Hence $635_8 = 3123_5$

7. **(A)**
$$\begin{array}{r} 5410 \text{ (base 6)} \\ -\ \underline{2535} \\ \underline{2431} \end{array}$$
[As a check, $2535_6 + 2431_6 = 5410_6$]

8. **(D)**
$$\begin{array}{r} 11010 \\ +\ \underline{10011} \\ 101101 \\ -\underline{11011} \\ \underline{10010} \end{array}$$

9. **(B)**
$$\begin{array}{r} 313 \text{ (base 4)} \\ +\ \underline{333} \\ 1312 \\ -\ \underline{131} \\ \underline{1121} \end{array}$$

10. **(B)** $x_{10} = 101010_2 - 1111_2$
$= (2^5 + 2^3 + 2) - (2^3 + 2^2 + 2 + 1)$
$= 42 - 15 = 27_{10}$

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11. **(A)** $(x_2)^3 = 11011_2 = 2^4 + 2^3 + 2 + 1 = 27_{10}$
$\therefore x_2 = 3_{10} = 11_2$

12. **(C)** $(y_3)^2 = 301_4 = 3(4^2) + 1 = 49_{10}$
$y_3 = 7_{10}$, or $y = 21$ in base 3

13. **(B)** $1Y14_5 = 5^3 + Y(5^2) + 5 + 4$
$= (25Y + 134)_{10}$
$\therefore 25Y + 134 = 184$
$Y = 2$

14. **(C)** $32_x = 3x + 2 = 20_{10}$
$3x = 18$, or $x = 6$

15. **(A)** $45y = 4y + 5 = 1(2^5) + 1 = 33$
$4y = 28$, or $y = 7$

16. **(D)** $3a^2 + 1 = 193$
$3a^2 = 192$, $a^2 = 64$ and $a = 8$

17. **(D)** $P = 11110_2 \div 101_2$

$$
\begin{array}{r|l}
 & 110 \\
\hline
101 & 11110 \\
 & \underline{101} \\
 & 101 \\
 & \underline{101} \\
 & 000 \\
 & \underline{000}
\end{array}
$$

$\therefore P = 110_2$

18. **(B)** $(1000 - 11)y = 101 \times 11$
$101y = 101 \times 11$
$y = \dfrac{101 \times 11}{101} = 11_2$

19. **(A)** $5(7) + x = 37$
$35 + x = 37$, or $x = 2$

20. **(C)** $4(5^2) + 4 = 104_{10}$
$150_x = 1(x^2) + 5x$
$\therefore x^2 + 5x = 104$
$x^2 + 5x - 104 = 0$
$(x - 8)(x + 13) = 0$, or $x = 8$.

21. **(C)** The housewife spent $(34 \times 5)_6 + (5 \times 3)_6$
$= 252_6 + 23_6 = 315_6$

22. **(B)** $(6x^2 + 4x + 1) + (x^2 + 5x + 6) = 8x^2 + 7$
$7x^2 + 9x + 7 = 8x^2 + 7$
$x^2 - 9x = 0$
$x(x - 9) = 0$, or $x = 9$

23. **(C)** The trader made a profit of C12 on each orange.
Total profit on 132 oranges is $(132 \times 12)_4$
$= C2310_4$

24. **(A)** $14_{12} = 1(12)) + 4 = 16_{10}$
$16_{10} = 20_8$

25. **(C)** $1260 = 2 \times 2 \times 3 \times 3 \times 5 \times 7$
Prime factors are 2, 3, 5 and 7

26. **(B)** Prime numbers between 1 and 15 are 2, 3, 5, 7, 11 and 13, which add up to 41.

27. **(A)** Prime numbers between 1 and 10 are 2, 3, 5 and 7, with a least common multiple of $2 \times 3 \times 5 \times 7$, or 210.

28. **(B)** Prime numbers between 40 and 50 are 41, 43 and 47, with a sum of 131.
Prime numbers between 1 and 6 are 2, 3 and 5, which add up to 10.
$131 \div 10 = 13.1$

29. **(C)** $36 = 2 \times 2 \times 3 \times 3$
$54 = 2 \times 3 \times 3 \times 3$
$90 = 2 \times 3 \times 3 \times 5$
$LCM = 2 \times 2 \times 3 \times 3 \times 3 \times 5$
$HCF = 2 \times 3 \times 3$
$\therefore \dfrac{LCM}{HCF} = \dfrac{2 \times 2 \times 3 \times 3 \times 3 \times 5}{2 \times 3 \times 3} = 30$

30. **(D)** $180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$
To obtain a perfect square, it must be multiplied by 5 to give $2^2 \times 3^2 \times 5^2$.

31. **(B)** This is the *least common multiple* (*LCM*) of 30 and 36
$30 = 2 \times 3 \times 5$
$36 = 2 \times 2 \times 3 \times 3$
$LCM = 2 \times 2 \times 3 \times 3 \times 5 = 180$ cm

32. **(D)** Multiply the numerator in each case by 4.
The one which is less than 1 is (D)

33. **(A)** $^5/_6 = 0.833$; $^{11}/_{13} = 0.846$
In ascending order, the numbers are $^5/_6$, 0.837, $^{11}/_{13}$.

34. **(A)** $3^1/_2 \div 2^1/_3 - {}^3/_4 = {}^7/_2 \times {}^3/_7 - {}^3/_4$
$= {}^3/_2 - {}^3/_4 = {}^3/_4$.

35. **(D)** $4^1/_3 - (2^1/_2 - 1^3/_5) = 4^1/_3 - {}^9/_{10}$
$= 4\dfrac{10 - 27}{30} = 3\dfrac{13}{30}$

36. **(B)** ${}^1/_4 \div [{}^2/_3(2^7/_8 - 1 + {}^3/_4)]$
$= {}^1/_4 \div {}^2/_3(2^5/_8)$
$= {}^1/_4 \div {}^2/_3 \times {}^{21}/_8$
$= {}^1/_4 \div {}^7/_4 = {}^1/_4 \times {}^4/_7 = {}^1/_7$

37. **(C)** $\dfrac{2^1/_4}{4 \div {}^1/_8 \text{ of } 2^2/_3} = \dfrac{2^1/_4}{4 \div ({}^1/_8 \times {}^8/_3)}$

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$$= \frac{2\,^1/_4}{4 \div {}^1/_3} = \frac{2\,^1/_4}{4 \times 3}$$

$$= \frac{9}{4} \times \frac{1}{12} = \frac{3}{16}$$

38. **(B)** $1\,^1/_2 - 1 \div {}^4/_5 + {}^1/_4$
$= 1\,^1/_2 - {}^5/_4 + {}^1/_4$
$= {}^1/_2$.

39. **(A)** $\dfrac{({}^3/_4 - {}^2/_5) - {}^1/_3 \text{ of } {}^3/_{10}}{2 - \frac{1}{1\,^3/_7}} = \dfrac{{}^7/_{20} - {}^1/_{10}}{2 - {}^7/_{10}}$

$$= \frac{5/20}{13/10} = \frac{5}{26}$$

40. **(D)** $\dfrac{1}{4} + \dfrac{1}{1 - \frac{1}{1 - \frac{1}{4}}} = \dfrac{1}{4} + \dfrac{1}{1 - \frac{4}{3}}$

$$= \frac{1}{4} + \frac{1}{-{}^1/_3}$$
$$= 1/4 - 3 = -2\,^3/_4$$

41. **(B)** Reciprocal of $\dfrac{{}^1/_4}{{}^1/_3 + {}^3/_4}$ is

$$\frac{{}^1/_3 + {}^3/_4}{{}^1/_4} = \frac{{}^{13}/_{12}}{{}^1/_4} = \frac{13}{12} \times \frac{4}{1} = \frac{13}{3} = 4\,{}^1/_3$$

42. **(C)** First girl got $^2/_5$
Second girl got $^5/_9 \times {}^3/_5 = {}^1/_3$
Third girl got $1 - ({}^2/_5 + {}^1/_3) = 1 - {}^{11}/_{15} = {}^4/_{15}$
$\therefore {}^4/_{15}$ of the total number = 8
Total number = $8 \times {}^{15}/_4 = 30$ sweets

43. **(D)** All of the statements are correct

44. **(C)** The fourth digit is greater than 5, hence this is rounded to zero while the third digit is increased by one.

45. **(B)** Sixth digit after decimal is greater than 5, hence increase fifth digit by one.

46. **(D)** $\sqrt{42{,}000} = \sqrt{4.2 \times 10{,}000} = 100\sqrt{4.2}$
$= 100 \times 2.0494 = 204.94$
$= 205$ to three significant figures

47. **(C)** Because the number starts with zero, the first significant figure is 4, at the third decimal place.

48. **(A)** $39 \times 0.054 = 2.106$
$= 2.1$ to two significant figures.

49. **(B)** $\dfrac{0.31744}{0.0407} = \dfrac{3174.4}{407} = 7.7995$
$= 7.8$

50. **(C)** 0.003286 is 0.00329 to three significant figures and 0.0033 to four decimal places.
The difference is $0.0033 - 0.00329$
$= 0.00001$

51. **(D)** $\dfrac{4512}{0.05} = \dfrac{451200}{5} = 90240$
$= 90200$ to 3 significant figures

52. **(B)** $\dfrac{8.25 - 3.64}{3.47 - 3.01} = \dfrac{4.61}{0.46} = \dfrac{461}{46} = 10.02$
$= 10$, to two significant figures

53. **(C)** Radius of ball, $r = 10$ cm
Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 10^3$
$= \frac{88}{21} \times 10^3 = 4190 \text{ cm}^3$
$= 4200 \text{ cm}^3$ to two significant figures.

54. **(A)** The given expression is equal to
$50.00 + 0.02 + 0.006 + 0.005 = 50.031$
$= 50.03$, to two decimal places.

55. **(C)** $\dfrac{0.35 \times 0.84}{0.0105} = \dfrac{35 \times 84}{105} = 28.0$

56. **(B)** By multiplying numerator and denominator by 100000, the expression becomes
$\dfrac{48 \times 63 \times 56}{28 \times 12 \times 18 \times 100} = \dfrac{28}{100} = 0.28$

57. **(B)** To two significant figures, the expression becomes
$\dfrac{0.044 \times 2.4}{0.048} = \dfrac{44 \times 24}{480} = 2.2$

58. **(A)** % error $= \dfrac{4.0 - 3.8}{3.8} \times 100 = \dfrac{20}{3.8}$
$= \dfrac{200}{38} = \dfrac{100}{19} = 5\,^5/_{19}$

59. **(D)** With $r = 5$ cm, $A = \pi r^2 = \pi(5^2) = 25\pi \text{ cm}^2$
With $r = 5.1$ cm, $A = \pi(5.1^2) = 26.01\pi \text{ cm}^2$
% error in area $= \dfrac{26.01\pi - 25\pi}{25\pi} \times 100 = 4$

60. **(C)** Area of stool top as ordered
$= 25 \times 30 = 750 \text{ cm}^3$
Area of stool top as made by the carpenter
$= 24 \times 32 = 768 \text{ cm}^3$
% error $= \dfrac{768 - 750}{750} \times 100 = \dfrac{1800}{750} = 2.4$

61. **(B)** Perimeter as ordered $= 2(25 + 30) = 110$ cm
Perimeter as made $= 2(24 + 32) = 112$ cm
% error $= \dfrac{112 - 110}{110} \times 100 = 1.82$

62. **(A)** The correct value is 0.025
His answer is 0.03

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$$\% \text{ error} = \frac{0.03 - 0.025}{0.025} \times 100$$
$$= \frac{0.5}{0.025} = \frac{500}{25} = 20$$

63. **(D)** Actual length = 50 cm

$$\text{Measured length} = 50 + \frac{5}{100}(50) = 50 + 2.5$$
$$= 52.5 \text{ cm.}$$

64. **(C)** $\% \text{ error} = \frac{31.7 - 30}{30} \times 100 = \frac{170}{30} = 5.6666$
= 5.7, to two significant figures.

65. **(D)** The actual measurement could range from 3.695 cm (an error of 0.00 5 cm) to 3.704 cm (an error of 0.004 cm)
Maximum error = 3.70 – 3.695 = 0.005 cm

66. **(A)** Let L = actual length of the bar
$\frac{20 - L}{L} \times 100 = 10$
$20 - L = 0.1L$
$L = 20/1.1 = 18.2$ cm.

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2 Ratio, Proportion and Averages

2.1 Ratio and Proportion

- A **ratio** or proportion is used to express the relative amount of a certain quantity with respect to another quantity.
- If, for instance, there are 20 girls and 30 boys in a class, the ratio of girls to boys in that class is 2 to 3, written as 2 : 3 or $^2/_3$.
- If a certain quantity Q is divided into two parts in the ration $a : b$, the two parts are

$$\frac{a}{a+b}Q \quad \text{and} \quad \frac{b}{a+b}Q$$

e.g. dividing 35 in the ratio 2 : 5 gives

$$\frac{2}{2+5}(35) = 10, \text{ and } \frac{5}{2+5}(35) = 25$$

- If the quantity Q is divided into three parts in the ratio $a : b : c$, the parts are

$$\frac{a}{a+b+c}Q, \frac{b}{a+b+c}Q \text{ and } \frac{c}{a+b+c}Q$$

- If k is a real, non-zero number, the ratio $a : b$ is also equal to $ka : kb$, or

$$\frac{a}{b} = \frac{ka}{kb}$$

- If $\frac{a}{b} = m$ and $\frac{a}{c} = n$, then

$$\frac{b}{c} = \frac{b}{a} \times \frac{a}{c} = \frac{1}{m} \times n = \frac{n}{m}$$

EXAMPLE 2 - 1

Given that $a : b = 1 : 3$ and $b : c = 2 : 3$, what is $a : b : c$?

Solution

$\frac{a}{b} = \frac{1}{3}$ and $\frac{b}{c} = \frac{2}{3}$

$\frac{a}{c} = \frac{a}{b} \times \frac{b}{c} = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$, or

$a : c = 2 : 9$.

Since we can also write $a : b$ as 2 : 6, it follows that

$a : b : c = 2 : 6 : 9$.

EXAMPLE 2 - 2

Increase 44 in the ratio 2:5

Solution

Let the new number be x

$44 : x = 2 : 5$, or

$\frac{44}{x} = \frac{2}{5}$, $x = 110$

EXAMPLE 2 - 3

If $a : b = 1 : 3$, evaluate $\frac{4a-b}{a+b}$

Solution

$\frac{a}{b} = \frac{1}{3}$, or $\frac{b}{a} = 3$

Dividing the numerator and the denominator of the given expression by a gives

$$\frac{4a-b}{a+b} = \frac{4-b/a}{1+b/a} = \frac{4-3}{1+3} = \frac{1}{4}$$

DRILL 1

(a) Find the value of x if

(i) $3 : x = 18 : 42$ (ii) $x : 7 = 1 : 5$

(b) Divide (i) 50 in the ratio 2 : 3
(ii) 80 in the ratio 3 : 7
(iii) 72 in the ratio 3 : 4 : 5

(c) The number 120 is divided into three parts 24, x and y in the ratio 2 : 3 : 5. What are the values of x and y?

(d) Reduce 65 in the ratio 5 : 2

2.2 Rates and Averages

- When a certain quantity A changes as a result of a change in the value of another quantity B, the **rate** of change of A relative to B is expressed as

$$\frac{\text{change in } A}{\text{change in } B}$$

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- If a car, for example, covers a distance of 100 km in 2 hours, the rate of change of the distance with respect to the time, or the *speed*, is 100/2, or 50 km per hour.
- The **average** or **mean** of a set of quantities is the sum of the quantities divided by the number of quantities, e.g. the average of the numbers 2, 5, 8, 12 and 23 is

$$\frac{2+5+8+12+23}{5} = \frac{50}{5} = 10.$$

- Certain quantities do not change at a uniform rate, e.g. the distance covered by a moving car within a specified time could vary appreciably. In such cases an average speed is determined as

$$\text{Average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

EXAMPLE 2 - 4

A car burns a tankful (50 litres) of fuel within a distance of 400 km. What is the rate of fuel consumption ?

Solution

Rate of fuel consumption

$$= \frac{\text{Quantity of fuel consumed}}{\text{Distance covered}}$$

$$= \frac{50 \text{ litre}}{400 \text{ km}}$$

$$= 0.12 \text{ litre per km.}$$

EXAMPLE 2 - 5

A motorist drove from town *A* to town *B*, a distance of 40 km, in 30 minutes, stopped at *B* for 20 minutes and drove from town *B* to town *C*, 20 km away, in 25 minutes. What is his average speed for the journey ?

Solution:

Total distance covered

= distance from *A* to *C* = 40 + 20 = 60 km

Total time taken = 30 + 20 + 25 = 75 min

= 1.25 hr.

$$\text{Average speed} = \frac{60}{1.25} = 48 \text{ km per hr.}$$

DRILL 2

(a) Find the average of the following numbers:

(i) 15, 25, 32, 44, 58 and 96

(ii) 3.2, 5.1, 7.8, 9.2 and 11.7

(b) What is the average speed of a vehicle which covers a distance of 20 km at 80 km/hr and a further 10 km at 100 km/hr?

2.3 Percentages

- A percentage (%) is a ratio in which the denominator is 100, e.g.

$\frac{1}{4} = \frac{25}{100}$, written as 25 percent or 25%.

- To change a decimal number to a percentage, the decimal point is shifted two places to the right, e.g.

0.375 = 37.5%

2.187 = 218.7%

0.004 = 0.4%

- To change a fraction to a percentage, the fraction is first converted to a decimal, e.g.

$^1/_8 = 0.125 = 12.5\%$

$5^1/_4 = 5.25 = 525\%$

- When a commodity is purchased at a certain price and then sold at a different price, a **profit** or **loss** is incurred, depending on which price is higher.
- Profits and losses are usually expressed as percentages of the cost price. Thus

Profit or Loss

$$= \frac{\text{selling price} - \text{cost price}}{\text{cost price}} \times 100\%$$

EXAMPLE 2 - 6

A trader sold a crate of soft drinks for N300 and made a profit of 20 percent on the sale.

(a) What is her cost price ?

(b) What would have been her percentage loss if she sold it for N225 ?

Solution

Let s and c represent the selling and cost prices respectively

(a) Profit $(\%) = \frac{s-c}{c} \times 100 = 20$, where $s = 300$.

$$300 - c = 0.2c$$

$$c = \frac{300}{1.2} = \text{N250}$$

(b) Loss $(\%) = \frac{c-s}{c} \times 100 = \frac{250-225}{250} \times 100$

$$= 10\%$$

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DRILL 3

(a) What percentage of (i) 40 is 25 ?
(ii) 1.50 is 6 ?

(b) Find the cost price if
(i) a profit of 50% is made on a sale ot N45.
(ii) a loss of 25% is made on a sale of N1200.

(c) Find the selling price if
(i) a commodity bought for N80 is sold at 10% loss.
(ii) an item which cost N500 is sold at 30% profit.

2.4 Simple Interest

- When money is deposited into a savings account with a commercial bank, it yields interest at a certain rate. If, for instance, a sum of N500 in a savings account yields interest at the rate of 12 percent per annum, the interest at the end of one year is 12% of N500, or N60. The amount due to the depositor thus becomes N500 + N60, or N560.
- Money borrowed from a bank also gathers interest at a rate specified by the bank, so that the borrower ends up paying more than he actually borrowed.
- The simple interest (*S.I.*) on a borrowed (or lent) sum of money is calculated as

$$S.I. = \frac{PRT}{100}$$

where
P = Principal, or the money borrowed,
R = Rate of interest (*percent*),
T = Time for the which money is borrowed (in *years*).

EXAMPLE 2 - 7

A man deposits a sum of N2500 into his savings account. If simple interest is paid at the rate of 8 percent per annum, how much is due to him at the end of 3 years ?

Solution

P = N2500, R = 8 and T = 3

$$S.I. = \frac{PRT}{100} = \frac{2500 \times 8 \times 3}{100} = N600$$

Amount due = N2500 + N600 = N3100

DRILL 4

(a) Find the S.I. on a sum of N200 at the rate of 10 percent after (i) 1 yr. (ii) 6 months (iii) $2\frac{1}{2}$ years.

(b) Find the rate on a sum of N800 which yields a simple interest of N192 at the end of 2 years.

ANSWERS TO DRILLS

1. (a) (i) 7 (ii) $1\frac{2}{5}$
(b) (i) 20, 30 (ii) 24, 56 (iii) 18, 24, 30
(c) 24, 36, 60
(d) 26

2. (a) (i) 45 (ii) 7.4
(b) 85.7 km/hr

3. (a) (i) 62.5 (ii) 400
(b) (i) N30 (ii) N1600
(c) (i) N72 (ii) N650

4. (a) (i) N20 (ii) N10 (iii) N50
(b) 12 percent.

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SSCE/JME - TYPE OBJECTIVE QUESTIONS

1. If $a : b = {}^1/_4 : {}^3/_8$ and $b : c = {}^1/_3 : {}^4/_9$, then $a : c$ is
   A. 8 · 9 **B.** 9 : 8 **C.** 1 : 2 **D.** 3 : 8

2. A number of sweets were shared out among three children X, Y and Z in the ratio 2 : 3 : 4 respectively. If Z got 8, how many sweets were shared out ?
   A 12 **B.** 18 **C.** 24 **D.** 36

3. If $a : b = 3 : 4$ and $m : n = 2 : 5$, find ${}^a/_m : {}^b/_n$
   A 3 : 10 **B.** 10 : 3 **C.** 8 : 15 **D.** 15 : 8

4. A bag contains white, blue and red ping-pong balls in the ratio 8 : 3 : 2. There are 10 red balls. If 10 white and 10 blue balls are removed from the bag, the new ratio is
   A. 7 : 2 : 1 **B.** 6 : 1 : 2
   C. 6 : 1 : 1 **D.** 7 : 1 : 2

5. A sum of N50,000 was invested by two partners in a project which yielded N90,000. If the profit was shared in the ratio 3 : 5, the difference between the amounts received by the partners is
   A. N10,000 **B.** N15,000
   C. N25,000 **D.** N40,000

6. Two brothers X and Y owned a business in which the ratio of shares was 3 : 2 respectively. Y later sold ${}^1/_2$ of his share to X for N12,000. The value of the business was
   A. N24,000 **B.** N48,000
   C. N60,000 **D.** N84,000

7. A sum of N420 is shared among three children in the ratio 6 : 7 : 8. The largest share is
   A. N120 **B.** N140 **C.** N160 **D.** N180

8. The ratio of the age of a father to that of his son is 9 : 2. If the son is now 8 years old, the ratio of their ages in 4 years' time will be
   A. 9 : 4 **B.** 5 : 1 **C.** 3 : 1 **D.** 10 : 3

9. The profit on a business was shared between two partners P and Q in the ratio 3 : 4. If Q received N8,000 more than P, the profit was
   A. N56,000 **B.** N42,000
   C. N32,000 **D.** N24,000

10. Three numbers a, b and c are in the ratio 3 : 2 : 5. The value of
    $$\frac{4a - c}{a + 2b}$$ is
    A. ${}^1/_2$ **B.** $2{}^1/_3$ **C.** ${}^3/_7$ **D.** 1

11. A rectangle is formed from a square by increasing the latter's length by 50% while reducing its width by 50%. The ratio of the area of the rectangle to that of the square is
    A 4 : 3 **B.** 3 : 4 **C.** 3 : 2 **D.** 2 : 3

12. The ratio of the price of a tin of milk to that of a packet of sugar is $m : p$. If the price of milk now goes up by 25% while that of sugar goes up by 15%, the new ratio of the prices is
    A. $25m : 23p$ **B.** $23m : 25p$
    C. $5p : 3m$ **D.** $3p : 5m$

13. The baking recipe for a piece of cake is given as 1.5 cups of sugar and 3.5 cups of flour. If a cake is baked, based on this recipe with a total of 40 cups of sugar and flour, the number of cups of sugar used is
    A. 20 **B.** 15 **C.** 12 **D.** 8

14. The speed of 144 kilometres per hour, expressed in *metres per second* is
    A. 72 **B.** 40 **C.** 36 **D.** 20

15. A school has 43 students in class 3*A* and 47 students in 3*B*. The average score in a test is 55 for class 3*A* and 60 for class 3*B*. The average score for both classes, correct to one decimal place, is
    A. 57.5 **B.** 57.6 **C.** 57.8 **D.** 58.1

16. If 45% of the Nigerian naira (N) is equal to 35% of a foreign currency $, the conversion rate of $ to N is $1 =
    A. N${}^7/_9$ **B.** N$1{}^2/_7$ **C.** N$2{}^2/_9$ **D.** N$2{}^6/_7$

17. In a test taken by 40 boys and x girls, the mean scores were 5.5 and 6 respectively for the boys and girls. If the total score was 412, the value of x is
    A. 38 **B.** 36 **C.** 34 **D.** 32

18. The journey from Lagos to Ibadan usually takes a motorist 1 hour 30 minutes. By increasing his average speed by 20 km/hr., the motorist saves 15 minutes. His usual speed, in km/hr is
    A. 100 **B.** 90 **C.** 85 **D.** 80

19. A labourer's daily wage is N80 for the first 10 days and N100 for the next 9 days. Find the daily wage for the remaining 6 working days of the month, if his average daily wage for the month is N90.80.
    A. N110 **B.** N105 **C.** N95 **D.** N92.50

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20. Two men and six boys can cut a field in 3 hours. If the men work at $^3/_2$ times that of the boys, how many men are required to cut the field in 2 hrs ?
A. 4 **B**. 6 **C**. 8 **D**. 9

21. A motorist drives from P to Q at an average speed of 80 km/hr and immediately returns from Q to P through the same route at an average speed of 50 km/hr. The average speed for the round-trip journey is
A. 53.3 km/hr **B**. 60.0 km/hr
C. 62.5 km/hr **D**. 65.0 km/hr.

22. A trader sold an article for N660.00 at a profit of 20%. In order to make a profit of 30%, the article should have been sold for
A. N990 **B**. N858 **C**. N792 **D**. N715

23. X sold a radio set to Y at a profit of 10% and Y sold it for N2612.50 at a loss of 5%. The cost of the radio to X was
A. N2500 **B**. N2488 **C**. N2375 **D**. N2272

24. A grocer sells an article for N54 at a profit of 8%. What is her percentage profit or loss if she sells the same article for N44 ?
A. 12% profit **B**. 12% loss
C. 18.5% profit **D**. 18.5% loss

25. A dealer makes a profit of 20% by selling a crate of soft drinks for N600.00. The cost of the crate of soft drinks to the dealer was
A. N480.00 **B**. N500.00
C. N520.00 **D**. N540.00

26. A man bought a wrist watch for N750 and re-sold it for N600. His percentage loss was
A. 25% **B**. 22.5% **C**. 20% **D**. 15%

27. A businessman invested a total of N200,000 in two companies which paid dividends of 5% and 7% respectively. If he received a total of N11,600, how much did he invest at 7% ?
A. N160,000 **B**. N140,000
C. N100,000 **D**. N80,000

28. A worker is allowed 10% of his income tax-free and pays 15% on the remainder. If his tax amounts to N2,970, his income is
A. N28,000 **B**. N25,000
C. N22,000 **D**. N20,000

29. A worker's salary on appointment is N30,000 per annum. At an annual increment rate of 10%, his salary at the beginning of the third year is
A. N36,000 **B**. N36,300
C. N39,000 **D**. N39,930

30. After getting a pay rise of 20%, a man's new annual salary is N21,600. His salary before the pay rise was
A. N18,000 **B**. N17,700
C. N17,280 **D**. N16,680

31. A man and his wife contribute 10% and 7% of their respective incomes into a savings account. If they earn N35,000 and N22,000 per annum respectively, the total amount saved in the account after one year is
A. N4,845 **B**. N5,000 **C**. N5,040 **D**. N9,690

32. Two friends T and K went out to lunch at a restaurant. If T contributed 40% of the cost and K contributed the balance , which is N150, the cost of the lunch was
A. N375 **B**. N300 **C**. N280 **D**. N250

33. When the food bill for a family increased by 20%, the family head reduced the quantity of food bought for the family by 15%. The fractional increase in the food bill is now
A. $^7/_{20}$ **B**. $^7/_{40}$ **C**. $^1/_{20}$ **D**. $^1/_{50}$

34. To increase the surface area of a sphere by 69% the percentage increase in diameter should be
A. 69% **B**. 34.5% **C**. 30% **D**. 23%

35. A bank customer deposited N1500.00 in her bank account. At the end of five years the simple interest on the account amounted to N550. The rate per annum of interest was
A. $7^1/_3$ % **B**. 7% **C**. 5% **D**. $4^1/_2$ %

36. A sum of money invested at 6% per annum simple interest increased to N14,750 after 3 years. The sum invested was
A. N12,000 **B**. N12,500
C. N12,800 D. N13,000

37. The sum invested (question 36) will amount to N17,750 in
A. 10 yr. **B**. 8 yr. **C**. 7 yr. **D**. 6 yr.

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38. The simple interest rate per annum at which a sum of N10,000 accumulates to N12,400 in three years is
A. 8% **B**. $8^1/_2$% **C**. 9% **D**. 11%

39. How long will it take for a sum of money invested at 8% simple interest to double the original sum ?
A. 8 yr. **B**. 10.5 yr. **C**. 12 yr. **D**. 12.5 yr.

40. A sum of money invested at a simple interest rate R increases to $1^2/_3$ times the original sum after 5 years. The value of R is
A. 13.3% **B**. 10.5% **C**. 9.0% **D**. 6.0%

ANSWER KEY

1. C	**8. D**	**15. B**	**22. D**	**29. B**	**36. B**
2. **B**	9. **A**	16. **B**	23. **A**	30. **A**	37. **C**
3. **D**	10. **D**	17. **D**	24. **B**	31. **C**	18. **A**
4. **B**	11. **B**	18. **A**	25. **B**	32. **D**	39. **D**
5. **A**	12. **A**	19. **C**	26. **C**	33. **D**	40. **A**
6. **C**	13. **C**	20. **D**	27. **D**	34. **C**	
7. **C**	14. **C**	21. **A**	28. **C**	35. **A**	

SOLUTIONS TO OBJECTIVE QUESTIONS

1. (C) $\frac{a}{b} = \frac{^1/_4}{^3/_8} = \frac{1}{4} \times \frac{8}{3} = \frac{2}{3}$

$\frac{b}{c} = \frac{^1/_3}{^4/_9} = \frac{1}{3} \times \frac{9}{4} = \frac{3}{4}$

$\frac{a}{c} = \frac{a}{b} \times \frac{b}{c} = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

i.e. $a : c = 1 : 2$.

2. **(B)** Z's share = $\frac{4}{2+3+4} = \frac{4}{9}$, which is equivalent to 8 sweets
Total number of sweets = 9/4 x 8 = 18.

3. **(D)** $\frac{a}{b} = \frac{3}{4}$ and $\frac{m}{n} = \frac{2}{5}$

$\frac{a/m}{b/n} = \frac{a}{m} \times \frac{n}{b} = \frac{a}{b} \times \frac{n}{m}$

$= \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$

4. **(B)** If there are 10 red balls, the number of white and blue balls initially in the bag must be 40 and 15. (The ratio 8 : 3 : 2 is the same as 40 : 15 : 10). On removing 10 white and 10 blue balls, the number left is 30 : 5 : 10, or 6 : 1 : 2.

5. **(A)** Profit = N90,000 – N50,000 = N40,000
One partner received $^3/_8$ x 40,000 = N15,000
The second partner received N25,000
Difference = N25,000 – N15,000 = N10,000

6. **(C)** Y initially owned $^2/_5$ of the business.
$^1/_2$ of $^2/_5 = ^1/_5$, which is equivalent to N12,000 in value. Total value of the business
= 5 x N12,000 = N60,000.

7. **(C)** Largest share = $\frac{8}{6+7+8}$ x 420
$= \frac{8}{21}$ x 420 = N160

8. **(D)** The ratio 9 : 2 is also equal to 36 : 8, so the father is presently 36 years old. In another 4 years, the father will be 40 while the son will be 12 and the new ratio will be 40 : 12, or 10 : 3.

9. **(A)** Let x = profit
P received $\frac{3x}{7}$, Q received $\frac{4x}{7}$
Difference = $\frac{4x}{7} - \frac{3x}{7} = \frac{x}{7} = 8{,}000$
x = N56,000

10. **(D)** Substitute the values 3, 2 and 5 for a, b and c respectively to give
$\frac{4a-c}{a+2b} = \frac{4(3)-5}{3+2(2)} = \frac{7}{7} = 1$

11. **(B)** Let length of the square = L
Area of the square = L^2

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Length of the rectangle $= 1.5L$
Width of the rectangle $= 0.5\ L$
Area of the rectangle $= (1.5L)(0.5L)$
$= 0.75L^2$
Required ratio $= 0.75L^2 : L^2$
$= 0.75 : 1 = 3 : 4$

12. (**A**) The new ratio is $1.25m : 1.15p$,
$= 125m : 115p$, or $25m : 23p$.

13. (**C**) Ratio of sugar : flour = 1.5 : 3.5 = 3 : 7
Amount of sugar = $^3/_{10}$ x 40 = 12 cups

14. (**C**) Since 1 kilometre $= 10^3$ metres and
1 hour = 3,600 sec.
144 kilometres per hour =
$$\frac{144 \text{ kilometres}}{1 \text{ hour}} = \frac{144 \times 10^3 \text{ metres}}{3600 \text{ sec.}}$$
= 40 metres per second

15. (**B**) Total mark scored by students in class $3A$
= total number of students x average mark
= 43 x 55 = 2365
Total mark scored by students in class $3B$
= 47 x 60 = 2820
Total mark scored by both classes = 5185
Number of students in both classes = 90
Average score $= \frac{5185}{90} = 57.6.$

16. (**B**) \$0.35 = N0.45
$\$\ 1 = \text{N}\frac{0.45}{0.35} = \text{N}\frac{9}{7} = \text{N}1\frac{2}{7}$

17. (**D**) Total scores for boys = 40 x 5.5 = 220
Total score for girls = $6x$
$6x + 220 = 412$
$6x = 192$, or $x = 32$

18. (**A**) Let d = distance from Lagos to Ibadan
and s = usual speed, in km/hr
Usual time taken = $d/s = 1.5$ ..... (i)
With the speed increased by 20 km/hr,
time taken $= \frac{d}{s+20} = 1.25$ ..... (ii)
Dividing (i) by (ii) gives $(s+20)/s = 1.5/1.25$,
or $s = 100$ km/hr

19. (**C**) Let x = daily wage for the last 6 days
Total wage received
$= 80(10) + 100(9) + 6x = 1700 + 6x$
This is also equal to the average daily wage x
total number of days = N90.80 x 25 = N2270
$1700 + 6x = 2270$, or x = N95

20. (**D**) Each boy works at $^2/_3$ times the rate for each man.
6 boys put in the work equivalent of 6 x $^2/_3$,
4 men.
In 3 hours, the field can be cut by 2 men and
6 boys, or (2 + 4) men = 6 men
In 2 hours, the field can be cut by $\frac{6 \times 3}{2}$
or 9 men.

21. (**A**) Let d = distance from P to Q
Time to drive from P to Q is $t = \frac{d}{80}$ hr.
Time to drive from Q to P is $t_2 = \frac{d}{40}$ hr
Total time taken $= \frac{d}{80} + \frac{d}{40} = \frac{3d}{80}$ hr.
Average speed $= \frac{\text{total distance}}{\text{total time}} = \frac{2d}{3d/80}$
$= \frac{160}{3} = 53.3$ km/hr

22. (**D**) Let c = cost of the article
$\frac{660 - c}{c} = 20\% = 0.20$
$c = \frac{660}{1.2} =$ N550
If s = selling price for a profit of 30%,
then $\frac{s - 550}{550} = 0.3$
$s = 550 \times 1.3 =$ N715

23. (**A**) Let c_x, c_y be cost of the radio to X and to Y respectively.
$\frac{c_y - 2612.50}{c_y} = 0.05$
$c_y = \frac{2612.50}{0.95} =$ N2750 = selling price for X
$\frac{2750 - c_x}{c_x} = 0.10$
$c_x = \frac{2750}{1.10} =$ N2500

24. (**B**) Let c = cost of the article
$\frac{54 - c}{c} = 0.08$
$c = \frac{54}{1.08} =$ N50
When she sells the article for N44;
Profit = $\frac{44 - 50}{50}$
= –0.12 or –12%
(The negative sign indicates a *loss)*

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25. (**B**) $\frac{600 - c}{c} = 0.20$

$$c = \frac{600}{1.20} = \text{N}500$$

26. (**C**) $\text{Loss} = \frac{750 - 600}{750} \times 100\% = 20\%$

27. (**D**) Let x = amount invested at 7% and $200{,}000 - x$ at 5%
Dividend received at 7% $= 0.07x$
Dividend received at 5% $= 0.05(200{,}000 - x)$
$0.07x + 0.05(200{,}000 - x) = 11{,}600$
$0.02x = 1{,}600$, or $x = \text{N}80{,}000$

28. (**C**) Let x = worker's income
Taxable income $= 0.9x$
At 15%, his tax is $0.9x \times 0.15 = \text{N}2970$

$$x = \frac{2970}{0.9 \times 0.15} = \text{N}22{,}000$$

29. (**B**) His salary at the beginning of the second year is $30{,}000 + (0.10 \times 30{,}000) = \text{N}33{,}000$, and at the beginning of the third year is $33{,}000 + (0.10 \times 33{,}000) = \text{N}36{,}300$

30. (**A**) Let x = original salary
New salary $= 1.2x = 21{,}600$

$$x = \frac{21{,}600}{1.2} = \text{N}18{,}000$$

31. (**C**) Man's contribution
$= 10\%$ of $\text{N}35{,}000 = \text{N}3{,}500$
Wife's contribution
$= 7\%$ of $\text{N}22{,}000 = \text{N}1{,}540$
Total contribution
$= 3{,}500 + 1{,}540 = \text{N}5{,}040$

32. (**D**) K contributes 60% which is N150

$$\text{Total cost} = 150 \times \frac{100}{60} = \text{N}250$$

33. (**D**) As the unit cost of food increases from c to $1.2c$, the quantity is reduced from q to $0.85q$. The food bill hence increases from cq to $(1.2 \times 0.85)cq$, or $1.02cq$.
This represents a fractional increase of

$$\frac{(1.02 - 1)cq}{cq} = \frac{2}{100} = \frac{1}{50}$$

34. (**C**) Let A_o, A_f = original and final areas, and
D_o, D_f = original and final diameters
Surface area of sphere $= 4\pi r^2 = \pi D^2$
$A_o = \pi D_o^2$ and $A_f = \pi D_f^2$

$$\frac{A_f - A_o}{A_o} = 0.69 \Rightarrow \frac{A_f}{A_o} = 1.69$$

Hence $\frac{D_f^2}{D_o^2} = 1.69$, or $\frac{D_f}{D_o} = 1.3$
i.e. D_f is 30% higher than D_o.

35. (**A**) $I = \frac{PRT}{100}$,

$$R = \frac{100I}{PT} = \frac{100 \times 550}{1500 \times 5} = 7\tfrac{1}{3}\%$$

36. (**B**) $I = \frac{PRT}{100} = \frac{P \times 6 \times 3}{100} = 0.18P$
Total amount after 3 years $= P + 0.18P$
$= 1.18P$
$\therefore 1.18P = 14750$

$$P = \frac{14750}{1.18} = \text{N}12{,}500$$

37. (**C**) Interest $= 17{,}750 - 12{,}500 = \text{N}5{,}250$

$$T = \frac{100I}{PR} = \frac{100 \times 5{,}250}{12{,}500 \times 6} = 7 \text{ yr.}$$

38. (**A**) Interest $= 12{,}400 - 10{,}000 = \text{N}2{,}400$

$$R = \frac{100I}{PT} = \frac{100 \times 2{,}400}{10{,}000 \times 3} = 8\%$$

39. (**D**) When the sum invested is doubled, the interest must be equal to the original principal.

$$\therefore I = \frac{PRT}{100} = P$$

$RT = 100$, or $T = \frac{100}{R} = \frac{100}{8} = 12.5$ yr.

40. (**A**) Interest $= \frac{2}{3}P$

$$R = \frac{100I}{PT} = \frac{100 \times \frac{2}{3}P}{P \times 5} = 13.3\%$$

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3 Indices, Logarithms and Surds

3.1 Indices

- The expression a^n means a is multiplied by itself n times, e.g.

  $2^4 = 2 \times 2 \times 2 \times 2 = 16$ and $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$.

- **Laws of Indices**

		Example
(i)	$a^m \times a^n = a^{m+n}$	$2^5 \times 2^2 = 2^{5+2} = 2^7$
(ii)	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{3^5}{3^3} = 3^{5-3} = 3^2$
(iii)	$(a^m)^n = a^{mn}$	$(2^2)^3 = 2^{2\times3} = 2^6$
(iv)	$\frac{1}{a^n} = a^{-n}$	$\frac{1}{3^2} = 3^{-2}$
(v)	$a^0 = 1$	$5^0 = 1$
(vi)	$a^{1/n} = \sqrt[n]{a}$	$8^{1/3} = \sqrt[3]{8}$
(vii)	If $a^x = a^y$, then $x = y$	If $2^m = 2^n$, then $m = n$

EXAMPLE 3–1

Evaluate

(a) $9^{-1/2}$ (b) $81^{1/4}$ (c) $(1/4)^{-1/2}$

Solutions

(a) $9^{-1/2} = (3^2)^{-1/2} = 3^{-1} = \frac{1}{3}$ [Laws (iii) & iv)]

(b) $81^{1/4} = (3^4)^{1/4} = 3^1 = 3$ [Law (iii)]

(c) $(1/4)^{-1/2} = 1/(4^{-1/2}) = 4^{1/2} = \pm 2$ [Laws (iv) & vi)]

EXAMPLE 3–2

Simplify

(a) $\frac{16}{8^{-2/3}}$ (b) $\frac{27^{2/3} \times 81^{1/4}}{9^{3/2}}$

Solutions

(a) $\frac{16}{8^{-2/3}} = 16(8^{2/3}) = 2^4(2^3)^{2/3}$

$= 2^4 \times 2^2 = 2^6 = 64$ [Laws (i), (iii) & iv)]

(b) $\frac{27^{2/3} \times 81^{1/4}}{9^{3/2}} = \frac{(3^3)^{2/3} \times (3^4)^{1/4}}{(3^2)^{3/2}}$

$= \frac{3^2 \times 3^1}{3^3} = 3^0 = 1$

[Laws (i), (ii), (iii) & (v)]

EXAMPLE 3–3

Simplify

(i) $(2a^2b)^2(-a^2b)^3$ (ii) $(2xy)^2 + (-y)(4xy)(-2x)$

Solutions

(i) $(2a^2b)^2(-a^2b)^3 = (4a^4b^2)(-a^6b^3)$

$= -4a^{10}b^5$

[Laws (i) & (iii)]

(ii) $(2xy)^2 + (-y)(4xy)(-2x) = 4x^2y^2 + 8x^2y^2$

$= \mathbf{12x^2y^2}$

[Laws (i) & (iii)]

EXAMPLE 3–4

Solve for x:

(a) $3^{2x+1} = 243$ (b) $2^{x+2} = 8^{x-2}$

Solutions

(a) $3^{2x+1} = 3^5$

$2x + 1 = 5,\ x = 2$ [Law (vii)]

(b) $2^{x+2} = (2^3)^{x-2} = 2^{3x-6}$

$x + 2 = 3x - 6,\ x = 4$ [Laws (iii) & (vii)]

DRILL 1

Simpify:

(a) $27^{\frac{1}{3}}$ (b) $125^{-\frac{1}{3}}$ (c) $25^{-\frac{3}{2}}$

(d) $16^{-\frac{1}{4}}$ (e) $81^{\frac{3}{4}}$ (f) $(8^{\frac{1}{3}})^{-2}$

(g) $\left(\frac{1}{4}\right)^{-\frac{3}{2}}$ (h) $(x^3)^{-\frac{2}{3}}$

(i) $y^{1/2} \times y^{-1} \times y^{\frac{3}{2}}$ (j) $(z^0)^{1/4}$

3.2 Standard Notation

- A number is expressed in **standard form** as $a \times 10^n$ where $1 \le a < 10$ and n is an integer.
- To express a number in standard form:
  (i) Write the number as $a \times$ (multiples of 10) or $a \div$ (multiples of 10).
  (ii) Express the multiples of 10 as 10^n, meaning that 10 is multiplied by itself n times.

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- n is equal to the number of places by which the decimal point is shifted. It is positive if the decimal point is shifted to the left and negative if it is shifted to the right.

EXAMPLE 3–5

Express in standard form

(a) 7890100 (b) 0.00513 (c) 0.0000207

Solutions

(a) $7890100.0 = 7.8901 \times 1000000$
$= 7.8901 \times 10^6$
= [decimal shifted 6 places to the left]

(b) $0.00513 = 5.13/1000$
$= 5.13 \times 10^{-3}$
= [decimal shifted 3 places to the right]

(c) $0.0000207 = 2.07/100000$
$= 2.07 \times 10^{-5}$
= [decimal shifted 5 places to the right]

DRILL 2

Express in standard form:

(a) 3,617,000 (b) 0.0003617 (c) $\frac{1}{10,000}$

(d) 40 (e) 0.2 (f) 5

3.3 Logarithms

- The **logarithm** of a number to a certain base is the power to which the base must be raised to obtain the number, i.e.

$$\text{If } \log_b N = x, \text{ then } N = b^x$$

EXAMPLE 3–6

Evaluate

(a) $\log_{10}1000$ (b) $\log_3 81$

(c) $\log_2 0.125$ (d) $\log_5 0.2$

Solutions

(a) Let $\log_{10}1000 = x$, then $10^x = 1000 = 10^3$, and $x = 3$.

(b) Let $\log_3 81 = x$, then $3^x = 81 = 3^4$, and $x = 4$.

(c) Let $\log_2 0.125 = x$, then $2^x = 0.125 = \frac{1}{8} = 2^{-3}$, and $x = -3$.

(d) Let $\log_5 0.2 = x$, then $5^x = 0.2 = \frac{1}{5} = 5^{-1}$, and $x = -1$.

DRILL 3

Evaluate:

(a) $\log_5 5$ (b) $\log_2\left(\frac{1}{2}\right)$

(c) $\log_4 64$ (d) $\log_{10}\sqrt{10}$

(e) $\log_3\left(\frac{1}{81}\right)$ (f) $\log_7 343$

- **Laws of Logarithms**

(i) $\log_b xy = \log_b x + \log_b y$
e.g. $\log_{10} 72 = \log_{10}(10 \times 7.2)$
$= \log_{10}10 + \log_{10} 7.2$

(ii) $\log_b(x/y) = \log_b x - \log_b y$
e.g. $\log_{10}0.3 = \log_{10}(3/10)$
$= \log_{10}3 - \log_{10}10$

(iii) $\log_b x^m = m\log_b x$
e.g. $\log_{10}10^3 = 3\log_{10}10$

(iv) $\log_b a = \log_k a/\log_k b$
e.g. $\log_3 7 = \log_{10}7/\log_{10}3$

(v) If $\log_b x = \log_b y$ then $x = y$
e.g. If $\log_{10}8 = \log_{10}N$,
then $N = 8$

(vi) $\log_b b = 1$
e.g. $\log_{10}10 = 1$

(vii) $\log_b 1 = 0$
e.g. $\log_{10}1 = 0$

EXAMPLE 3–7

Simplify:

(a) $\frac{\log\sqrt{27}}{\log 9}$

(b) $\log_{10}\sqrt{20} + \log_{10}5 - \log_{10}\sqrt{5}$

(c) $\log_5 12 - \log_3 27$

Solutions

(a) $\frac{\log\sqrt{27}}{\log 9} = \frac{\log 27^{1/2}}{\log 9} = \frac{\log 3^{3/2}}{\log 3^2}$
$= \frac{\frac{3}{2}\log 3}{2\log 3} = \frac{3}{4}$

(b) $\log_{10}\sqrt{20} + \log_{10}5 - \log_{10}\sqrt{5}$
$= \log_{10}(2\sqrt{5}) + \log_{10}5 - \log_{10}\sqrt{5}$
$= \log_{10}\left[\frac{2\sqrt{5} \times 5}{\sqrt{5}}\right] = \log_{10}10 = 1$

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(c) $\log_5 125 - \log_3 27$

$= \log_5 5^3 - \log_3 3^3$

$= 3\log_5 5 - 3\log_3 3$

$= 3 - 3 = 0$

[Note that $\log_5 5 = \log_3 3 = 1$, Law (vi)]

EXAMPLE 3–8

Evaluate x if

(a) $\log_{10} 10^x = 3x + 2$

(b) $\log_4 (x + 3)(x - 3) = 2$

Solutions

(a) $10^{3x+2} = 10^x$ (or $x \log_{10} 10 = 3x + 2$)

$3x + 2 = x$, or $x = -1$

(b) $\log_4 (x + 3)(x - 3) = 2$

$(x^2 - 9) = 4^2 = 16$

$x^2 = 25$, or $x = \pm 5$.

DRILL 4

Simplify

(a) $\log_{10} 25 + \log_{10} 8 - \log_{10} 2$

(b) $\log_2 8 - \log_2 4$

(c) $\log_4 2 - \log_{25} 5$

(d) $\dfrac{\log_{10} 64}{\log_{10} 16}$

(e) $2\log_6 9 - \log_6 54 + \log_6 24$

(f) $\log_3 9 + \log_3 15 - \log_3 5$

3.4 Table of Logarithms

- The table of logarithms is used to find the logarithms of numbers to base 10.
- The log of a number consists of the **characteristic** and the **mantissa**, e.g.

$\log 36.74 = 1.5652$

characteristic — mantissa

- To find the log of a number such as 36.74, the number is written in standard form:

$36.74 = 3.674 \times 10^1$

$\log 36.74 = \log 10 + \log 3.674$

$= 1 + \log 3.674 = 1.5652$

where the mantissa is read directly off the log tables.

Thus $\log 367.4 = \log(10^2 \times 3.674)$

$= 2 + \log 3.674 = 2.5652.$

Also $\log 3674 = \log(10^3 \times 3.674)$

$= 3 + \log 3.674 = 3.5652.$

- If the number whose log is to be found is less than 1, the *characteristic is a negative number* while the *mantissa is always positive*, e.g.

$\log 0.03674 = \log(10^{-2} \times 3.674)$

$= -2 + \log 3.674 = \bar{2}.5652$

$\log 0.3674 = \log(10^{-1} \times 3.674)$

$= -1 + \log 3.674 = \bar{1}.5652$

[Note that the number $\bar{1}.5652$ means $-1 + 0.5652$, and **not** -1.5652].

- The antilog of a number X is the number whose log is X.

If N is the antilog of the number 1.5652, then $\log_{10} N = 1.5652$, or $N = 10^{1.5652} = 10^1 \times 10^{0.5652} = 10 \times 10^{0.5652}$. The term $10^{0.5652}$ is the antilog of 0.5652, which is read directly off the antilog table as 3.674.

Hence $N = 10 \times 3.674 = 36.74$

Similarly, antilog of $2.5652 = 10^2 \times 3.674 = 367.4$ and antilog of $3.5652 = 10^3 \times 3.674 = 3674$.

- A number having a negative characteristic has an antilog which is less than 1.0, e.g. antilog of $\bar{2}.5652 = 10^{-2} \times 3\ 6754 = 0.03674$ and antilog of $\bar{3}.5652 = 10^{-3} \times 3.674 = 0.003674$.
- The following rules are followed in performing arithmetical operations involving logarithms:

(i) If two numbers are to be multiplied, their logarithms are added. The antilog of the resulting number is the answer desired.

(ii) If a number is used to divide another number, the logarithm of the divisor is subtracted from that of the dividend.

(iii) If a number is to be raised to a power, the logarithm of the number is multiplied by the power.

(iv) If the nth root of a number is to be found, the logarithm of the number is divided by n.

(v) To find the sum or the difference of two logarithms, one (or both) of which has a negative characteristic, the characteristics should be added separately and the mantissas separately,

e.g. $\bar{2}.7261 + 1.3474 = (\bar{2} + 1) + (0.7261 + 0.3474)$

$= \bar{1} + (1.0735) = 0.0735.$

EXAMPLE 3–9

If $\log x = \bar{1}.2014$ and $\log y = 2.8451$ find

(a) $\log x + \log y$ (b) $\log x - \log y$

(c) $\log y - \log x$ (d) $\frac{1}{4} \log x$

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Solutions

(a)

		Explanation
	$\bar{1}.2014$	
(+)	$\underline{2.8451}$	$\bar{1}.2014 + 2.8451$
	$\underline{2.0465}$	$= (\bar{1} + 2) + 1.0465$
		$= 2.0465$

(b)

		Explanation
	$\bar{1}.2014$	
(–)	$\underline{2.8451}$	$\bar{1}.2014 - 2.8451$
	$\underline{\bar{4}.3563}$	$= (2 + 1.2014) - (2 + 0.8451)$
		$= (\bar{2} - 2) + (1.2014 - 0.8451)$
		$= \bar{4}.3563$

(c) $\log y - \log x = 2.8451 - \bar{1}.2014$

$= 2 + 0.8451 - (\bar{1} + 0.2014)$

$= 3.6437$

(d) $\frac{1}{4}\log x = \frac{1}{4} \times \bar{1}.2014$

$= \frac{1}{4} \times (\bar{4} + 3.2014)$

$= \bar{1} + 0.8004$

$= \bar{1}.8004$

EXAMPLE 3–10

Evaluate using log tables:

(a) 0.0754×8.3120 (b) $0.00384 \div 0.5167$

(c) $\sqrt[3]{0.0326}$ (d) $(0.7401)^4$

(e) $\dfrac{(0.8233)^4 \times 7.152}{69.021}$

Solutions

(a)

No.		Log
0.0754		$\bar{2}.8774$
x 8.3120	(+)	0.9197
0.6267		$\bar{1}.7971$

Ans: 0.6267

(b)

No.		Log
0.00384		$\bar{3}.5843$
÷ 0.5167	(–)	$\bar{1}.7132$
0.007432		$\bar{3}.8711$

Ans: 0.007432

(c)

No.	Log
$0.0326^{1/3}$	$\bar{2}.5133 \div 3$
0.3195	$= \bar{1}.5044$

Ans: 0.3195

(d)

No.	Log
$(0.7401)^4$	$\bar{1}.8693 \times 4$
0.3000	$= \bar{1}.4772$

Ans: 0.3000

(e)

No.	Log
$(0.8233)^4$	$\bar{1}.9156 \times 4$
	$= \bar{1}.6624$
x 7.152	(+) 0.8544
	0.5168
÷ 69.021	(–) 1.8389
0.04763	$\bar{2}.6779$

Ans: 0.04763

DRILL 5

If $\log A = \bar{2}.1784$ and $\log B = 1.8321$, find

(a) $\log A + \log B$ (b) $\log A - \log B$

(c) $\log B - \log A$ (c) $\frac{1}{3}\log A$

(e) $\frac{1}{4}\log A - \log B$ (f) $\log B + 4 \log A$

3.6 Surds

- A **surd** is a root of a rational number, e.g. $\sqrt{2}$, $\sqrt{7}$.
- An expression containing a surd can be simplified by first factorizing the number inside the root sign, where it is factorizable.
- If a and b are real (positive) numbers, then

  $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

  e.g. $\sqrt{15} = \sqrt{3 \times 5} = \sqrt{3} \cdot \sqrt{5}$, and $\sqrt{\frac{16}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3}$
- Expressions in which both the numerator and the denominator contain surds are simplified by rationalizing the denominator. This is done by multiplying both numerator and denominator by the same factor.

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- The difference of two squares: $a^2 - b^2 = (a+b)(a-b)$ is often found useful in solving problems on surd.

EXAMPLE 3–13

Simplify

(a) $5\sqrt{27} - 3\sqrt{12} - 4\sqrt{3}$ (b) $\dfrac{3}{2-\sqrt{2}}$

(c) $\dfrac{1}{\sqrt{2}-1} - \dfrac{1}{\sqrt{2}+1}$

Solutions

(a) $5\sqrt{27} - 3\sqrt{12} - 4\sqrt{3}$

$= 5(\sqrt{9} \cdot \sqrt{3}) - 3(\sqrt{4} \cdot \sqrt{3}) - 4\sqrt{3}$

$= 5(3\sqrt{3}) - 3(2\sqrt{3}) - 4\sqrt{3}$

$= 15\sqrt{3} - 6\sqrt{3} - 4\sqrt{3}$

$= (15 - 6 - 4)\sqrt{3}$

$= 5\sqrt{3}$

(b) $\dfrac{3}{2-\sqrt{2}} = \dfrac{3}{2-\sqrt{2}} \cdot \dfrac{2+\sqrt{2}}{2+\sqrt{2}}$

$= \dfrac{3(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})} = \dfrac{3(2+\sqrt{2})}{4-2}$

$= \dfrac{3}{2}(2+\sqrt{2}) = 3 + \dfrac{3\sqrt{2}}{2}$

(c) $\dfrac{1}{\sqrt{2}-1} - \dfrac{1}{\sqrt{2}+1}$

$= \dfrac{\sqrt{2}+1-(\sqrt{2}-1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$= \dfrac{\sqrt{2}+1-\sqrt{2}+1}{2-1} = 2$

DRILL 6

If $\sqrt{2} = 1.414$ and $\sqrt{3} = 1.732$ evaluate without using tables

(a) $\dfrac{3}{\sqrt{3}}$ (b) $\sqrt{4.5}$ [Hint: write as $\sqrt{\frac{9}{2}}$]

(c) $\sqrt{50}$ (d) $\sqrt{12} + \sqrt{27}$

(e) $\sqrt{48} - \dfrac{6}{\sqrt{3}}$ (f) $2\sqrt{3} \times 5\sqrt{6}$

(g) $(5\sqrt{7})^2$ (h) $\sqrt{3}(\sqrt{12} - \sqrt{3})$.

ANSWERS TO DRILLS

1. (a) 3 (b) $\frac{1}{5}$ (c) $\frac{1}{125}$ (d) $\frac{1}{2}$ (e) 27
   (f) $\frac{1}{4}$ (g) $\frac{1}{8}$ (h) $\frac{1}{x^2}$ (i) y (j) 1

2. (a) 3.617×10^6 (b) 3.617×10^{-4} (c) 1×10^4 (d) 4×10^1
   (e) 2×10^{-1} (f) 5×10^0

3. (a) 1 (b) –1 (c) 3
   (d) $\frac{1}{2}$ (e) –4 (f) 3

4. (a) 2 (b) 2 (c) 0
   (d) $\frac{3}{2}$ (e) 2 (f) 3

5. (a) 0.0105 (b) $\bar{4}.3463$ (c) 3.6537
   (d) $\bar{1}.3928$ (e) $\bar{3}.7125$ (f) $\bar{6}.5457$

6. (a) 1.732 (b) 2.121 (c) 7.07 (d) 8.66
   (e) 3.464 (f) 42.42 (g) 175 (h) 3

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SSCE/JME–TYPE OBJECTIVE QUESTIONS

1. Simplify: $\frac{9^{-1/2} \times 81^{3/4}}{9^{1/2}}$

   A. $\frac{1}{9}$ **B.** $\frac{1}{3}$ **C.** 3 **D.** 9

2. Simplify: $2^{-3} \times 16^{3/4} \times 2^0$

   A. 0 **B.** 1 **C.** 2 **D.** $\frac{1}{2}$

3. Which of the following is equal to $\frac{72}{49}$?

   A. $2^2 \times 3^3 \times 7^{-2}$ **B.** $2^3 \times 3^2 \times 7^{-2}$
   C. $2^3 \times 3^3 \times 7^2$ **D.** $2^3 \times 3^3 \times 7^{-2}$

4. The value of x which satisfies the equation $\frac{9^{2x}}{9^{3x}} = 3$ is

   A. $-\frac{1}{2}$ **B.** –2 **C.** $\frac{1}{2}$ **D.** 2

5. Find the value of x if $4^{(x-1/2)} = 2^{x^2}$

   A. $\frac{1}{2}$ **B.** 1 **C.** 2 **D.** 3

6. Solve the equation $3x^{-2} = \frac{4}{27}$

   A. $\frac{4}{81}$ **B.** $\frac{81}{4}$ **C.** $\frac{2}{9}$ **D.** $\frac{9}{2}$

7. Evaluate $\frac{16^{3/4} - 8^{1/3}}{2^2 \times 3}$

   A. $\frac{1}{3}$ **B.** $\frac{1}{6}$ **C.** $\frac{1}{2}$ **D.** 2

8. Evaluate: $\frac{16^{-1/2}}{64^{2/3}}$

   A. $\frac{1}{64}$ **B.** $\frac{1}{8}$ **C.** $\frac{1}{4}$ **D.** $\frac{1}{2}$

9. Evaluate: $81^{-1/4} \times 36^{1/2} \times 10^0$

   A. 0 **B.** 1 **C.** 2 **D.** 20

10. Simplify: $4 \times 4^{-n} \times 2^{2n-2}$

    A. 2^n **B.** 2 **C.** 2^{n-1} **D.** 1

11. Simplify: $(0.064)^{-1/3}$

    A. 2 **B.** $2\frac{1}{2}$ **C.** 4 **D.** $\frac{1}{4}$

12. Simplify: $\sqrt{\frac{54a^{-4}}{6a^{-8}}}$

    A. $3a^{-2}$ **B.** $3a^4$ **C.** $3a^2$ **D.** $9a^4$

13. The expression $\left(3\frac{3}{8}\right)^{-2/3}$ is equal to

    A. $\frac{4}{9}$ **B.** $\frac{2}{3}$ **C.** $\frac{2}{3}$ **D.** $\frac{9}{4}$

14. Solve for x: $5(2^{x-3}) = 160$

    A. 6 **B.** 7 **C.** 8 **D.** 10

15. Solve for x: $27^{x/2} = 3^{3/4} \times 9^{-1/4}$

    A. 6 **B.** $\frac{1}{6}$ **C.** 3 **D.** $\frac{1}{3}$

16. If $\left(\frac{2}{3}\right)^{a-1} = \frac{27}{8}$, the value of a is

    A. –2 **B.** –4 **C.** 2 **D.** 4

17. Find n if $9^{1+n} \times 3 = 27^{-n}$

    A. $-1/3$ **B.** $1/3$ **C.** $3/5$ **D.** $-3/5$

18. Simplify: $\sqrt[3]{(729y^{-6})^{1/2}}$

    A. $\frac{1}{3y}$ **B.** $3y$ **C.** $\frac{y}{3}$ **D.** $\frac{3}{y}$

19. Evaluate: $\frac{6^{2/3} \times 8^{1/3}}{12^{2/3}}$

    A. $\frac{1}{3}$ **B.** 3 **C.** $\sqrt[3]{2}$ **D.** $\sqrt[3]{6}$

20. Without using tables, evaluate $(0.027)^{-1/3} \times (0.09)^{3/2}$

    A. 0.9 **B.** 0.3 **C.** 0.03 **D.** 0.09

21. Simplify: $\frac{2^x - 2^{x-1}}{2^3 \times 2^x - 8 \times 2^{x-1}}$

    A. $1/8$ **B.** $1/4$ **C.** 2^x **D.** 2^{x-2}

22. If $\left(\frac{3}{4}\right)^x \left(\frac{2}{3}\right)^y = \frac{32}{27}$, the values of x and y are respectively

    A. 2, 2 **B.** 1, 2 **C.** –2, 1 **D.** 2, –2

23. Simplify: $\frac{3^{2a+1} \times 9^a \times 4^{2a}}{18^a \times 2^a \times 6^{2a}}$

    A. 3^a **B.** 2^a **C.** 1 **D.** 3

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24. Evaluate $\left(3^0 - 9^{-1/2}\right)^{-3}$

A. $\frac{3}{2}$ **B.** $\frac{27}{8}$ **C.** $\frac{2}{3}$ **D.** $\frac{1}{9}$

25. Which of the following is not a factor of $13^4 - 5^4$?

A. 26 **B.** 32 **C.** 97 **D.** 144

26. Solve for n if $\sqrt{5^n} = \sqrt[3]{25}$

A. $\frac{3}{4}$ **B.** $\frac{4}{3}$ **C.** $\frac{2}{3}$ **D.** $\frac{1}{2}$

27. Solve for m: $9^{2m+1} = \frac{81^{m+2}}{3^m}$

A. 2 **B.** 4 **C.** 6 **D.** –4

28. Solve for n: $4^{n-1} \times 5^{2n-2} \times 10^n = 1$

A. $\frac{2}{3}$ **B.** $\frac{3}{4}$ **C.** 1 **D.** $\frac{5}{4}$

29. Express $\frac{3.50}{0.0025}$ in standard form

A. 1.4×10^{-3} **B.** 1.4×10^2
C. 1.4×10^3 **D.** 1.4×10^4

30. Simplify: $\frac{(2.50 \times 10^4) \times (2.0 \times 10^{-3})}{12.5 \times 10^5}$

A. 4.0×10^{-4} **B.** 4.0×10^{-5}
C. 5.0×10^{-4} **D.** 5.0×10^{-3}.

31. The product of 0.0022 and 0.017 expressed in standard form is

A. 3.74×10^{-3} **B.** 3.74×10^{-4}
C. 3.74×10^{-5} **D.** 3.74×10^{-6}

32. Evaluate $\frac{0.0035}{0.210}$, leaving your answer in standard form.

A. 1.67×10^{-3} **B.** 1.67×10^{-2}
C. 1.67×10^{-1} **D.** 1.67×10^2

33. Evaluate $\sqrt[3]{\frac{0.0024 \times 35000}{0.0105}}$, leaving your answer in standard form.

A. 2×10^1 **B.** 2×10^2
C. 1×10^2 **D.** 1×10^1

34. A 500-page book, without its covers, is 12 mm thick. The thickness of each leaf of the book is

A. 2.4×10^{-5} m **B.** 4.8×10^{-5} m
C. 2.4×10^{-4} m **D.** 4.8×10^{-4} m

35. Evaluate $(0.05)^3 - (0.03)^3$

A. 9.8×10^{-5} **B.** 9.8×10^{-6}
C. 9.8×10^{-7} **D.** 9.8×10^{-8}

36. Evaluate $\log_3 27 + \log_3 6 - \log_3 54$

A. 0 **B.** $\log_3 2$ **C.** $\frac{1}{3}$ **D.** 1

37. If $\log_{10} 2 = 0.3010$, the number whose logarithm is $\bar{2}.9030$ is

A. 0.06 **B.** 0.006 **C.** 0.008 **D.** 0.08

38. If $a = x^{1/n}$, $\log_a x^n$ is

A. n^2 **B.** n **C.** n^{-1} **D.** n^{-2}

39. If $2\log_4 p + \log_4 q^2 = 0$, then p is given by

A. $2/q$ **B.** q **C.** $q/2$ **D.** $1/q$

40. Evaluate $\frac{\log_{10} 0.04 + \log_{10} 0.25}{\log_5 100 - \log_5 4}$

A. 1 **B.** –1 **C.** 2 **D.** –2

41. The value of $4^{3/2} + \log_5 0.04 + \log_3 81$ is

A. 2 **B.** 8 **C.** 10 **D.** 12

42. If $\log_{10} x = 0.5$, the value of x is

A. 5.0 B. $\sqrt{10}$ C. $\sqrt{5}$ D. 0.05

43. Find the value of x if
$4\log x + 5\log x - 7\log x = \log 16$

A. 2 **B.** 4 **C.** 8 **D.** 16

44. Express x in terms of p and q if
$\log p = 0.5 \log x + 2\log q$

A. p^2/q^4 **B.** p/q^2 **C.** pq^2 **D.** $\sqrt{p}/q$

45. Solve for y if $5\log(y+3) = \log 32$

A. –1 **B.** 0 **C.** 1 **D.** 2

46. Evaluate $\log(0.75)^3$ if $\log 7.5 = 0.875$

A. 2.625 **B.** 0.625 **C.** $\bar{1}.625$ **D.** $\bar{2}.625$

47. Solve for a: $2\log_8 a - 5\log_8 a = 2$

A. 4 **B.** 2 **C.** $\frac{1}{2}$ **D.** $\frac{1}{4}$

48. Find x if $\log_2 x = \log_9 3$

A. 2/3 **B.** 1/2 **C.** $\sqrt{2}$ **D.** 4

49. Find the value of x if $\log_3 x^2 = -4$

A. 1/9 **B.** 3/4 **C.** 9/16 **D.** $1/\sqrt{3}$

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50. If $\log_{10}4 = 0.6021$, evaluate $\log_{10}40 + 3\log_{10}400$
A. 7.4084 **B.** 8.4084
C. 9.4084 **D.** 10.4084

51. Using the information in question 50, the expression $\log_{10}40 - 3\log_{10}400$ is equivalent to
A. –6.2042 **B.** –7.7958
C. –6.7958 **D.** –7.2042

52. Given that $\log_b 2 = 0.693$ and $\log_b 5 = 1.609$, evaluate $\log_b 62.5$
A. 1.016 **B.** 2.202 **C.** 4.134 **D.** 5.520

53. Evaluate $\log_3 324 - 2\log_3 6$
A. 1 **B.** 2 **C.** 3 **D.** 4

54. Find the value of x if $\frac{1}{2}\log_{10}x = 1$
A. $\sqrt{10}$ **B.** $1/\sqrt{10}$ **C.** 100 **D.** 0.01

55. Find the value of ab if $a = \log_2 3$ and $b = \log_3 8$
A. 2 **B.** 3 **C.** 4 **D.** 8

56. If $\log_{10}6 = 0.778$ and $\log_{10}3 = 0.477$, evaluate $\log_{10}54$
A. 1.732 **B.** 1.301 **C.** 2.732 **D.** 2.301

57. Evaluate $\log_{10}\sqrt{40} + \log_{10}\sqrt{2} - \log_{10}\sqrt{8}$
A. 0.5 **B.** 1 **C.** 2 **D.** $\sqrt{10}$

58. Solve for x: $\log_{10}(3x - 4) - \log_{10}2 = 1$
A. 5 **B.** 6 **C.** 7 **D.** 8

59. Evaluate $\dfrac{\log_3 27}{\log_3 \frac{1}{9}}$
A. –3/2 **B.** –2/3 **C.** 1/5 **D.** 5

60. Find the value of a if $\log a - \log(a - 1) = \log 3$
A. 0 **B.** 1 **C.** 1.5 **D.** 3

61. Evaluate $\dfrac{\log 9 - \log 4}{\log\sqrt{3} - \frac{1}{2}\log 2}$
A. 1/4 **B.** 4 **C.** 2 **D.** 1/2

62. Evaluate $\log_{10}0.225$ if $\log_{10}3 = 0.477$ and $\log_{10}5 = 0.699$
A. –0.352 **B.** –0.648 **C.** –1.352 **D.** –1.648

63. Solve for n: $\log_2 5 + \log_2 6 - \log_2 n = 1$
A. 3 **B.** 10 **C.** 15 **D.** 30

64. Find the value of x if $\log_4 x = 3.5$
A. 256 **B.** 128 **C.** 64 **D.** 14

65. Evauate $\dfrac{\log_2 8 - \log_{1/3} 81}{\log_4 \frac{1}{64}}$
A. –7/3 **B.** 7/3 **C.** –3/2 **D.** 2/3

66. If $\log_{10}3 = 0.4771$, and $10^x = 0.3$, find x
A. –1.4771 **B.** –0.5229
C. 0.4771 **D.** 0.5229

67. Simplify $\log_5 25^x - \log_5(0.04)$
A. $2x + 1$ **B.** $2x + 2$
C. $x/2$ **D.** $2x - 2$

68. If $\log_{10}7 = 0.8451$, find the value of $\log_{10}\left(\frac{1}{70}\right)$
A. $\bar{1}.8451$ **B.** $\bar{2}.8451$ **C.** $\bar{1}.1549$ **D.** $\bar{2}.1549$

69. Evaluate $\dfrac{\ln\sqrt{60} - \ln\sqrt{20} - \ln\sqrt{2}}{\ln 9 - \ln 4}$
A. –4 **B.** – 1/4
C. 1/4 **D.** ln 3 – ln 2

70. Evaluate $\log_3 40.5 - \log_3\left(\frac{1}{2}\right)$ without using tables
A. 4 **B.** 3 **C.** 1/4 **D.** 1/3

71. Simplify: $5\sqrt{12} - 4\sqrt{75} + 3\sqrt{48}$
A. $\sqrt{3}$ **B.** $2\sqrt{3}$ **C.** $3\sqrt{2}$ **D.** $4\sqrt{3}$

72. If $\sqrt{3} = 1.732$, find the value of $2/\sqrt{3}$ without using tables
A. 0.732 **B.** 1.346 **C.** 1.155 **D.** 3.464

73. Simplify: $3\sqrt{18} - \dfrac{16}{\sqrt{2}} + \sqrt{50}$
A. $6\sqrt{2}$ **B.** $2\sqrt{6}$ **C.** $4\sqrt{2}$ **D.** $3\sqrt{2}$

74. The square of $3 - \sqrt{2}$ is
A. $6\sqrt{2}$ **B.** 7
C. $13 - 6\sqrt{2}$ **D.** $11 - 6\sqrt{2}$

75. Simplify $(\sqrt{5} + 2)(\sqrt{5} - 2)$
A. 1 **B.** 3 **C.** $2\sqrt{5}$ **D.** $9 + 4\sqrt{5}$

76. Simplify $\dfrac{1}{2 - \sqrt{3}} - \dfrac{1}{2 + \sqrt{3}}$
A. 4 **B.** $4\sqrt{3}$ **C.** $2\sqrt{3}$ **D.** 2

77. Simplify $\dfrac{\sqrt{8} - \sqrt{2}}{\sqrt{8} + \sqrt{2}}$
A. 1/2 **B.** 1/3 **C.** 2/3 **D.** $\sqrt{2}$

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78. Simplify $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

A. $\frac{1}{2}\sqrt{15}$ **B.** $8+2\sqrt{15}$

C. $4+\sqrt{15}$ **D.** $4-\sqrt{15}$

79. Simplify $\sqrt{18x^2+\sqrt{48x^4+\sqrt{x^8}}}$

A. $4x$ **B.** $2x\sqrt{3}$ **C.** $5x\sqrt{2}$ **D.** $5x$

80. Simplify $2-\frac{1}{\sqrt{2}-1}$

A. $\sqrt{2}$ **B.** $1-\sqrt{2}$ **C.** $\sqrt{2}-1$ **D.** $2\sqrt{2}+1$

81. If $\frac{1}{\sqrt{3}}\left(m-\frac{9}{\sqrt{3}}\right)=2\sqrt{3}-3$, the value of m is

A. 2 **B.** 4 **C.** 6 **D.** 8

82. Simplify $\frac{3\sqrt{15}\times 2\sqrt{48}}{\sqrt{32}\times 4\sqrt{45}}$

A. $\frac{3\sqrt{2}}{4}$ **B.** $\frac{3}{2}$ **C.** $\frac{2\sqrt{3}}{3}$ **D.** $\frac{2\sqrt{2}}{3}$

83. Simplify $\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{1}{\sqrt{3}-\sqrt{2}}\right]$

A. $-2\sqrt{2}$ **B.** $\sqrt{2}/2$ **C.** -2 **D.** $\sqrt{2}$

84. Simplify: $(\sqrt{3}-\sqrt{2})^2(\sqrt{3}+\sqrt{2})^2$

A. 0 **B.** 1 **C.** $\sqrt{6}$ **D.** 6

85. Simplify: $\frac{1}{\sqrt{5}-\sqrt{3}}-\frac{3}{\sqrt{5}+\sqrt{3}}$

A. $\sqrt{5}-\sqrt{3}$ **B.** $\frac{1}{2}(\sqrt{3}-\sqrt{5})$

C. $2\sqrt{3}-\sqrt{5}$ **D.** $\sqrt{3}-\sqrt{5}$

86. Rationalize: $\frac{\sqrt{1+x}+\sqrt{x}}{\sqrt{1+x}-\sqrt{x}}$

A. $\sqrt{x(1+x)}$ **B.** $2x+1$

C. $1+2x+2\sqrt{x(x+1)}$ **D.** $2x-2\sqrt{x(x+1)}$

87. If $a\sqrt{5}+b\sqrt{2}$ is a square root of $95-30\sqrt{10}$, the value of a and b are, respectively

A. 5, 2 **B.** 2, –5 **C.** –5, 3 **D.** 3, –5

88. Simplify: $\sqrt{3}+2-\frac{1}{2-\sqrt{3}}$

A. $\sqrt{3}-1$ **B.** $2\sqrt{3}$ **C.** $\sqrt{3}$ **D.** 0

89. If $x=2-\sqrt{2}$, evaluate $x^2+\frac{4}{x^2}$

A. 12 **B.** 24

C. $6-4\sqrt{2}$ **D.** $3-2\sqrt{2}$

ANSWER KEY

1.	C	16.	A	31.	C	46.	C	61.	B	76.	C
2.	B	17.	D	32.	B	47.	D	62.	B	77.	B
3.	B	18.	D	33.	A	48.	C	63.	C	78.	C
4.	A	19.	C	34.	B	49.	A	64.	B	79.	D
5.	B	20.	D	35.	A	50.	C	65.	A	80.	B
6.	D	21.	A	36.	D	51.	A	66.	B	81.	C
7.	C	22.	C	37.	D	52.	C	67.	D	82.	A
8.	A	23.	D	38.	A	53.	B	68.	D	83.	C
9.	C	24.	B	39.	D	54.	C	69.	C	84.	B
10.	D	25.	A	40.	B	55.	B	70.	A	85.	C
11.	B	26.	B	41.	C	56.	A	71.	B	86.	C
12.	C	27.	C	42.	B	57.	A	72.	C	87.	D
13.	A	28.	A	43.	B	58.	D	73.	A	88.	D
14.	C	29.	C	44.	A	59.	A	74.	D	89.	A
15.	B	30.	B	45.	A	60.	C	75.	A		

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SOLUTIONS TO OBJECTIVE QUESTIONS

1. **(C)** $\dfrac{9^{-1/2} \times 81^{3/4}}{9^{1/2}} = \dfrac{9^{-1/2} \times 9^{3/2}}{9^{1/2}} = 9^{-1} \times 9^{3/2}$
$= 9^{1/2} = 3$

2. **(B)** $2^{-3} \times 16^{3/4} \times 2^0 = 2^{-3} \times 2^3 \times 2^0 = 2^0 = 1$

3. **(B)** $\dfrac{72}{49} = \dfrac{8 \times 9}{7^2} = \dfrac{2^3 \times 3^2}{7^2} = 2^3 \times 3^2 \times 7^{-2}$

4. **(A)** $\dfrac{9^{2x}}{9^{3x}} = 9^{-x} = 3^{-2x}$
$3^{-2x} = 3^1$, or $2x = -1$, and $x = -1/2$

5. **(B)** $4^{(x-1/2)} = 2^{2(x-1/2)} = 2^{x^2}$
$2x - 1 = x^2$, or $x^2 - 2x + 1 = 0$
$(x-1)^2 = 0$, $x = 1$

6. **(D)** $3x^{-2} = \dfrac{4}{27} \Rightarrow x^{-2} = \dfrac{4}{81}$
$x^2 = \dfrac{81}{4}$, and $x = \dfrac{9}{2}$

7. **(C)** $\dfrac{16^{3/4} - 8^{1/3}}{2^2 \times 3} = \dfrac{(2^4)^{3/4} - (2^3)^{1/3}}{2^2 \times 3} = \dfrac{2^3 - 2}{12}$
$= \dfrac{6}{12} = \dfrac{1}{2}$

8. **(A)** $\dfrac{16^{-1/2}}{64^{2/3}} = \dfrac{(2^4)^{-1/2}}{(2^6)^{2/3}} = \dfrac{2^{-2}}{2^4}$
$= 2^{-6} = \dfrac{1}{64}$

9. **(C)** $81^{-1/4} \times 36^{1/2} \times 10^0 = (3^4)^{-1/4} \times 6 \times 1$
$= 3^{-1} \times 6 = \dfrac{6}{3} = 2$

10. **(D)** $4 \times 4^{-n} \times 2^{2n-2} = 2^2 \times 2^{-2n} \times 2^{2n-2}$
$= 2^{2-2n+2n-2} = 2^0 = 1$

11. **(B)** $(0.064)^{-1/3} = (64 \times 10^{-3})^{-1/3} = 64^{-1/3} \times 10^1$
$= \dfrac{10}{4} = 2\dfrac{1}{2}$

12. **(C)** $\sqrt{\dfrac{54a^{-4}}{6a^{-8}}} = \sqrt{9a^4} = 3a^2$

13. **(A)** $\left(3\dfrac{3}{8}\right)^{-2/3} = \left(\dfrac{27}{8}\right)^{-2/3} = \dfrac{27^{-2/3}}{8^{-2/3}}$
$= \dfrac{(3^3)^{-2/3}}{(2^3)^{-2/3}} = \dfrac{3^{-2}}{2^{-2}} = \dfrac{4}{9}$

14. **(C)** $5(2^{x-3}) = 160$
$2^{x-3} = 32 = 2^5$
$x - 3 = 5$, or $x = 8$

15. **(B)** $27^{x/2} = 3^{3/4} \times 9^{-1/4}$
$3^{3x/2} = 3^{3/4} \times 3^{-1/2} = 3^{1/4}$
$\dfrac{3x}{2} = \dfrac{1}{4}$, or $x = \dfrac{1}{6}$

16. **(A)** $\left(\dfrac{2}{3}\right)^{a-1} = \dfrac{27}{8} = \left(\dfrac{3}{2}\right)^3 = \left(\dfrac{2}{3}\right)^{-3}$
$a - 1 = -3$, or $a = -2$

17. **(D)** $9^{1+n} \times 3 = 27^{-n}$
$3^{2(1+n)} \times 3^1 = 3^{-3n} \Rightarrow 2 + 2n + 1 = -3n$,
or $n = -3/5$

18. **(D)** $\sqrt[3]{(729y^{-6})^{1/2}} = \sqrt[3]{27y^{-3}} = (27y^{-3})^{1/3}$
$= 3y^{-1} = \dfrac{3}{y}$

19. **(C)** $\dfrac{6^{2/3} \times 8^{1/3}}{12^{2/3}} = \dfrac{6^{2/3} \times 2}{6^{2/3} \times 2^{2/3}} = 2^{1/3} = \sqrt[3]{2}$

20. **(D)** $(0.027)^{-1/3} \times (0.09)^{3/2}$
$= (27 \times 10^{-3})^{-1/3} \times (9 \times 10^{-2})^{3/2}$
$= \dfrac{1}{3} \times 10 \times 27 \times 10^{-3} = 9 \times 10^{-2}$
$= 0.09$

21. **(A)** $\dfrac{2^x - 2^{x-1}}{2^3 \times 2^x - 8 \times 2^{x-1}} = \dfrac{2^x[1 - \frac{1}{2}]}{2^x[8 - \frac{8}{2}]} = \dfrac{1}{8}$

22. **(C)** $\left(\dfrac{3}{4}\right)^x \left(\dfrac{2}{3}\right)^y = \dfrac{32}{27}$

$\dfrac{3^x}{2^{2x}} \cdot \dfrac{2^y}{3^y} = \dfrac{2^5}{3^3}$

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$$\frac{2^{y-2x}}{3^{y-x}} = \frac{2^5}{3^3}$$

$\therefore\ y - 2x = 5$ and $y - x = 3$

Solving the two equations simultaneously gives $x = -2$ and $y = 1$

23. **(D)** $$\frac{3^{2a+1} \times 9^a \times 4^{2a}}{18^a \times 2^a \times 6^{2a}} = \frac{3^{2a+1} \times 3^{2a} \times 2^{4a}}{2^a \times 3^{2a} \times 2^a \times 2^{2a} \times 3^{2a}}$$

$$= \frac{3^{4a+1} \times 2^{4a}}{3^{4a} \times 2^{4a}} = 3$$

24. **(B)** $$(3^0 - 9^{-1/2})^{-3} = \left(1 - \frac{1}{3}\right)^{-3} = \left(\frac{2}{3}\right)^{-3}$$

$$= \frac{27}{8}$$

25. **(A)** $13^4 - 5^4 = (13^2 - 5^2)(13^2 + 5^2)$

$= (13 + 5)(13 - 5)(194)$

$= 18 \times 8 \times 97 \times 2$

[26 is **not** a factor]

26. **(B)** $\sqrt{5^n} = 25^{1/3} = 5^{2/3}$

$5^{n/2} = 5^{2/3} \Rightarrow \frac{n}{2} = \frac{2}{3}$, or $n = \frac{4}{3}$

27. **(C)** $9^{2m+1} = \frac{81^{m+2}}{3^m}$

$3^{2(2m+1)} = \frac{3^{4(m+2)}}{3^m} = 3^{4m+8-m}$

$\therefore\ 4m + 2 = 3m + 8$, or $m = 6$

28. **(A)** $4^{n-1} \times 5^{2n-2} \times 10^n = 1$

$2^{2n-2} \times 5^{2n-2} \times 10^n = 1$

$\therefore\ 10^{2n-2} \times 10^n = 10^0$

$2n - 2 + n = 0$, or $n = \frac{2}{3}$

29. **(C)** $$\frac{3.50}{0.0025} = \frac{35 \times 10^{-1}}{25 \times 10^{-4}} = 1.4 \times 10^3$$

30. **(B)** $$\frac{2.50 \times 10^4 \times 2.0 \times 10^{-3}}{12.5 \times 10^5}$$

$$= \frac{25 \times 10^3 \times 2 \times 10^{-3}}{12.5 \times 10^5}$$

$$= \frac{50}{12.5} \times 10^{-5} = 4 \times 10^{-5}$$

31. **(C)** $0.0022 \times 0.017 = 2.2 \times 10^{-3} \times 1.7 \times 10^{-2}$

$= 3.74 \times 10^{-5}$

32. **(B)** $$\frac{0.0035}{0.210} = \frac{35 \times 10^{-4}}{21 \times 10^{-2}} = 1.67 \times 10^{-2}$$

33. **(A)** $$\sqrt[3]{\frac{0.0024 \times 35000}{0.0105}}$$

$$= \left[\frac{24 \times 10^{-4} \times 35 \times 10^3}{105 \times 10^{-4}}\right]^{1/3}$$

$= (8 \times 10^3)^{1/3} = 2 \times 10^1$

34. **(B)** Total thickness of book $= 12$ mm

$= 12 \times 10^{-3}$ m

Number of leaves $= 500/2 = 250$

Thickness of each leaf $= \frac{12 \times 10^{-3}}{250}$

$= \frac{120 \times 10^{-4}}{25 \times 10}$ m

$= 4.8 \times 10^{-5}$ m

35. **(A)** $(0.05)^3 - (0.03)^3$

$= (5 \times 10^{-2})^3 - (3 \times 10^{-2})^3$

$= 125 \times 10^{-6} - 27 \times 10^{-6}$

$= 98 \times 10^{-6} = 9.8 \times 10^{-5}$

36. **(D)** $\log_3 27 + \log_3 6 - \log_3 54 = \log_3 \frac{27 \times 6}{54}$

$= \log_3 3 = 1$

37. **(D)** $0.9030 = 3\log_{10} 2 = \log_{10} 2^3 = \log_{10} 8$

$\Rightarrow \bar{2}.9030 = \log_{10} 0.08$

38. **(A)** If $\log_a x^n = y$, then $a^y = x^n$

Since $a = x^{1/n}$, $a^y = x^{y/n} = x^n$

or $y/n = n \Rightarrow y = n^2$

39. **(D)** $2\log_4 p + \log_4 q^2 = 0$

$\log_4 p^2 + \log_4 q^2 = 0$

$\log_4(p^2 q^2) = 0$

$p^2 q^2 = 4^0 = 1$, $pq = 1$ and $p = 1/q$

40. **(B)** $$\frac{\log_{10} 0.04 + \log_{10} 0.25}{\log_5 100 - \log_5 4}$$

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$$= \frac{\log_{10}(0.04 \times 0.25)}{\log_5 (100/4)}$$

$$= \frac{\log_{10} 0.01}{\log_5 25} = \frac{-2}{2} = -1$$

41. **(C)** $4^{3/2} + \log_5 0.04 + \log_3 81$

$= 2^3 + \log_5 (5^{-2}) + \log_3 (3^4)$

$= 8 - 2 + 4 = 10$

42. **(B)** $\log_{10} x = 0.5$

$x = 10^{0.5} = 10^{1/2} = \sqrt{10}$

43. **(B)** $4 \log x + 5 \log x - 7 \log x = \log 16$

$(4 + 5 - 7) \log x = \log 16$

$2 \log x = \log 16$

$\log x^2 = \log 16$

$x^2 = 16$, and $x = 4$

44. **(A)** $\log p = 0.5 \log x + 2 \log q$

$\log x^{1/2} = \log p - \log q^2 = \log \frac{p}{q^2}$

$x^{1/2} = p/q^2$

$x = p^2/q^4$

45. **(A)** $\log(y+3)^5 = \log 2^5$

$(y+3)^5 = 2^5$

$y + 3 = 2$, or $y = -1$

46. **(C)** $\log (0.75)^3 = 3 \log 0.75 = 3 \times \bar{1}.875$

$= \bar{1}.625$

47. **(D)** $\log_8 a^2 - \log_8 a^5 = 2$

$\log_8 \left(\frac{a^2}{a^5}\right) = 2$

$\log_8 a^{-3} = 2$

$a^{-3} = 8^2 = 64$

$a^3 = \frac{1}{64}$, or $a = \frac{1}{4}$

48. **(C)** $\log_2 x = \log_9 3 = \log_9 9^{1/2} = \frac{1}{2}$

$x = 2^{1/2} = \sqrt{2}$

49. **(A)** $\log_3 x^2 = -4$

$x^2 = 3^{-4}$

$x = 3^{-2} = \frac{1}{9}$

50. **(C)** $\log_{10} 40 = 1.6021$

$3 \log_{10} 400 = 3 \times 2.6021 = 7.8063$

$\log_{10} 40 + 3 \log_{10} 400 = 9.4084$

51. **(A)** $\log_{10} 40 - 3 \log_{10} 400$

$= 1.6021 - 7.8063$

$= -6.2042$

[Note that this is the same as $\bar{7}.7958$, which is **not** equal to -7.7958.]

52. **(C)** $\log_b 62.5 = \log_b \left(\frac{125}{2}\right) = \log_b 125 - \log_b 2$

$= \log_b 5^3 - \log_b 2$

$= 3 \log_b 5 - \log_b 2$

$= 3(1.609) - 0.693$

$= 4.134$

53. **(B)** $\log_3 324 - 2 \log_3 6 = \log_3 324 - \log_3 6^2$

$= \log_3 \frac{324}{36} = \log_3 9 = 2$

54. **(C)** $\frac{1}{2} \log_{10} x = 1 \Rightarrow \log_{10} x^{1/2} = 1$

$x^{1/2} = 10^1$, $x = 100$

55. **(B)** Using the law $\log_3 8 = \frac{\log_2 8}{\log_2 3}$,

$ab = \log_2 3 \times \frac{\log_2 8}{\log_2 3} = \log_2 8 = 3$

56. **(A)** $\log_{10} 54 = \log_{10} (6 \times 3^2)$

$= \log_{10} 6 + 2 \log_{10} 3$

$= 0.778 + 2(0.477)$

$= 1.732$

57. **(A)** $\log_{10} \frac{\sqrt{40} \times \sqrt{2}}{\sqrt{8}} = \log_{10} \sqrt{10}$

$= \log_{10} 10^{1/2} = \frac{1}{2}$

58. **(D)** $\log_{10} (3x - 4) - \log_{10} 2 = 1$

$\log_{10} \frac{3x-4}{2} = 1$

$\frac{3x-4}{2} = 10$

$3x - 4 = 20$, $x = 8$

59. **(A)** $\frac{\log_3 27}{\log_3 (1/9)} = \frac{\log_3 3^3}{\log_3 3^{-2}} = -\frac{3}{2}$

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60. **(C)** $\log \frac{a}{a-1} = \log 3$

$\frac{a}{a-1} = 3$

$a = 3(a-1) = 3a - 3$

$2a = 3$, or $a = 1.5$

61. **(B)** $\frac{\log 9 - \log 4}{\log \sqrt{3} - \frac{1}{2}\log 2} = \frac{\log \frac{9}{4}}{\frac{1}{2}\log \frac{3}{2}}$

$= \frac{2\log \frac{3}{2}}{\frac{1}{2}\log \frac{3}{2}} = 4$

62. **(B)** $\log_{10} 0.225 = \log_{10}(225 \times 10^{-3})$

$= \log_{10}(3^2 \times 5^2 \times 10^{-3})$

$= \log_{10} 3^2 + \log_{10} 5^2 + \log_{10} 10^{-3}$

$= 2\log_{10} 3 + 2\log_{10} 5 - 3$

$= 2(0.477) + 2(0.699) - 3$

$= -0.648$

63. **(C)** $\log_2 \left(\frac{5 \times 6}{n}\right) = 1$

$\frac{30}{n} = 2^1$, or $n = \frac{30}{2} = 15$

64. **(B)** $\log_4 x = 3.5$

$x = 4^{3.5} = 4^3 \times 4^{0.5}$

$= 64 \times 4^{1/2} = 128$

65. **(A)** Let $a = \log_2 8 = 3$

$b = \log_{1/3} 81$

$\Rightarrow \left(\frac{1}{3}\right)^b = 81$, or $3^{-b} = 3^4$

$b = -4$

$c = \log_4 \frac{1}{64} = \log_4 4^{-3} = -3$

$\frac{a-b}{c} = \frac{3+4}{-3} = -\frac{7}{3}$

66. **(B)** $10^x = 0.3$

$x \log 10 = \log 0.3$, or

$x = \log 0.3 = \bar{1}.4771$

$= -1 + 0.4771 = -0.5229$

67. **(D)** $\log_5 25^x - \log_5 0.04$

$= \log_5 (5^2)^x - \log_5 \left(\frac{1}{25}\right)$

$= \log_5 5^{2x} - \log_5 5^{-2}$

$= 2x - 2$

68. **(D)** $\log_{10}\left(\frac{1}{70}\right) = \log_{10} 1 - \log_{10} 70$

$= -\log_{10} 70 = -1.8451$

$= -2 + 0.1549$

$= \bar{2}.1549$

69. **(C)** Numerator $= \ln \frac{\sqrt{60}}{\sqrt{20} \times \sqrt{2}}$

$= \ln \sqrt{\frac{60}{40}}$

$= \ln \sqrt{\frac{3}{2}} = \frac{1}{2} \ln \frac{3}{2}$

Denominator $= \ln \frac{9}{4} = \ln \left(\frac{3}{2}\right)^2 = 2 \ln \frac{3}{2}$

The given expression $= \frac{\frac{1}{2} \ln \frac{3}{2}}{2 \ln \frac{3}{2}} = \frac{1}{4}$

70. **(A)** $\log_3 40.5 - \log_3 0.5$

$= \log_3 \frac{40.5}{0.5} = \log_3 81 = 4$

71. **(B)** $5\sqrt{12} - 4\sqrt{75} + 3\sqrt{48}$

$= 5 \times 2\sqrt{3} - 4 \times 5\sqrt{3} + 3 \times 4\sqrt{3}$

$= 10\sqrt{3} - 20\sqrt{3} + 12\sqrt{3}$

$= 2\sqrt{3}$

72. **(C)** $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{2\sqrt{3}}{3}$

$= \frac{2 \times 1.732}{3} = 1.155$

73. **(A)** $3\sqrt{18} - \frac{16}{\sqrt{2}} + \sqrt{50}$

$= 3 \times 3\sqrt{2} - \frac{16\sqrt{2}}{2} + 5\sqrt{2}$

$= 9\sqrt{2} - 8\sqrt{2} + 5\sqrt{2}$

$= 6\sqrt{2}$

74. **(D)** $(3 - \sqrt{2})^2 = 3^2 - 6\sqrt{2} + (\sqrt{2})^2$

$= 9 - 6\sqrt{2} + 2$

$= 11 - 6\sqrt{2}$

75. **(A)** $(\sqrt{5} + 2)(\sqrt{5} - 2) = (\sqrt{5})^2 - (2)^2$

$= 5 - 4 = 1$

76. **(C)** $\frac{1}{2 - \sqrt{3}} - \frac{1}{2 + \sqrt{3}} = \frac{2 + \sqrt{3} - 2 + \sqrt{3}}{(2 - \sqrt{3})(2 + \sqrt{3})}$

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$$= \frac{2\sqrt{3}}{2^2 - (\sqrt{3})^2} = \frac{2\sqrt{3}}{4-3}$$

$$= 2\sqrt{3}$$

77. **(B)** Multiply numerator and denominator by $\sqrt{8} - \sqrt{2}$ to give

$$\frac{\sqrt{8}-\sqrt{2}}{\sqrt{8}+\sqrt{2}} \times \frac{\sqrt{8}-\sqrt{2}}{\sqrt{8}-\sqrt{2}}$$

$$= \frac{8 - 2\sqrt{2}\sqrt{8} + 2}{8-2}$$

$$= \frac{10-8}{6} = \frac{1}{3}$$

78. **(C)** $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$= \frac{5 + 2\sqrt{3}\sqrt{5} + 3}{5-3}$$

$$= \frac{8 + 2\sqrt{15}}{2} = 4 + \sqrt{15}$$

79. **(D)** Start with the innermost $\sqrt{}$ sign:

$$\sqrt{18x^2 + \sqrt{48x^4 + x^4}}$$

$$= \sqrt{18x^2 + \sqrt{49x^4}}$$

$$= \sqrt{18x^2 + 7x^2} = \sqrt{25x^2}$$
$$= 5x$$

80. **(B)** $2 - \dfrac{1}{\sqrt{2}-1} = \dfrac{2\sqrt{2} - 2 - 1}{\sqrt{2}-1}$

$$= \frac{2\sqrt{2}-3}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \frac{2 \times 2 + 2\sqrt{2} - 3\sqrt{2} - 3}{2-1}$$

$$= 1 - \sqrt{2}$$

81. **(C)** Multiply both sides by $\sqrt{3}$:

$$m - \frac{9}{\sqrt{3}} = 6 - 3\sqrt{3}$$

$$m - \frac{9\sqrt{3}}{3} = 6 - 3\sqrt{3}$$

$m - 3\sqrt{3} = 6 - 3\sqrt{3}$, or
$m = 6$

82. **(A)** $\dfrac{3\sqrt{3}\sqrt{5} \times 2 \times 4\sqrt{3}}{4\sqrt{2} \times 4 \times 3\sqrt{5}} = \dfrac{3}{2\sqrt{2}}$

$$= \frac{3\sqrt{2}}{2 \times 2} = \frac{3\sqrt{2}}{4}$$

83. **(C)** $\dfrac{1}{\sqrt{2}}\left[\dfrac{\sqrt{3} - \sqrt{2} - \sqrt{3} - \sqrt{2}}{3-2}\right]$

$$= -\frac{2\sqrt{2}}{\sqrt{2}} = -2$$

84. **(B)** The easiest approach to this problem is to re-write the expression as

$$\left[(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})\right]^2$$
$$= (3-2)^2 = 1$$

85. **(C)** $\dfrac{\sqrt{5} + \sqrt{3} - 3\sqrt{5} + 3\sqrt{3}}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})}$

$$= \frac{4\sqrt{3} - 2\sqrt{5}}{5-3}$$
$$= 2\sqrt{3} - \sqrt{5}$$

86. **(C)** $\dfrac{\sqrt{1+x} + \sqrt{x}}{\sqrt{1+x} - \sqrt{x}} \times \dfrac{\sqrt{1+x} + \sqrt{x}}{\sqrt{1+x} + \sqrt{x}}$

$$= \frac{1 + x + 2\sqrt{x(1+x)} + x}{1 + x - x}$$

$$= 1 + 2x + 2\sqrt{x(1+x)}$$

87. **(D)** $(a\sqrt{5} + b\sqrt{2})^2 = 95 - 30\sqrt{10}$

$5a^2 + 2b^2 + 2ab\sqrt{10} = 95 - 30\sqrt{10}$

Hence $2ab = -30$, which eliminates options A and B. Since $5a^2 + 2b^2 = 95$, the correct option is (D).

88. **(D)** $\dfrac{(\sqrt{3}+2)(2-\sqrt{3}) - 1}{2 - \sqrt{3}}$

$$= \frac{2\sqrt{3} - 3 + 4 - 2\sqrt{3} - 1}{2 - \sqrt{3}} = 0$$

89. **(A)** If $x = 2 - \sqrt{2}$

$$x^2 = (2 - \sqrt{2})^2 = 4 + 2 - 4\sqrt{2}$$
$$= 6 - 4\sqrt{2}$$

The expression becomes

$$6 - 4\sqrt{2} + \frac{4}{6 - 4\sqrt{2}}$$

$$= \frac{36 - 48\sqrt{2} + 32 + 4}{6 - 4\sqrt{2}}$$

$$= \frac{72 - 48\sqrt{2}}{6 - 4\sqrt{2}} = \frac{12(6 - 4\sqrt{2})}{6 - 4\sqrt{2}} = 12$$

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4 Sets and Venn Diagrams

4.1 Set Terminology

- A **set** is a group of well-defined objects or numbers, e. g. the set of integers can be written as

  $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

  while the set of numbers on a die is

  $D = \{1, 2, 3, 4, 5, 6\}$.

- Any member of a set is referred to as an **element** of the set, e.g. 3 is an **element** of the set D, written as $3 \in D$. On the other hand, 8 is not an element of D, i.e. $8 \notin D$.
- An **infinite** set is one which contains an infinite number of elements, e.g. set Z above, while a **finite** set contains a definite number of elements, e.g. set D above.
- If every element of a set A is also an element of a set B, then A is a **subset** of B, written as $A \subset B$, e.g. $D \subset Z$ in the example above.
- An **empty set** or a **null set**, written as $\varnothing$ or $\{\}$, is a set which has no elements, e.g. the set of integers between 1 and 4 which are divisible by 5 is $\varnothing$.
- A **universal set** U or ξ is defined as the set of **all** elements under consideration.

EXAMPLE 4 - 1

If U = {positive integers less than 20}, list the following :

(a) $\{x : x > 5\}$
(b) $\{x : x$ is even$\}$
(c) $\{x : 10 < x \leq 14\}$
(d) $\{x : 10 < x < 18$ and x is odd$\}$
(e) $(x : x$ is a prime factor of 210$\}$

Solutions

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$

(a) {6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19}
(b) {2, 4, 6, 8, 10, 12, 14, 16, 18}
(c) {11, 12, 13, 14}
(d) {11, 13, 15, 17}
(e) {2, 3, 5, 7}

DRILL 1

List the following sets:

(a) {integers $x : 3 < x \leq 15$}
(b) {negative integers x: x is even and $|x| < 12$}
(c) {English vowels}
(d) {prime numbers $x : 20 < x < 50$}
(e) {integers $x : 0 < x < 30$ and a multiple of 4}
(f) {integers $x : 0 < x^2 < 20$}

4.2 Operations on Sets

- The **union** of two sets A and B, written as $A \cup B$, consists of elements which are contained either in set A or in set B or in both. In set notation.

  $A \cup B = \{x : x \in A$, or $x \in B\}$

- If $A = \{2, 3, 5, 7, 9, 10\}$ and $B = \{1, 4, 7, 10\}$, then $A \cup B = \{1, 2, 3, 4, 5, 7, 9, 10\}$.
- The **intersection** of two sets A and B, written as $A \cap B$, consists of elements which belong to both A and B, i.e.

  $A \cap B = \{x : x \in A$ and $x \in B\}$

  In the example above, $A \cap B = \{7, 10\}$
- The **complement** of set A, written as A' or A^c, consists of elements which are contained in the universal set U but not in A, i.e.

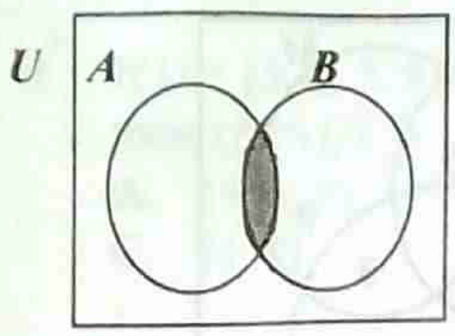
  $A' = \{x : x \in U, x \notin A\}$

- If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then $A' = \{1, 4, 6, 8\}$ and $B' = \{2, 3, 5, 6, 8, 9\}$.

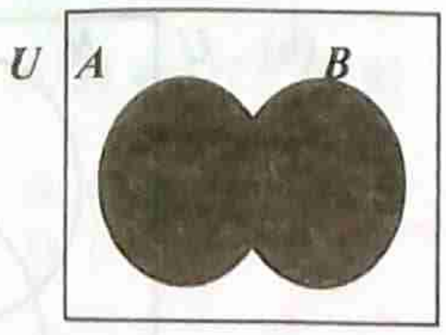
4.3 Venn Diagrams

- A **Venn diagram** is used to illustrate problems on sets. A rectangle is used to represent the universal set while circles within the rectangle denote subsets of the universal set.
- Fig. 4.1 shows Venn diagrams illustrating simple operations on subsets A and B of a universal set U.

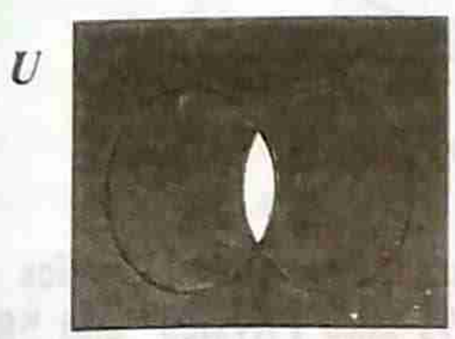
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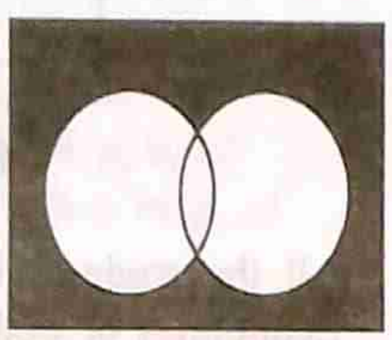
(a) $A \cap B$



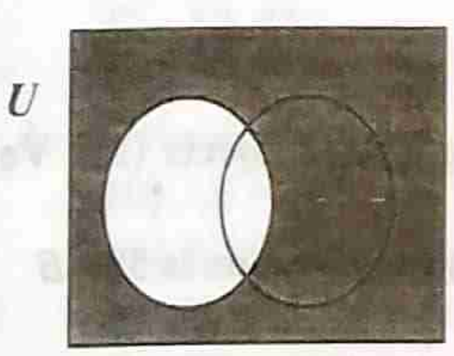
(b) $A \cup B$



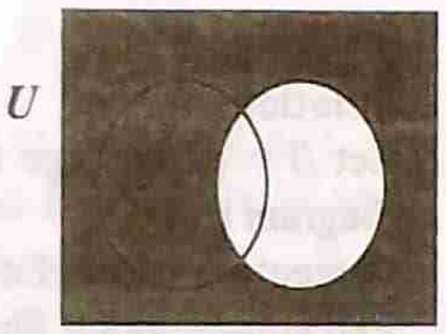
(c) $(A \cap B)'$



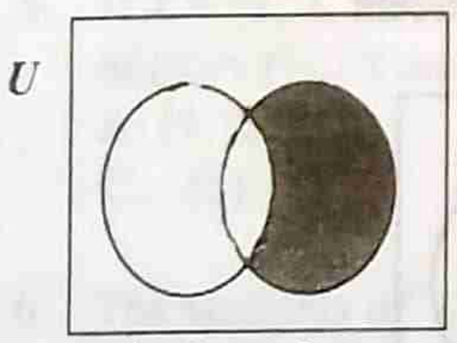
(d) $(A \cup B)'$



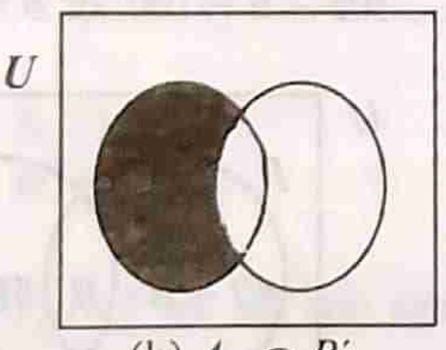
(e) A'



(f) B'



(g) $A' \cap B$



(h) $A \cap B'$

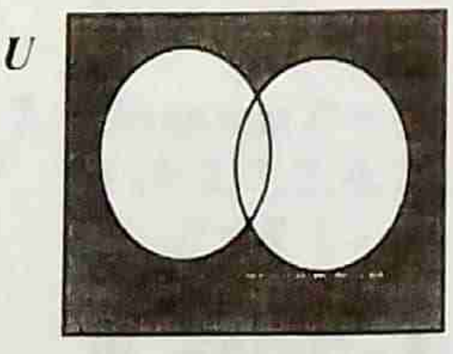


Fig. 4.1

(i) $A' \cap B'$

- Fig. 4.2 shows the subsets $P = \{2, 4, 6, 8, 10\}$ and $Q = \{7, 8, 9, 10\}$ within the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

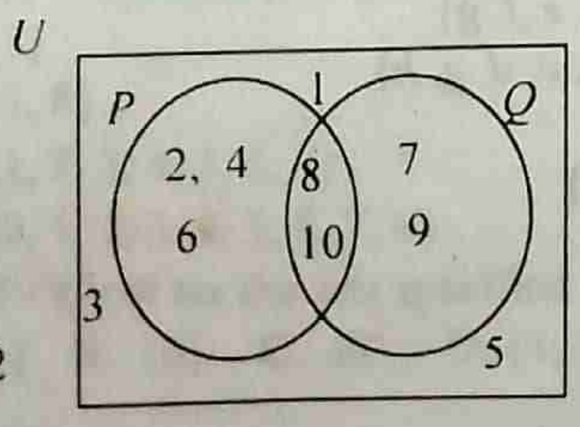


Fig. 4.2

From the diagram:
$P' = \{1, 3, 5, 7, 9\}$ and $Q' = \{1, 2, 3, 4, 5, 6\}$

$P \cap Q = \{8, 10\}$
$P \cup Q = \{2, 4, 6, 7, 8, 9, 10\}$
$P' \cap Q$ = elements belonging to Q but not to P
$= \{7, 9\}$
$n(P \cup Q)$ = number of elements in the set $P \cup Q = 7$
$n(P' \cap Q) = 2$.

DRILL 2

With reference to the Venn diagram in Fig. 4.2, list the following:

(a) $Q' \cup P$ (b) $(P \cap Q)'$
(c) $(P \cup Q)'$ (d) $P' \cap Q'$
(e) $(P \cap Q) \cup Q$ (f) $(P \cap Q)' \cap P$
(g) $n(P' \cap Q')$ (h) $n(Q' \cap P)$.

- Fig. 4.3 shows a Venn diagram illustrating three subsets X, Y and Z of a universal set U.

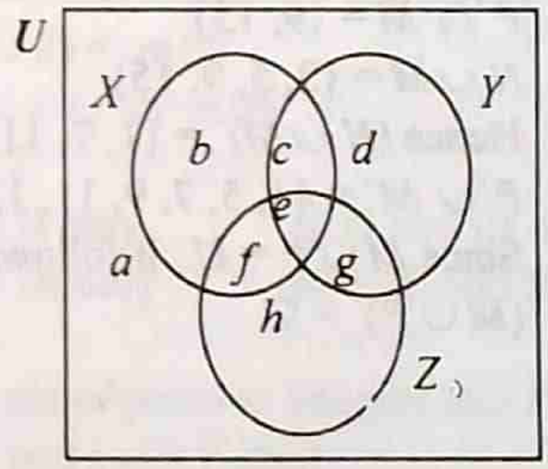


Fig. 4.3

From the Diagram:
$U = \{a, b, c, d, e, f, g, h\}$
$X = \{b, c, e, f\}$, $X' = \{a, d, g, h\}$
$Y = \{c, d, e, g\}$, $Y' = \{a, b, f, h\}$
$Z = \{e, f, g, h,\}$, $Z' = \{a, b, c, d\}$
$X \cap Y \cap Z$ = set of elements which belong to X and Y and $Z = \{e\}$
$X \cap Y \cap Z'$ = set of elements which belong to X and Y but not to $Z = \{c\}$
$(X \cap Y) \cup (X' \cap Z) = \{c, e\} \cup \{g, h\}$
$= \{c, e, g, h\}$

DRILL 3

Referring to Fig. 4.3 , list the following sets:
(a) $X \cup Y$ (b) $Y \cup Z$
(c) $X \cap Z$ (d) $X \cap Y$
(e) $X \cup (Y \cap Z)$ (f) $(X \cup Y) \cap (X \cup Z)$
(g) $(X \cap Z)' \cup Y$ (h) $X \cap Z' \cap Y'$
(i) $(X' \cup Y) \cap (Y' \cup Z)$ (j) $X' \cap Z' \cap Y'$

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EXAMPLE 4 - 2

If N = {odd numbers less than 20, divisible by 5}
M = {odd numbers less than 20, divisible by 3};
P = {odd, prime numbers less than 20};
U = {universal set of odd numbers less than 20}

(a) Find

(i) $N \cup M$, $M \cup P$, $N \cap M$, $N \cap P$, $N \cap M \cap P$

(ii) $N \cup M'$, $P' \cap M$, $(N \cup M)'$, $P' \cup M'$, $(M \cup P)'$

(b) Sketch a Venn diagram.

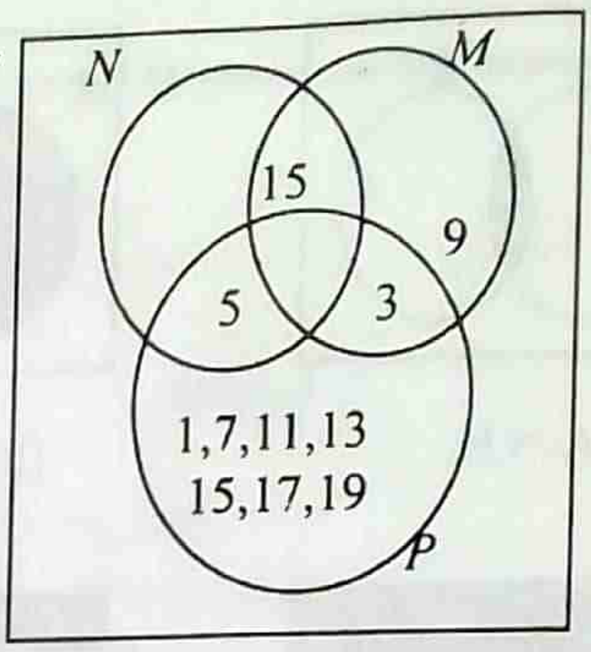
Solution

N = {5, 15}, M = {3, 9, 15},
P = {1, 3, 5, 7, 11, 13, 17, 19}
U = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}

(a) (i) $N \cup M$ = {3, 5, 9, 15}
$M \cup P$ = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19} = U
$N \cap M$ = {15}
$N \cap P$ = {5}
$N \cap M \cap P = \varnothing$

(ii) M' = {1, 5, 7, 11, 13, 17, 19},
P' = {9, 15}
$N \cup M'$ = {1, 5, 7, 11, 13, 15, 17, 19}
$P' \cap M$ = {9, 15}
$N \cup M$ = {3, 5, 9, 15}
Hence $(N \cup M)'$ = {1, 7, 11, 13, 17, 19}
$P' \cup M'$ = {1, 5, 7, 9, 11, 13, 15, 17, 19}
Since $M \cup P = U$, it follows that
$(M \cup P)' = \varnothing$

(b)

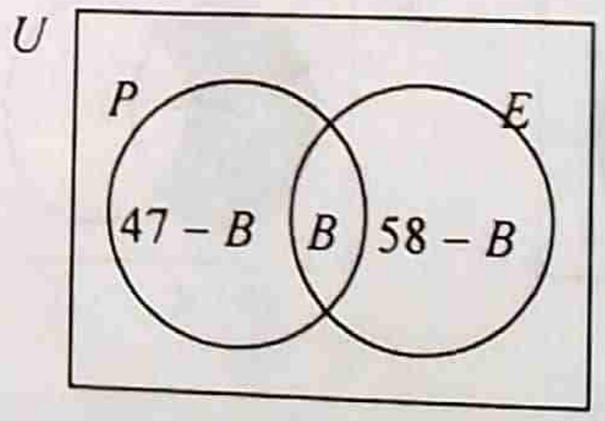


EXAMPLE 4 - 3

All the students in a class take either Physics
Economics or both. If 47% take Physics and 58
take Economics, what percentage take both subje

Solution

Let B = percentage taking both subjects (see Ve
diagram below)
Percentage taking Economics (E) alone is $58 - B$
Percentage taking Physics (P) alone is $47 - B$
Total percentage of students = 100
Hence $B + (58 - B) + (47 - B) = 100$
$105 - B = 100$, or $B = 5$ percent



ANSWERS TO DRILLS

1. (a) {4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
(b) {−10, −8, −6, −4, −2}
(c) {a, e, i, o, u}
(d) {23, 29, 31, 37, 41, 43, 47}
(e) {4, 8, 12, 16, 20, 24, 28}
(f) {−4, −3, −2, −1, 0, 1, 2, 3, 4}

2. (a) {1, 2, 3, 4, 6}
(b) {1, 2, 3, 4, 5, 6, 7, 9}
(c) {1, 3, 5}
(d) {1, 3, 5}
(e) {7, 8, 9, 10} = Q
(f) {2, 4, 6}
(g) 3
(h) 3

3. (a) {b, c, d, e, f, g}
(b) {c, d, e, f, g, h}
(c) {e, f}
(d) {c, e}
(e) {b, c, e, f, g}
(f) {b, c, e, f, g}
(g) {a, b, c, d, g, h}
(h) {b}
(i) {a, h}
(j) {a}

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SSCE/JME - TYPE OBJECTIVE QUESTIONS

1. If $U = \{5, 6, 7, 8\}$, $P = \{6, 7\}$ and $Q = \{6, 8\}$ then, $(P \cap Q)'$ is
   A. $\{5, 6, 7\}$ **B.** $\{5, 7, 8\}$
   C. $\{6, 7\}$ **D.** $\{5, 8\}$

2. The subsets X, Y and Z of a universal set are defined as
   $X = \{a, e, m, p\}$; $Y = \{a, e, i, o, u\}$ and
   $Z = \{l, m, n, o, p, q, r, s, t, u\}$.
   The elements of $X \cup (Y \cap Z)$ are
   A. $\{a, e, m, o, p, u\}$ **B.** $\{m, p, o, u\}$
   C. $\{a, e, m, o, p\}$ **D.** $\{a, e, m, p, u\}$

3. Given that $P = \{1, 3, 4\}$ and $Q = \{2, 3, 4\}$,then $P \cap Q$ is
   A. $\{1, 2\}$ **B.** $\{2, 3\}$
   C. $\{2, 4\}$ **D.** $\{3, 4\}$

4. If the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $X = \{2, 3, 6, 9\}$ and $Y = \{1, 2, 4, 6, 10\}$, which of the following is equal to $(X \cup Y)'$?
   A. $X' \cup Y'$ **B.** $(X \cap Y)'$
   C. $X' \cap Y'$ **D.** $X \cup Y$

5. If $X = \{1, 2, 3, 4\}$ and $Y = \{3, 5, 6\}$, the elements of $(X \cap Y) \cup X$ are
   A. $\{1, 2, 3, 4\}$ **B.** $\{3, 5, 6\}$
   C. $\{3\}$ **D.** $\{1, 2, 4\}$

6. The elements of $(X \cup Y) \cap Y$ for the sets specified in question 5 are:
   A. $\{1, 2, 3, 4, 5, 6\}$ **B.** $\{1, 2, 4\}$
   C. $\{3, 5, 6\}$ **D.** $\{2, 4, 6\}$

7. Given that $P = \{1, 2, 3, 4\}$; $Q = \{5, 6, 7, 8\}$ and $R = \{1, 2, 3, 2, 4, 2\}$, it follows that
   I. $P = R$
   II. $n(R) = 6$
   III. $n(P \cup R) = 4$
   Which of the statements above is (are) true ?
   A. I only **B.** II only
   C. III only **D.** I and III only

8. If $P = \{3, 1, 0, 5\}$; $Q = \{2, 3, 8, 1, 4\}$ and $R = \{7, 6, 5\}$, then $P \cup Q \cup R$ is
   A. $\{5\}$
   B. $\{3, 8\}$
   C. $\{1, 2, 3, 4, 5, 6, 7\}$
   D. $\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

9. Find $P \cap Q \cap R$ for the sets specified in question 8
   A. $\{1\}$ **B.** $\{0\}$ **C.** $\varnothing$ **D.** $\{1, 3, 5\}$

10. If $X = \{1, 2, 3, 4, 5, 6\}$, $Y = \{2, 4, 5, 7\}$and $Z = \{1, 4, 5\}$,which of the following is (are) correct?
    I. $n(X \cup Y \cup Z) = 13$
    II. $(X \cap Y) \cup Z = X \cap (Y \cup Z)$
    III. $n(X \cap Y \cap Z) = 2$
    A. I only **B.** II only
    C. I and III only **D.** II and III only

11. Given that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $P = \{1, 3, 4, 6, 8, 9, 10\}$, $Q = \{1, 3, 4, 5, 6, 9\}$ and $R = \{3, 4, 5, 6, 10\}$, the elements of $P \cap Q \cap R'$ are
    A. $\{1, 2\}$ **B.** $\{1\}$
    C. $\{1, 9\}$ **D.** $\{2, 9\}$

12. If $X = (a, b, c)$, $Y = (a, b, c, d, e)$ and $Z = (a, b, c, d, e, f, g)$ then $(X \cup Y) \cap (X \cup Z)$ is
    A. (a, b, c) **B.** (a, b, c, d, e)
    C. (a, b, c, d, e, f, g) **D.** $\varnothing$

13. If $U = \{c, a, t, a, l, o, g, u, e\}$; $X = \{c, a, t\}$ $Y = \{a, a, o, u, e\}$ and $Z = \{c, t, l, g\}$, which of the following statements are correct?
    I. $n(Y \cup Z) = n(U)$
    II. $(X \cup Y \cup Z)' = \varnothing$
    III. $Y' = Z$
    A. I and II only **B.** II and III only
    C. I and III only **D.** I, II, and III.

14. If I is the set of positive integers and $P = \{x : x \in I, x^2 < 10 \text{ and } x \neq 0\}$, then
    A. $P = \{0, 1\}$ **B.** $P = \{0, 1, 2, 3\}$
    C. $P = \{1, 2, 3\}$ **D.** P is an infinite set.

The universal set U is the set of all integers and the subsets P, Q and R of U are given as
$P = \{x : x > 0\}$
$Q = \{x : x \text{ is odd}\}$
$R = \{-5 \leq x < 5\}$
Use this information to answer questions **15** to **17**

15. $Q \cap R$ is given by
    A. $\{1, 3, 5\}$ **B.** $\{-5, -3, -1, 1, 3\}$
    C. $\{-5, -3, -1, 1, 3, 5\}$ **D.** $\{0, 1, 3, 5\}$

16. $P' \cap R$ is given by
    A. $\{-5, -4, -3, -2, -1, 0\}$
    B. $\{-5, -4, -3, -2, -1\}$
    C. $\{1, 2, 3, 4\}$
    D. $\{0, 1, 2, 3, 4, 5\}$

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17. $(P \cap Q)'$ is
A. {0, 1, 2, 3, 4, 5} **B.** {–10, –8, –6, –2, 0}
C. ∅ **D.** an infinite set

18. If U = {integers ≤ 20}, X = {multiples of 4}, Y = {multiples of 3}. The elements of $X' \cap Y$ are
A. {12} **B.** {3, 6, 9, 12, 15, 18}
C. {3, 6, 9, 15, 18} **D.** {4, 8, 16, 20}

The universal set U consists of all integers. Subsets of U are defined as:
$P = \{x : x \leq 3\}$
$Q = \{x : -5 < x < 12\}$
$R = \{x : -2 \leq x < 5)$
Use the information to answer questions **19** to **21**

19. $P \cap Q$ is
A. {–4, –3 –2, –1, 0, 1, 2, 3}
B. {0, 1, 2, 3}
C. {1, 2, 3}
D. an infinite set.

20. $Q \cup R$ is the same as
A. R **B.** Q **C.** P **D.** R'

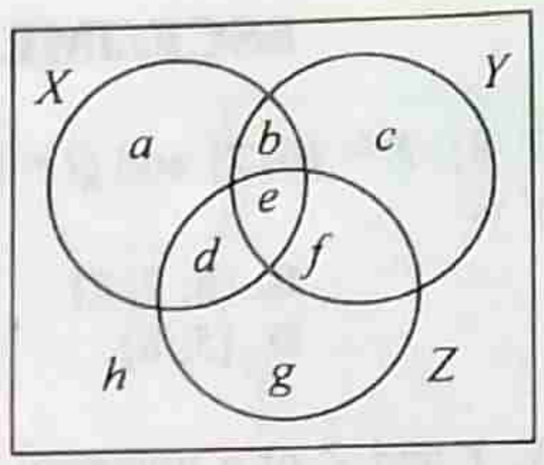
21. $P \cap (Q \cup R)'$ is
A. $\{x < -4\}$ **B.** $\{-4 \leq x \leq 3\}$
C. ∅ **D.** $\{x < 0\}$

22. If X = {prime factors of 84} and Y = {prime factors of 315}, the elements of $X \cup Y$ and $X \cap Y$ are, respectively
A. {3, 4, 5, 7, 9} and {3, 7}
B. {3, 4, 5, 7} and {5, 7}
C. {2, 3, 5, 7} and {3, 7}
D. {2, 3, 5, 7} and {2, 7}

23. Given the universal set U = {2, 3, 4, 5, 6, 7, 8, 9} and subsets P = {2, 4, 6, 8} and $Q = \{x : x^2 < 50, x$ is odd}, find $(P \cap Q)'$
A. {9} **B.** {0} **C** ∅ **D.** U

24. The universal set U has subsets M and N such that $M \subseteq N$. The set of $M \cap (M \cap N)'$ is
A. M **B.** ∅ **C.** N **D.** U

25. If $R = [x : x^2 = 16, x > 5\}$, then R is equal to
A. 0 **B.** {0} **C.** ∅ **D.** {∅}



Use the Venn diagram shown above to answer questions **26** to **30**. X, Y and Z are subsets of the universal set U.

26. The region $f + g$ in the diagram is
A. $Y \cap Z$ **B.** $Y' \cap Z$
C. $X' \cap Z$ **D.** $(X \cup Y)' \cap Z$

27. The region $d + e + f$ is
A. $X \cap Y \cap Z$ **B.** $(X \cap Z) \cup (Y \cap Z)$
C. $X \cup Y \cap Z'$ **D.** $(X \cup Z) \cap (Y \cup Z)$

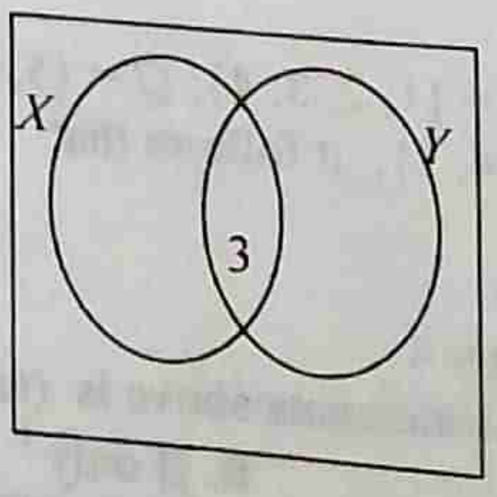
28. The region $g + h$ is
A. $(X \cup Y)'$ **B.** $(X \cup Y \cup Z)'$
C. $X' \cup Z \cap Y$ **D.** $Y \cup Z \cup X'$

29. The region represented by $X' \cap Y \cap Z'$ is
A. a **B.** g **C.** h **D.** c

30. The region represented by $X' \cap Z'$ is
A. $c + h$ **B.** $b + c$ **C.** $c + f$ **D.** h

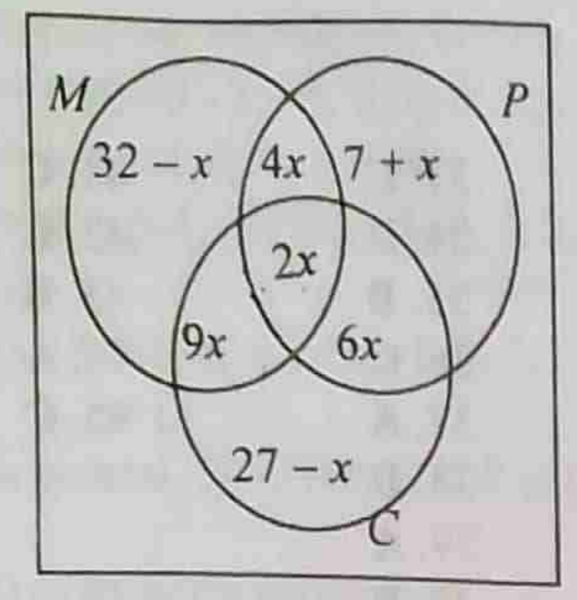
31 P and Q are two sets such that $n(P \cup Q)$ is 50, $n(P)$ is 40 and $n(Q)$ is 18. What is $n(P \cap Q)$?
A. 108 **B.** 58 **C.** 32 **D.** 8

32.



X and Y are subsets of the universal set U shown in the diagram above. If $n(X \cap Y) = 3$, $n(X \cup Y) = 22$ and $n(U) = 27$, what is $n(X' \cap Y')$?
A. 5 **B.** 9 **C.** 19 **D.** 24

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The Venn diagram above shows the number of students offering Mathematics (M), Physics (P) and Chemistry (C) in a final year class in a secondary school. There are 166 students altogether. Use this information to answer questions **33** to **36**.

33. How many students offer both mathematics and chemistry ?
A. 30 **B**. 42 **C**. 55 **D**. 72

34. How many students offer at least one of physics or chemistry ?
A. 60 **B**. 85 **C**. 128 **D**. 139

35. How many students offer both mathematics and physics but **not** chemistry ?
A. 10 **B**. 20 **C**. 32 **D**. 48

36. How many students offer at least two of the three subjects ?
A. 85 **B**. 100 **C**. 105 **D**. 125

A school registers 300 candidates for an examination. 180 of the candidates offer Economics and 70 offer Geography. 80 candidates offer neither Economics nor Geography. Use this information to answer questions **37** and **38**.

37. The number of candidates offering both Economics and Geography is
A. 30 **B**. 40 **C**. 50 **D**. 60

38. The number of candidates offering at least one of Economics or Geography is
A. 200 **B**. 220 **C**. 230 **D**. 250

39. In a class of 58 students, 35 offer Biology, 28 offer Economics and 15 offer Physics. Each student offers at least one of the three subjects. If 12 students offer both Biology and Economics, 7 students offer both Biology and Physics and 5 students offer both Economics and Physics, how many students offer all three subjects ?
A. 4 **B**. 6 **C**. 7 **D**. 8

40. Referring to question 39, how many students offer only one of the three subjects ?
A. 30 **B**. 42 **B**. 50 **D**. 58

41. 33 of the students in a class play either football or table tennis. If 22 students play football and 18 play table tennis, how many students play football only ?
A. 7 **B**. 11 **C**. 15 **D**. 17

42. A school went into a sports fiesta with a contingent which includes 11 footballers and 8 sprinters. If 3 of the footballers are also sprinters, how many of the students are either footballers or sprinters ?
A. 11 **B**. 14 **C**. 16 **D**. 19

43. In a class of 60 students, 40 speak Hausa while 19 speak Igbo and 7 students speak both languages. How many students speak neither of the languages ?
A. 11 **B**. 8 **C**. 7 **D**. 6

44. In a certain class, 37 students take at least one of Chemistry, Economics and Government. 8 students take Chemistry, 19 take Economics and 25 take Government. 12 students take Economics and Government but nobody takes Chemistry and Economics. How many students take both Chemistry and Government?
A. 3 **B**. 4 **C**. 5 **D**. 6

45. In a class of 45 students, every student has to offer either Chemistry or Economics or both. If 25 students offer Chemistry while 34 offer Economics, how many students offer both subjects ?
A. 20 **B**. 15 **C**. 14 **D**. 11

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ANSWER KEY

1. **B**	9. **C**	17. **D**	25. **C**	33. **C**	41. **C**
2. **A**	10. **D**	18. **C**	26. **C**	34. **D**	42. **C**
3. **D**	11. **C**	19. **A**	27. **B**	35. **B**	43. **B**
4. **C**	12. **B**	20. **B**	28. **A**	36. **C**	44. **A**
5. **A**	13. **D**	21. **A**	29. **D**	37. **A**	45. **C**
6. **C**	14. **C**	22. **C**	30. **A**	38. **D**	
7. **B**	15. **B**	23. **D**	31. **D**	39. **A**	
8. **D**	16. **A**	24. **B**	32. **A**	40. **B**	

SOLUTIONS TO OBJECTIVE QUESTIONS

1. **(B)** $P \cap Q = \{6\}$ and $(P \cap Q)' = \{5, 7, 8\}$

2. **(A)** $Y \cap Z = \{o, u\}$
   $X \cup (Y \cap Z) = \{a, e, m, p, o, u\}$

3. **(D)** $P \cap Q$ = elements common to both P and Q
   $= \{3, 4\}$

4. **(C)** $X \cup Y = \{1, 2, 3, 4, 6, 9, 10\}$
   $(X \cup Y)' = \{5, 7, 8\}$
   $X' = \{1, 4, 5, 7, 8, 10\}$
   $Y' = \{3, 5, 7, 8, 9\}$
   $X' \cap Y' = \{5, 7, 8\} = (X \cup Y)'$

5. **(A)** $X \cap Y = \{3\}$
   $(X \cap Y) \cup X = \{1, 2, 3, 4\}$

6. **(C)** $X \cup Y = \{1, 2, 3, 4, 5, 6\}$
   $(X \cup Y) \cap Y = \{3, 5, 6\}$

7. **(B)** $P \neq R$ since $n(P) = 4$ while $n(R) = 6$
   $P \cup R = \{1, 2, 3, 2, 4, 2\}$
   $n(P \cup R) = 6$

8. **(D)** $P \cup Q = \{3, 1, 0, 5, 2, 8, 4\}$
   $P \cup Q \cup R = \{3, 1, 0, 5, 2, 8, 4, 7, 6\}$
   $= \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

9. **(C)** $P \cap Q = \{3, 1\}$
   $P \cap Q \cap R = \varnothing$
   [Note that $\varnothing$ is not the same as $\{0\}$:
   $\varnothing$ is an empty set while $\{0\}$ is a member of the set P].

10. **(D)** $X \cup Y \cup Z = \{1, 2, 3, 4, 5, 6, 7\}$
    $n(X \cup Y \cup Z) = 7$
    $(X \cap Y) \cup Z = \{1, 2, 4, 5\} = X \cap (Y \cup Z)$
    $X \cap Y \cap Z = \{4, 5\}$, or $n(X \cap Y \cap Z) = 2$

11. **(C)** $P \cap Q = \{1, 3, 4, 6, 9\}$
    $R' = \{1, 2, 7, 8, 9\}$
    $P \cap Q \cap R' = \{1, 9\}$

12. **(B)** $X \cup Y = (a, b, c, d, e)$
    $X \cup Z = (a, b, c, d, e, f, g)$
    $(X \cup Y) \cap (X \cup Z) = (a, b, c, d, e)$

13. **(D)** $Y \cup Z = \{a, a, o, u, e, c, t, l, g\} = U$
    $\therefore n(Y \cup Z) = n(U) = 9$
    $X \cup Y \cup Z = U \Rightarrow (X \cup Y \cup Z)' = \varnothing$
    $Y' = \{c, t, l, g\} = Z$

14. **(C)** The given statement means that *the elements of P are positive integers such that the square of each element is less than 10 and none of the elements is zero.* The set of elements which satisfies these conditions is $\{1, 2, 3\}$.

15. **(B)** $Q = \{\ldots -9, -7, -5, -3, -1, 1, 3, 5, 7, 9, \ldots\}$
    $R = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$
    $Q \cap R = \{-5, -3, -1, 1, 3\}$

16. **(A)** $P' = \{x : x \leq 0\} = \{\ldots -4 -3, -2, -1, 0\}$
    $P' \cap R = \{-5, -4, -3, -2, -1, 0\}$

17. **(D)** $P \cap Q$ = all negative, odd integers
    $= \{\ldots, -9, -7, -5, -3, -1\}$
    $(P \cap Q)'$ = {all negative even integers, zero, and all positive integers} = infinite set

18. **(C)** $X = \{4, 8, 12, 16, 20\}$
    $Y = \{3, 6, 9, 12, 15, 18\}$
    $X' = \{1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19\}$
    $X' \cap Y = \{3, 6, 9, 15, 18\}$

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19. **(A)** $P = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3\}$
$Q = \{-4, -3 -2, -1, 0, 1, 2, 3, \ldots, 11\}$
$R = \{-2, -1, 0, 1, 2, 3, 4\}$
$P \cap Q = \{-4, -3, -2, -1, 0, 1, 2, 3\}$

20. **(B)** $Q \cup R = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots 11\}$
$= Q$
[In general, $Q \cup R = Q$ if $R \subseteq Q$]

21. **(A)** $(Q \cup R)' = Q'$ = all integers not contained in Q
$= \{x < -4$, and $x > 11\}$
$P \cap (Q \cup R)' = \{x < -4\}$

22. **(C)** Since $84 = 2 \times 2 \times 3 \times 7$, $X = \{2, 3, 7\}$
Also $315 = 3 \times 3 \times 5 \times 7$, hence $Y = \{3, 5, 7\}$
$\Rightarrow X \cup Y = \{2, 3, 5, 7\}$
$X \cap Y = \{3, 7\}$

23. **(D)** $U = \{2, 3, 4, 5, 6, 7, 8, 9\}$
$P = \{2, 4, 6, 8\}$
$Q = \{3, 5, 7\}$
$P \cap Q = \varnothing$ and $(P \cap Q)' = U$.

24. **(B)** If $M \subseteq N$, then $M \cap N = M$
$\therefore M \cap (M \cap N)' = M \cap M' = \varnothing$

25. **(C)** If $x^2 = 16$, then $x = \pm 4$, Since $x > 5$, set R is an empty set $\varnothing$. Note that options (A) and (D) have no meaning in set notation while (B) gives a set with the number '0' as a member.

26. **(C)** $Y \cap Z$ = region $e + f$
$Y' \cap Z$ = region $d + g$
$X' \cap Z$ = region $f + g$
$(X \cup Y)'$ = region $g + h$
$(X \cup Y)' \cap Z$ = region g

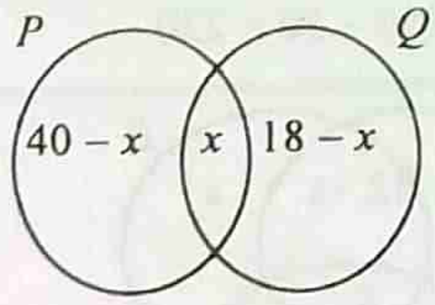
27 **(B)** $X \cap Y \cap Z$ = region e
$X \cap Z = d + e$, and $Y \cap Z = e + f$
$(X \cap Z) \cup (Y \cap Z) = d + e + f$
$X \cup Y \cap Z' = a + b + c$
$(X \cup Z) \cap (Y \cup Z) = b + e + f$

28. **(A)** $X \cup Y = a + b + c + d + e + f$
$(X \cup Y)' = g + h$
$(X \cup Y \cup Z)' = h$
$X' \cup Z \cap Y = f$
$Y \cup Z \cup X' = b + c + d + e + f + g + h$

29. **(D)** $X' \cap Y = c + f$
$X' \cap Y \cap Z' = c$

30. **(A)** $X' = c + f + g + h$, $Z' = a + b + c + h$
$X' \cap Z' = c + h$

31. **(D)**



The sets P and Q are illustrated in the diagram above, where $x = n(P \cap Q)$. From the figure:
$n(P \cup Q) = 40 - x + x + 18 - x = 50$,
or $x = 8$

32. **(A)** Referring to the diagram,
$n(X' \cap Y') = n(U) - n(X \cup Y)$
$= 27 - 22 = 5$

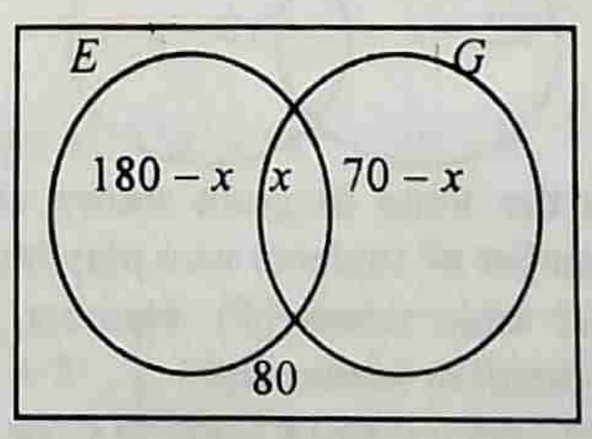
33. **(C)** Total number of students = $(32 - x) + 4x$
$+ (7 + x) + 2x + 9x + 6x + (27 - x) = 166$
$20x = 100$, or $x = 5$
Number of students offering both M and C is
$M \cap C = 9x + 2x = 11x = 11 \times 5 = 55$

34. **(D)** This is given by $P \cup C$
$= 4x + 2x + (7 + x) + 9x + 6x + (27 - x)$
$= 21x + 34 = 105 + 34 = 139$
[*Alternatively*: This is the difference between the total number of students (166), and the number offering Mathematics only (i.e. $32 - x = 27$). Hence $166 - 27 = 139$]

35. **(B)** This is $(M \cap P) \cap C' = 4x = 4(5) = 20$.

36. **(C)** These are students offering either mathematics and physics, or mathematics and chemistry, or physics and chemistry, i.e.
$(M \cap P) \cup (M \cap C) \cup (P \cap C)$
$= 4x + 2x + 9x + 6x = 21x = 21 \times 5 = 105$.

37. **(A)**



The Venn diagram is illustrated above. Number offering both E and G is x.
$(180 - x) + x + (70 - x) + 80 = 300$,
or $x = 30$

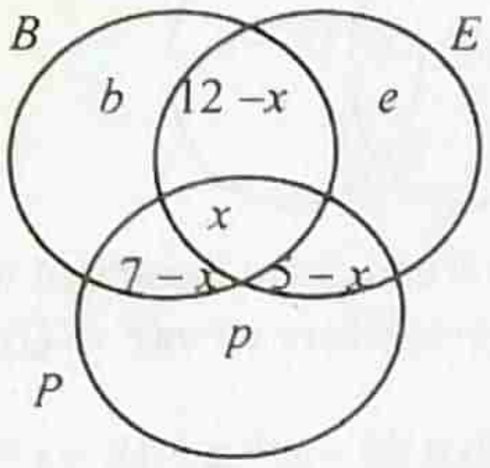
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38. **(D)** Number offering at least one of Economics or Geography is

$E \cup G = (180 - x) + x + (70 - x) = 250 - x$

$= 250 - 30 = 220$

39. **(A)**



The information supplied is displayed in the Venn diagram shown above, where x = number of students offering all three subjects and b, e, and p are respectively the numbers offering Biology only, Economics only and Physics only.

Total number offering Biology

$= b + (12 - x) + x + (7 - x) = 35$, or

$b = 16 + x$

Number offering Economics

$= e + (12 - x) + x + (5 - x) = 28$, or

$e = 11 + x$

Number offering Physics

$= p + (7 - x) + x + (5 - x) = 15$, or

$p = 3 + x$

Total number of students

$= (16 + x) + (12 - x) + x + (7 - x) +$

$(11 + x) + (5 - x) + 3 + x = 58$, or

$x = 4$.

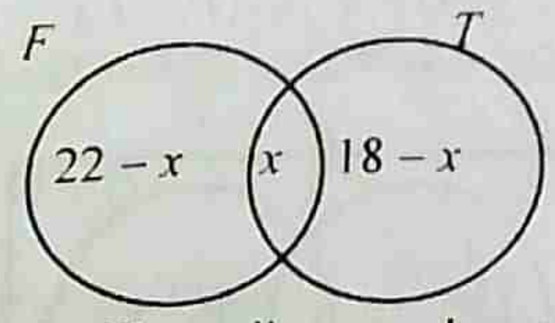
40. **(B)** Number offering only one of the three subjects

$= b + e + p = (16 + x) + (11 + x) + (3 + x)$

$= 30 + 3x$

$= 30 + 12 = 42$

41. **(C)**

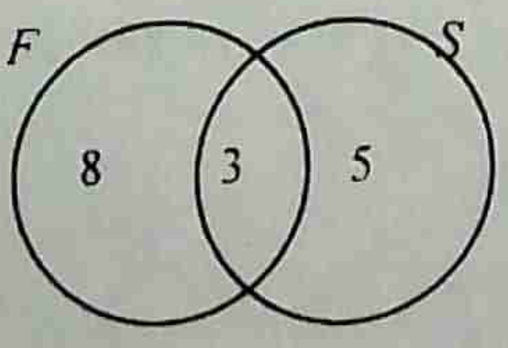


In the Venn diagram shown above, x is the number of students who play both football (F) and table tennis (T). Number playing either football or tennis is

$(22 - x) + x + (18 - x) = 33$, or $x = 7$

Number playing only football $= 22 - x = 15$

42. **(C)**



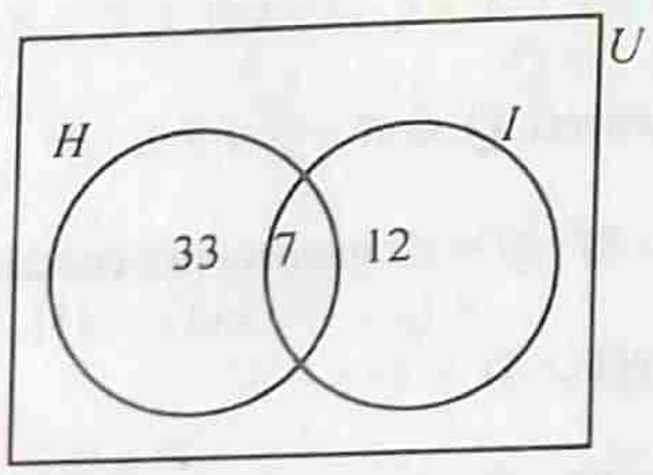
The Venn diagram is illustrated above.

Number playing football (F) only $= 11 - 3 = 8$

Number sprinting only (S) $= 8 - 3 = 5$

Number either playing football or sprinting $= 8 + 3 + 5 = 16$

43. **(B)**



See Venn diagram above:

Number who speak Hausa (H) alone

$= 40 - 7 = 33$

Number who speak Igbo (I) alone

$= 19 - 7 = 12$

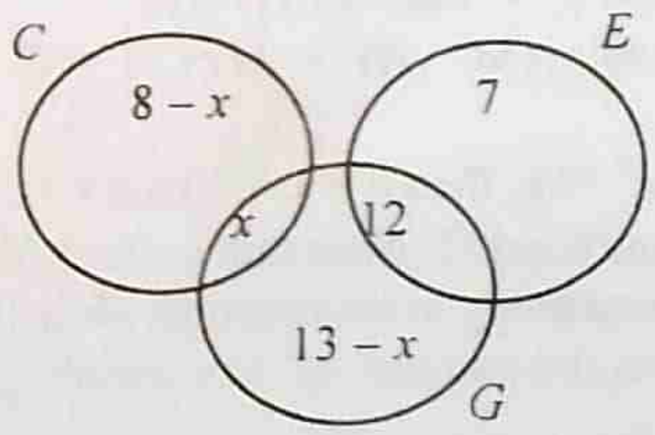
Number who speak either Hausa or Igbo

$= 33 + 7 + 12 = 52$

Number who speak neither language

$= 60 - 52 = 8$

44. **(A)**



The Venn diagram is shown above, where x = number taking both Chemistry (C) and Government (G).

Number taking Chemistry only $= 8 - x$

Number taking Government only $= 13 - x$

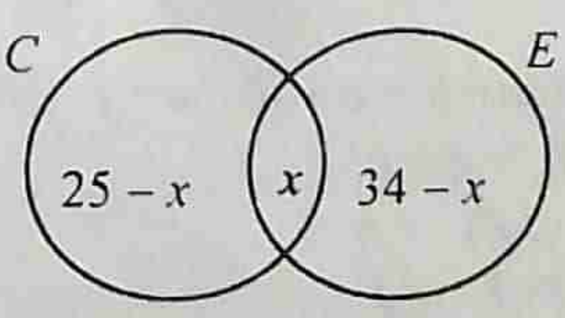
Number taking Economics only $= 19 - 12 = 7$

Total number $= (8 - x) + x + (13 - x) + 12 + 7$

$= 40 - x$

$40 - x = 37$, or $x = 3$.

45. **(C)**



Refer to the Venn diagram shown above

Total number of students is

$(25 - x) + x + (34 - x) = 45$

$59 - x = 45$, or

$x = 14$.

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5 Algebraic Expressions

5.1 Evaluation of Algebraic Expressions

- An **algebraic expression** consists of a numeral and a variable, e.g. $3x$, which is a product of the numeral 3 and the variable x, or the product of a numeral and two or more variables, e.g. $7xy$, $2abc^2$.
- A **polynomial** is an algebraic expression which is the sum (or difference) of two or more terms, e.g. $3a^2 - 7$, $y^2 + 5y - 1$.
- An algebraic expression can be evaluated when values are assigned to the variables in the expression, e.g. the expression $2z^2 - 3$ has the value $2(2)^2 - 3$, or 5, when $z = 2$.

DRILL 1

(a) Find the values of the following expressions:

(i) $xy^2 - yz^2$, when $x = 2$, $y = -2$ and $z = \frac{1}{2}$

(ii) $x^3 + 2x^2 - 5x + 7$, when $x = -3$

(iii) $(a^2 - 3b^2)(ab + b^2)$, when $a = 1$ and $b = -1$.

(b) Find the value of x for which

(i) $\frac{1}{3}(x + 7) = \frac{1}{4}(7x - 2)$

(ii) $\frac{5}{2 + x} = \frac{3}{x - 2}$

5.2 Multiplication and Division of Polynomials

- To find the product of a polynomial $ax^2 + bx + c$ and a second polynomial $dx^2 + ex + f$, each term of one polynomial is multiplied by each term of the second polynomial.
- The terms are arranged in vertical columns so that each column contains terms having the same powers of x.
- The final result is obtained by adding up the terms under each column.

EXAMPLE 5–1

Multiply $3x^2 - x + 6$ by $2x^2 + 5x - 7$

Solution

$$\begin{array}{r} 3x^2 - x + 6 \\ \times \quad 2x^2 + 5x - 7 \\ \hline -21x^2 + 7x - 42 \\ 15x^3 - 5x^2 + 30x \\ 6x^4 - 2x^3 + 12x^2 \\ \hline 6x^4 + 13x^3 - 14x^2 + 37x - 42 \end{array}$$

DRILL 2

Do the following multiplications:

(a) $2x^2 - x + 3$ by $x^2 - 5$

(b) $2x + 3y - z$ by $5x + 2z$

(c) $2a^2 - 4a + 3$ by $1 - 8a$

- If a polynomial P is to be divided by a second polynomial Q, P is called the **dividend** and Q the **divisor**.
- To divide P by Q, arrange both P and Q in descending order of the powers of the variable, e.g. $5x^2 - x + x^3$ is written as $x^3 + 5x^2 - x$.
- Write any missing power as a term with a coefficient of 0, e.g. the term containing x^2 is missing in the expression $x^3 + 3x + 7$, which is now written as $x^3 + 0x^2 + 3x + 7$.
- Follow the same procedure as for long division (see Example 5–2 below).

EXAMPLE 5–2

Divide $30x - 9 + 25x^3$ by $5x^2 + 2x + 7$

Solution

Write the dividend as $25x^3 + 0x^2 + 30x - 9$

$$\begin{array}{r|l} & 5x - 2 \\ \hline 5x^2 + 2x + 7 & 25x^3 + 0x^2 + 30x - 9 \\ & 25x^3 + 10x^2 + 35x \\ \hline & -10x^2 - 5x - 9 \\ & -10x^2 - 4x - 14 \\ \hline \text{remainder} \longrightarrow & -x + 5 \end{array}$$

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Hence the answer is $5x - 2$ (called the **quotient**) with a remainder of $-x + 5$. Note that you cannot carry out any further division when the highest order of the remainder (x in this case) is lower than the highest order in the divisor (x^2).

DRILL 3

Perform the following divisions:

(a) $(y^4 + 2y^2 + y + 2) \div (y^2 - y + 2)$

(b) $(3x^4 + 19x^3 - 15x^2 + 35x - 2) \div (3x^2 - 2x + 5)$

(c) $(3 - 5a^2 + a^4) \div (a^2 + a + 1)$

5.3 Change of Subject of a Formula

- A **formula** is an equation which states the relationship between two or more variables, e.g.

$$s = ut + \frac{1}{2}at^2$$

- In the equation stated above, s is the **subject** of the formula u, t and a are called **independent variables** while the subject, s, is a **dependent variable**.
- To change the subject of a formula, some or all of the following steps must be followed:

(i) Clear all fractions on both sides of the equation.
(ii) Expand all factors inside brackets.
(iii) Eliminate all root signs by suitable multiplications.
(iv) Rearrange the formula by bringing all terms containing the new subject to the left hand side (LHS) and writing all other terms on the right hand side (RHS) of the equation.
(v) Where the new subject is contained in more than one term, make the new subject a factor of an expression by bringing it outside a bracket.
(vi) Divide or multiply both sides of the equation by an appropriate factor to isolate the new subject.

EXAMPLE 5–3

Make x the subject of the formula

$$y = \frac{a + bx}{c + dx}$$

Solution

Clear fractions ..... $y(c + dx) = a + bx$
Expand bracket ..... $yc + ydx = a + bx$
Bring terms containing x to LHS ..... $ydx - bx = a - yc$
Bring x outside bracket ..... $x(yd - b) = a - yc$
Divide both sides by $(yd - b)$: $x = \frac{a - yc}{yd - b}$

EXAMPLE 5–4

Make c the subject of the formula

$$T = 2\pi \sqrt{\frac{l^2 + c^2}{2gl}}$$

Solution

Eliminate square-root sign

$$T^2 = 4\pi^2 \left[\frac{l^2 + c^2}{2gl}\right]$$

Bring term containing c to LHS

$$l^2 + c^2 = \frac{2glT^2}{4\pi^2}$$

$$c^2 = \frac{2glT^2}{4\pi^2} - l^2$$

Take the square root of both sides

$$c = \sqrt{\frac{glT^2}{2\pi^2} - l^2}$$

5.4 Variation

- **Direct variation** between one quantity and another implies that an *increase* in one leads to a corresponding *increase* in the other. If y varies **directly** as x, it is written mathematically as

$$y \propto x, \text{ or } y = kx$$

where k is a constant.

- **Inverse variation** between two quantities means that an *increase* in one results in a corresponding *decrease* in the other. If y varies inversely as x, then

$$y \propto \frac{1}{x}, \text{ or } y = \frac{k}{x}$$

- **Joint variation** occurs when a certain quantity varies as the product of two or more variables. If z varies jointly as x and y, then

$$z \propto xy, \text{ or } z = kxy$$

- **Partial variation** occurs when a quantity depends on two or more terms. If the distance s covered by a moving object varies partly as the time t and partly as the square of t, then

$$s \propto t \text{ and } s \propto t^2, \text{ or } s = at + bt^2$$

where a and b are constants.

- In solving problems on variation:

(i) Write out the general relation between the variables in terms of the proportionality constant.
(ii) Find the constant by fitting the given values of the variables into the general equation.

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(iii) Write down the specific relation, using the value of the constant obtained in (ii).
(iv) Solve for the unknown variable using the data provided.

EXAMPLE 5–5

If y varies inversely as x and if $y = 15$ when $x = 3$, find the value of y when $x = 9$.

Solution

General equation: $y = \frac{k}{x}$

Solve for k: $k = yx = (15)(3) = 45$

Specific equation: $y = \frac{45}{x}$

For $x = 9$, $y = \frac{45}{9} = 5$.

EXAMPLE 5–6

If r varies jointly as x and the square of y, and if $r = 40$ when $x = 2$ and $y = 4$, find x when $r = 240$ and $y = 8$.

Solution

General equation: $r = kxy^2$

Solve for k: $k = \frac{r}{xy^2} = \frac{40}{2(4)^2} = \frac{5}{4}$

Specific equation: $r = 5xy^2/4$

For $r = 240$ and $y = 8$, solve for x.

$$x = \frac{4r}{5y^2} = \frac{(4)(240)}{(5)(8^2)} = 3$$

DRILL 4

Write down the general equations to illustrate the relationships between the variables as specified below:

(a) y varies inversely as x^2
(b) x varies jointly as the square of x and the square root of y.
(c) y is partly constant and varies partly as the square of x.
(d) a is directly proportional to b and inversely proportional to c^3.

ANSWERS TO DRILLS

1. (a) (i) $8\frac{1}{2}$ (ii) 13 (iii) 0
   (b) (i) 2 (ii) 8
2. (a) $2x^4 - x^3 - 7x^2 + 5x - 15$ (b) $10x^2 + 15xy - xz + 6yz - 2z^2$
   (c) $-16a^3 + 34a^2 - 28a + 3$
3. (a) $y^2 + y + 1$ (b) $x^2 + 7x - 2$, R. $8 - 4x$
   (c) $a^2 + a - 5$, R. $8 - 6a$
4. (a) $y = k/x^2$ (b) $z = kx^2\sqrt{y}$
   (c) $y = k_1 + k_2x^2$ (d) $a = kb/c^3$

SSCE/JME–TYPE OBJECTIVE QUESTIONS

1. If $4x - 2 = 3x - 5$, the value of $7x$ is
   A. 7 **B.** –3 **C.** –21 **D.** –49

2. Solve the equation $6(y - 4) + 3(y + 7) = 6$
   A. $\frac{3}{2}$ **B.** 1 **C.** $\frac{2}{3}$ **D.** 0

3. Simplify: $8a - 3(2a - 3b)$
   A. $2a + 9b$ **B.** $5a - 9b$
   C. $2a - 9b$ **D.** $2a + 3b$

4. If $3y + 4 - \frac{y}{3} = y + 9$, then y is
   A. 3 **B.** 4 **C.** 5 **D.** 6

5. If $y = \frac{2}{3}x + 1$ and $z = \frac{2}{3}y + 1$, then the value of z when $x = 66$ is
   A. $15\frac{1}{2}$ **B.** 21 **C.** 31 **D.** 61

6. If $P = \frac{R}{R+2} \div \frac{R+2}{R}$, where R is a number which is neither 0 nor –2, then
   A. $P \leq 0$ **B.** $P \geq 0$ **C.** $P = 1$ **D.** $P > 0$

7. The value of the expression $\frac{9}{4}x^3 - \frac{9}{2}x^2 - \frac{8}{27}x^{-3}$ when is $x = -\frac{2}{3}$ is

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A. $-1\frac{2}{3}$ **B.** $2\frac{1}{3}$ **C.** $-3\frac{2}{3}$ **D.** 6

8. Evaluate $(xy^2 - yz^2)(x^2z - xyz)$ when $x = 2$, $y = -2$ and $z = -\frac{1}{2}$

A. –34 **B.** –30 **C.** –28 **D.** 0

9. Given that $a^2 + b^2 + c^2 = 50$ and $a = 5$, $\sqrt{b} = 2$, find c

A. 2 **B.** 3 **C.** $3\sqrt{2}$ **D.** $4\sqrt{2}$

10. If $a = 2$ and $b = 3$, solve for y in the equation

$$\frac{a}{a-y} = \frac{b}{y-b}$$

A. $\frac{2}{3}$ **B.** $2\frac{1}{3}$ **C.** $2\frac{2}{5}$ **D.** 12

11. Five years ago, a father was four times as old as his son. In five years' time the father will be twice as old as the son. If x and y are the present ages of the father and the son respectively, the equations relating x and y are:

A. $4y - x = 0;\ x - 2y = 0$
B. $4y - x = 15;\ x - 2y = 5$
C. $4y - x = 25;\ x - 2y = 5$
D. $x - 4y = 15;\ y - 2x = 5$

12. The cost (y) of feeding x people for a week is given as $y = 250 + 80x$, where y is expressed in naira (₦). The cost of feeding two additional persons is

A. ₦660 **B.** ₦410 **C.** ₦160 **D.** ₦80

13. A father and his son can do a piece of work in 10 days. The father can do it alone in x days while the son alone will take five days longer. The equation which is satisfied by x is

A. $x^2 - 15x + 10 = 0$ **B.** $x^2 - 10x + 5 = 0$
C. $x^2 + 35x - 50 = 0$ **D.** $x^2 - 15x - 50 = 0$

14. A mother is now twice as old as her daughter. Ten years ago the mother was four times as old as her daughter. The present ages of the daughter and the mother are, respectively

A. 15, 30 **B.** 20, 40 **C.** 25, 50 **D.** 30, 60

15. John was 40 years old in 1995 while his father was 65. In what year was John exactly half his father's age?

A. 1990 **B.** 1985 **C.** 1980 **D.** 1978

16. Expand $(3x + y + 7)(x^2 - 4y + 5)$

A. $3x^3 + x^2y - 11xy + 7x^2 + 15x - 4y^2 - 28y + 35$
B. $3x^3 + x^2y - 12xy + 7x^2 + 15x - 4y^2 - 23y + 35$
C. $3x^3 + 2x^2y - 12xy + 7x^2 + 15x - y^2 - 23y + 35$
D. $3x^3 + x^2y - 12xy + 7x^2 + 15x - 4y^2 + 23y + 35$

17. Expand $(2x - 3y + z)(3x - y + 4z)$

A. $6x^2 - 2xy + 3y^2 + 4z^2 + 11xz - 13yz$
B. $6x^2 - 11xy - 3y^2 + 4z^2 + 3xz - 12yz$
C. $6x^2 - 9xy + 3y^2 + 4z^2 + 8xz - 12yz$
D. $6x^2 - 11xy + 3y^2 + 4z^2 + 11xz - 13yz$

18. The product of $(2\sqrt{x} - 3x)$ and $(3x + 2\sqrt{x})$ is

A. $4x + 9x^2$ **B.** $4x + x^2$
C. $4x - 9x^2$ **D.** $-4x - 9x^2$

19. Multiply $a^2 + a + 1$ by $a^2 - a + 1$

A. $a^4 + 3a^2 + a + 1$ **B.** $a^4 + a^2 + 1$
C. $a^4 + 4a^2 - 6a + 1$ **D.** $a^4 - 6a^2 - 4a + 1$

20. Divide $y^3 - 2y^2 - 5y + 6$ by $(y - 1)$

A. $y^2 - y - 6$ **B.** $y^2 - 5y + 6$
C. $y^2 - 7y + 6$ **D.** $y^2 - 5y - 6$

21. Divide $a^3 - a - 7 + 7a^2$ by $(-1 + a^2)$

A. $-a^3 + 7a^2 - a - 7$ **B.** $-a^2 - 7a + 7$
C. $a - 7$ **D.** $a + 7$

22. Make y the subject of the formula $x = \frac{a+y}{a-y}$

A. $y = \frac{2a - x}{a + x}$ **B.** $y = \frac{ax - 1}{x + 1}$
C. $y = \frac{a(x - 1)}{x + 1}$ **D.** $y = \frac{ax}{a + 1}$

23. Make b the subject of the formula

$$d = \sqrt{\frac{ab}{c} - c^2b}$$

A. $b = \frac{cd^2}{a - c^3}$ **B.** $b = \frac{d^2}{a - c^2}$
C. $b = \frac{cd}{a - c^3}$ **D.** $b = \frac{a - c^3}{cd^2}$

24. Make z the subject of the equation

$$x(y + z) + \frac{5}{a} - 2 = 0$$

A. $z = \frac{2a - 5 - y}{ax}$ **B.** $z = \frac{5 - 2a - y}{ax}$
C. $z = \frac{5 - 2a - axy}{ax}$ **D.** $z = \frac{2a - 5 - axy}{ax}$

25. Make z the subject of the formula $a = 1 - \frac{x}{5}\left(y + \frac{3}{7}z\right)$

A. $z = -\left[\frac{7y}{3} + \frac{5}{x}(a - 1)\right]$

B. $z = -\frac{7}{3}\left[y + \frac{5}{x}(a - 1)\right]$

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C. $z=\frac{7}{3}\left[y+\frac{5}{x}(a-1)\right]$

D. $z=-\frac{7}{3}\left[y+\frac{5a-1}{x}\right]$

26. Make Y the subject of the equation:

$$\frac{ax}{1-x}=\sqrt[3]{\frac{2x+Y}{a+2Y}}$$

A. $Y=\frac{3ax}{1-x}$

B. $Y=\frac{2x(1-x)^2-a^3x^2}{2a^2x^2-(1-x)^2}$

C. $Y=\frac{2x(1-x)^2-a^4x^2}{2a^2x^2+(1-x)^2}$

D. $Y=\frac{2x(1-x)^3-a^4x^3}{2a^3x^3-(1-x)^3}$

27. If $xy+1=y^2$ and $z=\frac{1}{x}-\frac{1}{xy}$, express z in terms of y

A. $\frac{1}{x-y}$

B. $\frac{1}{y-1}$

C. $\frac{1}{y+1}$

D. $1+y$

28. Make x the subject of the formula

$$d=\sqrt{\frac{6}{x}-\frac{y}{2}}$$

A. $x=\frac{6}{d^2}+\frac{12}{y}$

B. $x=\frac{12}{2d^2-y}$

C. $x=\frac{12}{y}-2d^2$

D. $\frac{12}{2d^2+y}$

29. Make b the subject of the formula

$$c=a^2+\frac{1}{b^3}$$

A. $b=\frac{1}{(c-a^2)^3}$

B. $b=\frac{1}{(c+a^2)^{1/3}}$

C. $b=\frac{1}{(c-a^2)^{1/3}}$

D. $b=\frac{1}{\sqrt[3]{c}-\sqrt[3]{a^2}}$

30. Express z in terms of x and y in the equation

$$\frac{x}{\sqrt{2}}=\sqrt{\frac{z}{z+y}}$$

A. $z=\frac{x^2y}{2-x^2}$

B. $z=\frac{xy^2}{2-y^2}$

C. $z=\frac{x^2y^2}{2-xy}$

D. $z=\frac{x}{y(2-x)}$

31. If $s=ut+\frac{1}{2}at^2$, then t is equal to

A. $\frac{1}{a}\left[u\pm\sqrt{u^2-2as}\right]$

B. $\frac{1}{a}\left[-u\pm\sqrt{u^2+2as}\right]$

C. $\frac{1}{a}\left[u\pm\sqrt{u^2+2as}\right]$

D. $\frac{1}{a}\left[-u\pm\sqrt{u^2-2as}\right]$

32. Make y the subject of the relation

$$\frac{1+ay}{1-ay}=\frac{c}{d}$$

A. $\frac{c+d}{a(c-d)}$

B. $\frac{c-d}{a(c+d)}$

C. $\frac{c-d}{acd}$

D. $\frac{cd}{a(c-d)}$

33. If $Y=2a\sqrt{\pi(XT+K)}$, then T is equal to

A. $\frac{Y^2}{2\pi a^2X}-\frac{K}{X}$

B. $\frac{Y^2}{2\pi a^2X}-K$

C. $\frac{Y^2}{4\pi a^2X}-\frac{K}{X}$

D. $\frac{Y^2}{4\pi a^2X}-K$

34. Make f the subject of the formula

$$T=\sqrt{\frac{u}{\left(\frac{1}{f}+\frac{1}{g}\right)}}$$

A. $\frac{gT^2}{gu-T^2}$

B. $\frac{gu-T^2}{gT^2}$

C. $\frac{u}{\frac{1}{T^2}-\frac{1}{g}}$

D. $\frac{gu}{T^2-g}$

35. If $a=x+yb^3$, then b is equal to

A. $\frac{a^{1/3}-x^{1/3}}{y}$

B. $\left[\frac{x-a}{y}\right]^{1/3}$

C. $\left[\frac{a-x}{y}\right]^{1/3}$

D. $\left[\frac{a-x}{y}\right]^3$

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36. If $x \propto \frac{1}{y^2}$ and $z = 4$ when $y = 3$, find the value of z when $y = 2$.
A. 18 **B.** 9 **C.** 3 **D.** 1

37. P varies directly as x and inversely as the square of y. If $P = 3$ when $x = 2$ and $y = 1$, then P can be expressed as
A. $P = \frac{2x}{3y}$ **B.** $P = \frac{3x}{2y^2}$
C. $P = \frac{3x^2}{2y^2}$ **D.** $P = \frac{x^2}{2y}$

38. If p varies inversely as $\frac{qr}{q-r}$, and $p = 5$ when $q = 7$ and $r = 2$, then p is equal to
A. $\frac{14(q-r)}{qr}$ **B.** $\frac{q-r}{14qr}$
C. $\frac{25(q-r)}{14qr}$ **D.** $\frac{14(q-r)}{25qr}$

39. If x varies directly as the square root of n, and $x = 9$ when $n = 9$, the value of x when $n = 1\frac{7}{9}$ is
A. $1\frac{1}{2}$ **B.** $1\frac{7}{9}$ **C.** 2 **D.** 4

40. P is directly proportional to m and inversely proportional to n. If $P = 9$ when $m = 24$ and $n = 8$, find the value of P when $m = 5$ and $n = 6$.
A. $1\frac{1}{4}$ **B.** $2\frac{1}{2}$ **C.** $3\frac{3}{5}$ **D.** 11

41. Given that a varies inversely as b, and b varies directly as the square of c, and $a = 1$ when $c = 3$, find the value of a when $c = \frac{1}{3}$.
A. 81 **B.** 27 **C.** $\frac{1}{9}$ **D.** $\frac{1}{27}$

42. Q is jointly proportional to the cube of x and the fourth power of y. If x is halved and y doubled, then the value of Q is
A. increased in the ratio 1 : 4
B. decreased in the ratio 4 : 1
C. increased in the ratio 1 : 2
D. left unchanged

43. Y varies partly as the square of m and partly as the inverse of the square root of m. If $Y = 2$ when $m = 1$ and $Y = 6$ when $m = 4$, then Y can be expressed as
A. $\frac{10m^2}{31} + \frac{52}{31\sqrt{m}}$ **B.** $m^2 + \frac{1}{\sqrt{m}}$
C. $\frac{10}{31}\left[m^2 + \frac{1}{\sqrt{m}}\right]$ **D.** $\frac{m^2}{31} + \frac{1}{31\sqrt{m}}$

44. A quantity q varies inversely as m and n varies directly as q^3. Express n in terms of m given that $n = 1$ when $m = 2$.
A. $n = 8m^3$ **B.** $n = \frac{2}{m^{1/3}}$
C. $n = \frac{m^{1/3}}{2}$ **D.** $n = \frac{8}{m^3}$

45. The cost of providing a plate of lunch to a certain group of people is partly constant and partly inversely proportional to the number of people. If it costs ₦30 per plate to feed 100 people and ₦60 per plate to feed 40 people, find the cost per plate for 50 people.
A. ₦20 **B.** ₦40 **C.** ₦45 **D.** ₦50

46. X varies inversely as y and y varies directly as t^2. Find the relationship between X and t given that $t = 7$ when $X = 2$.
A. $X = 98t^2$ **B.** $Xt^2 = 98$
C. $98Xt^2 = 1$ **D.** $X = t^2/98$

47. P varies inversely as the cube root of R and $P = 1$ when $R = 8$. Find R when $P = 3$.
A. 2/3 **B.** 1/3 **C.** 8/27 **D.** 4/9

48. M varies inversely as N^2 and N varies directly as P^2. If k is a constant, the relationship between M and P can be expressed as
A. $P^4M = k$ **B.** $P^2M = k$
C. $M = kP^3$ **D.** $M = kP^4$

49. The quantity Q varies inversely as p; p varies directly as the square root of r, and r varies directly as $1/t^2$. Then
A. Q varies inversely as t^2
B. Q varies directly as t^2
C. Q varies directly as t
D. Q varies inversely as t

50. z is partly constant and partly varies inversely as the square of d. When $d = 1$, $z = 11$ and when $d = 2$, $z = 5$. Find the value of z when $d = 4$.
A. 2 **B.** 3.5 **C.** 5 **D.** 5.5

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ANSWER KEY

1.	C	10.	C	19.	B	28.	D	37.	B	46.	B
2.	B	11.	B	20.	A	29.	C	38.	A	47.	C
3.	A	12.	C	21.	C	30.	A	39.	D	48.	A
4.	A	13.	D	22.	C	31.	B	40.	B	49.	C
5.	C	14.	A	23.	A	32.	B	41.	A	50.	B
6.	D	15.	C	24.	D	33.	C	42.	C		
7.	A	16.	B	25.	B	34.	A	43.	A		
8.	A	17.	D	26.	D	35.	C	44.	D		
9.	B	18.	C	27.	C	36.	B	45.	D		

SOLUTIONS TO OBJECTIVE QUESTIONS

1. **(C)** $4x - 2 = 3x - 5,\ x = -5 + 2 = -3$
$7x = 7(-3) = -21$

2. **(B)** $6y - 24 + 3y + 21 = 6$
$9y = 9,$ or $y = 1$

3. **(A)** $8a - 3(2a - 3b) = 8a - 6a + 9b = 2a + 9b$

4. **(A)** $3y - y - \frac{y}{3} = 9 - 4$
$\frac{5y}{3} = 5,$ or $y = 3$

5 **(C)** $y = \frac{2}{3}(66) + 1 = 45$
$z = \frac{2}{3}(45) + 1 = 31$

6. **(D)** $P = \frac{R}{R+2} \div \frac{R+2}{R} = \frac{R}{R+2} \times \frac{R}{R+2}$
$= \left[\frac{R}{R+2}\right]^2$
Since the square of any (non-zero) number must be greater than zero, $P > 0$.

7. **(A)** The value of the given expression is
$\frac{9}{4}\left(-\frac{2}{3}\right)^3 - \frac{9}{2}\left(-\frac{2}{3}\right)^2 - \frac{8}{27}\left(-\frac{2}{3}\right)^{-3}$
$= \frac{9}{4}\left(-\frac{8}{27}\right) - \frac{9}{2}\left(\frac{4}{9}\right) - \frac{8}{27}\left(-\frac{27}{8}\right)$
$= -\frac{2}{3} - 2 + 1 = -1\frac{2}{3}$

8. **(A)** $xy^2 - yz^2 = (2)(-2)^2 - (-2)\left(-\frac{1}{2}\right)^2$
$= 8 + \frac{1}{2} = 8\frac{1}{2}$
$x^2z - xyz = 2^2\left(-\frac{1}{2}\right) - (2)(-2)\left(-\frac{1}{2}\right)$
$= -2 - 2 = -4$
$(x^2y - yz^2)(x^2z - xyz) = 8\frac{1}{2}(-4)$
$= -34$

9. **(B)** $a^2 = 25,\ b = 4$ and $b^2 = 16$
$c^2 = 50 - a^2 - b^2 = 50 - 25 - 16 = 9,$ or
$c = 3$

10. **(C)** $\frac{2}{2-y} = \frac{3}{y-3}$
$2(y - 3) = 3(2 - y)$
$2y - 6 = 6 - 3y,$ or $y = \frac{12}{5} = 2\frac{2}{5}$

11. **(B)** Five years ago, father's age was $x - 5$ and son's age was $y - 5$.
Hence $x - 5 = 4(y - 5)$,
or $4y - x = 15$
In five years' time, father's age will be $x + 5$ and son's age will be $y + 5$, hence $(x + 5) = 2(y + 5)$,
or $x - 2y = 5$.

12. **(C)** Cost of feeding $(x + 2)$ persons is
$y = 250 + 80(x + 2) = 250 + 80x + 160$
Cost of feeding two additional persons is
$(250 + 80x + 160) - (250 + 80x) = ₦160.$

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13. **(D)** In 1 day, father and son can do $\frac{1}{10}$ of the work.

In 1 day, father alone can do $\frac{1}{x}$ of the work.

In 1 day, son alone can do $\frac{1}{x+5}$ of the work.

Hence $\frac{1}{x}+\frac{1}{x+5}=\frac{1}{10}$

$\frac{2x+5}{x(x+5)}=\frac{1}{10}$

$20x+50=x^2+5x$, or

$x^2-15x-50=0$

14. **(A)** Although an equation can be set up to solve this problem, it is most quickly solved by inspection. Subtracting 10 from each of the answers given, option (A) gives the ages 10 years ago as 5 and 20, which is the correct answer.
[*Alternate Solution:* Let x, $2x$ be the present ages of daughter and mother respectively. Then $2x-10=4(x-10)$, or $x=15$ and $2x=30$.]

15. **(C)** Suppose John was half his father's age x years before 1995.

$40-x=\frac{1}{2}(65-x)$,

$80-2x=65-x$, or $x=15$

Fifteen years before 1995 was 1980.

16. **(B)** $(3x+y+7)(x^2-4y+5)$

$=3x(x^2-4y+5)+y(x^2-4y+5)$
$+7(x^2-4y+5)$

$=3x^3-12xy+15x+yx^2-4y^2$
$+5y+7x^2-28y+35$

$=3x^3+x^2y-12xy+7x^2+15x-4y^2-23y+35.$

17. **(D)** $(2x-3y+z)(3x-y+4z)$

$=2x(3x-y+4z)-3y(3x-y+4z)$
$+z(3x-y+4z)$

$=6x^2-2xy+8xz-9xy+3y^2-12yz$
$+3xz-yz+4z^2$

$=6x^2-11xy+3y^2+4z^2+11xz-13yz$

18. **(C)** $(2\sqrt{x}-3x)(3x+2\sqrt{x})$

$=2\sqrt{x}(3x+2\sqrt{x})-3x(3x+2\sqrt{x})$

$=6x\sqrt{x}+4x-9x^2-6x\sqrt{x}$

$=4x-9x^2$

19. **(B)** $(a^2+a+1)(a^2-a+1)$

$=a^2(a^2-a+1)+a(a^2-a+1)+1(a^2-a+1)$

$=a^4-a^3+a^2+a^3-a^2+a+a^2-a+1$

$=a^4+a^2+1$

20. **(A)**

$$y-1\overline{)\,y^3-2y^2-5y+6}\;\;y^2-y-6$$

y^3-y^2

$-y^2-5y$

$-y^2+y$

$-6y+6$

$-6y+6$

21. **(C)**

$$a^2-1\overline{)\,a^3+7a^2-a-7}\;\;a-7$$

a^3+0a^2-a

$7a^2-7$

$7a^2-7$

22. **(C)** $x=\frac{a+y}{a-y}$

$ax-xy=a+y$

$y(1+x)=a(x-1)$

$y=\frac{a(x-1)}{1+x}$

23. **(A)** $d^2=\frac{ab}{c}-c^2b$

$cd^2=ab-c^3b=b(a-c^3)$

$b=\frac{cd^2}{a-c^3}$

24. **(D)** $x(y+z)+\frac{5}{a}-2=0$

$ax(y+z)+5-2a=0$

$axz=2a-5-axy$

$z=\frac{2a-5-axy}{ax}$

25. **(B)** $a=1-\frac{x}{5}\left(y+\frac{3}{7^z}\right)$

$5a=5-x\left(y+\frac{3}{7^z}\right)$

$y+\frac{3}{7^z}=\frac{5-5a}{x}=\frac{5(1-a)}{x}$

$\frac{3}{7^z}=-y+\frac{5(1-a)}{x}$

$z=\frac{7}{3}\left[-y+\frac{5}{x}(1-a)\right]$

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$$= -\frac{7}{3}\left[y + \frac{5}{x}(a-1)\right]$$

26. **(D)** $\dfrac{a^3x^3}{(1-x)^3} = \dfrac{2x+Y}{a+2Y}$

$$a^4x^3 + 2a^3x^3Y = 2x(1-x)^3 + Y(1-x)^3$$

$$Y[2a^3x^3 - (1-x)^3] = 2x(1-x)^3 - a^4x^3$$

$$Y = \frac{2x(1-x)^3 - a^4x^3}{2a^3x^3 - (1-x)^3}$$

27. **(C)** $x = \dfrac{(y^2-1)}{y}$ and $xy = y^2 - 1$

$$z = \frac{y}{y^2-1} - \frac{1}{y^2-1} = \frac{y-1}{y^2-1}$$

$$= \frac{y-1}{(y+1)(y-1)} = \frac{1}{y+1}$$

28. **(D)** $d^2 = \dfrac{6}{x} - \dfrac{y}{2}$

$$\frac{6}{x} = d^2 + \frac{y}{2} = \frac{2d^2+y}{2}$$

$$x = \frac{12}{2d^2+y}$$

29. **(C)** $c = a^2 + \dfrac{1}{b^3}$

$$\frac{1}{b^3} = c - a^2 \Rightarrow b^3 = \frac{1}{c-a^2}$$

$$b = \frac{1}{(c-a^2)^{1/3}}$$

30. **(A)** $\dfrac{x^2}{2} = \dfrac{z}{z+y}$

$$zx^2 + yx^2 = 2z$$

$$z(2-x^2) = yx^2$$

$$z = \frac{yx^2}{2-x^2}$$

31. **(B)** $s = ut + \dfrac{1}{2}at^2 \Rightarrow 2s = 2ut + at^2$

$$at^2 + 2ut - 2s = 0$$

This is a quadratic equation with roots given by

$$t = \frac{-2u \pm \sqrt{4u^2 + 8as}}{2a} = \frac{1}{a}\left[-u \pm \sqrt{u^2 + 2as}\right]$$

32. **(B)** $d(1+ay) = c(1-ay)$

$$d + day = c - cay$$

$$y(ad + ac) = c - d$$

$$y = \frac{c-d}{a(c+d)}$$

33. **(C)** $Y = 2a\sqrt{\pi(XT+K)}$

$$Y^2 = 4a^2\pi(XT+K)$$

$$4a^2\pi XT = Y^2 - 4a^2\pi K$$

$$T = \frac{Y^2}{4\pi a^2 X} - \frac{K}{X}$$

34. **(A)** $T^2 = \dfrac{u}{\frac{1}{f}+\frac{1}{g}} = \dfrac{u}{(f+g)/fg} = \dfrac{ufg}{f+g}$

$$T^2f + T^2g = ufg$$

$$f(gu - T^2) = T^2g$$

$$f = \frac{gT^2}{gu - T^2}$$

35. **(C)** $yb^3 = a - x$

$$b^3 = \frac{a-x}{y}$$

$$b = \left[\frac{a-x}{y}\right]^{1/3}$$

36. **(B)** $z = k/y^2$

$4 = k/(3)^2 \Rightarrow k = 36$

$$z = \frac{36}{y^2}$$

For $y = 2$, $z = \dfrac{36}{2^2} = 9$

37. **(B)** $P = kx/y^2$

$3 = k(2)/1^2$, or $k = 3/2$

$$p = \frac{3x}{2y^2}$$

38. **(A)** $p = k\left[\dfrac{qr}{q-r}\right]^{-1} = \dfrac{k(q-r)}{qr}$

$$5 = \frac{k(7-2)}{(7)(2)}, \text{ or } k = 14$$

$$\Rightarrow p = \frac{14(q-r)}{qr}$$

39. **(D)** $x = k\sqrt{n}$

$9 = k\sqrt{9} = 3k$, or $k = 3$

$\Rightarrow x = 3\sqrt{n}$

For $n = 16/9$, $x = 3\sqrt{16/9} = 3(4/3) = 4$

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40. **(B)** $P = km/n$

$9 = k(24)/8$, or $k = 3$

$P = 3m/n$

When $m = 5$ and $n = 6$

$P = 3(5)/6 = 2\frac{1}{2}$

41. **(A)** $a \propto \frac{1}{b}$ and $b \propto c^2$

$\Rightarrow a \propto \frac{1}{c^2}$, or $a = \frac{k}{c^2}$

$1 = k/3^2$, or $k = 9$

$\Rightarrow a = 9/c^2$

when $c = \frac{1}{3}$, $a = \frac{9}{(1/3)^2} = 81$

42. **(C)** $Q = kx^3y^4$

If x is halved and y doubled, new Q is

$Q_1 = k(x/2)^3(2y)^4 = 2kx^3y^4 = 2Q$

i.e. Q is doubled

43. **(A)** $Y = k_1m^2 + k_2/\sqrt{m}$

$2 = k_1(1)^2 + k_2/\sqrt{1} = k_1 + k_2$ ... (i)

$6 = k_1(4)^2 + k_2/\sqrt{4} = 16k_1 + k_2/2$ ...(ii)

Multiply (ii) by (2) and subtract (i) from (ii):

$31k_1 = 10$, $k_1 = 10/31$, and from (i),

$k_2 = 2 - k_1 = 2 - \frac{10}{31} = \frac{52}{31}$

$Y = \frac{10}{31}m^2 + \frac{52}{31\sqrt{m}}$

44. **(D)** $q \propto \frac{1}{m}$ and $n \propto q^3 \Rightarrow n \propto \frac{1}{m^3}$, or

$n = \frac{k}{m^3}$

$1 = \frac{k}{2^3}$, or $k = 8$

Hence $n = 8/m^3$

45. **(D)** $c = k_1 + \frac{k_2}{n}$, where k_1, k_2 are constants

For $n = 100$, $c = 30 \Rightarrow 30 = k_1 + \frac{k_2}{100}$ ...(i)

For $n = 40$, $c = 60 \Rightarrow 60 = k_1 + \frac{k_2}{40}$ ...(ii)

Solving equations (i) and (ii) simultaneously gives $k_1 = 10$ and $k_2 = 2000$. Hence we can write

$c = 10 + \frac{2000}{n}$

For $n = 50$, $c = 10 + \frac{2000}{50} = 10 + 40 = ₦50$

46. **(B)** $X \propto \frac{1}{y}$ and $y \propto t^2 \Rightarrow X \propto \frac{1}{t^2}$

or $X = \frac{k}{t^2}$ where k is a constant.

$X = 2$ when $t = 7 \Rightarrow 2 = \frac{k}{7^2}$, or $k = 98$

Hence $X = \frac{98}{t^2}$, or $Xt^2 = 98$.

47. **(C)** $P = \frac{k}{R^{1/3}}$

When $R = 8$, $P = 1 \Rightarrow 1 = \frac{k}{8^{1/3}}$, or $k = 2$

Hence $P = \frac{2}{R^{1/3}}$

For $P = 3$, $R^{1/3} = \frac{2}{3}$, and $R = (2/3)^3 = \frac{8}{27}$

48. **(A)** $M \propto \frac{1}{N^2}$ and $N \propto P^2 \Rightarrow M \propto \frac{1}{P^4}$

$M = \frac{k}{P^4}$ or $MP^4 = k$

49. **(C)** $r \propto \frac{1}{t^2}$ and $p \propto \sqrt{r} \Rightarrow p \propto \frac{1}{t}$

But $Q \propto \frac{1}{p} \Rightarrow Q \propto \left(\frac{1}{t}\right)^{-1}$

i.e. $Q \propto t$ [Q varies directly as t].

50. **(B)** $z = k_1 + \frac{k_2}{d^2}$

$11 = k_1 + k_2/(1)^2 \Rightarrow k_1 + k_2 = 11$ ...(i)

$5 = k_1 + k_2/(2)^2 \Rightarrow 4k_1 + k_2 = 20$ ...(ii)

(ii)–(i) gives $3k_1 = 9$, or $k_1 = 3$

From (i), $k_2 = 11 - k_1 = 11 - 3 = 8$

Hence $z = 3 + \frac{8}{d^2}$

For $d = 4$, $z = 3 + \frac{8}{4^2} = 3.5$

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6 Factorization of Polynomial Expressions

6.1 Factorization

- A **polynomial** in x of degree n is an equation of the type

$$y = ax^n + bx^{n-1} + \ldots + zx + k$$

in which a, b, ..., z and k are constants and n is a positive integer.
When $n = 3$, it is a **cubic** equation, e.g.
$y = 5x^3 + 15x^2 - 11x + 3$.
When $n = 2$, it is a **quadratic** equation, e.g.
$y = 3x^2 + x + 4$.
When $n = 1$, it is a **linear** equation, e.g.
$y = 4x + 3$.

- Factorization of polynomial expressions involves the identification of factors which, when multiplied together, give the original expression.
- To factorize a polynomial in simple form, find the **highest common factor (HCF)** of all the terms and express the polynomial as the product of the HCF and another polynomial.

EXAMPLE 6–1

Factorize the following expressions:
(i) $a^3 - 3a^2 + a$ (ii) $5r^3 - 15r + 60$
(iii) $3ax^2 - 9a^2x$ (iv) $ac + ad + bc + bd$
(v) $x^2 - xy + x - y$

Solutions

(i) The highest common factor is a. We can write the function as $a(a^2) - a(3a) + a(1)$, or $a(a^2 - 3a + 1)$.

(ii) HCF of all terms is 5. We can write the function as $5(r^3) - 5(3r) + 5(12)$ or,
$5(r^3 - 3r + 12)$.

(iii) We can write this expression as
$3ax(x) - 3ax(3a)$, or $3ax(x - 3a)$.

(iv) a is common factor to the first two terms while b is common to the last two terms. We can write the function as $a(c+d) + b(c+d)$, which can be considered to be made up of two terms with a common factor $c + d$. Hence we can write it as $(c + d)(a + b)$.

(v) The expression can be written as
$x(x) - x(y) + x - y = x(x-y) + 1(x-y)$,
or $(x + 1)(x - y)$.

DRILL 1

Factorize: (a) $15xyz - 30x^2y$
(b) $a + ab - ab^2$
(c) $14a^4b^3 - 21a^4b^2 + 35a^3b^4$
(d) $y^3 - 3y^2 + 6y - 18$.

6.2 Factorization of Quadratic Functions

- A quadratic function is a polynomial of the form

$$ax^2 + bx + c, \quad (a \neq 0)$$

- For the purposes of factorizing such functions, two different cases are considered: $a = 1$ and $a \neq 1$.
- For $a = 1$, the quadratic function can be written as

$$x^2 + bx + c$$

- If $x + p$ and $x + q$ are the factors of this quadratic, then $(x + p)(x + q) = x^2 + bx + c$, or
$x^2 + (p + q)x + pq = x^2 + bx + c$. Hence

$$pq = c \quad \ldots\ldots\ldots\ldots (1)$$
and
$$p + q = b \quad \ldots\ldots\ldots\ldots (2)$$

- In factorizing a quadratic function with $a = 1$, it is only necessary to find the values of p and q which satisfy conditions (1) and (2) above.

EXAMPLE 6–2

Factorize: (i) $x^2 + 6x - 7$
(ii) $x^2 - x - 6$
(iii) $y^2 + 7y + 12$
(iv) $4 - 3z - z^2$

Solutions

(i) The factors are $x + p$ and $x + q$ such that

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$pq = -7$ .....(1)
and $p + q = 6$ .....(2)
Possible combinations of (p, q) satisfying (1) are (1,–7) and (7, –1). Only (7, –1) satisfies (2).
Hence $p = 7, q = -1$, i.e. $(x + 7)$ and $(x - 1)$ are the factors.
Hence $x^2 + 6x - 7 = (x - 1)(x + 7)$

(ii) $pq = -6$ .....(1)
$p + q = -1$ .....(2)
Possible combinations for (1) are (2, –3), (–2, 3), (1, –6), and (–1, 6). Only (2, –3) satisfies (2).
Hence the factors are $x + 2$ and $x - 3$
i.e. $x^2 - x - 6 = (x + 2)(x - 3)$.

(iii) $pq = 12$ .....(1)
$p + q = 7$ .....(2)
Possible combinations for (1) are (6, 2), (–6, –2), (3, 4), (–3, –4), (12, 1) and (–12, –1). Only (3, 4) satisfies (2). Hence factors are $y + 3$ and $y + 4$, i.e.
$y^2 + 7y + 12 = (y + 3)(y + 4)$

(iv) $4 - 3z - z^2 = -(z^2 + 3z - 4)$
For the function inside the parenthesis,
$pq = -4$ .....(1)
$p + q = 3$ .....(2)
Possible combinations for (1) are (2, –2), (4, –1) and (–4, 1).
Combination satisfying (2) is (4, –1), hence $z + 4$ and $z - 1$ are factors.
i.e. $4 - 3z - z^2 = -(z + 4)(z - 1) = (4 + z)(1 - z)$

- To factorize a quadratic $ax^2 + bx + c$ in which $a \neq 1$, follow the procedure outlined below:
  (i) Multiply the quadratic term (ax^2) by the constant (c) to give acx^2,
  (ii) Factorize acx^2 into px and qx such that $pq = ac$ and $px + qx = bx$
  (iii) Rewrite the original quadratic function, replacing bx with $px + qx$
  (iv) Factorize the resulting function by grouping.

EXAMPLE 6 – 3

Factorize (a) $20x^2 - 2x - 6$
(b) $2x^2 - 3x + 1$
(c) $75 - 49x + 8x^2$
(d) $12a^2b^2 + 11ab - 5$
(e) $24x^2 + xy - 23y^2$

Solutions

(a) Step (i): $20x^2(-6) = -120x^2$
Step (ii): Possible factors of $120x^2$ are $(5x, -24x)$, $(-5x, 24x)$, $(10x, -12x)$ $(-10x, 12x)$, $(8x, -15x)$, $(-8x, 15x)$, etc. By inspection, only $(10x, -12x)$ add up to $-2x$.
Step (iii): Rewrite quadratic as
$20x^2 + 10x - 12x - 6$
$= 10x(2x + 1) - 6(2x + 1)$
$= (10x - 6)(2x + 1)$
$= 2(5x - 3)(2x + 1)$

(b) $2x^2(1) = 2x^2$
Factors of $2x^2$ which add up to $-3x$ are $-2x$ and $-x$
$2x^2 - 3x + 1 = 2x^2 - 2x - x + 1$
$= 2x(x - 1) - 1(x - 1)$
$= (2x - 1)(x - 1)$

(c) The function can be written as
$8x^2 - 49x + 75$
$8x^2(75) = 600x^2$
Factors of $600x^2$ which add up to $-49x$ are $-25x$ and $-24x$. Hence $8x^2 - 49x + 75$
$= 8x^2 - 25x - 24x + 75$
$= x(8x - 25) - 3(8x - 25) = (x - 3)(8x - 25)$

(d) $12a^2b^2(-5) = -60a^2b^2$
Factors of $-60a^2b^2$ adding up to $11ab$ are $15ab$ and $-4ab$.
Hence $12a^2b^2 + 11ab - 5$
$= 12a^2b^2 + 15ab - 4ab - 5$
$= 3ab(4ab + 5) - 1(4ab + 5)$
$= (3ab - 1)(4ab + 5)$

(e) $24x^2(-23y^2) = -1052x^2y^2$
Factors adding up to xy are $24xy$ and $-23xy$.
Hence $24x^2 + xy - 23y^2$
$= 24x^2 + 24xy - 23xy - 23y^2$
$= 24x(x + y) - 23y(x + y)$
$= (24x - 23y)(x + y)$

DRILL 2

Factorize: (a) $x^2 + 6x + 5$
(b) $x^2 - 4x - 21$
(c) $6y^2 - 7y - 5$
(d) $5a^3 + 11a^2 - 12a$

6.3 Algebraic Identities

- The following identities are often found useful in factorizing quadratic polynomials:

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$a^2 - b^2 \equiv (a - b)(a + b)$ ..... (1)
$(a + b)^2 \equiv a^2 + 2ab + b^2$ ..... (2)
$(a - b)^2 \equiv a^2 - 2ab + b^2$ ..... (3)

- The following identities are also useful for cubic polynomials:

$a^3 + b^3 \equiv (a + b)(a^2 - ab + b^2)$ ..... (4)
$a^3 - b^3 \equiv (a - b)(a^2 + ab + b^2)$ ..... (5)
$(a + b)^3 \equiv a^3 + 3a^2b + 3ab^2 + b^3$ ..... (6)
$(a - b)^3 \equiv a^3 - 3a^2b + 3ab^2 - b^3$ ..... (7)

EXAMPLE 6–4

Factorize the following expressions:

(i) $x^2 - 4$ (ii) $12x^2 - 3y^2$
(iii) $a^4 - b^4$ (iv) $x^2 - 4x + 4$
(v) $x^2 + 6x + 9$ (vi) $x^6 - a^6$
(vii) $z^3 + 27$ (viii) $x^3 - 125$

Solutions

(i) $x^2 - 4 = x^2 - 2^2 = (x - 2)(x + 2)$ ..... identity (1)

(ii) $12x^2 - 3y^2 = 3(4x^2 - y^2)$
$= 3[(2x)^2 - y^2]$
$= 3(2x - y)(2x + y)$ ..... identity (1)

(iii) $a^4 - b^4 = (a^2)^2 - (b^2)^2$
$= (a^2 - b^2)(a^2 + b^2)$ ... identity (1)
$= (a - b)(a + b)(a^2 + b^2)$ ..... identity (1)

(iv) $x^2 - 4x + 4 = x^2 - 2(x)(2) + 2^2$
$= (x - 2)^2$ ..... identity (3)

(v) $x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2$
$= (x + 3)^2$ ..... identity (2)

(vi) $x^6 - a^6 = (x^3)^2 - (a^3)^2$
$= (x^3 - a^3)(x^3 + a^3)$ ..... identity (1)
$= (x - a)(x^2 + ax + a^2)(x^3 + a^3)$
..... identity (5)
$= (x - a)(x^2 + ax + a^2)(x + a)(a^2 - ax + a^2)$
..... identity (4)

[Alternatively, $x^6 - a^6 = (x^2)^3 - (a^2)^3$. As an exercise the candidate should check that it gives the same result as above].

(vii) $z^3 + 27 = z^3 + 3^3$
$= (z + 3)(z^2 - 3z + 9)$. ..... identity (4)

(viii) $x^3 - 125 = x^3 - 5^3$
$= (x - 5)(x^2 + 5x + 25)$
..... identity (5)

DRILL 3

(a) Factorize: (i) $32a^4 - 2b^4$
(ii) $(x + 2)^2 - y^2$
(iii) $1 - 81x^4$
(iv) $25 - y^2$

(b) Simplify: (i) $10.2^2 - 9.8^2$
(ii) $520^2 - 480^2$

6.3 The Perfect Square

- The expression $x^2 + 4x + 4$ is called a **perfect square** because it can be expressed as the square of a polynomial, i.e. $(x + 2)^2$.
- In general, if the quadratic equation $a^2x^2 + bx + c^2$ is a perfect square $(ax + c)^2$, then $a^2x^2 + bx + c^2 = a^2x^2 + 2acx + c^2$, or $b = 2ac$.
i.e. *the coefficient of the middle term must be equal to twice the square root of the product of the first and the last terms.*

EXAMPLE 6–5

Find the third term to complete the perfect squares in the following expressions:

(i) $x^2 + 6x$ (ii) $x^2 - {}^1/_2x$
(iii) $x^2 + 9$ (iv) $4y^2 + 4y$

Solutions

(i) By comparing the given expression to the perfect square quadratic $a^2x^2 + 2acx + c^2$, it is clear that a^2 = coefficient of $x^2 = 1$, or $a = 1$.
$2ac$ = coefficient of $x = 6 \Rightarrow c = 6/2a = 6/2 = 3$, and $c^2 = 3^2 = 9$. The perfect square is $x^2 + 6x + 9 = (x + 3)^2$.

(ii) $x^2 - {}^1/_2x + c^2 \equiv a^2x^2 + 2acx + c^2$
Equate coefficients of x^2 : $a = 1$
Equate coefficients of x : $2ac = -{}^1/_2$,
$c = -1/4a = -{}^1/_4$, and $c^2 = {}^1/_{16}$.
The perfect square is $x^2 - {}^1/_2x + {}^1/_{16} = (x - {}^1/_4)^2$

(iii) $x^2 + 2acx + 9 \equiv a^2x^2 + 2acx + c^2$,
$a = 1$ and $c^2 = 9$, or $c = \pm 3$.
$\Rightarrow 2ac = \pm 2(1)(3) = \pm 6$.
The perfect square is $x^2 + 6x + 9 = (x + 3)^2$
or $x^2 - 6x + 9 = (x - 3)^2$.

(iv) $4y^2 + 4y + c^2 \equiv a^2y^2 + 2acy + c^2$
$a^2 = 4$, or $a = 2$
$2ac = 4$, or $c = 4/2a = 4/4 = 1$ and $c^2 = 1$.
The perfect square is $4y^2 + 4y + 1 = (2y + 1)^2$.

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DRILL 4

Find the third term which will make the following expressions perfect squares:

(a) $x^2 + 14x$ (b) $x^2 + 64$

(b) $25y^2 + 9$ (d) $49x^2 + 36y^2$

6.4 The Remainder and Factor Theorems

- If a polynomial function (called the **dividend**) is divided by a linear expression (called the **divisor**), we obtain a **quotient** (which may be another polynomial) plus a **remainder** which is a constant.
- The **remainder theorem** states that if a polynomial function $F(x)$ is divided by $x - a$, we obtain a quotient $Q(x)$, and the remainder is $F(a)$. We can write

$$F(x) = (x - a)Q(x) + R$$

where $Q(x)$ is the quotient and R the remainder. Substituting $x = a$ in the above equation gives

$F(a) = R$.... **(Remainder Theorem)**

The **factor theorem** states that if $x - a$ is a factor of the polynomial function $F(x)$, then $F(a) = 0$. In other words the remainder is *zero* if $x = a$ is substituted into $F(x)$.

EXAMPLE 6 – 6

(a) Find the remainder when $2x^3 - 9x^2 - 3x - 90$ is divided by $x - 7$.

(b) Show that $x - 6$ is a factor of the expression.

Solution

(a) $F(x) = 2x^3 - 9x^2 - 3x - 90$

Using the *remainder theorem*, the remainder, R is given by $R = F(7) = 2(7)^3 - 9(7)^2 - 3(7) - 90 = 686 - 441 - 21 - 90 = 134$.

(b) By the *factor theorem*, $F(6) = 0$ if $x - 6$ is a factor. $F(6) = 2(6)^3 - 9(6)^2 - 3(6) - 90$

$= 432 - 324 - 18 - 90 = 0$

$\Rightarrow x - 6$ is a factor

EXAMPLE 6 – 7

(i) Show that $x - 1$ is a factor of $x^3 - 1$

(ii) Find the other factor or quotient.

Solution

(i) $F(x) = x^3 - 1$

$F(1) = 1^3 - 1 = 0$

$\Rightarrow x - 1$ is a factor.

(ii) The quotient is obtained by dividing $x^3 - 1$ by $x - 1$. On performing the division, the quotient is $x^2 + x + 1$.

DRILL 5

Determine whether or not the linear expression is a factor of the quadratic (or cubic) expression in each of the following cases:

(a) $x - 1$ and $x^2 + 5x - 6$

(b) $y - 2$ and $y^3 - 6y^2 + 12y - 8$

(c) $x + 3$ and $x^2 + 4x - 5$

(d) $y + 2$ and $3y^2 - 2y - 10$

ANSWERS TO DRILLS

1. (a) $15xy(z - 2x)$
   (b) $a(1 + b - b^2)$
   (c) $7a^3b^2(2ab - 3a + 5b^2)$
   (d) $(y^2 + 6)(y - 3)$

2. (a) $(x + 5)(x + 1)$
   (b) $(x - 7)(x + 3)$
   (c) $(3y - 5)(2y + 1)$
   (d) $a(5a + 4)(a - 3)$

3. (a) (i) $2(2a + b)(2a - b)(4a^2 + b^2)$
   (ii) $(x + y + 2)(x - y + 2)$
   (iii) $(1 + 3x)(1 - 3x)(1 + 9x^2)$
   (iv) $(5 + y)(5 - y)$
   (b) (i) $(10.2 + 9.8)(10.2 - 9.8) = 20(0.4) = 8$
   (ii) $(520 + 480)(520 - 480) = 1000 \times 40 = 40{,}000$

4. (a) 7 (b) $16x$
   (c) $30y$ (d) $84xy$

5. (a) Yes, $F(1) = 0$
   (b) Yes, $F(2) = 0$
   (c) No, $F(-3) = -8$
   (d) No, $F(-2) = 6$

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SSCE/JME - TYPE OBJECTIVE QUESTIONS

1. Factorize the expression $x(p-q)+y(q-p)$
   A. $(p-q)(y-x)$ **B.** $(p-q)(x-y)$
   C. $(p+q)(x-y)$ **D.** $(p+q)(x+y)$

2. Factorize: $pqx^2 + 12y - 4qx - 3pxy$
   A. $(px-4)(qx-3y)$ **B.** $(px+q)(x-12y)$
   C. $(px-3y)(qx-4)$ **D.** $(qx-4)(px+3y)$

3. Factorize: $y^2 + 2a + ay + 2y$
   A. $(y+2a)(y+1)$ **B.** $(y+2a)(y-1)$
   C. $(y^2-1)(y-a)$ **D.** $(y+2)(y+a)$

4. Factorize: $a^2 + 4a - 192$
   A. $(a-4)(a+48)$ **B.** $(a-48)(a+4)$
   C. $(a-12)(a+16)$ **D.** $(a-12)(a-16)$

5. Factorize: $2y^2 + y - 15$
   A. $(2y+5)(y-3)$ **B.** $(2y-5)(y+3)$
   C. $(2y-5)(y-3)$ **D.** $(2y-3)(y+5)$

6. Factorize: $2x^2 - 3x + 1$
   A. $(x+1)(2x+1)$ **B.** $(2x+3)(x-2)$
   C. $(2x-1)(x-1)$ **D.** $(2x-3)(x-1)$

7. Factorize: $3x^2 - 11x + 6$
   A. $(3x-2)(x-3)$ **B.** $(2x-2)(x-3)$
   C. $(3x-2)(x+3)$ **D.** $(3x+2)(x-3)$

8. Factorize: $35 - 2x - x^2$
   A. $(35-2x)(x-1)$ **B.** $(7+x)(5-x)$
   C. $(3+7x)(5-x)$ **D.** $(35-x)(3x+7)$

9. Factorize: $5x^2 + 2ax - 3a^2$
   A. $(5x-a)(x+3a)$ **B.** $(5x+a)(x-3a)$
   C. $(x-a)(5x+3a)$ **D.** $(x+a)(5x-3a)$

10. Factorize: $2z^2 + xz - 3x^2$
    A. $(2z-x)(2z+x)$ **B.** $(3x-2z)(x-z)$
    C. $(2z+3x)(z-x)$ **D.** $(x-z)(2z+3x)$

11. Expand $(2y-5)(y-3)$
    A. $2y^2 - 11y + 15$ **B.** $2y^2 - 5y + 15$
    C. $2y^2 - y + 15$ **D.** $2y^2 + y - 15$

12. Factorize: $6a - 5 - a^2$
    A. $-(a+3)(a+2)$ **B.** $(a-5)(a-1)$
    C. $(5-a)(a-1)$ **D.** $-(5-a)(1-a)$

13. Factorize: $6y^2 - 14y - 12$
    A. $2(y+3)(3y-2)$ **B.** $6(y-2)(y+1)$
    C. $2(y-3)(3y+2)$ **D.** $(3y-4)(2y+3)$

14. Find the values of a and b respectively if $\frac{1}{2}(3z-4y)^2 = 8y^2 + ayz + bz^2$
    A. $-6, 9$ **B.** $-12, 9/2$ **C.** $6, 9$ **D.** $12, 9/2$

15. Factorize completely: $pqx^2 + 6y - 3px - 2qxy$
    A. $(px-2y)(qx-3)$ **B.** $(qx+3)(2y-px)$
    C. $(qx+3)(px-2y)$ **D.** $(px+2y)(qx-3)$

16. If $19x = 240^2 - 235^2$, the value of x is
    A. 25 **B.** 95 **C.** 125 **D.** 205

17. Evaluate: $27.63^2 - 22.37^2$
    A. 27.7 **B.** 189.0 **C.** 241.3 **D.** 263.0

18. The following are factors of $2x^2 - 2$ **except**
    A. $x-1$ **B.** $x+1$ **C.** $2x-2$ **D.** $2x+1$

19. Factorize: $32a^3 - 8ab^2$
    A. $4(4a+b)(2a-b)$ **B.** $8a(2a+b)(2a-b)$
    C. $8a(2a-b)$ **D.** $4(2a+b)(4a-b)$

20. Which of the following polynomials has $(a+b)$ as a factor ?
    A. $a^2 + b^2$
    B. $a^3 - b^3$
    C. $2a^3 + 2a^2b - ab + 3a - b^2 + 3b$
    D. $2a^2 - 3ab + b^2 - a + 1$

21. What is the common factor to the expressions $y^2 - y$, $2y^2 + y - 1$ and $y^2 - 1$?
    A. y **B.** $y-1$ **C.** $y+1$ **D.** None

22. Factorize completely: $81x^4 - 16y^4$
    A. $(3x-2y)(3x+2y)(9x^2+4y^2)$
    B. $(3x+2y)(2x-3y)(9x^2+4y^2)$
    C. $(3x-2y)(2x-3y)(4x^2+9y^2)$
    D. $(3x-2y)(2x-3y)(9x^2+4y^2)$

23. What is the common factor to the binomial expressions $4x^2 - 9y^2$, $8x^3 + 27y^3$ and $(4x+6y)^2$?
    A. $4x+6y$ **B.** $4x-6y$
    C. $2x+3y$ **D.** $2x-3y$

24. Factorize: $9 - (a^2 - 3a - 1)^2$
    A. $(a-4)(a-2)(a-1)(a+1)$
    B. $-(a-4)(a+1)(a-1)(a-2)$

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C. $-(a-2)(a+1)(a+2)(a+4)$
D. $(a-4)(a-3)(a-2)(a+1)$

25. Factorize completely: $8p + 125py^3$
A. $(2p + 5y^2)(4 + 25py)$
B. $p(2 + 5y)(4 - 10y + 25y^2)$
C. $(2p + 5y)(4 - 10py + 25y^2)$
D. $p(2 + 5y)(4 + 10y + 25y^2)$

26. Factorize: $(4x + 3)^2 - (3x - 2)^2$
A. $(x + 1)(x + 5)$ B. $(x - 5)(7x - 1)$
C. $(x + 5)(7x + 1)$ D. $x(7x + 1)$

27. Factorize completely: $(z^2 + z)^2 - (2z + 2)^2$
A. $(z + 1)(z + 2)(z - 2)$
B. $(z + 1)^2(z - 2)^2$
C. $(z + 1)^2 (z + 2)^2$
D. $(z + 1)^2(z + 2)(z - 2)$

28. Factorize: $6^{2m+1} + 7(6^m) - 5$
A. $[3(6^m) + 5][2(6^m) - 1]$
B. $[3(6^m) - 5][2(6^m) + 1]$
C. $[3(6^m) - 5][3(6^m) + 1]$
D. $[2(6^m) + 5][3(6^m) - 1]$

29. Factorize completely: $b^3 - 4ab + ab^3 - 4b$
A. $b(1 + a)(b + 2)(b - 2)$
B. $b(1 - a)(b + 2)(b - 2)$
C. $(b + ab)(b + 2)(b - 2)$
D. $(b - ab)(b + 2)(b - 2)$

30. The factors of $y^3 - 8^{-1}$ are $y - 2^{-1}$ and
A. $y^2 - 2^{-1}y - 4^{-1}$ B. $y^2 + 2^{-1}y + 4^{-1}$
C. $y^2 + 4^{-1}y + 2^{-1}$ D. $y^2 - 2^{-1}y + 4^{-1}$

31. Factorize: $4x^2 - 12xy - z^2 + 9y^2$
A. $4x(x - 3y) + (3y - z)^2$
B. $(2x + 3y - z)(2x + 3y + z)$
C. $(2x - 3y - z)(2x - 3y + z)$
D. $4x(x - 3y) + (3y + z)$

32. Factorize: $9(a + b)^2 - 4(a - b)^2$
A. $(a + b)(5a + b)$ B. $(a + b)^2$
C. $5(a + b)^2$ D. $(a + 5b)(5a + b)$

33. If $x^2 + y^2 = 16$ and $2xy = 7$, the possible values of $x - y$ are
A. 2, –2 B. 3, –3 C. 1, –1 D. 3, –1

34. Factorize completely: $64a^3c + 27b^3c$
A. $(4ac + b)(16a^2 + 27b^2c)$
B. $c(4a + 3b)(16a^2 + 12ab + 9b^2)$
C. $c(4a + 3b)(16a^2 - 12ab + 9b^2)$
D. $c(4a - 3b)(16a^2 + 12ab - 9b^2)$

35. Factorize: $9x^2 - y^2 + 6yz - 9z^2$
A. $(3x - y + 3z)(3x + y - 3z)$
B. $(3x - 3y + z)(3x - y - 9z)$
C. $(6x - 3y - 3z)(3y - x - 4z)$
D. $(3y - x + 3z)(3y - x + 3z)$

36. Simplify $(a + {}^1/a + 1)^2 - (a - {}^1/a - 1)^2$
A. $4a^2$ B. $({}^2/a + 2)^2$ C. 4 D. $4(1 + a)$

37. Evaluate $83.726^2 - 16.274^2$ correct to 3 significant figures
A. 6,740 B. 6,750 C. 6,700 D. 674

38. Factorize: $a^2 - b^2 + (a + b)^2$
A. $2a^2$ B. $2a(a - b)$
C. $2a(a + b)$ D. $2b(b - a)$

39. If $a^2 - 8a + k$ is a perfect square, the value of k is
A. 2 B. 4 C. 8 D. 16

40. What must be added to the expression $y^2 - 18y$ to make it a perfect square ?
A. 81 B. 36 C. 9 D. 2

41. Given that the quadratic expression $3m^2 - 7m + K$ is a perfect square, the value of K is
A. $\frac{49}{24}$ B. $\frac{49}{12}$ C. $\frac{49}{36}$ D. $\frac{49}{4}$

42. Find the values of a which will make the expression $y^2 + 2(a + 1)y + a + 3$, a perfect square.
A. –1, 1 B. 2, –2 C. 1, –2 D. –1, 2

43. To make the expression $4a^2 + 5a + R$ a perfect square, the value of R is
A. 25/16 B. 25/64 C. 5/4 D. 5/8

44. What must be added to $4z^2 - 4$ to make it a perfect square ?
A. $-1/z^2$ B. $1/z^2$ C. 1 D. $1/z$

45. If $4a^2 - 18ab + P$ is a perfect square, the value of P is
A. 9 B. $9b^2/4$ C. $81b^2$ D. $81b^2/4$

46. The expression $y^4 - ay^3 + 10y^2 + by - 3$ is exactly divisible by $y - 1$. When it is divided by $y + 2$, the remainder is 27. The values of the constants a and b are, respectively
A. –7, –15 B. –15, –7
C. –21/5, –61/5 D. –5/3, 19/3

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47. If a function is defined as $F(x + 2) = 3x^2 - x + 5$, then $F(0)$ is
A. 5 B. 0 C. 15 D. 19

48. If $f(x - 3) = x^2 + 2x + 3$, find $f(1)$
A. 3 B. 6 C. 11 D. 27

49. If $f(x) = 2x^2 - 4x + 5$, find $f(x + 1)$
A. $2x^2 + 8x + 3$ B. $2x^2 - 8x + 3$
C. $2x^2 + 3$ D. $2x^2 + 8x + 11$

50. If $f(y - 4) = y^2 + 2y + 3$, then $f(2)$ is
A. 66 B. 51 C. 11 D. 3

51. Factorize: $3y^3 + 4y^2 - 13y + 6$ completely, if $y - 1$ is a factor
A. $(y - 1)(y - 3)(y + 2)$
B. $(y - 1)(y + 3)(y - 2)$
C. $(y - 1)(y + 3)(3y - 2)$
D. $(y - 1)(y + 3)(3y + 2)$

52. The expression $px^3 + 2qx^2 + 24$ has $x - 1$ and $x+2$ as factors. The values of p and q are respectively
A. −6, −9 B. −2, 1 C. −2, −1 D. 0, 1

53. If $f(y) = 2(y - 3)^2 + 3(y - 3) + 4$ and $g(x) = \sqrt{x + 5}$, find $f[g(4)]$ and $g[f(3)]$
A. −3 and 4 B. 3 and 4 C. −3 and −4 D. 3 and −4

54 The expression $x^3 + ax^2 + bx + 1$ has $x - 2$ and $x + 1$ as factors. The value of $a + b$ is
A. −3 B. 3 C. −17/3 D. 0

55. If $y - 3$ is a factor of the expression $y^3 - py - 6$, find the other two factors.
A. $y - 2, y + 1$ B. $y + 2, y + 1$
C. $y + 3, y + 5$ D. $y - 3, y - 5$

56. Find the remainder when $3x^5 - 4x^3 - 5x + 2$ is divided by $x - 2$
A. 28 B. −28 C. −56 D. 56

57. If $x + 1$ is factor of the expression $x^3 + 2x^2 + kx - 5$, the value of k is
A. −4 B. 4 C. −2 D. 2

58. Which of the following is a factor of $15 + 7y - 2y^2$?
A. $y - 3$ B. $y + 3$ C. $y - 5$ D. $y + 5$

59. The expression $2x^2 - kx + k$ leaves a remainder of 6 when divided by $x - k$. The positive value of k is
A. 1 B. 2 C. 3 D. 4

60. If $x + 1$ and $x - 1$ are factors of the expression $x^3 + cx^2 + kx + 6$, the values of c and k are, respectively
A. −6, −1 B. 6, 1 C. 1, −1 D. 6, −6

ANSWER KEY

1. B	11. A	21. D	31. C	41. B	51. C
2. A	12. D	22. A	32. D	42. C	52. A
3. D	13. C	23. C	33. B	43. A	53. B
4. C	14. B	24. B	34. C	44. B	54. A
5. B	15. A	25. B	35. A	45. D	55. B
6. C	16. C	26. C	36. D	46. A	56. D
7. A	17. D	27. D	37. B	47. D	57. A
8. B	18. D	28. A	38. C	48. D	58. C
9. D	19. B	29. A	39. D	49. C	59. B
10. C	20. C	30. B	40. A	50. B	60. A

SOLUTIONS TO OBJECTIVE QUESTIONS

1. **(B)** $x(p - q) + y(q - p) = x(p - q) - y(p - q)$
$= (x - y)(p - q)$

2. **(A)** $pqx^2 + 12y - 4qx - 3pxy$
$= px(qx - 3y) - 4(qx - 3y)$
$= (px - 4)(qx - 3y)$

3. **(D)** $y^2 + 2a + ay + 2y = y(y + 2) + a(y + 2)$
$= (y + 2)(y + a)$

4. **(C)** This type of question is best solved by inspection of the options provided. Considering the

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middle term, option A gives $44a$, option B, $-44a$ and option C, $4a$.

5. **(B)** The middle term is $6y - 5y = y$.

6. **(C)** Options (B) and (D) can be eliminated since their last terms are different from +1. Middle term for option A is $2x + x$, or $+3x$.

7. **(A)** Middle term for option A is $-9x - 2x = -11x$.

8. **(B)** Options A, C and D can be eliminated immediately because the x^2 term is, in each case, different from that of the given expression.

9. **(D)** Middle terms: option A ($+14ax$), option B ($-14ax$), option C ($-2ax$). Note that the first and the last terms are correct in all the options.

10. **(C)** Middle term for option C is $+xz$. Note that the z^2 term has the wrong sign in option D.

11. **(A)** $(2y - 5)(y - 3) = 2y(y - 3) - 5(y - 3)$
$= 2y^2 - 6y - 5y + 15$
$= 2y^2 - 11y + 15$

12. **(D)** $-(5 - a)(1 - a) = -(5 - 6a + a^2) = 6a - 5 - a^2$

13. **(C)** Since 2 is a factor, options B and D can be eliminated right away. The expression can be written as $2(3y^2 - 7y - 6)$, which is consistent with option C.

14. **(B)** $(3z - 4y)^2 = 16y^2 + 2ayz + 2bz^2$
$9z^2 - 24zy + 16y^2$
$= 16y^2 + 2ayz + 2bz^2$
Equating coefficients of z^2:
$2b = 9$, or $b = 9/2$
Equating coefficients of yz:
$-24 = 2a$, or $a = -12$

15. **(A)** $pqx^2 - 3px + 6y - 2qxy$
$= px(qx - 3) - 2y(qx - 3)$
$= (px - 2y)(qx - 3)$

16. **(C)** $19x = (240 + 235)(240 - 235) = 475 \times 5$
$x = 475 \times 5/19 = 125.$

17. **(D)** $27.63^2 - 22.37^2$
$= (27.63 + 22.37)(27.63 - 22.37)$
$= 50 \times 5.26 = 263$

18. **(D)** $2x^2 - 2 = 2(x^2 - 1) = 2(x - 1)(x + 1)$
Note that $2x - 2 = 2(x - 1)$, which is also a factor.

19. **(B)** $32a^3 - 8ab^2 = 8a(4a^2 - b^2)$
$= 8a(2a + b)(2a - b)$

20. **(C)** $a^2 + b^2$ has no factor.
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
$2a^3 + 2a^2b - ab + 3a - b^2 + 3b$
$= 2a^2(a + b) - b(a + b) + 3(a + b)$
$= (2a^2 - b + 3)(a + b)$

21. **(D)** $y^2 - y = y(y - 1)$
$2y^2 + y - 1 = (2y - 1)(y + 1)$
$y^2 - 1 = (y + 1)(y - 1)$
The expressions have no common factor.

22. **(A)** $81x^4 - 16y^4 = (9x^2)^2 - (4y^2)^2$
$= (9x^2 - 4y^2)(9x^2 + 4y^2)$
$= (3x + 2y)(3x - 2y)(9x^2 + 4y^2)$

23. **(C)** $4x^2 - 9y^2 = (2x - 3y)(2x + 3y)$
$8x^3 + 27y^3 = (2x + 3y)(4x^2 - 6xy + 9y^2)$
$(4x + 6y)^2 = 4(2x + 3y)^2$
$\Rightarrow (2x + 3y)$ is a common factor.

24. **(B)** $9 - (a^2 - 3a - 1)^2$
$= [3 - (a^2 - 3a - 1)][3 + (a^2 - 3a - 1)]$
$= -(a^2 - 3a - 4)(a^2 - 3a + 2)$
$= -(a - 4)(a + 1)(a - 1)(a - 2)$

25. **(B)** $8p + 125py^3 = p(8 + 125y^3)$
$= p[2^3 + (5y)^3]$
$= p(2 + 5y)[2^2 - (2)(5y) + (5y)^2]$
$= p(2 + 5y)(4 - 10y + 25y^2)$

26. **(C)** $(4x + 3)^2 - (3x - 2)^2$
$= [(4x + 3) - (3x - 2)][(4x + 3) + (3x - 2)]$
$= (x + 5)(7x + 1)$

27. **(D)** $(z^2 + z)^2 - (2z + 2)^2$
$= [(z^2 + z) - (2z + 2)][(z^2 + z) + (2z + 2)]$
$= (z^2 - z + 2)(z^2 + 3z + 2)$
$= (z - 2)(z + 1)(z + 1)(z + 2)$

28. **(A)** $6^{2m+1} = 6^{2m} \times 6^1 = 6(6^{2m}) = 6(6^m)^2$
Let $x = 6^m$, then the given expression becomes $6x^2 + 7x - 5 = (3x + 5)(2x - 1)$
Substituting for x gives
$[3(6^m) + 5][2(6^m) - 1)]$

29. **(A)** $b^3 - 4ab + ab^3 - 4b = b(b^2 - 4a + ab^2 - 4)$
$= b[b^2(1 + a) - 4(1 + a)]$

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$= b(b^2 - 4)(1 + a)$
$= b(b + 2)(b - 2))(1 + a)$

30. **(B)** $y^3 - 8^{-1} = y^3 - {}^1/_8 = y^3 - ({}^1/_2)^3$
$= (y - {}^1/_2)(y^2 + {}^1/_2 y + {}^1/_4)$
$= (y - 2^{-1})(y^2 + 2^{-1}y + 4^{-1})$

31. **(C)** $4x^2 - 12xy - z^2 + 9y^2$
$= 4x^2 - 12xy + 9y^2 - z^2$
$= (2x - 3y)^2 - z^2$
$= (2x - 3y + z)(2x - 3y - z)$

32. **(D)** $9(a + b)^2 - 4(a - b)^2$
$= [3(a + b) - 2(a - b)][3(a + b) + 2(a - b)]$
$= (a + 5b)(5a + b)$

33. **(B)** $(x - y)^2 = x^2 + y^2 - 2xy = 16 - 7 = 9$
$x - y = +3$, or -3

34. **(C)** $64a^3c + 27b^3c = c(64a^3 + 27b^3)$
$= c[(4a)^3 + (3b)^3]$
$= c[4a + 3b][(4a)^2 - (4a)(3b) + (3b)^2]$
$= c(4a + 3b)(16a^2 - 12ab + 9b^2)$

35. **(A)** $9x^2 - y^2 + 6yz - 9z^2 = 9x^2 - (y^2 - 6yz + 9z^2)$
$= 9x^2 - (y - 3z)^2$
$= [3x - (y - 3z)][3x + (y - 3z)]$
$= (3x - y + 3z)(3x + y - 3z)$

36. **(D)** $(a + {}^1/a + 1)^2 - (a - {}^1/a - 1)^2$
$= (a + {}^1/a + 1) - (a - {}^1/a - 1)]$ x
$[(a + {}^1/a + 1) + (a - {}^1/a - 1)]$
$= ({}^2/a + 2)(2a) = 4 + 4a$
$= 4(1 + a)$

37. **(B)** $83.726^2 - 16.274^2$
$= (83.726 - 16.274)(83.726 + 16.274)$
$= 67.452$ x $100 \doteq 6{,}745.2$
$= 6{,}750$ to 3 significant figures

38. **(C)** $a^2 - b^2 + (a + b)^2$
$= (a + b)(a - b) + (a + b)^2$
$= (a + b)[a - b + a + b] = 2a(a + b)$

39. **(D)** Let $a^2 - 8a + k = (a - \sqrt{k}))^2$
$= y^2 - 2a\sqrt{k} + k$
Equate coefficients of a: $8 = 2\sqrt{k}$,
$\sqrt{k} = 4$ and $k = 16$.

40. **(A)** Let k^2 be the added term
$y^2 - 18y + k^2 = (y - k)^2 = y^2 - 2ky + k^2$
Equate coefficients of y: $2k = 18$,
$k = 9$ and $k^2 = 81$.

41. **(B)** $3m^2 - 7m + K = (m\sqrt{3} - \sqrt{K})^2$
$= 3m^2 - 2m\sqrt{3K} + K$
Equate coefficients of m: $2\sqrt{3K} = 7$
$\sqrt{3K} = {}^7/_2$, $3K = {}^{49}/_4$ and $K = {}^{49}/_{12}$.

42. **(C)** $y^2 + 2(a + 1)y + a + 3$
$= (y + \sqrt{a + 3})^2$
$= y^2 + 2\sqrt{a + 3}y + a + 3$
Equate coefficients of y:
$2(a + 1) = 2\sqrt{a + 3}$
$(a + 1)^2 = a + 3$
$a^2 + 2a + 1 = a + 3$
$a^2 + a - 2 = 0$
$(a + 2)(a - 1) = 0$; $a = -2$ or 1

43. **(A)** $4a^2 + 5a + R$
$= (2a + \sqrt{R})^2 = 4a^2 + 4a\sqrt{R} + R$
Equate coefficients of a: $5 = 4\sqrt{R}$
$\sqrt{R} = 5/4$, or $R = 25/16$

44. **(B)** Let k^2 be the added term
$4z^2 - 4 + k^2 = (2z - k)^2 = 4z^2 - 4kz + k^2$
Equate middle terms: $4 = 4kz$, or $k = {}^1/z$
and $k^2 = {}^1/z^2$

45. **(D)** $4a^2 - 18ab + P = (2a - \sqrt{P})^2$
$= 4a^2 - 4a\sqrt{P} + P$
Equate middle terms: $18ab = 4a\sqrt{P}$
$\sqrt{P} = 9b/2$, or $P = 81b^2/4$

46. **(A)** $f(y) = y^4 - ay^3 + 10y^2 + by - 3$
$f(1) = (1)^4 - a(1)^3 + 10(1)^2 + b(1) - 3 = 0$
$\Rightarrow b - a = -8$ .....(i)
$f(-2) = (-2)^4 - a(-2)^3 + 10(-2)^2 + b(-2) - 3 = 27$
$8a - 2b = -26$ .....(ii)
Solving eqs. (i) and (ii) simultaneously gives
$a = -7$ and $b = -15$.

47. **(D)** $F(x + 2) = 3x^2 - x + 5$
$F(0) = F(-2 + 2)$, which is obtained by substituting $x = -2$ in the given function.
Thus $F(0) = 3(-2)^2 - (-2) + 5 = 12 + 2 + 5$
$= 19$
[Note that if you substitute $x = 0$, you'd be be calculating $F(0 + 2)$, or $F(2)$, rather than $F(0)$].

48. **(D)** $f(x - 3) = x^2 + 2x + 3$
$f(1) = f(4 - 3) = (4)^2 + (2)(4) + 3 = 27$

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49. **(C)** $f(x) = 2x^2 - 4x + 5$

$f(x+1) = 2(x+1)^2 - 4(x+1) + 5$
$= 2(x^2 + 2x + 1) - 4x - 4 + 5$
$= 2x^2 + 3$

50. **(B)** $f(y-4) = y^2 + 2y + 3$

$f(2) = f(6-4) = (6)^2 + 2(6) + 3 = 51$

51. **(C)** Divide $3y^3 + 4y^2 - 13y + 6$ by $y - 1$:

$$
\begin{array}{r|l}
y-1 & 3y^3 + 4y^2 - 13y + 6 \\
 & \underline{3y^3 - 3y^2} \\
 & \quad 7y^2 - 13y \\
 & \quad \underline{7y^2 - 7y} \\
 & \quad\quad -6y + 6 \\
 & \quad\quad \underline{-6y + 6}
\end{array}
\qquad 3y^2 + 7y - 6
$$

The result is $3y^2 + 7y - 6 = (3y - 2)(y + 3)$

Hence the given expression is

$(y - 1)(3y - 2)(y + 3)$.

52. **(A)** Let $f(x) = px^3 + 2qx^2 + 24$

$f(1) = p + 2q + 24 = 0$ ..... (i)

$f(-2) = -8p + 8q + 24 = 0$ .....(ii)

We can re–write eqs (i) and (ii) as

$p + 2q = -24$, and

$-p + q = -3$

which, added together, gives $3q = -27$, or

$q = -9$, and $p = q + 3 = -6$.

53. **(B)** $f(y) = 2(y-3)^2 + 3(y-3) + 4$

$g(x) = \sqrt{x+5}$

$g(4) = \sqrt{4+5} = 3$

$f(3) = 2(0)^2 + 3(0) + 4 = 4$

$f[g(4)] = f(3) = 4$

$g[f(3)] = \sqrt{4+5} = 3$

54. **(A)** $f(x) = x^3 + ax^2 + bx + 1$

$f(2) = 8 + 4a + 2b + 1 = 0$

$4a + 2b = -9$ ..... (i)

$f(-1) = -1 + a - b + 1 = 0$

$a - b = 0$ .....(ii)

From (i) and (ii) $a = b = -1.5$

$a + b = -1.5 - 1.5 = -3$

55. **(B)** $f(y) = y^3 - py - 6$

$f(3) = 27 - 3p - 6 = 0$, or $p = 7$

$\Rightarrow f(y) = y^3 - 7y - 6$

Divide $y^3 - 7y - 6$ by $y - 3$:

$$
\begin{array}{r|l}
y-3 & y^3 + 0y^2 - 7y - 6 \\
 & \underline{y^3 - 3y^2} \\
 & \quad 3y^2 - 7y \\
 & \quad \underline{3y^2 - 9y} \\
 & \quad\quad 2y - 6 \\
 & \quad\quad \underline{2y - 6}
\end{array}
\qquad y^2 + 3y + 2
$$

Since $y^2 + 3y + 2 = (y + 2)(y + 1)$,

$y^3 - 7y - 6 = (y - 3)(y + 2)(y + 1)$.

56. **(D)** $f(x) = 3x^5 - 4x^3 - 5x + 2$

$f(2) = 3(2)^5 - 4(2)^3 - 5(2) + 2$
$= 96 - 32 - 10 + 2 = 56.$

57. **(A)** $f(x) = x^3 + 2x^2 + kx - 5$

$f(-1) = (-1)^3 + 2(-1)^2 + k(-1) - 5 = 0$,

$\Rightarrow -1 + 2 - k - 5 = 0$, or $k = -4$

58. **(C)** $15 + 7y - 2y^2 = (5 - y)(3 + 2y)$
$= -(y - 5)(3 + 2y)$

$\Rightarrow y - 5$ is a factor

59. **(B)** $f(x) = 2x^2 - kx + k$

$f(k) = 2k^2 - k^2 + k = 6$

$k^2 + k - 6 = 0$

$(k + 3)(k - 2) = 0$, $k = -3$ or 2

(The positive value is 2)

60. **(A)** $f(x) = x^3 + cx^2 + kx + 6$

$f(-1) = -1 + c - k + 6 = 0$

$c - k = -5$ .....(i)

$f(1) = 1 + c + k + 6 = 0$, or

$c + k = -7$ .....(ii)

Add (i) and (ii): $2c = -12$, $c = -6$

From (i), $k = c + 5 = -1$

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7 Quadratic Equations

7.1 Solution of Quadratic Equations

- A **quadratic equation** is an equation of the form

$$ax^2 + bx + c = 0$$

where a, b and c are constants and $a \neq 0$.

- The values of x that satisfy the quadratic equation are called the **roots** of the equation.
- The roots of a quadratic equation can be found by
  (a) factorization,
  (b) completing the square, or
  (c) using the quadratic formula.

(a) Factorization

- The quadratic function is simply factorized and set equal to zero.
  Given the quadratic equation $ax^2 + bx + c = 0$, if $(px + q)$ and $(rx + s)$ are the factors, then
  $(px + q)(rx + s) = 0$.
  Either $px + q = 0$, i.e. $x = -q/p$
  or $rx + s = 0$, i.e. $x = -s/r$
  $\Rightarrow$ Required roots are $-q/p$ and $-s/r$.

EXAMPLE 7 – 1

Solve the equations
(i) $x^2 + 6x - 7 = 0$ (ii) $x^2 - x - 6 = 0$
(iii) $3y^2 + 13y + 12 = 0$ (iv) $4 - 3z - z^2 = 0$

Solutions
The given functions are factorized and set equal to zero :

(i) $(x - 1)(x + 7) = 0$; $x = 1$ and -7
(ii) $(x + 2)(x - 3) = 0$; $x = -2$ and 3
(iii) $(3y + 4)(y + 3) = 0$; $y = -4/3$ and -3
(iv) $(4 + z)(1 - z) = 0$; $z = -4$ and 1

DRILL 1

Solve the following equations:
(a) $4x^2 - 36 = 0$ **(b)** $x^2 - 5x - 14 = 0$
(c) $15 + 7x - 2x^2 = 0$ **(d)** $48 - 3x^2 = 0$

(b) Completing the Square

- A quadratic of the form $(x + p)^2$ is called a **perfect square.**
- This can be expanded to give $x^2 + 2px + p^2$.
- For a quadratic to be expressible as a perfect square, the constant term must be **the square of half of the coefficient of x**, e.g. for the above quadratic, constant term $= p^2 = (2p/2)^2$.
- To solve a quadratic equation by completing the square:
  (i) Move the constant term to the RHS of the equation.
  (ii) Divide both sides by the coefficient of x^2.
  (iii) Add the square of half the coefficient of x to both sides.
  (iv) Take square roots of both sides.
  (v) Solve for x.

EXAMPLE 7 – 2

Solve each of the following equations for x by completing the squares:
(a) $x^2 + 6x + 4 = 0$
(b) $3x^2 + 4x - 4 = 0$
(c) $20x^2 - 2x - 6 = 0$

Solutions
(a) Step (i)..... $x^2 + 6x = -4$
(ii)..... $x^2 + 6x = -4$
(iii).... $x^2 + 6x + (^6/_2)^2 = -4 + (^6/_2)^2$
$(x + 3)^2 = 5$
(iv)..... $x + 3 = \pm\sqrt{5}$
(v)..... ..... $x = -3 \pm \sqrt{5}$

(b) $3x^2 + 4x = 4$
$x^2 + {}^4/_3 x = 4$
$x^2 + {}^4/_3 x + [{}^1/_2 \cdot {}^4/_3]^2 = {}^4/_3 + [{}^1/_2 \cdot {}^4/_3]^2$
$x^2 + {}^4/_3 x + ({}^2/_3)^2 = {}^4/_3 + {}^4/_9 = {}^{16}/_{9=}$
$(x + {}^2/_3)^2 = {}^{16}/_9$
$x + {}^2/_3 = \pm {}^4/_3$
$x = -{}^2/_3 + {}^4/_3$ or $-{}^2/_3 - {}^4/_3$
$x = {}^2/_3$ or -2.

(c) Divide the equation through by 2 to give
$10x^2 - x - 3 = 0$
$10x^2 - x = 3$

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$$x^2 - \frac{1}{10}x = \frac{3}{10}$$

$$x^2 - \frac{1}{10}x + \left(\frac{1}{20}\right)^2 = \frac{3}{10} + \left(\frac{1}{20}\right)^2$$

$$\left(x - \frac{1}{20}\right)^2 = \frac{121}{400}$$

$$x - \frac{1}{20} = \pm\frac{11}{20}$$

$$x = \frac{1}{20} + \frac{11}{20} \text{ or } \frac{1}{20} - \frac{11}{20}$$

$$= \frac{3}{5} \text{ or } -\frac{1}{2}$$

DRILL 2

What must be added to both sides in each of the following equations to make the left hand side a perfect square ?

(a) $x^2 + 4x = 5$ (b) $y^2 + {}^2/_3\, y = 2$

(c) $x^2 + 15x = 1$ (d) $y^2 + {}^3/_7\, y = 4$

(c) The Quadratic Formula

- The quadratic formula makes it possible to express the roots of a general quadratic equation of the form $ax^2 + bx + c = 0$ in terms of the coefficients a, b and c.
- The procedure is exactly the same as in "completing the square" discussed above:

Step (i)..... $ax^2 + bx = -c$

(ii)..... $x^2 + \frac{b}{a}x = -\frac{c}{a}$

(iii).. $x^2 + \frac{b}{a}x + \left(\frac{1}{2}\cdot\frac{b}{a}\right)^2 = -\frac{c}{a} + \left(\frac{1}{2}\cdot\frac{b}{a}\right)^2$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

(iv)..... $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$

(v)..... $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Hence the roots are

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

- The equation in step (v), called the **quadratic formula,** can be used to solve any quadratic equation.
- The value of $b^2 - 4ac$ determines the nature of the roots of the equation:

(i) If $b^2 - 4ac$ is positive, the two roots are **real** and **different**;

(ii) If $b^2 - 4ac$ is zero, the two roots are **real** and **both equal** to $-b/2a$;

(iii) If $b^2 - 4ac$ is negative, the quadratic equation has no real roots, i.e. the roots are **imaginary**.

EXAMPLE 7 – 3

Use the quadratic formula to find the roots of the following equations:

(a) $x^2 - 4x + 3 = 0$

(b) $x^2 + 4x - 9 = 0$

(c) $3y^2 + 8y + 2 = 0$

(d) $\frac{1}{x+3} + \frac{1}{x+1} = 1$

Solutions

(a) Substituting $a = 1$, $b = -4$ and $c = 3$ into the quadratic formula gives:

$$x = \frac{4 \pm \sqrt{16 - (4 \times 1 \times 3)}}{2 \times 1} = \frac{4 \pm \sqrt{4}}{2}$$

$$= \frac{4 \pm 2}{2} = 2 \pm 1$$

$$= 3 \text{ or } 1$$

(b) $a = 1, b = 4, c = -9$

$$x = \frac{-4 \pm \sqrt{16 - (4 \times 1 \times -9)}}{2 \times 1} = \frac{-4 \pm \sqrt{52}}{2}$$

$$= -\frac{4}{2} \pm \frac{2\sqrt{13}}{2} = -2 \pm \sqrt{13}$$

$$= -2 + \sqrt{13} \text{ or } -2 - \sqrt{13}$$

(c) $a = 3, b = 8, c = 2$

$$y = \frac{-8 \pm \sqrt{64 - (4 \times 3 \times 2)}}{2 \times 3} = \frac{-8 \pm \sqrt{40}}{6}$$

$$= -\frac{8}{6} \pm \frac{2\sqrt{10}}{6} = \frac{-4}{3} \pm \frac{\sqrt{10}}{3}$$

$$= \frac{-4 + \sqrt{10}}{3} \text{ or } \frac{-4 - \sqrt{10}}{3}$$

(d) $\frac{1}{x+3} + \frac{1}{x+1} = 1$

$$\frac{x + 1 + x + 3}{(x+3)(x+1)} = 1$$

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$2x + 4 = x^2 + 4x + 3$
$x^2 + 2x - 1 = 0$
$a = 1, b = 2, c = -1$
$x = \frac{-2 \pm \sqrt{4 - (4 \times 1 \times -1)}}{2 \times 1} = \frac{-2 \pm \sqrt{8}}{2}$

$= \frac{2}{2} \pm \frac{2\sqrt{2}}{2} = 1 \pm \sqrt{2}$
$= -1 + \sqrt{2}$ or $-1 - \sqrt{2}$

DRILL 3

Using the quadratic formula, find the roots of the following equations:

(a) $2x^2 - 2x - 1 = 0$
(b) $x^2 + x - 1 = 0$
(c) $3 - 5z - 2z^2 = 0$
(d) $3y^2 + 3y - 1 = 0$

7.2 Determining the Quadratic Equation from the Roots

- If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, it can be shown that

$\alpha + \beta = -b/a$ and $\alpha\beta = c/a$

- As an example, the equation $x^2 - 4x + 3 = 0$ (Example 7 – 3(a)) has roots $\alpha = 3$, $\beta = 1$. For the given equation $a = 1$, $b = -4$ and $c = 3$:
$\alpha + \beta = 3 + 1 = 4 = -b/a$.
Also $\alpha\beta = 3 = c/a$

- Once the roots α and β are known, the quadratic equation can then be written as
$x^2 + (b/a)x + c/a = 0$, or
$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

EXAMPLE 7 – 4

If one of the roots of the equation $x^2 + bx + 8 = 0$ is 2, find the other root and the value of b.

Solution
$\alpha = 2, \beta = ?, b = ?$
$\alpha\beta = c/a = 8/1 = 8$
$\Rightarrow \beta = 8/\alpha = 8/2 = 4$
$-b/a = \alpha + \beta = 2 + 4 = 6$
$\Rightarrow b = -6 \times a = -6 \times 1 = -6$

EXAMPLE 7–5

The roots of the quadratic equation $2x^2 - 3x - 2 = 0$ are α and β. Find the equations whose roots are
(a) $1/\alpha$ and $1/\beta$
(b) α^2 and β^2

Solution
The given equation can be written as $x^2 - {}^3/_2 x - 1 = 0$.
Hence $\alpha + \beta = -b/a = {}^3/_2$, and $\alpha\beta = c/a = -1$

(a) Let the equation whose roots are $1/\alpha$ and $1/\beta$ be

$$x^2 + \frac{b_1}{a_1}x + \frac{c_1}{a_1} = 0$$

Product of the roots $= \frac{1}{\alpha}\frac{1}{\beta} = \frac{1}{\alpha\beta} = -1 = \frac{c_1}{a_1}$

Sum of the roots
$= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{3/2}{-1} = -\frac{3}{2} = -\frac{b_1}{a_1}$
The required equation is $x^2 + {}^3/_2 x - 1 = 0$, or
$2x^2 + 3x - 2 = 0$.

(b) Let the equation whose roots are α^2 and β^2 be

$$x^2 + \frac{b_2}{a_2}x + \frac{c_2}{a_2} = 0$$

Product of the roots
$= (\alpha^2)(\beta^2) = (\alpha\beta)^2 = (-1)^2 = 1 = c_2/a_2$.
Sum of the roots
$= \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (3/2)^2 + 2$
$= 17/4 = -b_2/a_2$
The required equation is
$x^2 - {}^{17}/_4 x + 1 = 0$, or
$4x^2 - 17x + 4 = 0$.

DRILL 4

(a) Find the sums and products of the roots of the following equations:
(i) $x^2 + 4x + 3 = 0$
(ii) $x^2 - {}^1/_3 x + {}^1/_6 = 0$
(iii) $3 - 7x - x^2 = 0$
(iv) $5x - 4 - 3x^2 = 0$

(b) Write down the quadratic equations for the following:
(i) sum of roots $= -2$, product of roots $= {}^2/_3$
(ii) sum of roots $= 6$, product of roots $= -7$
(iii) sum of roots $= {}^1/_2$, product of roots $= -2$

7.3 Solution of Cubic Equations

- Cubic equations are of the form

$ax^3 + bx^2 + cx + d = 0$

where a, b, c, and d are constants.

- A cubic equation has **three** roots.
- To solve a cubic equation:
(i) Identify one of the factors (or roots), usually by trial and error using the *factor theorem*.

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(ii) Divide the cubic expression by this factor to yield the quotient, which is a quadratic equation.

(iii) Solve the quadratic equation using any of the methods outlined above. This gives the two other roots.

EXAMPLE 7 - 6

Find the three roots of the equations

(a) $3x^3 - 13x^2 + 13x - 3 = 0$

(b) $2x^3 + x^2 - 7x - 6 = 0$

Solutions

(a) Step *(i)*:

Check if $x - 1$ is a factor (i.e. if $x = 1$ is a root):

$F(1) = 3(1)^3 - 13(1)^2 + 13(1) - 3$

$= 3 - 13 + 13 - 3 = 0$

Hence $(x - 1)$ is a factor.

Step *(ii)*:

Divide $3x^3 - 13x^2 + 13x - 3$ by $x - 1$

$$
\begin{array}{r|l}
 & 3x^2 - 10x + 3 \\
\hline
x - 1 & 3x^3 - 13x^2 + 13x - 3 \\
 & \underline{3x^3 - 3x^2} \\
 & \quad -10x^2 + 13x \\
 & \quad \underline{-10x^2 + 10x} \\
 & \qquad 3x - 3 \\
 & \qquad \underline{3x - 3}
\end{array}
$$

Hence the quotient is $3x^2 - 10x + 3$

Step *(iii)*

Solve the quadratic $3x^2 - 10x + 3 = 0$

By factorizing, this gives $(3x - 1)(x - 3) = 0$

$\Rightarrow$ the roots are 1, 1/3 and 3.

[*Alternate method for step (ii)*

Write the equation as

$3x^3 - 13x^2 + 13x - 3 = (x - 1)(ax^2 + bx + c)$

Equate coefficients of x^3 : $a = 3$

Equate constants: $-3 = -1(c)$, or $c = 3$

Equate coefficients of x^2: $-13 = -a + b$

$b = a - 13 = -10$

Hence the quotient is $3x^2 - 10x + 3$]

(b) *Step (i)*:

Try $x = 1$: $F(1) = 2 + 1 - 7 - 6 = -10$

$\Rightarrow x - 1$ is *not* a factor.

Try $x = -1$: $F(-1) = -2 + 1 + 7 - 6 = 0$

$\Rightarrow x + 1$ is a factor

Step (ii):

$$
\begin{array}{r|l}
 & 2x^2 - x - 6 \\
\hline
x + 1 & 2x^3 + x^2 - 7x - 6 \\
 & \underline{2x^3 + 2x^2} \\
 & \quad -x^2 - 7x \\
 & \quad \underline{-x^2 - x} \\
 & \qquad -6x - 6 \\
 & \qquad \underline{-6x - 6}
\end{array}
$$

$\Rightarrow$ the quotient is $2x^2 - x - 6$

[As an exercise, the candidate should try the alternate method as shown in (a) above]

Step (iii)

Solve $2x^2 - x - 6 = 0$

$(2x + 3)(x - 2) = 0$

$x = -3/2$ or 2

$\Rightarrow$ the roots are -1, $-3/2$ and 2

EXAMPLE 7 - 7

If $x = -1$ and $x = 2$ are roots of the equation $2x^3 + bx^2 + cx - 2 = 0$, find the values of b and c.

Solution

$F(-1) = 2(-1)^3 + b(-1)^2 + c(-1) - 2 = 0$

$\Rightarrow -2 + b - c - 2 = 0$, or

$b - c = 4$ .....(1)

$F(2) = 2(2)^3 + b(2)^2 + c(2) - 2 = 0$

$\Rightarrow 16 + 4b + 2c - 2 = 0$, or

$2b + c = -7$ .....(2)

Adding eqs. (1) and (2) gives $3b = -3$ or $b = -1$

From (1): $-1 - c = 4$, or $c = -5$, i.e. the cubic equation is $2x^3 - x^2 - 5x - 2 = 0$.

DRILL 5

Identify one of the roots of each of the following equations:

(a) $2x^3 + 3x^2 - 4x - 5 = 0$

(b) $x^3 + x^2 - 18x - 8 = 0$

(c) $x^3 - 2x^2 + x - 2 = 0$

(d) $x^3 + 2x^2 - 4x - 3 = 0$

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ANSWERS TO DRILLS

1. (a) $x = \pm 3$ (b) $x = 7$ or -2
   (c) $x = 5$ or $-3/2$ (d) $x = \pm 4$

2. (a) 4 (b) 1/9
   (c) 225/4 (d) 9/196

3. (a) $\frac{1 \pm \sqrt{3}}{2}$ (b) $\frac{-1 \pm \sqrt{5}}{2}$
   (c) -3 or $\frac{1}{2}$ (d) $\frac{-3 \pm \sqrt{21}}{6}$

4. (a) (i) sum = –4, product = 3
   (ii) sum = 1/3, product = 1/6
   (iii) sum = –7, product = –3
   (iv) sum = 5/3, product = 4/3

   (b) (i) $3x^2 + 6x + 2 = 0$
   (ii) $x^2 - 6x - 7 = 0$
   (iii) $2x^2 - x - 4 = 0$

5. (a) –1 (b) 4
   (c) 2 (d) –3

SSCE/JME – TYPE OBJECTIVE QUESTIONS

1. The roots of the equations $2y^2 - 3y - 2 = 0$ are
   A. –2, 1 **B.** –2, 1/2 **C.** –2, 2 **D.** –1/2, 2

2. Solve the equation $6z^2 - 7z - 5 = 0$
   A. $z = 1/2$ or $-5/2$ **B.** $z = 1/3$ or $5/2$
   C. $z = 5/3$ or $-1/2$ **D.** $z = -5/3$ or $1/2$

3. Solve the equation $5x^2 = 3x$
   A. $x = 0$ or $3/5$ **B.** $x = 0$ or 5
   C. $x = 3$ or 5 **D.** $x = 0$ or 3

4. Solve the equation $2x^2 - 3x - 27 = 0$
   A. 3/2, 9 **B.** –3, 9/2 **C.** –2/3, 9 **D.** 3, 9/2

5. Solve the equation $3x + 10 = x^2$
   A. 5 or –2 **B.** 5 or 2 **C.** –5 or 2 **D.** –5 or –2

6. Solve the equation $a^2 - 2a - 3 = 0$
   A. -3, 1 **B.** –1, 3 **C.** –1, –3 **D.** 3, 1

7. Find the roots of the equation
   $3y^2 + 25y - 18 = 0$
   A. –3, 2 **B.** –2, 3 **C.** $-9, {}^2/_3$ **D.** $-{}^2/_3, 9$

8. Solve the equation $(a - 7)(a + 2) = 0$
   A. $a = -2$ or 7 **B.** $a = 2$ or -7
   C. $a = -1$ or 8 **D.** $a = -4$ or 5

9. Which of the following is a root of the equation
   $y^2 + 6y = 0$?
   A. 3 **B.** 2 **C.** 1 **D.** 0

10. Find the smaller value of a for which
    $a^2 - 3a + 2 = 0$
    A. –2 **B.** –1 C. 1 **D.** 2

11. One of the roots of the equation $6y^2 = 5 - 7y$ is
    A. $-{}^1/_2$ **B.** $-{}^1/_3$ **C.** ${}^1/_2$ **D.** ${}^5/_3$

12. Solve the equation $2z^2 + 5z - 3 = 0$
    A. $z = 1$ or -3 **B.** $z = {}^1/_2$ or -3
    C. $z = 2$ or -3 **D.** $z = {}^5/_2$ or -3

13. One of the roots of the equation $x^2 + 9 = 0$ is
    A. 3 **B.** –3
    C. –9 **D.** none of these

14. The roots of the equation $10y^2 - 13y - 3$ are
    A. –1/5 or 3/2 **B.** 3/5 or –1/2
    C. 3/10 or –1 **D.** –3/10 or 1

15. The roots of the quadratic equation
    $5 + 3y - 2y^2 = 0$ are
    A. 5/2, 1 **B.** –5/2, 1 **C.** –5/2, –1 **D.** 5/2, –1

16. The largest root of the equation
    $(x - 1)^2 = 4x - 7$ is
    A. 2 **B.** 4 **C.** 6 **D.** 7

17. Solve the equation $(3a - 2)(5a - 4) = (3a - 2)^2$
    A. –2/3, 1 **B.** –1, 2/3 **C.** 2/3, 1 **D.** 2/3, 4/5

18. Solve the equation $(\log_3 a)^2 - 6\log_3 a + 9 = 0$
    A. 81 **B.** 27 **C.** 18 **D.** 9

19. Find the value of x for which $3^{2x} + 6(3^x) = 27$
    A. 3 **B.** –1 **C.** –3 **D.** 1

20. Find the values of y for which
    $16^y - 5 \times 4^y + 4 = 0$
    A. 1, 4 **B.** –2, 2 **C.** 0, 1 **D** –1, 0

21. Solve for x: $x^2 + ({}^2/_{m^2})x + {}^1/_{m^4} = 0$
    A. $-{}^1/_{m^2}$ **B.** ${}^1/_{m^2}$ **C.** m^2 **D.** ${}^1/_m$

22. Find the values of x for which
    $9^x - 4 \times 3^x + 3 = 0$
    A. 0, –1 **B.** 1, –1 **C.** 1, 3 **D.** 1, 0

23. Solve for y: $25^y + 3(5^y) = 4$
    A. 1 or 4 **B.** 0 **C.** –4 or 0 **D.** 1

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24. Find the quadratic equation whose roots are $y = 7$ and $y = -2$
A. $y^2 + 2y - 7 = 0$ **B.** $y^2 - 2y + 7 = 0$
C. $y^2 - 5y - 14 = 0$ **D.** $y^2 + 5y + 14 = 0$

25. The equation whose roots are 1/2 and −7 is
A. $2x^2 - 13x - 7 = 0$
B. $2x^2 + 13x - 7 = 0$
C. $2x^2 + 7x - 14 = 0$
D. $2x^2 + 14x - 7 = 0$

26. The equation whose roots are −1/4 and −2/3 is
A. $12a^2 + 11a + 2 = 0$
B. $12a^2 - 11a + 2 = 0$
C. $12a^2 - 11a - 2 = 0$
D. $a^2 - ({}^{11}/_{12})a + 2 = 0$

27. One of the roots of the equation $x^2 + kx - 15 = 0$ is 3. The values of k and the other root are, respectively
A. −2, 5 **B.** 5, 2 **C.** 2, −5 **D.** −5, −2

28. The roots of the equation $ay^2 + by + c = 0$ are 1/2 and −3. The values of a, b and c are, respectively
A. 1,5,3 **B.** 2,5,−3 **C.** 2,5,3 **D.** 2,−5,−3

29. The product of the present ages of a mother and her daughter is 432. Four years ago, the mother was exactly four times as old as the daughter. Their present ages are
A. 48, 9 **B.** 36, 12 **C.** 28, 14 **D.** 30, 18

30. A motorist drove a distance of 90 km at constant speed for the first 40 km and then drove 20 km/hr faster for the rest of the journey. If the whole journey took him one hour, his average speed for the first 40 km was
A. 40 km/hr **B.** 60 km/hr
C. 70 km/hr **D.** 80 km/hr.

31. When the price of eggs was raised by ₦2 an egg, the number of eggs which can be bought for ₦120 is reduced by 5. The present price of an egg is
A. ₦6 **B.** ₦7 **C.** ₦8 **D.** ₦10

32. A rectangle has a perimeter of 22 cm and an area of 24 cm^2. The dimensions of the rectangle are
A. 8 cm x 3 cm **B.** 6 cm x 4 cm
C. 12 cm x 2 cm **D.** 16 cm x 1.5 cm

33. The length of a rectangle exceeds its width by 2 cm. If the area of the rectangle is 80 cm^2, the length of the rectangle is
A. 16 cm **B.** 10 cm **C.** 8 cm **D.** 5 cm

34. When 11 times a certain integer is subtracted from twice the square of the integer, the result is 21. The integer is
A. 5 **B.** 6 **C.** 7 D. 9

35. If the equation $y^2 + 3y + k = 0$ has equal roots, the value of k is
A. 9/16 B. 9/8 C. 9/4 D. 9/2

36. The roots of the quadratic equation $2x^2 + 6x - 3 = 0$ are
A. $\frac{-3 \pm \sqrt{3}}{2}$ **B.** $\frac{-6 \pm \sqrt{12}}{4}$ **C.** $\frac{-6 \pm \sqrt{15}}{4}$ **D.** $\frac{-3 \pm \sqrt{15}}{2}$

37. The roots of the quadratic equation $cy^2 + ay + b = 0$ are
A. $y = \frac{-a \pm \sqrt{a^2 - 4bc}}{2c}$ **B.** $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
C. $y = \frac{a \pm \sqrt{a^2 - 4bc}}{2c}$ **D.** $y = \frac{-c \pm \sqrt{c^2 - 4ab}}{2b}$

38. The quadratic equation whose roots are $1 + \sqrt{6}$ and $1 - \sqrt{6}$ is
A. $x^2 + (1 - \sqrt{6})x + 1 + \sqrt{6} = 0$
B. $x^2 + (1 + \sqrt{6})x + 1 - \sqrt{6} = 0$
C. $x^2 - 2x + 6 = 0$
D. $x^2 - 2x - 5 = 0$

39. If the equation $y^2 - (k - 2)y + 2k + 1 = 0$ has equal roots, the values of k are
A. 12, 0 **B.** 1, 2 **C.** 4, 5 **D.** 3, 4

40. If the equation $px^2 - 6x + 1 = 0$ has equal roots, the value of p is
A. 3 **B.** 4 **C.** 6 **D.** 9

41. The positive root of the equation $4x^2 + 7x = 1$, correct to 3 decimal places is
A. 1.062 **B.** 10.623 **C.** 0.133 **D.** 0.322

42. The sum of the roots of a quadratic equation is 3/2 and the product of the roots is 2. The equation is
A. $2x^2 + 3x + 4 = 0$ **B.** $2x^2 - 3x + 4 = 0$
C. $2x^2 - 3x - 4 = 0$ **D.** $2x^2 - 2x + 3 = 0$

43. The sum and product of the roots of the equation $1 - 6x - 3x^2 = 0$ are respectively,
A. −2, −1/3 **B.** −2, 1/3 **C.** 2, 1/3 **D.** 2. −1/3

44. The roots of the equation $y + 2\sqrt{y} = 8$ are
A. 16, 4 **B.** 16, 1 **C.** 4, 2 **D.** 4, 1

45. The roots of the equation $z - 2 + \sqrt{z^2 - 4} = 0$ are
A. 4 **B.** 2 **C.** −2 **D.** −4

46. Solve for y if $y + 1 = \sqrt{y^2 + 9}$
A. 1 **B.** 2 **C.** 3 **D.** 4

47. Solve for x: $x - 3\sqrt{x + 2} + 2 = 0$
A. 7, 1 **B.** 2, 7 **C.** −2, 7 **D.** 0, 7

48. If $y = -2$ is a root of the equation $y^3 - 12y - 16 = 0$, then the equation has
A. three equal roots

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B. two equal roots and the third root different
C. only one root
D. three roots all different

49. If the equation $y^3 - 2y^2 - 5y + 6 = 0$ has $y = 1$ as a solution, the two other roots are
A. $-3, 2$ B. $-2, 2$ C. $1, 3$ D. $3, -2$

50. The positive root of the equation $2(x^3 - x^2 - 2x) = 0$ is
A. 4 B. 3 C. 2 D. 1

ANSWER KEY

1. **D**	10. **C**	19. **D**	28. **B**	37. **A**	46. **D**
2. **C**	11. **C**	20. **C**	29. **B**	38. **D**	47. **C**
3. **A**	12. **B**	21. **A**	30. **D**	39. **A**	48. **B**
4. **B**	13. **D**	22. **D**	31. **C**	40. **D**	49. **D**
5. **A**	14. **A**	23. **B**	32. **A**	41. **C**	50. **C**
6. **B**	15. **D**	24. **C**	33. **B**	42. **B**	
7. **D**	16. **B**	25. **B**	34. **C**	43. **A**	
8. **A**	17. **C**	26. **A**	35. **C**	44. **A**	
9. **D**	18. **B**	27. **C**	36. **D**	45. **B**	

SOLUTIONS TO OBJECTIVE QUESTIONS

1. **(D)** $(2y + 1)(y - 2) = 0$, $y = -1/2$ or 2

2. **(C)** $(3z - 5)(2z + 1) = 0$, $z = 5/3$ or $-1/2$

3. **(A)** $5x^2 - 3x = 0$, $x(5x - 3) = 0$
$x = 0$ or $3/5$

4. **(B)** $(2x - 9)(x + 3) = 0$, $x = 9/2$ or -3

5. **(A)** $x^2 - 3x - 10 = 0$
$(x - 5)(x + 2) = 0$, $x = 5$ or -2

6. **(B)** $(a + 1)(a - 3) = 0$, $a = -1$ or 3

7. **(D)** $(3y + 2)(y - 9) = 0$, $y = -2/3$ or 9

8. **(A)** $(a - 7)(a + 2) = 0$, $a = 7$ or -2

9. **(D)** $y(y + 6) = 0$, $y = 0$ or 6

10. **(C)** $(a - 2)(a - 1) = 0$; $a = 2$ or 1

11. **(C)** $6y^2 + 7y - 5 = 0$, $(3y + 5)(2y - 1) = 0$
$y = 1/2$ or $-5/3$

12. **(B)** $(2z - 1)(z + 3) = 0$, $z = 1/2$ or -3

13. **(D)** $x^2 = -9$, $x = \pm\sqrt{-9}$, which is *imaginary*

14. **(A)** $(2y - 3)(5y + 1) = 0$, $y = 3/2$ or $-1/5$

15. **(D)** $(5 - 2y)(1 + y) = 0$, $y = 5/2$ or -1

16. **(B)** Expand to give:
$x^2 - 2x + 1 - 4x + 7 = 0$, or $x^2 - 6x + 8 = 0$
$(x - 4)(x - 2) = 0$; $x = 2$ or 4

17. **(C)** $(3a - 2)^2 - (3a - 2)(5a - 4) = 0$
$(3a - 2)[3a - 2 - 5a + 4] = 0$
$(3a - 2)(-2a + 2) = 0$
$a = 2/3$ or 1

18. **(B)** Let $x = \log_3 a$. Hence the equation becomes
$x^2 - 6x + 9 = 0$
$(x - 3)^2 = 0$, or $x = 3$
$\Rightarrow \log_3 a = 3$ and $a = 3^3 = 27$

19. **(D)** Let $y = 3^x$. The equation becomes
$y^2 + 6y - 27 = 0$
$(y + 9)(y - 3) = 0$, $y = -9$ or 3
Since 3^x must be a positive number, the solution $y = -9$ is dropped. Hence $3 = 3^x$ and $x = 1$

20. **(C)** Let $x = 4^y$. The equation becomes
$x^2 - 5x + 4 = 0$
$(x - 4)(x - 1) = 0$; $x = 4$ or 1
For $4^y = 4$, $y = 1$
For $4^y = 1$, $y = 0$

21. **(A)** The equation is a perfect square which can be written as
$[x + 1/m^2]^2 = 0$, or
$x = -1/m^2$

22. **(D)** Let $3^x = y$, hence
$y^2 - 4y + 3 = 0$
$(y - 3)(y - 1) = 0$, $y = 3$ or 1
For $y = 3$, $x = 1$
For $y = 1$, $x = 0$

23. **(B)** Let $5^y = x$
$\Rightarrow x^2 + 3x - 4 = 0$
$(x + 4)(x - 1) = 0$, $x = -4$ or 1

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For $x = 1$, $y = 0$
For $x = -4$, y does not exist.

24. (C) The equation is $(y - 7)(y + 2) = 0$, or
$y^2 - 5y - 14 = 0$

25. (B) The equation is $(x - {}^1/_2)(x + 7) = 0$
$(2x - 1)(x + 7) = 0$, or
$2x^2 + 13x - 7 = 0$

26. (A) $(a + {}^1/_4)(a + {}^2/_3) = 0$
$(4a + 1)(3a + 2) = 0$
$12a^2 + 11a + 2 = 0$

27. (C) $f(3) = 3^2 + 3k - 15 = 0$, or $k = 2$
The equation is $x^2 + 2x - 15 = 0$, or
$(x - 3)(x + 5) = 0$
The other root is -5.

28. (B) Sum of the roots $= 1/2 - 3 = -5/2 = -b/a$
Product of the roots $= (1/2)(-3) = -3/2 = c/a$
The equation is $x^2 + (b/a)x + c/a = 0$, or
$x^2 + {}^5/_2 x - {}^3/_2 = 0$, hence $2x^2 + 5x - 3 = 0$
i.e. $a = 2$, $b = 5$ and $c = -3$.

29. (B) Let x, y be mother and daughter's present ages respectively.
$xy = 432$ .....(i)
$x - 4 = 4(y - 4)$, or
$x - 4y = -12$ .....(ii)
Substituting $x = 432/y$ in (ii) gives
$\frac{432}{y} - 4y = -12$
$432 - 4y^2 = -12y$, or $y^2 - 3y - 108 = 0$
$(y + 9)(y - 12) = 0$
$y = 12$ and $x = 432/12 = 36$.
[Note that this question could have been answered much more quickly by inspection. Options (C) and (D) are eliminated since their product is different from 432. Option (A) does not satisfy the second condition]

30. (D) Let x = average speed for first 40 km.
Time to cover first 40 km = $40/x$ hr.
Time to cover the next 50 km = $\frac{50}{x + 20}$ hr.

Total time taken = $\frac{40}{x} + \frac{50}{x + 20} = 1$

$40x + 800 + 50x = x^2 + 20x$
$x^2 - 70x - 800 = 0$
$(x - 80)(x + 10) = 0$, or $x = 80$ km/hr.

31. (C) Let x = original price of an egg
₦120 would buy $\frac{120}{x}$ eggs

At the new price of $x + 2$ for an egg,
₦120 would buy $\frac{120}{x + 2} = \frac{120}{x} - 5$

$120x = 120(x + 2) - 5x(x + 2)$
$120x = 120x + 240 - 5x^2 - 10x$
$5x^2 + 10x - 240 = 0$
$(5x - 30)(x + 8) = 0$, or $x = 30/5 = 6$
Present price $= 6 + 2 =$ ₦8

32. (A) Let x, y = dimensions of the rectangle
$2(x + y) = 22$, or $x + y = 11$ ..... (i)
$xy = 24$.. ..... (ii)
Setting $y = 24/x$ in eq.(i) gives
$x + 24/x = 11$, or $x^2 + 24 - 11x = 0$
$x^2 - 11x + 24 = 0$
$(x - 8)(x - 3) = 0$, or $x = 8$ or 3
The dimensions are 8 cm by 3 cm.

33. (B) Let length $= x$, and width $= x - 2$
Area $= x(x - 2) = 80$
$x^2 - 2x - 80 = 0$
$(x - 10)(x + 8) = 0$, or $x = 10$ cm

34. (C) $2x^2 - 11x = 21$
$2x^2 - 11x - 21 = 0$
$(2x + 3)(x - 7) = 0$, $\Rightarrow x = 7$ or $-3/2$
(Since the question specifies an *integer*, the second solution, $-3/2$ is ignored)

35. (C) For the equation $ax^2 + bx + c = 0$ to have equal roots, $b^2 = 4ac$.
For the equation $y^2 + 3y + k = 0$,
$3^2 = 4(1)(k)$, or $k = 9/4$

36. (D) Using the *quadratic formula*:

$$x = \frac{-6 \pm \sqrt{6^2 + 24}}{4} = \frac{-6 \pm \sqrt{60}}{4}$$
$$= \frac{-6 \pm 2\sqrt{15}}{4} = \frac{-3 \pm \sqrt{15}}{2}$$

37. (A) Note that the coefficients of y^2, y and the constant term in the given equation are c, a and b respectively, as compared to a, b and c which are conventionally used.

38. (D) Sum of the roots
$= 1 + \sqrt{6} + 1 - \sqrt{6} = 2 = -b/a$
Product of the roots
$= (1+\sqrt{6})(1-\sqrt{6}) = 1 - 6 = -5 = c/a$
The equation is $x^2 + (b/a)x + c/a = 0$
or $x^2 - 2x - 5 = 0$
[Alternatively, $[x - (1+\sqrt{6})][(x - (1-\sqrt{6})] = 0$, which upon expansion, gives $x^2 - 2x - 5 = 0$].

39. (A) $b^2 = 4ac$ for equal roots:
$(k - 2)^2 = 4(1)(2k + 1)$
$k^2 - 4k + 4 = 8k + 4$
$k^2 - 12k = 0$, or $k(k - 12) = 0$
$k = 0$ or 12

40. (D) $b^2 = 4ac \Rightarrow 6^2 = 4p(1)$. or
$p = 36/4 = 9$

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41. (C) $4x^2 + 7x - 1 = 0$
Using the *quadratic formula*

$$x = \frac{-7 \pm \sqrt{49 + 16}}{8} = \frac{-7 \pm \sqrt{65}}{8}$$

Since $\sqrt{65}$ is about 8, the solution must be close to $^1/_8$. The positive root is $(-7 + \sqrt{65})/8$, which is close to 0.133.

42. (B) Sum $= -b/a = 3/2$
Product $= c/a = 2$
The equation is $x^2 + (b/a)x + c/a = 0$,
i.e. $x^2 - (3/2)x + 2 = 0$
or $2x^2 - 3x + 4 = 0$

43. (A) Rewrite the equation as $3x^2 + 6x - 1 = 0$
Sum of the roots $= -b/a = -6/3 = -2$
Product of the roots $= c/a = -1/3$

44. (A) Move the term containing the square root sign to one side, and all other terms to the other side, i.e.

$$2\sqrt{y} = 8 - y$$

Square both sides: $(2\sqrt{y})^2 = (8 - y)^2$
$4y = 64 - 16y + y^2$
$y^2 - 20y + 64 = 0$
$(y - 16)(y - 4) = 0$
$y = 16$ or 4

45. (B) Write the equation as $\sqrt{z^2 - 4} = 2 - z$
Square both sides: $z^2 - 4 = (2 - z)^2$
$z^2 - 4 = 4 - 4z + z^2$
$4z = 8$, $z = 2$

46. (D) $(y + 1)^2 = y^2 + 9$
$y^2 + 2y + 1 = y^2 + 9$
$2y = 8$, or $y = 4$

47. (C) $x + 2 = 3\sqrt{x + 2}$
$(x + 2)^2 = 9(x + 2)$
$x^2 + 4x + 4 = 9x + 18$
$x^2 - 5x - 14 = 0$
$(x - 7)(x + 2) = 0$
$x = 7$ or -2

48. (B) Re–write the equation as
$y^3 - 12y - 16 = (y + 2)(ay^2 + by + c)$
Equate coefficients of y^3: $a = 1$
Equate constants: $-16 = 2c$, or $c = -8$
Equate coefficients of y^2: $0 = 2a + b$,
or $b = -2a = -2$
Hence the equation is
$(y + 2)(y^2 - 2y - 8) = 0$, or
$(y + 2)(y + 2)(y - 4) = 0$
The roots are -2, -2 and 4

49. (D) Re–write the equation as
$y^3 - 2y^2 - 5y + 6 = (y - 1)(ay^2 + by + c)$
Equate coefficients of y^3: $a = 1$
Equate constants: $6 = -c$, or $c = -6$
Equate coefficients of y^2: $-2 = -a + b$
or $b = a - 2 = -1$
Hence the equation is
$(y - 1)(y^2 - y - 6) = 0$, or
$(y - 1)(y - 3)(y + 2) = 0$
$y = 1$ (*given*), and 3, -2

50. (C) The equation is $2x(x^2 - x - 2) = 0$, or
$2x(x - 2)(x + 1) = 0$
$x = 0$, -1 or 2.
[Since only the positive root is required, ignore 0 and -1]

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8 Simultaneous Equations

- **Simultaneous equations** consist of a pair of equations, which could be of the form:
  (i) both equations linear, or
  (ii) one linear and the other quadratic.
- The two equations contain two variables whose values must satisfy both equations at the same time.
- Solutions of simultaneous equations can be found by one of three methods:
  (i) Substitution
  (ii) Elimination, and
  (iii) Graphically
- The methods of substitution and elimination are discussed below while the graphical method is presented in chapter 9.

8.1 Method of Substitution

- To solve two simultaneous equations by substitution:
  (i) Express one variable (e.g. x) in terms of the other (y) using one of the equations.
  (ii) Substitute the expression for x in (i) in the second equation.
  (iii) Solve for y.
  (iv) Substitute the value of y in the expression for x obtained in step (i).
  (v) Solve for x.

EXAMPLE 8 – 1

Solve the following simultaneous equations for x and y:

$16x + 3y = 63$ .....(1)
$3x + 5y = 34$ .....(2)

Solution

Step (i): Express x in term of y in eq. (1):

$$x = \frac{1}{16}(63 - 3y)$$

Step (ii): Substitute for x in (2):

$$\frac{3}{16}(63 - 3y) + 5y = 34$$

Step (iii): Solve for y:

$$3(63 - 3y) + 80y = 544$$
$$71y = 544 - 189 = 355$$
$$y = 5$$

Step (iv): Substitute for y in expression for x

$$x = \frac{1}{16}(63 - 15) = \frac{48}{16} = 3$$

$\Rightarrow (3, 5)$ is the solution desired.

EXAMPLE 8 – 2

Find the values of x and y which together satisfy the following equations simultaneously:

$x + 2y = 11$ .....(1)
$x^2 + y^2 = 26$ .....(2)

Solution

Where one linear and one quadratic equations are given, use the linear equation for step (i):

Step (i) ..... $x = 11 - 2y$

Step (ii) .....$(11 - 2y)^2 + y^2 = 26$
$121 - 44y + 4y^2 + y^2 = 26$
$5y^2 - 44y + 95 = 0$

Step (iii) $(5y - 19)(y - 5) = 0$
$y = 5$ or $^{19}/_5$

Step (iv) For $y = 5$; $x = 11 - 2(5) = 1$
For $y = {}^{19}/_5$; $x = 11 - {}^{38}/_5 = 17/5$

Hence the solutions are (1, 5) and $({}^{17}/_5, {}^{19}/_5)$.

DRILL 1

Solve the following simultaneous equations by substitution:

(a) $x + y = 1; x + 4y = -2$
(b) $4x - y = 3; 3x + 2y = 16$
(c) $xy = -3; x + y = 2$
(d) $x + 2y = 0; x^2 + y^2 = 20$

8.2 Method of Elimination

(i) Multiply either or both of the two equations by appropriate factors in such a way as to equalise the coefficients of x (or of y).
(ii) Eliminate the x term (or the y term) by adding the two equations, or by subtracting one from the other.
(iii) Solve for y (or for x)
(iv) Substitute the value of y (or x) in any of the given equations.
(v) Solve for x (or y).

EXAMPLE 8 – 3

Find the values of x and y which simultaneously satisfy the following pair of equations:

$x + y = 18$ ..... (1)
$x - y = 12$ ..... (2)

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Solution
The coefficients of x are both equal to 1, while those of y are 1 and -1. Hence step (i) is not necessary.
(ii) ....Add (1) and (2) ... $2x = 30$
(iii) ... $x = 15$
(iv) ... From eq. (1), $y = 18 - x = 18 - 15 = 3$
Hence desired solution is (15, 3).

EXAMPLE 8 – 4

Solve the following equations simultaneously for x and y by the method of elimination:

$2x + 3y = 23$ .....(1)
$15x + 8y = 129$ .....(2)

Solution
(i) ....Multiply (1) by 15 and (2) by 2 to give
$30x + 45y = 345$ .....(3)
$30x + 16y = 258$ .....(4)
(ii): (3) – (4)..... $29y = 87$
(iii): ..... $y = 3$
(iv): From eq. (1): $x = {}^1/_2(23 - 3y) = {}^1/_2(23 - 9) = 7$
$\Rightarrow$ solution is (7, 3)

DRILL 2

Solve the following pairs of simultaneous equations by elimination:
(a) $x - y = 7; x + y = 3$
(b) $3x - 4y = 7; 4x + 3y = 1$
(c) $5a + 2b = 9; 2a - 3b = 15$
(d) ${}^2/x + {}^1/y = 7; {}^5/x - {}^2/y = 4$

ANSWERS TO DRILLS

1. (a) 2, – 1 (b) 2, 5 (c) –3, 1 (d) (4, –2) or (–4, 2)
2. (a) 5, –2 (b) 1, –1 (c) 3, –3 (d) ${}^1/_2$, ${}^1/_3$

SSCE/JME – TYPE OBJECTIVE QUESTIONS

1. Solve the simultaneous equations $p = 3q$, $4p - 5q = 14$
A. –6, –2 B. 6, 2 C. –6, 2 D. 6, –2

2. If $2x - y = 6$, and $2x + 4y = 1$, the value of $4x + 3y$ is
A. 1 B. 3 C. 5 D. 7

3. Given that $a + b = 3/2$ and $a - b = 5/2$, the value of $2b + a$ is
A. 1 B. 1/2 C. –1 D. –2

4. Find the value of y if $3x - y = 6$ and $2x + 3y = 4$
A. 0 B. 1/2 C. 2/3 D. 1

5. If $2p + m = 7$ and $3p - 2m = 3$, what is the difference between $7p$ and 10?
A. 1 B. 3 C. 7 D. 10

6. Solve the equations $4u - v = 11; 5u + 2v = 4$
A. 6, 13 B. –2, –3 C. –2, 3 D. 2, –3

7. Solve the simultaneous equations $y + z + 6 = 0$, $2y - 4z = 8$
A. $-2{}^2/_3, -8{}^2/_3$ B. $1{}^2/_3, -2{}^2/_3$
C. $-2{}^2/_3, -3{}^1/_3$ D. $3{}^2/_3, -2{}^1/_3$

8. If $2m + 5n = 11$ and $7m + 4n = 2$; find the value of $27(m+n)$
A. 54 B. 39 C. –189 D. –27

9. If $4p + 3q = 5$ and $3p + 5q = 4$, the value of $11(p + q)$ is
A. –12 B. 14 C. 12 D. 11

10. Solve the equations $2a + 5b = 6{}^1/_2; 5a - 2b = 9$
A. $2, {}^1/_2$ B. –2, 2 C. $ {}^1/_2, -2$ D. $2, {}^3/_2$

11. Find $y - z$ if $4y - 3z = 7$ and $3y - 2z = 5$
A. 4 B. 0 C. 2 D. –2

12. If $3y - 5x - 3 = 0$ and $2x - 6y + 5 = 0$, the value of (x, y) is
A. $\frac{24}{19}, 8$ B. $\frac{24}{19}, -8$ C. $\frac{19}{24}, -\frac{1}{8}$ D. $-\frac{1}{8}, \frac{19}{24}$

13. Solve the equations
$4m - 3 = 3m + n = 2n + 5m - 12$
A. $m = 5, n = 2$ B. $m = 2, n = 5$
C. $m = -2, n = -5$ D. $m = 5, n = -2$

14. Solve the simultaneous equations
$2p - 3q + 10 = 10p - 6q = 5$
A. $p = 3{}^1/_3, q = 2{}^1/_2$ B. $p = 2{}^1/_2, q = 3{}^1/_3$
C. $p = 2{}^1/_4, q = 3$ D. $p = -2{}^1/_2, q = 0$

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15. Solve the simultaneous equations
$^1/x - {}^5/y = 7$ and $^2/x + {}^1/y = 3$
A. $x = -1, y = {}^1/_2$ **B.** $x = {}^1/_2, y = -1$
C. $x = 2, y = -1$ **D.** $x = {}^1/_2, y = -2$

16. Solve the pair of equations
$2y^{-1} - 3z^{-1} = 4; 4y^{-1} + z^{-1} = 1$
A. 1, 2 **B.** −1, 2 **C.** 2, 1 **D.** 2, −1

17. Find the set of values of p and q which satisfies the equations $p^2 - q - 1 = 0$ and $q - 2p + 2 = 0$
A. 1, 0 **B.** 1, 1 **C.** 2, 2 **D.** 0, 2

18. Solve for (y, x) in the equations:
$2y + x = 4; y^2 + xy = -12$
A. (3, −4); (−1, 4) **B.** (8, −4); (−1, 4)
C. (6, −8); (−2, 8) **D.** (−8, 6); (8, −2)

19. Solve the following simultaneous equations for y:
$y^2 + x - 8 = 0; x + 5y - 2 = 0$
A. −1, 7 **B.** 6, −1 **C.** 6, −28 **D.** −28, 7

20. Find the values of q which satisfy the simultaneous equations $p + q = 5; p^2 - 2q^2 = 1$
A. 12, −2 **B.** −12, 12 **C.** −12, 2 **D.** 2, −2

21. Solve the following simultaneous equations for all possible values of m and n:
$3m^2 + 2n = 7, 2m + n = 4$
A. (1, 2) **B.** $(^1/_3, 1)$
C. $(2, 1); (3^1/_3, {}^1/_3)$ **D.** $(1, 2); (^1/_3, 3^1/_3)$

22. Solve the following simultaneous equations for r:
$r^2 + s - 5 = 0, s - 7r + 3 = 0$
A. 1, −8 **B.** −2, 4 **C.** −1, 8 **D.** 2, 4

23 Solve the simultaneous equations
$\log_{10}a + \log_{10}b = 4$, and
$\log_{10}a + 2\log_{10}b = 3$
A. 1,000; $^1/_{10}$ **B.** 100,000; $^1/_{10}$
C. 10,000; $^1/_{100}$ **D.** 100; $^1/_{100}$

24. Solve the equations
$2^{m+n} = 32; 3^{3n-m} = 27$
A. 3, 2 **B.** −3, 2 **C.** 3, −2 **D.** −3, −2

25. Given that $3^{x+2y} = 3$ and $5^{x+3y} = 25$, find 3^{x+y}
A. 27 **B.** 3 **C.** 1 **D.** 0

26. The sum of the digits in a 2–digit number is 13. With the digits interchanged, the new number exceeds the original number by 27. The original number is
A. 49 **B.** 58 **C.** 67 **D.** 76

27. The sum of the ages of a father and his daughter is 85 years. Five years ago, the father was exactly four times as old as his daughter. The present age of the father is
A. 65 yr. **B.** 61 yr. **C.** 57 yr. **D.** 53 yr.

28. The length of a rectangular plot of land exceeds the width by 7 m. If the area of the plot is 198 m^2, the length is
A. 27 m **B.** 24 m **C.** 21 m **D.** 18 m

29. The difference between the length and the width of a rectangle is 5 cm. If the perimeter of the rectangle is 34 cm, the length is
A. 17 cm **B.** 15 cm **C.** 11 cm **D.** 9 cm

30. Three times the tens digit of a 2-digit number is 1 less than twice the units digit. When the digits are interchanged, the new number is 17 less than twice the original number. The original number is
A. 12 **B.** 35 **C.** 47 **D.** 54

ANSWER KEY

1. **B**	6. **D**	11. **C**	16. **D**	21. **D**	26. **B**
2. **D**	7. **C**	12. **D**	17. **A**	22. **A**	27. **A**
3. **A**	8. **B**	13. **A**	18. **C**	23. **B**	28. **D**
4. **A**	9. **B**	14. **B**	19. **B**	24. **A**	29. **C**
5. **C**	10. **A**	15. **B**	20. **C**	25. **C**	30. **B**

SOLUTIONS TO OBJECTIVE QUESTIONS

1. **(B)** Substituting for $p = 3q$ in the second equation gives
$4(3q) - 5q = 14$
$7q = 14, q = 2$
$p = 3q = 3(2) = 6$

2. **(D)** A quick inspection shows that it is not necessary to solve the equations completely for x and y:
$2x - y = 6$ ..... (i)
$2x + 4y = 1$ ..... (ii)
Adding (i) and (ii) gives $4x + 3y = 7$

3. **(A)** Adding the two equations gives $2a = 4$, or $a = 2$, and $b = {}^3/_2 - a = {}^3/_2 - 2 = -{}^1/_2$
$2b + a = 2(-{}^1/_2) + 2 \doteq 1$

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4. (A) To find y, eliminate x by multiplying the first equation by 2 and the second by 3
$6x - 2y = 12$ ..... (i)
$6x + 9y = 12$ ..... (ii)
(ii) – (i): $11y = 0$, or $y = 0$

5. (C) Multiply the first equation by 2 and add to the second equation:
$4p + 2m = 14$ ..... (i)
$3p - 2m = 3$ ..... (ii)
(i) + (ii): $7p = 17$, and $7p - 10 = 7$

6. (D) Multiply the first equation by 2:
$8u - 2v = 22$ ..... (i)
$5u + 2v = 4$ ..... (ii)
Add: $13u = 26$, $u = 2$
From (ii): $v = (4 - 5u)/2 = (4 - 10)/2 = -3$

7. (C) Multiply the first equation by 4 and transfer the constant term to the RHS:
$4y + 4z = -24$ ..... (i)
$2y - 4z = 8$ ..... (ii)
Add: $6y = -16$, $y = -2^2/_3$
From (i): $z = -6 - y = -6 + 2^2/_3 = -3^1/_3$

8. (B) Multiply the equations by 4 and 5 respectively:
$8m + 20n = 44$ ..... (i)
$35m + 20n = 10$ ..... (ii)
(ii) – (i): $27m = -34$, $m = -{}^{34}/_{27}$
$n = \dfrac{11 - 2m}{5} = \dfrac{11 + {}^{68}/_{27}}{5} = \dfrac{73}{27}$
$27(m + n) = -34 + 73 = 39$

9. (B) Multiply the equations by 5 and 3 respectively:
$20p + 15q = 25$ .....(i)
$9p + 15q = 12$ .....(ii)
(i) – (ii)..... $11p = 13$, $p = {}^{13}/_{11}$
$q = (5 - 4p)/3 = {}^{1}/_{11}$
$11(p + q) = 13 + 1 = 14$

10. (A) Multiply the equations by 2 and 5 respectively:
$4a + 10b = 13$ ..... (i)
$25a - 10b = 45$ ..... (ii)
Add: $29a = 58$, $a = 2$
From (ii): $b = \dfrac{5a - 9}{2} = \dfrac{10 - 9}{2} = \dfrac{1}{2}$

11. (C)
$4y - 3z = 7$ ..... (i)
$3y - 2z = 5$ ..... (ii)
(i) – (ii): $y - z = 2$

12. (D) The equations are $-5x + 3y = 3$ and $2x - 6y = -5$, Multiply the first equation by 2:
$-10x + 6y = 6$ ..... (i)
$2x - 6y = -5$ ..... (ii)
Add: $-8x = 1$, or $x = -1/8$
From (ii): $y = \dfrac{4x + 5}{6} = \dfrac{-{}^{1}/_{4} + 5}{6} = \dfrac{19}{24}$

13. (A) Combining the first two equations:
$m - n = 3$ ..... (i)
Combining the last two equations:
$2m + n = 12$ ..... (ii)
Add (i) and (ii): $3m = 15$, or $m = 5$
From (i): $n = m - 3 = 5 - 3 = 2$

14. (B) The equations are $2p - 3q = -5$ and $10p - 6q = 5$.
Multiply the first equation by 2:
$4p - 6q = -10$ ..... (i)
$10p - 6q = 5$ ..... (ii)
(ii) – (i): $6p = 15$, $p = 5/2 = 2^1/_2$
$q = \dfrac{2p + 5}{3} = \dfrac{2(5/2) + 5}{3} = \dfrac{10}{3} = 3^1/_3$

15. (B) Multiply the second equation by 5:
${}^{1}/_{x} - {}^{5}/_{y} = 7$ ..... (i)
${}^{10}/_{x} + {}^{5}/_{y} = 15$ ..... (ii)
Add: ${}^{11}/_{x} = 22$, or $x = 1/2$
From (i): $2 - {}^{5}/_{y} = 7$, ${}^{5}/_{y} = -5$, or $y = -1$
[*Alternate method*: Substitute $p = {}^{1}/_{x}$ and $q = {}^{1}/_{y}$, and solve for p and q]

16. (D) Multiply the second equation by 3
$2y^{-1} - 3z^{-1} = 4$ ..... (i)
$12y^{-1} + 3z^{-1} = 3$ ..... (ii)
Add: $14y^{-1} = 7$, $y^{-1} = {}^{1}/_{2}$ or $y = 2$
From (ii): $z^{-1} = 1 - 4y^{-1} = 1 - 4({}^{1}/_{2}) = -1$
or $z = -1$

17. (A) Re–write the equations as
$p^2 - q = 1$ ..... (i)
$-2p + q = -2$ ..... (ii)
Add: $p^2 - 2p = -1$, or $p^2 - 2p + 1 = 0$
$(p - 1)^2 = 0$ or $p = 1$
From (i): $q = p^2 - 1 = 1 - 1 = 0$

18. (C) Square both sides of the first equation:
$(2y + x)^2 = 16$, or
$4y^2 + 4xy + x^2 = 16$ ..... (i)
Multiply second equation by 4:
$4y^2 + 4xy = -48$ ..... (ii)
(i) – (ii): $x^2 = 64$, or $x = 8$ or -8
From (i): $y = \dfrac{4 - x}{2}$
For $x = 8$, $y = \dfrac{4 - 8}{2} = -2$
For $x = -8$, $y = \dfrac{4 + 8}{2} = 6$
$(y, x) = (-2, 8)$ or $(6, -8)$

19. (B) Re–arrange the equations as
$x + y^2 = 8$ ..... (i)
$x + 5y = 2$ ..... (ii)
(i) – (ii): $y^2 - 5y = 6$, $y^2 - 5y - 6 = 0$
$(y - 6)(y + 1) = 0$, $y = 6$ or -1

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20. **(C)** Eliminate p by writing $p = 5 - q$ (first equation) and substituting into the second equation:

$$(5-q)^2 - 2q^2 = 1$$
$$25 - 10q + q^2 - 2q^2 = 1$$
$$q^2 + 10q - 24 = 0$$
$$(q+12)(q-2) = 0$$
$$q = -12 \text{ or } 2$$

21. **(D)** Multiply the second equation by 2:

$$3m^2 + 2n = 7 \quad \ldots\ldots\ldots\ldots (i)$$
$$4m + 2n = 8 \quad \ldots\ldots\ldots\ldots (ii)$$

(i) – (ii): $3m^2 - 4m = -1$, or

$$3m^2 - 4m + 1 = 0$$
$$\therefore (3m-1)(m-1) = 0,\ m = {}^1/_3 \text{ or } 1$$
$$n = 4 - 2m = 3{}^1/_3 \text{ or } 2$$

22. **(A)** Write the equations as

$$r^2 + s = 5 \quad \ldots\ldots\ldots\ldots (i)$$
$$-7r + s = -3 \quad \ldots\ldots\ldots\ldots (ii)$$

(i) – (ii): $r^2 + 7r = 8$, or $r^2 + 7r - 8 = 0$

$$(r+8)(r-1) = 0,\ r = -8 \text{ or } 1$$

23. **(B)**

$$\log_{10}a + \log_{10}b = 4 \quad \ldots\ldots\ldots\ldots (i)$$
$$\log_{10}a + 2\log_{10}b = 3 \quad \ldots\ldots\ldots\ldots (ii)$$

(i) – (ii): $-\log_{10}b = 1$

$$\log_{10}b = -1 \text{ and } b = 10^{-1} = {}^1/_{10}$$

From (i): $\log_{10}a = 4 - \log_{10}b = 4 + 1 = 5$

$$a = 10^5 = 100{,}000$$

24. **(A)** $2^{m+n} = 2^5 \Rightarrow m + n = 5 \quad \ldots\ldots\ldots\ldots (i)$

$3^{3n-m} = 3^3 \Rightarrow 3n - m = 3 \quad \ldots\ldots\ldots\ldots (ii)$

(i) + (ii): $4n = 8,\ n = 2$

From (i): $m = 5 - n = 3$

25. **(C)** $3^{x+2y} = 3$, or $x + 2y = 1 \quad \ldots\ldots\ldots\ldots (i)$

$5^{x+3y} = 5^2$, or $x + 3y = 2 \quad \ldots\ldots\ldots\ldots (ii)$

(ii) – (i):

From (i): $x = -1$, and $x + y = -1 + 1 = 0$

$$3^{x+y} = 3^0 = 1$$

26. **(B)** This question, like most *objective* word problems on simultaneous equations, can be solved by an inspection of the options provided. However each of them is solved fully here to illustrate the applications of simultaneous equations.

Let the number be xy. Hence

$$x + y = 13 \quad \ldots\ldots\ldots\ldots (i)$$

The value of the number xy in the decimal system is $10x + y$. With the digits interchanged, the number becomes yx with the value $10y + x$. Thus

$$10y + x = (10x + y) + 27, \text{ or}$$

i.e. $9x - 9y = -27$, or

$$x - y = -3 \quad \ldots\ldots\ldots\ldots (ii)$$

Add (i) and (ii): $2x = 10$ and $x = 5$

From (i): $y = 13 - x = 8$

$\Rightarrow$ The number is 58.

27. **(A)** Let x, y = present ages of father and daughter, respectively.

$$x + y = 85 \quad \ldots\ldots\ldots\ldots (i)$$

Five years ago, father's age was $x - 5$ and daughter's age was $y - 5$. Hence

$$(x-5) = 4(y-5), \text{ or}$$
$$x - 4y = -15 \quad \ldots\ldots\ldots\ldots (ii)$$

(i) – (ii): $5y = 100$, $y = 20$ and $x = 65$

28 **(D)** Let x, y = length and width of the plot

$$x - y = 7 \quad \ldots\ldots\ldots\ldots (i)$$
$$xy = 198 \quad \ldots\ldots\ldots\ldots (ii)$$

From (i): $y = x - 7$, which upon substituting into (ii) gives

$$x(x-7) = 198$$
$$x^2 - 7x - 198 = 0$$
$$(x-18)(x+11) = 0, \text{ or}$$
$$x = 18 \text{ m}$$

29. **(C)** Let x, y = length and width of the rectangle respectively.

$$x - y = 5 \quad \ldots\ldots\ldots\ldots (i)$$

Perimeter $= 2(x + y) = 34$, or

$$x + y = 17 \quad \ldots\ldots\ldots\ldots (ii)$$

Add (i) and (ii): $2x = 22$, or $x = 11$ cm

30. **(B)** Let the original number be xy

$3x = 2y - 1$, or $3x - 2y = -1 \quad \ldots\ldots\ldots\ldots (i)$

When interchanged, the new number is yx.

Hence $10y + x = 2(10x + y) - 17$, or

$$19x - 8y = 17 \quad \ldots\ldots\ldots\ldots (ii)$$

Multiply equation (i) by 4:

$$12x - 8y = -4 \quad \ldots\ldots\ldots\ldots (iii)$$

(ii) – (iii): $7x = 21$, or $x = 3$

From (i): $2y = 3x + 1 = 10$, or $y = 5$

$\Rightarrow$ The number is 35.

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9 Graphs

9.1 General Procedure

- A **graph** illustrates pictorially the relationship between two variables, e.g. the variation of y as x varies in the equation

$$3x + 2y = 12$$

- Values of the **independent variable** (x) are selected for which the corresponding values of the **dependent variable** (y) are obtained by making y the subject of the formula, i.e.

$$y = 6 - \frac{3}{2}x$$

- A table is normally constructed listing the values of several pairs of variables (x, y) as obtained above.
- In the **cartesian coordinate** system two reference lines, called **axes**, are chosen which intersect at right angles at a point O called the **origin** of coordinates.
- The independent variable (whose values are selected) is plotted along the horizontal (x) axis while the dependent variable is plotted along the vertical (y) axis.
- Suitable scales are chosen on the graph sheet such that the plot is neither too big nor too tiny.
- The axes are graduated and labelled, including units where necessary.
- The x-coordinate of any given point on the graph is called the **abscissa** while the y-coordinate is the **ordinate**.

9.2 Linear Functions

- A linear function has an equation of the form

$$ax + by = k \qquad (9.1)$$

where a, b and k are constants.

- When values of y are plotted against values of x, the points lie on a straight line.
- Only **two points** are required to draw a linear graph and the two points usually chosen for convenience are the intersections of the graph with each of the axes, i.e. the **x-intercept** and the **y-intercept**. A third point is usually plotted only as a check.
- The **slope** or **gradient** of a straight line is a measure of the steepness of the line. A line which passes through two points (x_1, y_1) and (x_2, y_2) has its slope (m) defined as

$$m = \frac{\text{change in ordinate}}{\text{corresponding change in abscissa}}$$

or

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad (9.2)$$

- Eq. (9.1) can be written as

$$y = (-a/b)x + k/b,$$

or

$$y = mx + c \qquad (9.3)$$

with $m = -a/b$ and $c = k/b$

- In Eq. (9.3); m is the slope of the line and c is the intercept on the y-axis (Fig. 9.1).
- A line which rises from left to right has a positive slope ($m > 0$) while a line which falls from left to right has a negative slope ($m < 0$), Fig. 9.2.

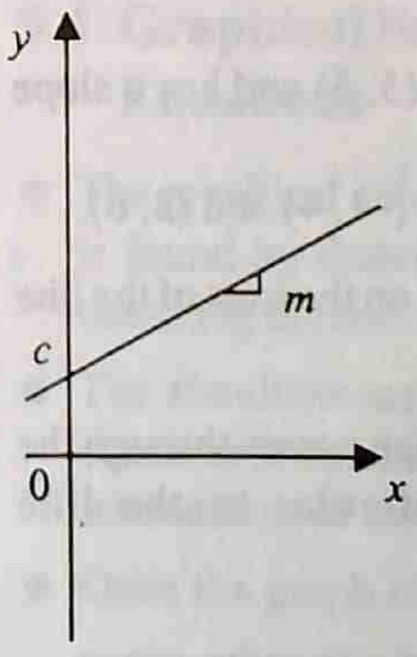


Fig. 9.1

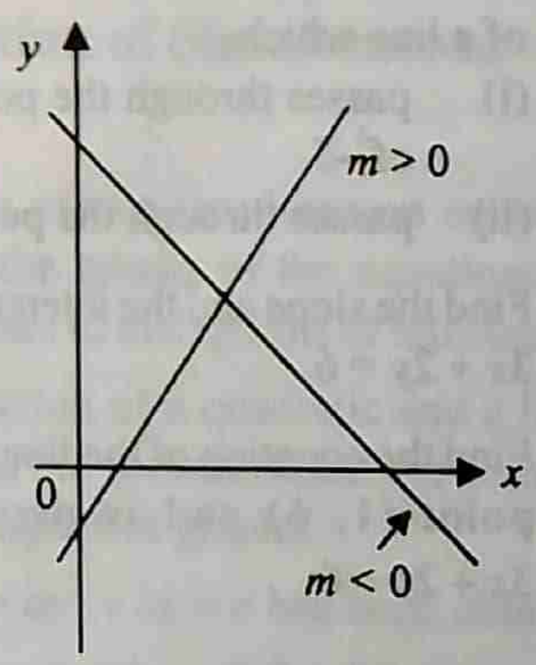


Fig. 9.2

- The slope of a horizontal line (i.e. one parallel to the x-axis) is zero ($m = 0$). The slope of a vertical line (i.e. one parallel to the y-axis) is infinite.
- Two lines are **parallel** if their slopes are equal and have the same sign, i.e. $m_1 = m_2$, e.g. the lines $y = 2x + 7$, $y = 2x - 10$ and $y = 2x + 1$ are all parallel. Also the lines

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$3x + 2y = 27$. $3x + 2y = -20$ and $3x + 2y = 10$ are all parallel.

- Two lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are **perpendicular** if the product of their slopes is equal to -1, i.e. if $m_1m_2 = -1$.

EXAMPLE 9–1

A line passes through the two points $(1, -2)$ and $(3, 6)$. Find

(a) its slope
(b) the equation of the line, and
(c) the intercept on the axes.

Solution

(a) $x_1 = 1, x_2 = 3, y_1 = -2, y_2 = 6$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-2)}{3 - 1} = 4$$

(b) The equation of the line is $y = 4x + c$. Substituting $x = 1, y = -2$ gives $c = y - 4x = -2 - 4 = -6$.
Hence $y = 4x - 6$.

(c) For the intercept on the x-axis,
$y = 0 \Rightarrow 4x - 6 = 0$, or $x = \frac{3}{2}$
For the intercept on the y-axis, $x = 0 \Rightarrow y = 6$

DRILL 1

(a) Find the equation and the intercepts on the two axes of a line which
  (i) passes through the point (5, 3) and has a slope of -5.
  (ii) passes through the points $(-1, 4)$ and $(3, 6)$.

(b) Find the slope and the intercepts on the axes of the line $3x + 2y = 6$.

(c) Find the equation of the line which passes through the point (1, 6) and is perpendicular to the line $3x + 2y = 6$.

EXAMPLE 9–2

Draw the graph of $5x + 2y = 10$ for values of x from -5 to $+5$ using a scale of 1 cm to 1 unit on each axis. Find

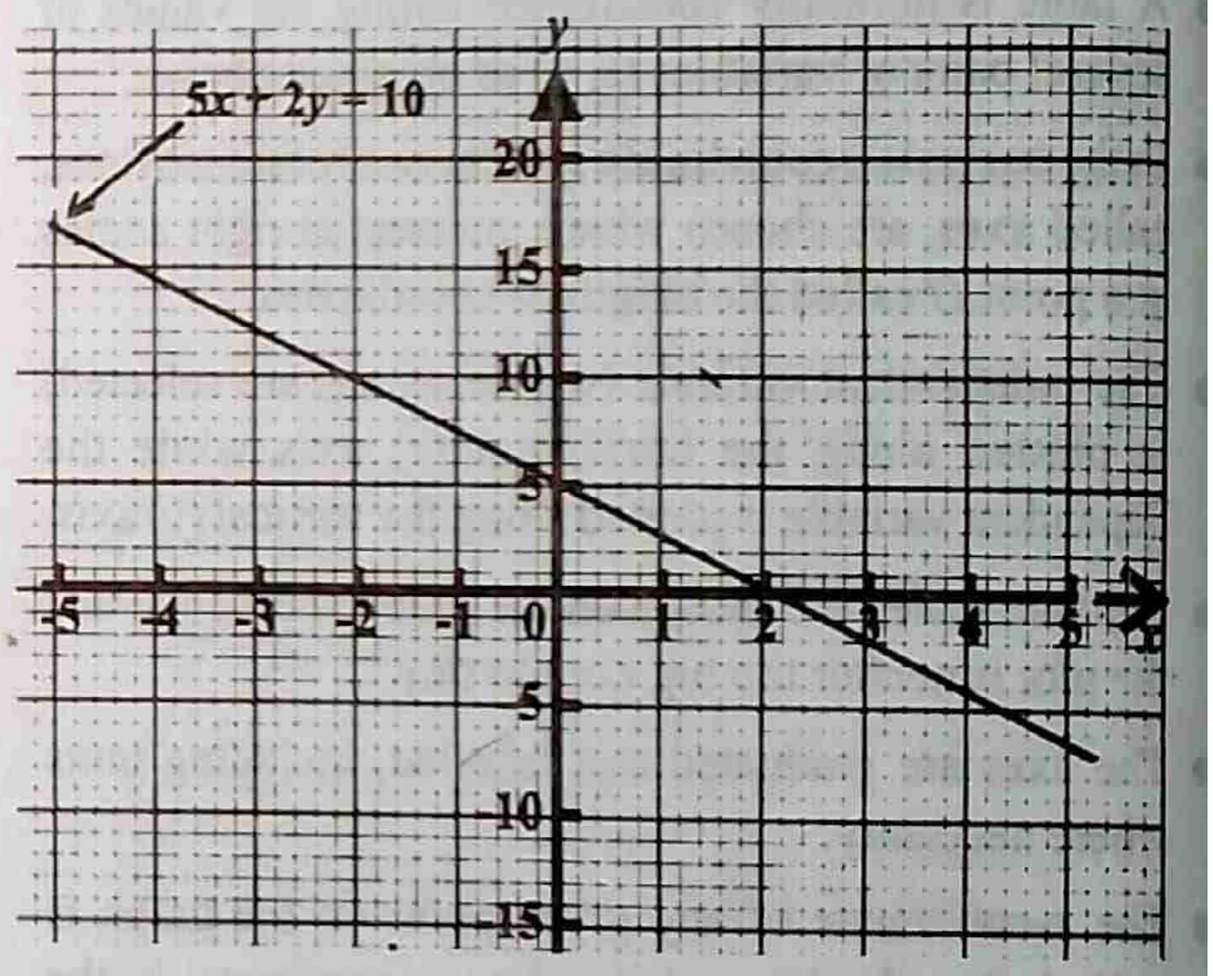
(a) the slope of the line
(b) the x and y intercepts
(c) x when $y = 2.5$
(d) y when $x = 3.0$

Solution

Rewrite the equation as $y = 5 - \frac{5}{2}x$

Table of values

x	$y = 5 - \frac{5}{2}x$
−5	17.5
−4	15
−3	12.5
−2	10
−1	7.5
0	5
1	2.5
2	0
3	−2.5
4	−5
5	−7.5



From the graph above:

(a) Slope $= \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$
Taking the intercepts as points 1 and 2, then $x_1 = 2, y_1 = 0; x_2 = 0, y_2 = 5$.
Hence slope $= \frac{5 - 0}{0 - 2} = -2.5$

(b) x-intercept = 2 and y-intercept = 5
(c) At $y = 2.5, x = 1$
(d) At $x = 3.0, y = 2.5$

9.3 Quadratic Functions

- A quadratic function has an equation of the form

$$y = ax^2 + bx + c \qquad (9.4)$$

where a, b and c are constants.

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- The plot of a quadratic function is a **parabola** which intersects the x-axis at no more than two points.
- The parabola opens upward if $a > 0$, in which case it has a lowest or **minimum** point (Fig. 9.3a).
- The parabola opens downward if $a < 0$, in which case it has a highest or **maximum** point (Fig. 9.3b).

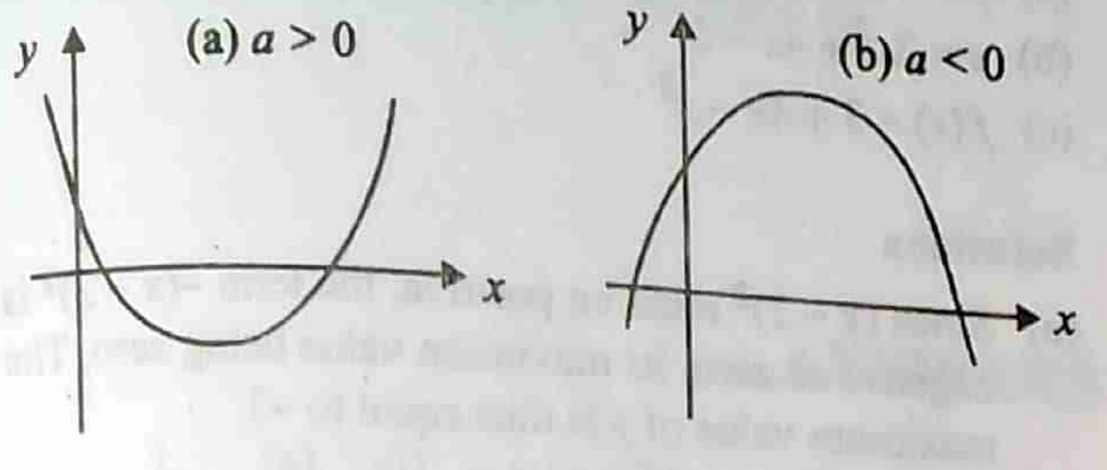


Fig. 9.3

- The **roots** of the quadratic equation $ax^2 + bx + c = 0$ are the x-coordinates of the points of intersection of the parabola with the x-axis ($y = 0$).
- The **gradient** of a quadratic function at a given point is the slope of the straight line which just touches the parabola (i.e. the tangent to the parabola) at that point.

EXAMPLE 9–3

Draw the graph of $y = 6 + x - x^2$. Use the graph to find

(a) the maximum value of y
(b) the roots of the equation $x^2 - x - 6 = 0$
(c) the gradient of the parabola at $x = -3$

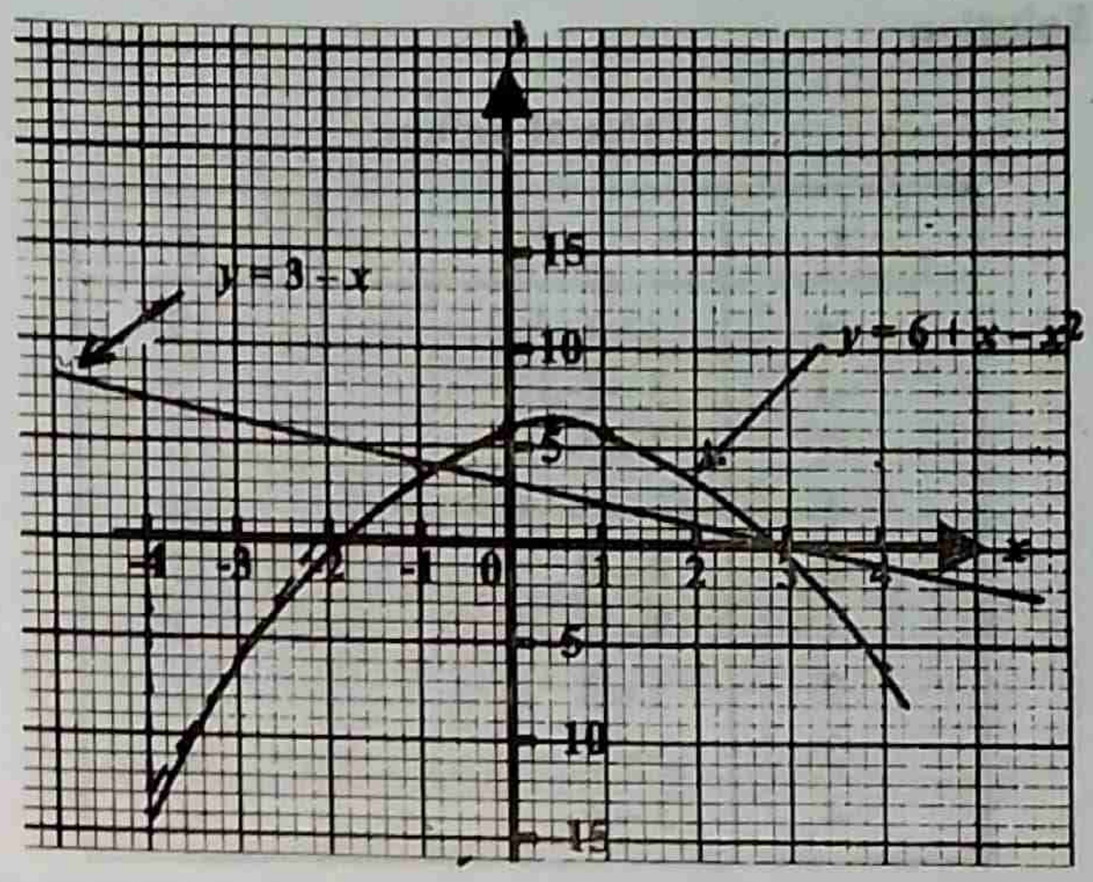
Solution

Table of Values

x	$6 + x - x^2$	y
–4	$6 + (-4) - (-4)^2$	–14
–3	$6 + (-3) - (-3)^2$	–6
–2	$6 + (-2) - (-2)^2$	0
–1	$6 + (-1) - (-1)^2$	4
0	$6 + 0 - 0^2$	6
1	$6 + 1 - 1^2$	6
2	$6 + 2 - 2^2$	4
3	$6 + 3 - 3^2$	0
4	$6 + 4 - 4^2$	–6

See figure below for graph.

(a) From the graph, the highest point on the curve occurs at $x = 0.5$ and has the value of $y_{max} = 6.25$.



(b) The equation $x^2 - x - 6 = 0$, when multiplied through by –1, becomes $6 + x - x^2 = 0$ with roots given by the points of intersection of the curve with the x-axis, i.e. $x = -2$ and $x = 3$.

(c) The slope of the tangent to the curve at $x = -3$ is

$$\frac{-13 - 0}{-2.1 - (-4)} = \frac{13}{1.9} \approx 7$$

DRILL 2

Prepare a table of values of y for integral values of x in the range $-4 \le x \le 4$ for the quadratic equation $y = 3x^2 - 2x + 5$..

9.4 Graphical Solution of Simultaneous Equations

- The graphical solution of two simultaneous equations is found by drawing the graphs of the equations and obtaining the coordinates of their points of intersection.
- The simultaneous solution of a quadratic and a linear equation is obtained as the coordinates of the points of intersection of their respective graphs.
- Once the graph of $y = ax^2 + bx + c$ has been obtained, every other quadratic equation of the form $y = ax^2 + mx + k$ has its solution defined by the intersection of the parabola $y = ax^2 + bx + c$ and the straight line $y = (b - m)x + (c - k)$.

EXAMPLE 9–4

Draw the graph of $y = 3 - x$ on the same set of axes as the graph of $y = 6 + x - x^2$. Hence find the roots of the equation $3 + 2x - x^2 = 0$.

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Solution

Table of Values

x	$3-x$	y
–4	3 – (–4)	7
–3	3 – (–3)	6
–2	3 – (–2)	5
–1	3 – (–1)	4
0	3 – 0	3
1	3 – 1	2
2	3 – 2	1
3	3 – 3	0
4	3 – 4	–1

See graph under Example 9–3 for the plot of $y = 3 - x$. If we equate $6 + x - x^2$ to $3 - x$, we obtain

$$6 + x - x^2 = 3 - x, \text{ or } 3 + 2x - x^2 = 0.$$

Hence the roots of the equation $3 + 2x - x^2 = 0$ are given by the points of intersection of the curve $y = 6 + x - x^2$ and the line $y = 3 - x$. These are $x = -1$ and $x = 3$.

EXAMPLE 9–5

Find the equation of the line whose points of intersection with the curve $y = x^2 + 5x + 2$ will give the roots of the equation $x^2 - 3x - 5 = 0$.

Solution

The equation $x^2 - 3x - 5 = 0$ can be written as

$$x^2 + 5x + 2 - 8x - 7 = 0, \text{ or}$$
$$x^2 + 5x + 2 = 8x - 7$$

The equation of the line required is thus $y = 8x - 7$.

DRILL 3

Given the quadratic equation $y = x^2 - 2x - 3$, find the equation of the linear functions whose points of intersection with the parabola will give the roots of the equation:

(a) $x^2 + 2x - 3 = 0$
(b) $x^2 - 2x - 7 = 0$
(c) $x^2 - 5x + 3 = 0$

9.5 Maximum and Minimum Points

- By *completing the square*, the equation $y = ax^2 + bx + c$ can be written as

$$y = a\left[x + \frac{b}{2a}\right]^2 + \frac{4ac - b^2}{4a} \qquad (9.5)$$

- If $a > 0$, the *minimum* value of the function y occurs for $x = -b/2a$.
- If $a < 0$, the *maximum* value of the function y occurs for $x = -b/2a$.

EXAMPLE 9–6

Find the maximum or minimum values for the quadratic functions

(a) $y = -(x - 2)^2 - 3$
(b) $y = 2x^2 + 4x - 5$
(c) $f(x) = 3 + 4x - x^2$

Solutions

(a) Since $(x - 2)^2$ must be positive, the term $-(x - 2)^2$ is negative or zero, its maximum value being zero. The maximum value of y is thus equal to –3.

(b) Since $a = 2$ (positive), y has a *minimum* value for
$x = -b/2a = -4/4 = -1$
$y_{min} = 2(-1)^2 + 4(-1) - 5 = -7.$

(c) $a = -1$ (negative), $f(x)$ has a *maximum* value for
$x = -b/2a = -4/-2 = 2$
$f(x)_{max} = 3 + 4(2) - 2^2 = 7.$

DRILL 4

Find the maximum or minimum value for each of the following functions:

(a) $y = 3x^2 - 6x + 2$
(b) $f(x) = 3 - 2x - 3x^2$
(c) $y = (3 - x)^2 + 5$
(d) $f(x) = 4 - (x - 1)^2$

9.6 Cubic Functions

- A cubic function has an equation of the form

$$y = ax^3 + bx^2 + cx + d \qquad (9.6)$$

where a, b, c and d are constants.

- The graph of a cubic function is a curve which has no specific shape.
- It intersects the x-axis at no more than three points which are also the roots of the equation

$$ax^3 + bx^2 + cx + d = 0$$

- The gradient of the function at a point is the slope of the tangent of the curve at that point.

EXAMPLE 9–7

Draw the graph of $y = x^3 - x^2 - 2x$ for $-2 \le x \le 3$. Determine the roots of the cubic equation from the graph.

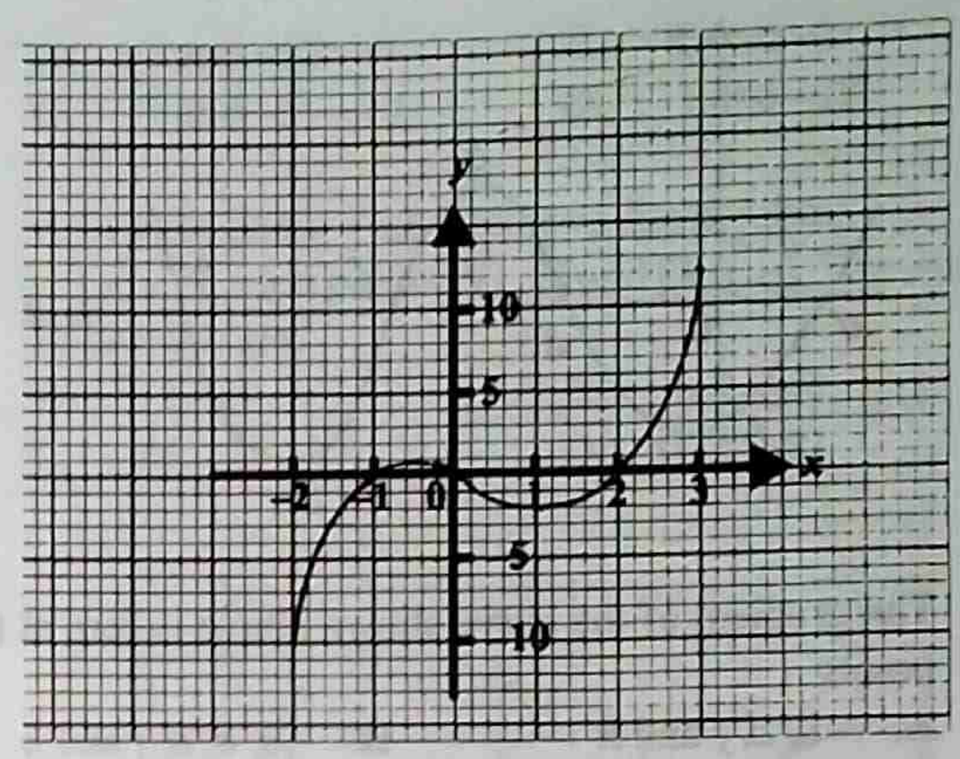
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Solution

Table of Values

x	–2	–1	0	1	2	3
$y = x^3 - x^2 - 2x$	–10	0	0	–2	0	12

From the graph (see figure), the roots of the equation are the intercepts on the x-axis, or $x = -1$, 0 and 2.



ANSWERS TO DRILLS

1. (a) (i) $y + 5x = 28$ (ii) $2y - x = 9$
   (b) Slope = –3/2 x-intercept = 2, y-intercept = 3 (c) $y - 2x = 16$

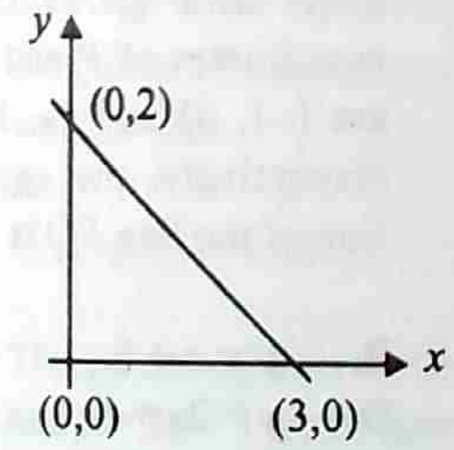
2.

x	–4	–3	–2	–1	0	1	2	3	4
y	61	38	21	10	5	6	13	26	45

3. (a) $y + 4x = 0$ (b) $y = 4$ (c) $y = 3x - 6$
4. (a) –1 (min) (b) 2 (max)
   (c) 5 (min) (d) 4 (max)

SSCE/JME–TYPE OBJECTIVE QUESTIONS

1. The intercept of the line $y = 3x$ on the x-axis is
   A. 3 **B.** ⅓ **C.** 1 **D.** 0

2. 

   The equation of the line indicated in the graph is
   A. $3y + 2x = 6$
   B. $3y - 2x = 6$
   C. $3y + 2x = -6$
   D. $2x - 3y = 6$

3. Which of the following lines is **not** parallel to the line $x - 2y = 5$?
   A. $x - 2y + 7 = 0$ **B.** $2x - 4y - 1 = 0$
   C. $x + 2y - 3 = 0$ **D.** $16x - 32y + 9 = 0$

4. Which of the following lines is perpendicular to the line $5x + 2y = 6$?
   A. $5y + 2x = -6$ **B.** $5y - 2x = 6$
   C. $5x - 2y = 6$ **D.** $2y - 5x = 6$

5. The point (1, –2) is located on the graph of a linear function. If y increases by 3 units for each unit increase in x, the equation of the function is
   A. $3y - x = 5$ **B.** $x - 3y = 5$
   C. $y - 3x = -5$ **D.** $y - 3x = 5$

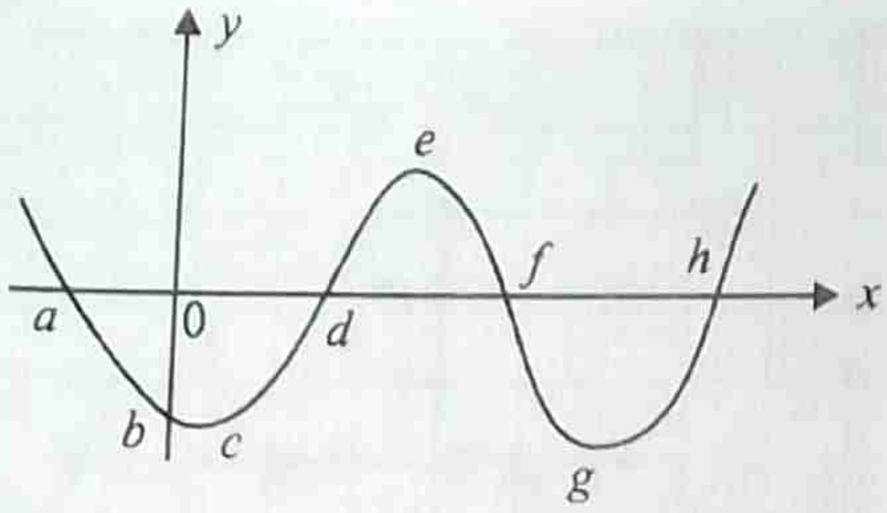
6. Which of the following points is located on the curve $y = 2x^2 - 3x - 5$?
   A. (0, 5) **B.** (1, 3) **C.** (2, –1) **D.** (3, 4)

7. The graph of the function $2y = x^2 - x - 6$ intersects the y-axis at the point
   A. (0, –3) **B.** (–2, 3) **C.** (0, –6) **D.** (–2, 0)

8. The points of intersection of the curve $y = -2x^2 - x + 3$ with the coordinate axes are:
   A. $(-1, 0), \left(\frac{3}{2}, 0\right)$ and (0, 3)
   B. $(1, 0), \left(-\frac{3}{2}, 0\right)$ and (0, 3)
   C. $\left(\frac{3}{2}, 0\right), (1, 0)$ and (0, –3)
   D. $(0, 0), \left(1, \frac{3}{2}\right)$ and (0, –3)

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9.

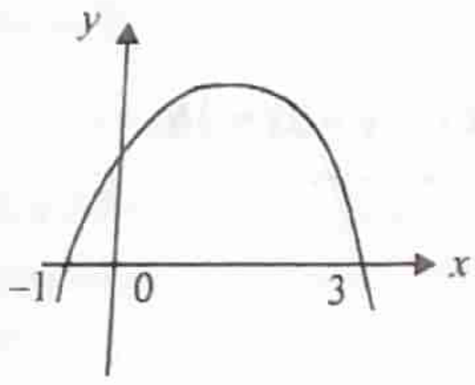


The gradient of the curve shown above is zero at the points

A. a, d, f and h **B.** a, b, d, f and h

C. c, e and g **D.** c and g only

10.



The function represented by the graph shown is

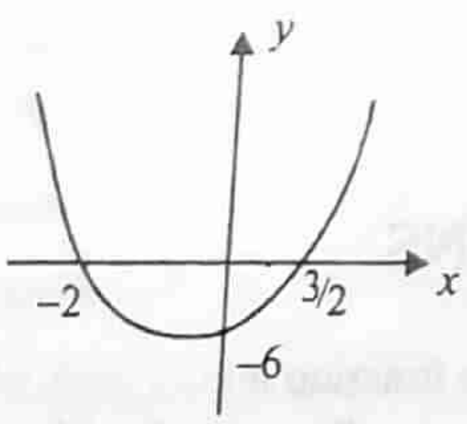
A. $f(x) = x^2 - 2x - 3$

B. $f(x) = 3 + 2x - x^2$

C. $f(x) = x^2 - 2x + 3$

D. $f(x) = -x^2 - 2x + 3$

11.



The curve shown is a sketch of the function $f(x) = ax^2 + bx + c$. The values of a, b and c are, respectively

A. -2, $\frac{3}{2}$ and -6 **B.** -4, 3 and -6

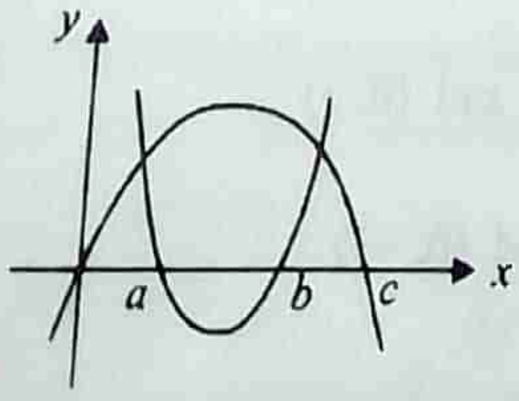
C. 2, $-\frac{3}{2}$ and -6 **D.** 2, 1 and -6

12. The graph of $f(x) = 3x^2 + 11x - 4$ crosses the x-axis at the points

A. $\left(\frac{1}{3}, 0\right)$ and $(-4, 0)$ **B.** $(3, 0)$ and $(0, -4)$

C. $(-4, 0)$ and $(0, -4)$ **D.** $(-4, 1)$ and $\left(\frac{1}{3}, 2\right)$

13.



Graphs of the functions $y = x(3 - x)$ and $y = (2x - 3)(x - 2)$ are sketched in the figure shown. The x-coordinates of points a, b and c, are respectively

A. 0, 2 and 3 **B.** 0, $\frac{3}{2}$ and 2

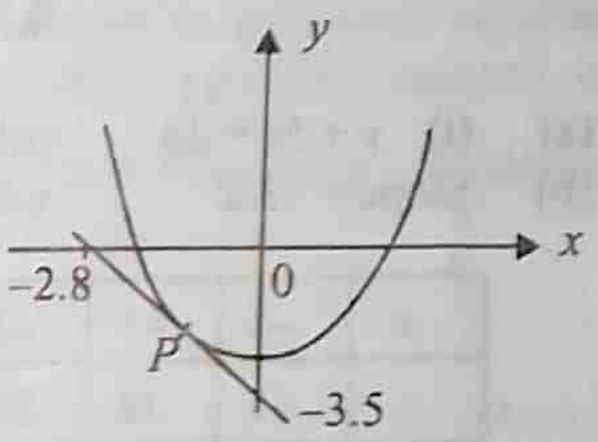
C. $\frac{3}{2}$, 2 and 3 **D.** 0, $\frac{3}{2}$ and 3

14. Find the values of a and b in the following table:

x	-3	-2	-1	0	1	2	3
$y = 2x^2 + 5x - 3$	0	-5	a	-3	4	b	30

A. -4, 15 **B.** -4, 18 **C.** -6, 15 **D.** -6, 18

15.

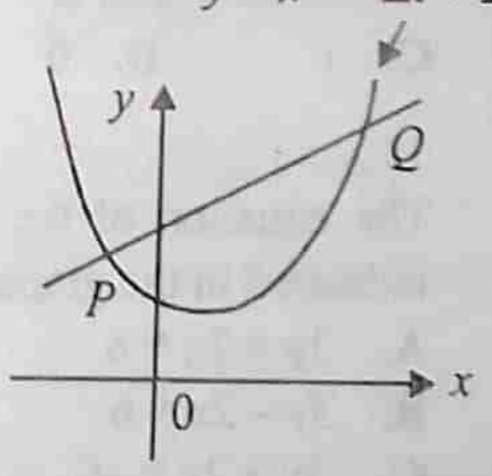


In the sketch above, the gradient of the curve at point P is

A. $-\frac{5}{4}$ **B.** $\frac{4}{5}$

C. $-\frac{4}{5}$ **D.** $\frac{5}{4}$

16.



In the sketch shown, the line PQ intersects the parabola $y = x^2 - 2x + 2$ at P and Q. If the coordinates of P and Q are $(-1, 5)$ and $(4, 10)$ respectively, the equation of the line PQ is

A. $y = x + 5$ **B.** $y = x + 6$

C. $y = 2x + 5$ **D.** $y = 2x + 6$

17. The y-intercepts of the line and the curve in the figure in question 16 are, respectively

A. 5 and 2 **B.** 6 and 1

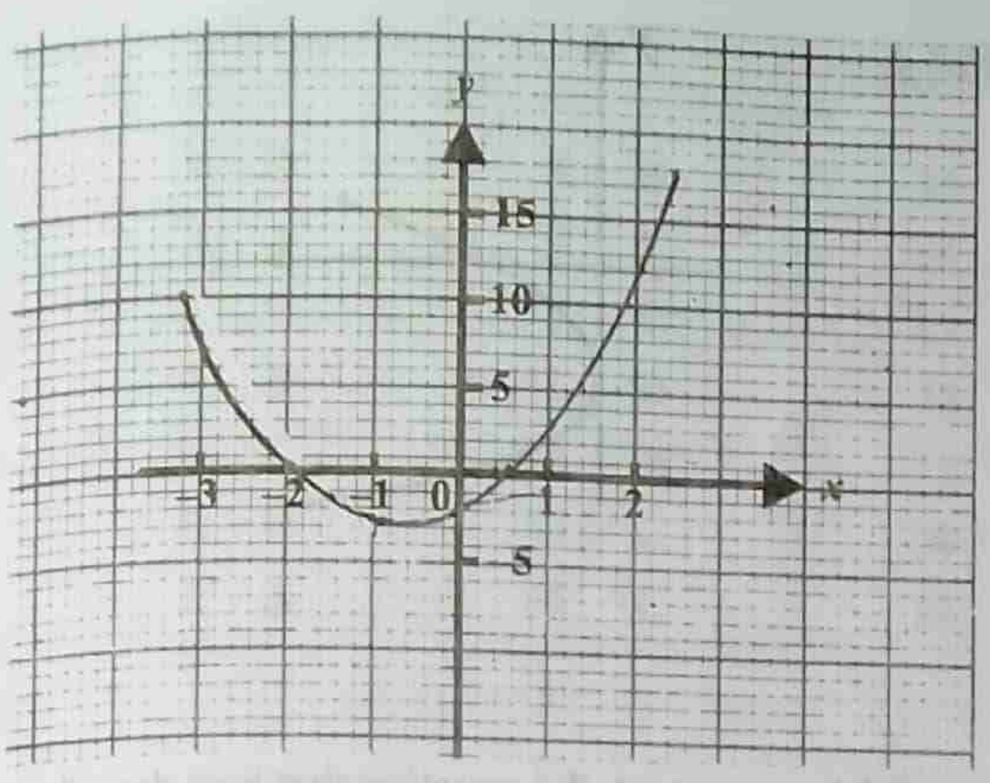
C. 5 and 1 **D.** 6 and 2

18. Which of the following equations has roots given by the points of intersection P and Q in question 16?

A. $x^2 - 3x - 4 = 0$ **B.** $x^2 - x - 8 = 0$

C. $x^2 - 3x - 8 = 0$ **D.** $x^2 - x - 4 = 0$

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The figure above shows the graph of the quadratic function $y = 2x^2 + 3x - 2$. Use the information provided by the figure to answer questions **19** to **22**.

19. The roots of the equation $2x^2 + 3x - 2 = 0$ are
A. 0.5, 2 B. –2, –2 C. 0.5, –2 D. 2, –0.5

20. The minimum value of y is approximately
A. –2 B. –0.5 C. –2 D. –3

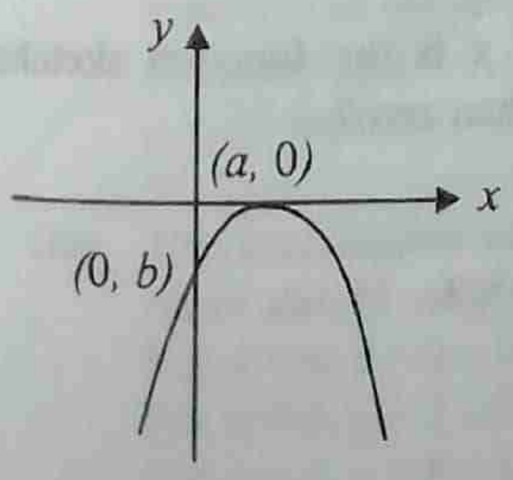
21. The values of x for which $y = 5$ are approximately
A. –2.0, 0.5 B. –2.8, 1.2
C. –0.8, –3 D. 0, 0

22. For what values of x is $2x^2 + 3x - 2 < 0$?
A. $x < -2$, or $x > 0.5$ B. $-2 \le x < 0.5$
C. $-2 < x < 0.5$ D. $-2 < x < 0$

23. If a graph of the function $y = 3x^2 - 5x - 7$ is provided, the roots of the equation $3x^2 - 2x - 5 = 0$ are given by the points of intersection of the curve $y = 3x^2 - 5x - 7$ and the line
A. $y = 3x - 2$ B. $y = -3x - 2$
C. $y = 7x - 12$ D. $y = -7x - 2$

24. The points of intersection of the graphs of $y = x^2 + 2$ and $y = 2x + 5$ when drawn on the same set of axes are
A. (3, 0) and (–1, 0) B. (1, 0) and (–3, 0)
C. (–1, 11) and (3, 3) D. (–1, 3) and (3, 11)

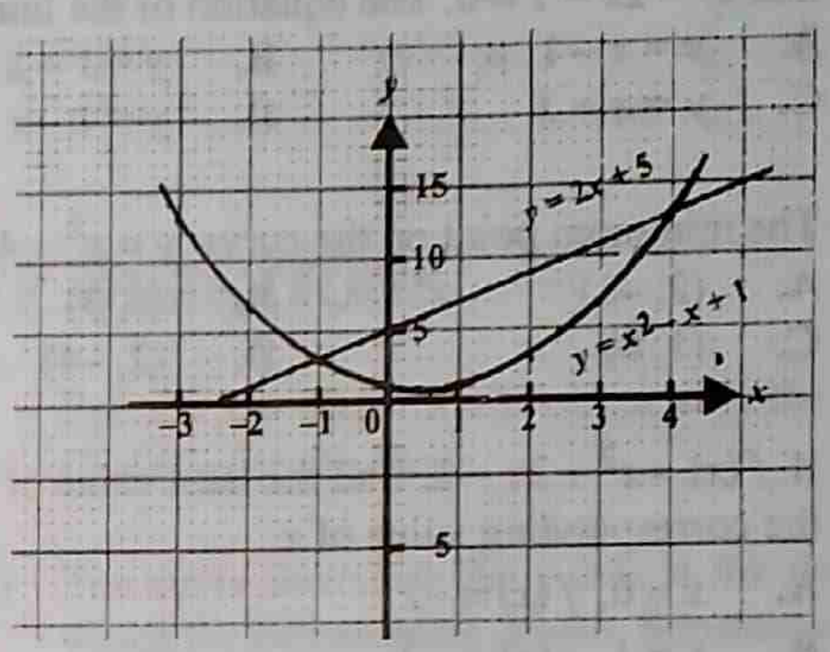
25.



The figure gives a sketch of the function $y = -3(x - 2)^2$. If the intercepts on the x and y axes are $(a, 0)$ and $(0, b)$, the values of a and b are, respectively
A. 1 and –12 B. 2 and –3
C. 1 and –3 D. 1 and –12

26. A straight line $y = ax$ intersects the parabola $y = x^2 - 5x + 6$ at points $P(1, 2)$ and $Q(m, n)$. The values of m and n are:
A. (3, 6) B. (–2, –4)
C. (6, 12) D. (–1, –2)

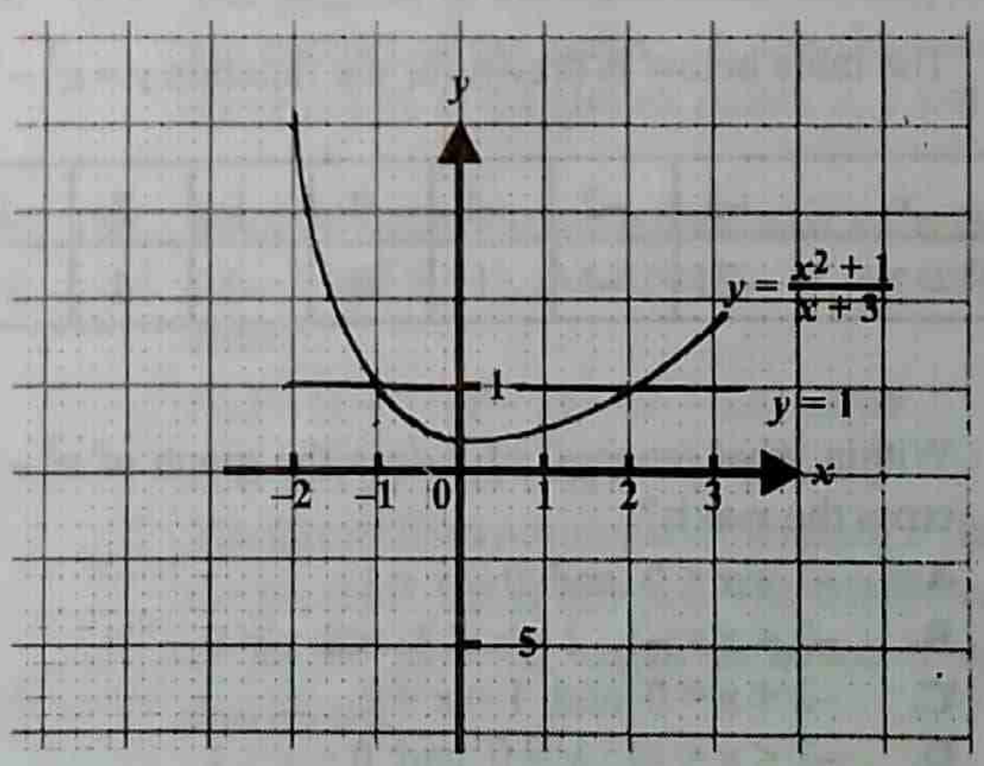
27.



The x-coordinates of the points of intersection of the two graphs above are:
A. –1, 3 B. 4, 13 C. 0, 5 D. –1, 4

28. The line $y = 3x + 1$ intersects the curve $y = x^2 - 2x + 7$ at the points
A. (2, 7) and (3, 10) B. (–2, –5) and (1, 4)
C. (1, 4) and (2, 7) D. (–1, –2) and (3, 10)

Use the following graph to answer questions **29** to **31**.



29. The roots of the equation $x^2 - x - 2 = 0$ are:
A. 1 and 2 B. –1 and 2
C. 1 and –2 D. imaginary

30. For what values of x is $x^2 - x - 2 > 0$?
A. $-1 < x < 2$ B. $-1 \le x \le 2$
C. $-3 < x < -1$ and $x > 2$ D. All x

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31. For what values of x is the function $y = \dfrac{x^2+1}{x+3}$ decreasing?

A. $-3 \le x < 0$ B. $-3 < x \le 0$
C. $-1 \le x \le 2$ D. $0 < x < 3$

32. The points of intersection between a straight line and the curve $y = -x^2 + x + 2$ give the roots of the equation $x^2 - 2x - 1 = 0$. The equation of the line is

A. $y = x - 1$ B. $y = 1 + x$
C. $y = x + 3$ D. $y = 1 - x$

33. The minimum point on the curve $y = x^2 - 4x + 3$ is

A. $(2, -1)$ B. $(0, 3)$
C. $(1, 0)$ D. $(2, -4)$

34. If $f(x) = x^2 + 2x - 2$, find the least value of $f(x)$ and the corresponding value of x.

A. $x = 0,\ f(x) = -2$
B. $x = 1,\ f(x) = -2$
C. $x = -1,\ f(x) = -3$
D. $x = -1,\ f(x) = -4$

35. The minimum value of $x^2 - 5x + 6$ is:

A. $-\frac{1}{4}$ B. $\frac{1}{4}$ C. $\frac{5}{2}$ D. $-\frac{5}{2}$

36. The maximum value of y in the equation $y = 1 + x - 4x^2$ is

A. 1 B. $1\frac{1}{16}$ C. 2 D. $2\frac{1}{8}$

37. The table below is drawn for the function $y = x^3 - 2x$.

x	–3	–2	–1	0	1	2	3
$x^3 - 2x$	–21	–4	1	0	–1	4	21

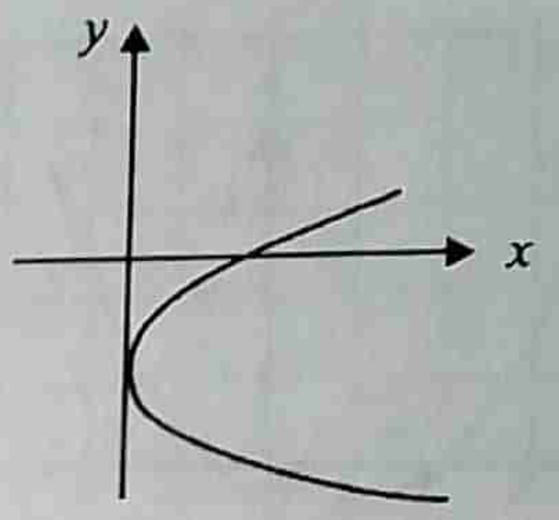
Within what range(s) of x does the graph of $x^3 - 2x$ cross the x-axis?

A. $-1 < x \le 0$ and $0 < x < 1$
B. $-2 < x < -1$, $1 < x < 2$ and $x = 0$
C. $-2 < x < 0$ and $1 < x < 2$
D. $-2 < x < -1$, $x = 0$ and $0 < x < 1$

38. Which of the following pairs of graphs **cannot** be used to find the solution to the cubic equation $x^3 - 3x^2 + 3x - 9 = 0$?

A. $y = x^3$ and $y = 3x^2 - 3x + 9$
B. $y = x^3 - 3x^2 + 3x$ and $y = 9$
C. $y = x^2 - 3x + 3$ and $y = 9/x$
D. $y = x^3 - 3x^2$ and $y = 3x - 9$

39.



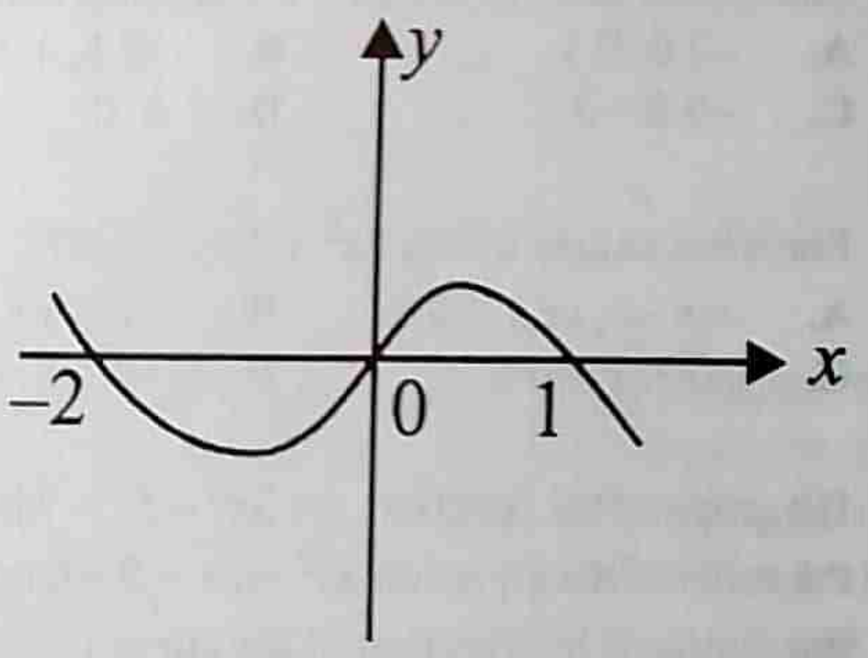
If k is a constant, the equation that best describes the parabola shown above is

A. $y = kx^2$ B. $y = k - x^2$
C. $x = (y + k)^2$ D. $x = k - y^2$

40. The graphs of the functions $y = x^3 - 9x$ and $y = 18 - 2x^2$ intersect at the points with the x-coordinates

A. –1, 0 and 3 B. –3, 1 and 3
C. 0, 2 and 3 D. –3, 2 and 3

41.



The graph of a cubic function is sketched above. The values of the independent variable for which the function vanishes are:

A. $-2 \le x \le 1$
B. –2, 0 and 1
C. $x \le -2$ and $x \ge 1$
D. All x except –2, 0 and 1

42. For what ranges of x is the function sketched in question 41 greater than zero?

A. $x < -2$ or $x > 1$
B. $-2 < x < 1$
C. $x < -2$ and $0 < x < 1$
D. $0 < x < 1$

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ANSWER KEY

1.	**D**	8.	**B**	15.	**A**	22.	**C**	29.	**B**	36.	**B**
2.	**A**	9.	**C**	16.	**B**	23.	**B**	30.	**C**	37.	**B**
3.	**C**	10.	**B**	17.	**D**	24.	**D**	31.	**B**	38.	**D**
4.	**B**	11.	**D**	18.	**A**	25.	**A**	32.	**D**	39.	**C**
5.	**C**	12.	**A**	19.	**C**	26.	**C**	33.	**A**	40.	**D**
6.	**D**	13.	**C**	20.	**D**	27.	**D**	34.	**C**	41.	**B**
7.	**A**	14.	**C**	21.	**B**	28.	**A**	35.	**A**	42.	**C**

SOLUTIONS TO OBJECTIVE QUESTIONS

1. **(D)** The intercept on the x-axis is the point at which $y = 0$, i.e. $3x = 0$, or $x = 0$.

2. **(A)** Slope of the line $= \frac{0-2}{3-0} = -\frac{2}{3}$

Equation of the line is $y = -\frac{2}{3}x + c$

Substituting $x = 3, y = 0$ gives $0 = -2 + c$, or $c = 2$. Hence $y = -\frac{2}{3}x + 2$, or $3y + 2x = 6$.

3. (C) Parallel lines have the same slope. The equation $x - 2y = 5$ can be written as $y = \frac{1}{2}x - \frac{5}{2}$. Hence the slope of the line $= \frac{1}{2}$. Slope of the lines indicated in (A), (B) and (D) is $\frac{1}{2}$ while that of (C) is $-\frac{1}{2}$.

4. **(B)** Slope of the line $5x + 2y = 6$ is $m_1 = -\frac{5}{2}$. Slope of the line perpendicular to this is $m_2 = -1/m_1 = 2/5$. Equation of the line is $y = \frac{2}{5}x + c$, or $5y - 2x =$ constant.

5. **(C)** $\frac{\text{increase in } y}{\text{corresponding increase in } x} = \frac{3}{1} = 3$

Equation of the line is $y = 3x + c$
Substituting $x = 1, y = -2$ gives $-2 = 3 + c$, or $c = -5$. Hence $y = 3x - 5$, or $y - 3x = -5$.

6. **(D)** The coordinates of any point located on the curve should satisfy the given equation.
For $x = 0, y = 0 - 0 - 5 = -5$
For $x = 1, y = 2 - 3 - 5 = -6$
For $x = 2, y = 8 - 6 - 5 = -3$
For $x = 3, y = 18 - 9 - 5 = 4$, i.e. only the point (3, 4) lies on the curve.

7. **(A)** The curve intersects the y-axis at the point at which the x-coordinate is zero. Substituting $x = 0$ in the equation gives $2y = 0 - 0 - 6$, or $y = -3$. Hence the point is $(0, -3)$.

8. **(B)** The curve intersects the x-axis at the point $y = 0$, i.e. $-2x^2 - x + 3 = 0$, or $2x^2 + x - 3 = 0$
$(2x + 3)(x - 1) = 0$, $x = -3/2$ or 1 i.e. points $\left(-\frac{3}{2}, 0\right)$ and (1, 0). Intercept on the y-axis occurs at $x = 0$ i.e. $y = 0 - 0 + 3$, or $y = 3$, i.e. (0, 3)

9. **(C)** The tangents to the curve at the *turning points* are parallel to the x-axis, i.e. the slope of the curve is zero at the turning points, c, e and g.

10. **(B)** Since the curve intersects the x-axis at the points $x = -1$ and $x = 3$, the equation of the curve is **either**
$f(x) = (x + 1)(x - 3) = x^2 - 2x - 3$, **or**
$f(x) = -(x + 1)(x - 3) = -x^2 + 2x + 3$
The former has a positive coefficient of x^2, hence the parabola opens upward. The coefficient of x^2 in the latter is negative, hence it opens downward.

11. **(D)** Since the parabola *opens upward*, a must be positive. This rules out options (A) and (B). The intercepts on the x-axis are -2 and $\frac{3}{2}$. Hence $f(x) = k(x + 2)(x - 3/2) = k(2x^2 + x - 6)$, where k is a constant. The family of curves of the type $f(x) = k(2x^2 + x - 6)$, all intersect the x-axis at -2 and $3/2$. However, their y-intercepts depend on the value of k, which in this case must be

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equal to 1 since the question specifies the value of c as –6. Hence $a = 2$, $b = 1$ and $c = -6$.

12. **(A)** The curve crosses the x-axis at the points where $f(x) = 0$, i.e.
$3x^2 + 11x - 4 = 0$
$(3x - 1)(x + 4) = 0$
$x = \frac{1}{3}$ or –4
Hence the points are $\left(\frac{1}{3}, 0\right)$ and (–4, 0)
[Note that the y-coordinates must be 0]

13. **(C)** Points a and b are the x-intercepts of the parabola which *opens upward*, i.e. $y = (2x - 3)(x - 2)$ which has a positive coefficient of x^2 if expanded. Hence $(2x - 3)(x - 2) = 0$, $x = \frac{3}{2}$ and 2 i.e. $a = \frac{3}{2}$ and $b = 2$. Point c is the non-zero intercept of the other parabola on the x-axis. Since $x(3 - x) = 0$, $x = 0$ or 3, hence $c = 3$.

14. **(C)** $a = 2(-1)^2 + 5(-1) - 3 = 2 - 5 - 3 = -6$
$b = 2(2)^2 + 5(2) - 3 = 8 + 10 - 3 = 15$

15. **(A)** Gradient of the curve at P = slope of the tangent. The intercepts of the tangent on the x and y-axes are (–2.8, 0) and (0, –3.5) respectively. Hence
$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3.5 - 0}{0 - (-2.8)}$$
$$= \frac{-3.5}{2.8} = -\frac{5}{4}$$

16. **(B)** Slope of line $PQ = \frac{10 - 15}{4 - (-1)} = 1$
Equation of PQ is $y = x + c$
Substituting $x = -1$ and $y = 5$ gives $c = 5 + 1 = 6$. Hence $y = x + 6$.

17. **(D)** y-intercept is the point at which $x = 0$.
For the line PQ: y-intercept $= 0 + 6 = 6$
For the curve: y-intercept $= 0 - 0 + 2 = 2$

18. **(A)** At the points of intersection of the line and the curve, $x + 6 = x^2 - 2x + 2$, or $x^2 - 3x - 4 = 0$.

19. **(C)** The roots of the equation are the points of intersection of the curve with the x-axis. These are read directly off the graph as $x = -2$ or 0.5.

20. **(D)** The minimum value of y is the lowest point on the curve. This is $y = -3$, approximately.

21. **(B)** Draw a line through the point 5 on the y-axis, parallel to the x-axis. This is the line $y = 5$. The points of intersection of the line and the curve are $x = -2.8$ and 1.2, approximately.

22. **(C)** This is true for all points on the curve lying below the x-axis, or $-2 < x < 0.5$.

23. **(B)** The equation $3x^2 - 2x - 5 = 0$ can be written as
$3x^2 - 5x - 7 + 3x + 2 = 0$, or
$3x^2 - 5x - 7 = -3x - 2$
Hence the roots of $3x^2 - 2x - 5 = 0$ are given by the points of intersection of $y = 3x^2 - 5x - 7$ and $y = -3x - 2$.

24. **(D)** At the points of intersection, $x^2 + 2 = 2x + 5$, or
$x^2 - 2x - 3 = 0$
$(x - 3)(x + 1) = 0$
$x = 3$, or -1
To find the corresponding values of y, substitute into any of the equations:
For $x = 3$, $y = 2x + 5 = 6 + 5 = 11$
For $x = -1$, $y = -2 + 5 = 3$
Hence the points are (3, 11) and (–1, 3).

25. **(A)** For intercept on the x-axis, $y = 0$, i.e. $-3(x - 2)^2 = 0$, or $x = 2 = a$.
For intercept on the y-axis, $x = 0$, or $y = -3(-2)^2 = -12 = b$.

26. **(C)** Since point (1, 2) lies on the line, substitute the values of x and y into the equation $y = ax$
$\Rightarrow 2 = a(1)$, or $a = 2$.
Hence the equation of the line is $y = 2x$.
At the points of intersection, $x^2 - 5x + 6 = 2x$, or $x^2 - 7x + 6 = 0$
$(x - 6)(x - 1) = 0$, $x = 1$ or 6
The value $x = 1$ corresponds to point P while $x = 6$ corresponds to point Q. Its y-coordinate is $y = 2x = 2(6) = 12$. Hence Q is (6, 12).

27. **(D)** These are read directly off the graph as $x = -1$ or 4.

28. **(A)** At the points of intersection
$x^2 - 2x + 7 = 3x + 1$, or $x^2 - 5x + 6 = 0$
$(x - 2)(x - 3) = 0$; $x = 2$ or 3.
For $x = 2$, $y = 3x + 1 = 3(2) + 1 = 7$
For $x = 3$, $y = 3(3) + 1 = 10$
Hence the points are (2, 7) and (3, 10).

29. **(B)** The intersection of the curve $y = \frac{x^2 + 1}{x + 3}$ with line $y = 1$ gives

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$\frac{x^2+1}{x+3} = 1$, or $x^2 + 1 = x + 3$

$\therefore\ x^2 - x - 2 = 0$

The roots of the equation $x^2 - x - 2 = 0$ are therefore given by the points of intersection. These are read off the graph as $x = -1$ and 2.

30. (C) $x^2 - x - 2 > 0$ implies that

$\frac{x^2+1}{x+3} > 1$

These are points on the curve lying *above* the line $y = 1$, i.e. $x < -1$ and $x > 2$. However, since the function is undefined at $x = -3$, the values are $-3 < x < -1$ and $x > 2$

31. (B) It can be seen from the graph that the value of y decreases for values of x lying between -3 (but $x \neq -3$) and 0, i.e. $-3 < x \leq 0$.

32. (D) The equation $x^2 - 2x - 1 = 0$ can be written as $-x^2 + 2x + 1 = 0$, or $(-x^2 + x + 2) + (x - 1) = 0$.
i.e. $-x^2 + x + 2 = 1 - x$
Hence the roots of $x^2 - 2x - 1 = 0$ are given by the points of intersection of $y = -x^2 + x + 2$ and $y = 1 - x$.

33. (A) The minimum point occurs at

$$x = -\frac{b}{2a} = -\left(\frac{-4}{2}\right) = 2$$

The corresponding value of y is

$y = x^2 - 4x + 3 = 2^2 - 4(2) + 3 = -1$

Hence the minimum point is $(2, -1)$.

34. (C) The least (or minimum) value of $f(x)$ occurs at

$$x = -\frac{b}{2a} = -\frac{2}{2} = -1$$

The corresponding value of $f(x)$ is

$f(-1) = (-1)^2 + 2(-1) - 2 = -3$

35. (A) The minimum point occurs at

$$x = -\frac{b}{2a} = \frac{5}{2}$$

and the value of the function is then

$$\left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) + 6 = -\frac{1}{4}$$

36. (B) The maximum point occurs at

$$x = -\frac{b}{2a} = -\left(\frac{1}{-8}\right) = \frac{1}{8}$$

The corresponding value of y is

$$y = 1 + \frac{1}{8} - 4\left(\frac{1}{8}\right)^2 = 1\frac{1}{16}$$

37. **(B)** The value of the function $y = x^3 - 2x$ changes from negative to positive in the range $-2 < x < -1$, from positive to negative at $x = 0$ and from negative back to positive in the range $1 < x < 2$.

38. **(D)** The equation $x^3 - 3x^2 + 3x - 9 = 0$ can be obtained by equating the two functions in options (A) to (C); but not in option (D).

39. **(C)** Parabolas which open to the left or to the right are of the general form $x = ay^2 + by + c$. When the y-axis is a tangent to the parabola, it is of the form $x = (y + k)^2$, with $y = -k$ at the point of contact.

40. (D) The points of intersection are found by equating the two functions, i.e. $x^3 - 9x = 18 - 2x^2$

$x^3 + 2x^2 - 9x - 18 = 0$

$x^2(x + 2) - 9(x + 2) = 0$

$(x^2 - 9)(x + 2) = 0$

$(x + 3)(x - 3)(x + 2) = 0$

$x = -3$, 2 and 3.

41. **(B)** The function vanishes when $y = 0$, i.e. where the curve intersects the x-axis. These are $x = -2$, 0 and 1.

42. (C) The function is greater than zero at all points where the curve lies above the x-axis. These are $x < -2$ and $0 < x < 1$.

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10 Rational Expressions

10.1 Simplification of Rational Expressions

- **A rational expression** is an algebraic fraction which is expressed as a quotient of two polynomials, e.g.

$$\frac{x}{x-2}, \quad \frac{x^2+3}{x^2-5}, \quad \frac{y^2+4}{y^3+3y^2+7}$$

- A rational expression is simplified (i.e. reduced to its lowest terms) by dividing the numerator and denominator by the highest common factor.

- A rational expression $P(x)/Q(x)$, when reduced to its lowest terms, is *undefined* for values of x for which the denominator $Q(x) = 0$, e.g. $x/(x-2)$ is undefined for $x - 2 = 0$, or $x = 2$.

EXAMPLE 10-1

Express in their lowest terms

(a) $\dfrac{x-1}{(1-x)^2}$ (b) $\dfrac{x^2y^2}{x^2y-xy^2}$ (c) $\dfrac{x^2-4}{x^2-x-6}$

Solutions

(a) $\dfrac{x-1}{(1-x)^2} = \dfrac{-(1-x)}{(1-x)^2} = -\dfrac{1}{1-x} = \dfrac{1}{x-1}$

(b) $\dfrac{x^2y^2}{x^2y-xy^2} = \dfrac{xy(xy)}{xy(x-y)} = \dfrac{xy}{x-y}$

(c) $\dfrac{x^2-4}{x^2-x-6} = \dfrac{(x+2)(x-2)}{(x+2)(x-3)} = \dfrac{x-2}{x-3}$

DRILL 1

Simplify the following expressions:

(a) $\dfrac{15x^4y^3}{90x^2y^5}$ (b) $\dfrac{-12a^2x}{-8a^2b^2}$

(c) $\dfrac{a-b}{a^2-b^2}$ (d) $\dfrac{m^3+mn^2-nm^2-n^3}{m^4-n^4}$

EXAMPLE 10-2

For what values of x are the following functions undefined?

(a) $\dfrac{x}{x+2}$ (b) $\dfrac{x+2}{x^2-9}$ (c) $\dfrac{(x-1)^2}{x(x^2-1)}$

Solutions

The fractions are undefined when the denominator is zero.

(a) The denominator $x + 2 = 0$ for $x = -2$.

(b) Factorize the denominator to obtain

$$\frac{x+2}{(x^2-9)} = \frac{x+2}{(x+3)(x-3)}$$

The denominator is 0 for $x = -3$ and $x = 3$.

(c) Reduce the expression to its lowest terms to give

$$\frac{(x-1)^2}{x(x-1)(x+1)} = \frac{x-1}{x(x+1)}$$

The denominator is 0 for $x = \mathbf{0}$ and $x = \mathbf{-1}$.

DRILL 2

Find the values of x for which the following expressions are undefined.

(a) $\dfrac{x^2+5}{x^2+6x+9}$ (b) $\dfrac{4x+6}{2x^2-5x-12}$

(c) $\dfrac{3x^2+4xy}{x+2y}$ (d) $\dfrac{x^2-7x}{3x}$

10.2 Sums and Differences

- To add or subtract algebraic fractions, express each fraction in terms of a common denominator which is equal to the least common multiple (LCM) of their denominators.

EXAMPLE 10-3

Simplify

(a) $\dfrac{3}{4x} - \dfrac{2}{3x}$ (b) $\dfrac{3}{x-5} + \dfrac{2}{5-x}$

(c) $\dfrac{1}{x-2} - \dfrac{x}{x^2-4}$

Solutions

(a) The LCM of $4x$ and $3x$ is $12x$

$$\frac{3}{4x} - \frac{2}{3x} = \frac{3(3) - 4(2)}{12x} = \frac{1}{12x}$$

(b) $\dfrac{3}{x-5} + \dfrac{2}{5-x} = \dfrac{3}{x-5} - \dfrac{2}{x-5} = \dfrac{1}{x-5}$

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(c) Factorize the denominator to give

$$\frac{1}{x-2} - \frac{x}{(x+2)(x-2)}$$

The LCM of the denominators is $(x + 2)(x - 2)$.

Hence the expression becomes

$$\frac{x+2-x}{(x+2)(x-2)} = \frac{2}{(x+2)(x-2)} = \frac{2}{x^2-4}$$

DRILL 3

Reduce each of the following expressions to a single algebraic fraction:

(a) $\frac{4}{ab} - \frac{a}{bc}$ (b) $\frac{x}{3y} - \frac{5}{6yz}$

(c) $\frac{7}{x-y} + \frac{2}{x^2-y^2}$ (d) $\frac{a^2}{b^2} - \frac{3c}{ab} + \frac{2a}{bc}$

10.3 Fractional Equations

- A *fractional equation* is an equation involving algebraic fractions on one or both sides of the equation.
- To solve a fractional equation:
  - (i) Simplify any side of the equation which has more than one term, using the procedure in section 10.2;
  - (ii) Find the LCM of the denominators on both sides of the equation and express each side as a fraction of the LCM;
  - (iii) Equate the numerators on both sides and solve the resulting equation.

EXAMPLE 10-4

Solve for x:

(a) $\frac{1}{x} - \frac{1}{3x} = \frac{1}{6}$ (b) $\frac{x+2}{x+3} = \frac{x}{x-1}$

(c) $\frac{1}{x^2-x-2} = \frac{x}{x-2} - 1$

Solutions

(a) LHS $= \frac{3-1}{3x} = \frac{2}{3x}$; $\frac{2}{3x} = \frac{1}{6}$, $3x = 12$, or $x = 4$

(b) LCM of $(x + 3)$ and $(x - 1)$ is $(x + 3)(x - 1)$

$$\frac{(x+2)(x-1)}{(x+3)(x-1)} = \frac{x(x+3)}{(x+3)(x-1)}$$

Equate numerators: $x^2 + x - 2 = x^2 + 3x$

$2x = -2$ or $x = -1$

(c) LHS $= \frac{1}{(x-2)(x+1)}$; RHS $= \frac{x-x+2}{x-2} = \frac{2}{x-2}$

$\Rightarrow \quad \frac{1}{(x-2)(x+1)} = \frac{2}{x-2}$

$\Rightarrow \quad 2(x + 1) = 1$, or $x = -\frac{1}{2}$.

DRILL 4

Solve for x:

(a) $\frac{x}{3} = \frac{5}{6}$ (b) $\frac{2}{x+10} = \frac{1}{3x}$

(c) $5 - \frac{9}{x+1} = x$ (d) $\frac{x+3}{2} = \frac{2}{x+3}$

10.4 Partial Fractions

- An algebraic fraction $P(x)/Q(x)$ can be resolved into its constituent **partial fractions**, e.g.

$$\frac{P(x)}{Q(x)} = \frac{A}{F_1(x)} + \frac{B}{F_2(x)}$$

where A and B are constants or simple functions of x and $F_1(x)$ and $F_2(x)$ are factors of $Q(x)$.

- To resolve an algebraic fraction into partial fractions:
  - (i) Find the factors of the denominator;
  - (ii) Write the equation as the sum of two partial fractions having constants A and B as numerators and each of the factors of $Q(x)$ as the denominator;
  - (iii) Multiply both sides of the equation by the product of the factors;
  - (iv) Solve for A by substituting a value of x for which the term containing B vanishes;
  - (v) Solve for B by substituting a value of x for which the term containing A vanishes.

EXAMPLE 10-5

Resolve into partial fractions:

(a) $\frac{2x - 9}{3x^2 + x - 4}$ (b) $\frac{6-x}{x^2-4}$

Solutions

(a) Factors of $2x^2 + x - 4$ are $(3x + 4)$ and $(x - 1)$

$$\frac{2x - 9}{3x^2 + x - 4} = \frac{A}{3x+4} + \frac{B}{x-1}$$

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Multiply both sides by $(3x + 4)(x - 1)$:

$2x - 9 = A(x - 1) + B(3x + 4)$

Substitute $x = -4/3$: $-8/3 - 9 = A(-4/3 - 1)$

$-35/3 = A(-7/3)$, or $A = 5$.

Substitute $x = 1$: $2 - 9 = B(3 + 4)$, or $B = -1$

Hence: $\frac{2x - 9}{3x^2 + x - 4} = \frac{5}{3x + 4} - \frac{1}{x - 1}$

(b) Factors of $x^2 - 4$ are $(x - 2)$ and $(x + 2)$

$\frac{6 - x}{x^2 - 4} = \frac{A}{x - 2} + \frac{B}{x + 2}$

$6 - x = A(x + 2) + B(x - 2)$

Substitute $x = 2$: $6 - 2 = A(2 + 2) \Rightarrow A = 1$

Substitute $x = -2$: $6 + 2 = B(-2 - 2) \Rightarrow B = -2$

Hence: $\frac{6 - x}{x^2 - 4} = \frac{1}{x - 2} - \frac{2}{x + 2}$

DRILL 5

Find the values of A and B if

(a) $\frac{2x + 10}{x^2 - 6x - 7} = \frac{A}{x - 7} + \frac{B}{x + 1}$

(b) $\frac{x + 9}{6x(x + 1)} = \frac{A}{x} + \frac{B}{x + 1}$

(c) $\frac{2x + 5}{3x^2} = \frac{A}{x} + \frac{B}{x^2}$

(d) $\frac{6}{x^2 - 9} = \frac{A}{x + 3} + \frac{B}{x - 3}$

ANSWERS TO DRILLS

1. (a) $\frac{x^2}{6y^2}$ (b) $\frac{3x}{b^2}$ (c) $\frac{7x + 7y + 2}{x^2 - y^2}$ (d) $\frac{a^3c - 3b^2c + 2a^2b}{ab^2c}$

   (c) $\frac{1}{a + b}$ (d) $\frac{1}{m + n}$

2. (a) -3 (b) 4

   (c) $-2y$ (d) none

3. (a) $\frac{4c - a^2}{abc}$ (b) $\frac{2xz - 5}{6yz}$

4. (a) 5/2 (b) 2

   (c) 2 (d) $-5, -1$

5. (a) $A = 3, B = -1$ (b) $A = 3/2, B = -4/3$

   (c) $A = 2/3, B = 5/3$ (d) $A = -1, B = 1$

SSCE/JME – TYPE OBJECTIVE QUESTIONS

1. Simplify $\frac{x^2 - y^2}{x^2 + xy}$

   A. $\frac{x - y}{x + y}$ **B.** $\frac{x - y}{x}$ **C.** $\frac{x + y}{x}$ **D.** $\frac{1}{1 + y}$

2. Simplify $\frac{1 - y^2}{y - y^2}$

   A. $\frac{1}{y}$ **B.** $\frac{1 - y}{y}$ **C.** $\frac{1 + y}{y}$ **D.** $\frac{1}{1 - y}$

3. Simplify $\frac{a - 7}{a^2 - 9} \times \frac{a^2 - 3a}{a^2 - 49}$

   A. $\frac{a}{(a - 3)(a + 7)}$ **B.** $\frac{(a + 3)(a + 7)}{a}$

   C. $\frac{a}{(a - 3)(a - 7)}$ **D.** $\frac{a}{(a + 3)(a + 7)}$

4. Simplify $\frac{a^2 - b^2}{2a^2 + ab - b^2}$

   A. $\frac{a + b}{2a + b}$ **B.** $\frac{a + b}{2a - b}$ **C.** $\frac{a - b}{2a - b}$ **D.** $\frac{a - b}{2a + b}$

5. Simplify $\frac{4x^2 - 49y^2}{2x^2 + 5xy - 7y^2}$

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A. $\frac{x-y}{zx-y}$ B. $\frac{2x+7y}{x-y}$

C. $\frac{2x-7y}{x+y}$ D. $\frac{2x-7y}{x-y}$

6. Simplify $\frac{324-4y^2}{2y+18}$

A. $-2(y-9)$ B. $2(y-9)$ C. $2(9+y)$ D. $81-y^2$

7. Simplify $\frac{(2x-y)^2-(x-2y)^2}{5x^2-5y^2}$

A. $\frac{3}{5}$ B. $\frac{2}{5}$ C. $\frac{2x-y}{5x+y}$ D. $\frac{x-2y}{x+5y}$

8. Simplify $\frac{a^2-1}{a^3+2a^2-a-2}$

A. $\frac{1}{a-2}$ B. $\frac{a-1}{a+1}$ C. $\frac{1}{a+2}$ D. $\frac{a-1}{a+2}$

9. If $\frac{x}{y}=\frac{z}{w}=c$, find the value of

$\frac{3x^2-xz+z^2}{3y^2-yw+w^2}$ in terms of c.

A. $3c^2$ B. $\frac{17c^2}{4}$ C. $2c-c^2$ D. c^2

10. A trader goes to Lagos for a days with x naira. He spends y naira per day for the first b days. The amount left for him to spend per day for the rest of his stay is

A. $\frac{x(a-b)}{a-b}$ B. $\frac{xa-yb}{a-b}$

C. $\frac{x-yb}{a-b}$ D. $\frac{x-yb}{b-a}$

11. Simplify $\frac{a-b}{a^{1/3}-b^{1/3}}$

A. a^2+ab-b^2 B. $a^{2/3}+a^{1/3}b^{1/3}+b^{2/3}$

C. $a^{2/3}-a^{1/3}b^{1/3}-b^{2/3}$ D. a^2-ab+b^2

12. For what value of x is the expression $\frac{x+2}{x^2-3x-10}$ undefined?

A. 0 B. –2 C. 3 D. 5

13. The expression $\frac{a^2+15a+50}{a-5}$ is not defined when a is equal to

A. 5 B. 0 C. –5 D. –10

14. The expression $\frac{5}{a^2-ab-6b^2}$ is undefined when b is equal to

A. $3a$ or $2a$ B. $-3a$ or $-2a$

C. $-\frac{a}{2}$ or $\frac{a}{3}$ D. $-\frac{a}{3}$ or $\frac{a}{2}$

15. The values of x for which the expression $\frac{(x-5)(x-2)}{x^2+x-2}$ is undefined are

A. 1, –2 B. –1, 2 C. 2, 3 D. –1, –2

16. If $x=-3, y=2, z=4$, evaluate $\frac{x^3-y^3-z^{1/2}}{x+y-z}$

A. 37 B. –37 C. –37/5 D. 37/5

17. Write as a single fraction $\frac{1}{1-y}+\frac{2}{1+y}$

A. $\frac{y+3}{1-y^2}$ B. $\frac{3-y}{(1-y)^2}$ C. $\frac{3-y}{1-y^2}$ D. $\frac{3-y}{1+y^2}$

18. Simplify $\left(\frac{3}{a}-\frac{15}{2b}\right)\div\frac{6}{ab}$

A. $\frac{5a-2b}{2}$ B. $\frac{2b-5a}{4}$

C. $\frac{9(2b-5a)}{a^2b^2}$ C. $\frac{a^2b^2}{18b-45a}$

19. Write as a single fraction $\frac{5}{6m}-\frac{3}{4m}$

A. $\frac{m}{12}$ B. $\frac{12}{m}$ C. $\frac{1}{6m}$ D. $\frac{1}{12m}$

20. Simplify $\frac{3}{x+2y}-\frac{2}{x-3y}$

A. $\frac{1}{(x+2y)(x-3y)}$ B. $\frac{3x-2y}{(x+2y)(x-3y)}$

C. $\frac{x-13y}{(x+2y)(x-3y)}$ D. $\frac{13x-y}{(x+2y)(x-3y)}$

21. Simplify: $\frac{5}{m-n}-\frac{4}{n-m}$

A. $\frac{9}{m-n}$ B. $\frac{9}{n-m}$ C. $\frac{1}{m-n}$ D. $\frac{1}{n-m}$

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22. Simplify: $\frac{1}{4x-2y} - \frac{1}{y-2x}$

A. $\frac{1}{2x+y}$ B. $\frac{5}{2(2x-y)}$ C. $\frac{3}{2x-y}$ D. $\frac{3}{4x-2y}$

23. Simplify: $\frac{2a}{a^2-b^2} + \frac{1}{b-a}$

A. $\frac{1}{a+b}$ B. $\frac{1}{(a+b)(a-b)}$

C. $\frac{1}{a-b}$ D. $\frac{1}{(a-b)(a-b)}$

24. Simplify: $\frac{x-y}{x+y} - \frac{x+y}{x-y}$

A. $\frac{4xy}{x^2-y^2}$ B. $\frac{-4xy}{x^2-y^2}$ C. $\frac{2xy}{x^2-y^2}$ D. $\frac{2xy}{(x-y)^2}$

25. Simplify: $\frac{a^2+b^2+ab}{a+b} - \frac{a^2+b^2-ab}{a-b}$

A. $\frac{a^2+b^2}{(a-b)^2}$ B. $\frac{2a^2-2b^2}{a^2+b^2}$

C. $\frac{2b^3}{b^2-a^2}$ D. $\frac{2a^2+2b^2}{a^2-b^2}$

26. If $\frac{5}{y+1} - \frac{3}{1-y} - \frac{7y-1}{y^2-1} = \frac{P}{y+1}$, then P equals

A. -1 B. $y-1$ C. $3(1-5y)$ D. 1

27. If $m = \frac{2y}{1-y}$ and $n = \frac{1+y}{1-y}$, then $m^2 - n^2$ in its simplest form is given by

A. $\frac{3y+1}{y-1}$ B. $\frac{3y^2-1}{(y-1)^2}$ C. $\frac{3y^2+1}{(1-y)^2}$ D. $\frac{5y^2+1}{(1-y)^2}$

28. Simplify: $\left[1+\frac{y-1}{\frac{1}{y+1}}\right]\left[y+2\right]$

A. $(y^2-1)(y+2)$ B. $\frac{y^2(y+2)}{y+1}$

C. $y^2(y+2)$ D. $2y(y+2)$

29. If $F = \frac{y}{y-3} + \frac{y}{y+4}$, find the value of F when $y = -2$

A. $\frac{3}{5}$ B. $-\frac{3}{5}$ C. $-\frac{7}{5}$ D. $-\frac{2}{5}$

30. Simplify: $\frac{1}{a-2} + \frac{1}{a+2} + \frac{2a}{a^2-4}$

A. $\frac{2a}{(a-2)(a+2)(a^2-4)}$ B. $\frac{2a}{a^2-4}$

C. $\frac{a}{a^2-4}$ D. $\frac{4a}{a^2-4}$

31. If $\frac{1}{p} = \frac{x^2+2xy+y^2}{x-y}$ and $\frac{1}{q} = \frac{x+y}{x^2-2xy+y^2}$, find $\frac{p}{q}$

A. $\frac{x+y}{x-y}$ B. $\frac{1}{x^2-y^2}$ C. $\frac{x-y}{x+y}$ D. x^2-y^2

32. Simplify: $\frac{a(a+1)^{-1/2} - (a+1)^{1/2}}{(a+1)^{1/2}}$

A. $\frac{1}{a+1}$ B. $\frac{1}{a}$ C. $\frac{-1}{a+1}$ D. $\frac{-1}{\sqrt{a+1}}$

33. Simplify: $\left[\frac{1}{a^{-1}} \div \frac{1}{b^{-1}}\right]^{-1}$

A. b/a B. a/b C. ab D. $(ab)^{-1}$

34. Simplify: $\left(\frac{1}{x} - \frac{1}{y}\right) \div \left(\frac{x}{y} - \frac{y}{x}\right)$

A. $\frac{1}{x-y}$ B. $-\frac{1}{x+y}$ C. $\frac{1}{xy}$ D. $\frac{1}{xy(x-y)}$

35. If $\frac{6m-n}{m+2n} = 2$, find the value of $\frac{m}{n}$

A. $\frac{3}{8}$ B. $\frac{5}{8}$ C. $\frac{4}{5}$ D. $\frac{5}{4}$

36. If $\frac{y}{y-2} - \frac{y+2}{y+3} = \frac{P}{(y-2)(y+3)}$, then the value of P is

A. $2y^2-3y-4$ B. $2y^2+3y-4$

C. $3y+4$ D. $3y-4$

37. Simplify: $1 + \frac{2}{1 - \frac{5}{5+\frac{2}{y}}}$

A. $\frac{2}{y}+2$ B. $y+1$ C. $10y+6$ D. $5y+3$

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38. Solve the equation $\frac{x}{3} + \frac{1}{2} = \frac{3}{4} + \frac{x}{4}$

A. 3 B. 2 C. –3 D. –2

39. Solve the equation $\frac{4}{x} + \frac{1}{5x} = 3$

A. $2\frac{1}{5}$ B. $1\frac{2}{5}$ C. $\frac{1}{3}$ D. $1\frac{1}{3}$

40. Solve the equation $\frac{y+1}{2} = y - 3 + \frac{5}{y}$

A. 5, –2 B. 2, –5 C. 5, 2 D. 10, 1

41. Solve the equation $\frac{1}{z+1} - \frac{1}{z+3} = \frac{1}{4}$

A. –1, 3 B. 1, 3 C. 1, –5 D. –1, 5

42. If $\frac{\frac{2}{y}}{\frac{1}{a^2} + \frac{1}{b^2}} = k$, then y is equal to

A. $\frac{2ab}{k(a+b)}$ B. $\frac{2a^2b^2}{k(a^2+b^2)}$

C. $\frac{2k(a^2+b^2)}{a^2b^2}$ D. $\frac{2}{k(a^2+b^2)}$

43. If $\frac{3x+y}{3y-x} = \frac{2}{5}$, find the value of $\frac{x+3y}{y-3x}$

A. $\frac{1}{3}$ B. 1 C. $\frac{5}{2}$ D. $\frac{26}{7}$

44. If $f(y) = \frac{1}{y-1} + \frac{y-1}{y^2-1}$, find $f(1-y)$

A. $-\frac{1}{y} - \frac{1}{y-2}$ B. $\frac{1}{y} + \frac{1}{y-2}$

C. $y + \frac{1}{2y-1}$ D. $-\frac{1}{y} - \frac{1}{2y-1}$

45. Solve the equation $\frac{2}{2x-1} - \frac{5}{3} = \frac{1}{x+2}$

A. –1, 5/2 B. 1, –5/2 C. 5/2, 1 D. 2, 1

46. If $a\left[\frac{m+1}{m-2} - \frac{m-1}{m+2}\right] = 6m$, find a in its simplest form

A. $m^2 - 1$ B. $m^2 + 1$ C. $m^2 + 4$ D. $m^2 - 4$

47. If $y + \frac{1}{y} = 4$, find $y^2 + \frac{1}{y^2}$

A. 9 B. 12 C. 14 D. 16

48. Solve for x: $\frac{1}{x-1} + \frac{2}{x+1} = \frac{3}{x}$

A. 3 B. 4 C. 5 D. 6

49. If $\frac{Q}{x^2+2x} = \frac{1}{2}\left[\frac{1}{x} + \frac{9}{x+2}\right]$, what is Q?

A. $10x + 2$ B. $x + 9$ C. $5x + 1$ D. $9x + 1$

50. Resolve $\frac{3}{y^2+y-2}$ into partial fractions

A. $\frac{1}{y-1} - \frac{1}{y+2}$ B. $\frac{1}{y+2} + \frac{1}{y-1}$

C. $\frac{1}{y+1} - \frac{1}{y-2}$ D. $\frac{1}{y-2} + \frac{1}{y+1}$

51. Find the value of M if

$$\frac{z-3}{(1-z)(z+2)} = \frac{M}{1-z} + \frac{N}{z+2}$$

A. $-\frac{5}{3}$ B. $\frac{5}{3}$ C. $-\frac{2}{3}$ D. $\frac{2}{3}$

52. Express $\frac{5y-12}{(y-2)(y-3)}$ in partial fractions.

A. $\frac{2}{y-2} - \frac{3}{y-3}$ B. $\frac{2}{y-2} + \frac{3}{y-3}$

C. $\frac{2}{y-3} - \frac{3}{y-2}$ D. $\frac{5}{y-3} + \frac{4}{y-2}$

53. If $\frac{5+2x}{(x+1)(x-2)} = \frac{P}{x-2} + \frac{Q}{x+1}$, the value of P is

A. –1 B. 1 C. 2 D. 3

54. If $f(x) = \frac{x-3}{11} + \frac{11}{x^2-9}$, what is $f(x+3)$?

A. $\frac{x}{11} + \frac{11}{x(x+3)}$ B. $\frac{x}{11} + \frac{11}{x(x+6)}$

C. $\frac{x+30}{11} + \frac{11}{x(x+3)}$ D. $\frac{x+3}{11} + \frac{11}{x(x-3)}$

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ANSWER KEY

1.	**B**	10.	**C**	19.	**D**	28.	**C**	37.	**D**	46.	**D**
2.	**C**	11.	**B**	20.	**C**	29.	**B**	38.	**A**	47.	**C**
3.	**D**	12.	**D**	21.	**A**	30.	**D**	39.	**B**	48.	**A**
4.	**C**	13.	**A**	22.	**D**	31.	**B**	40.	**C**	49.	**C**
5.	**D**	14.	**C**	23.	**A**	32.	**C**	41.	**C**	50.	**A**
6.	**A**	15.	**A**	24.	**B**	33.	**A**	42.	**B**	51.	**C**
7.	**A**	16	**D**	25.	**C**	34.	**B**	43.	**D**	52.	**B**
8.	**C**	17.	**C**	26.	**D**	35.	**D**	44.	**A**	53.	**D**
9.	**D**	18	**B**	27.	**A**	36.	**C**	45.	**B**	54.	**B**

SOLUTIONS TO OBJECTIVE QUESTIONS

1.(B) $\dfrac{x^2-y^2}{x^2+xy} = \dfrac{(x+y)(x-y)}{x(x+y)} = \dfrac{x-y}{x}$

2.(C) $\dfrac{1-y^2}{y-y^2} = \dfrac{(1-y)(1+y)}{y(1-y)} = \dfrac{1+y}{y}$

3.(D) $\dfrac{a-7}{a^2-9} \times \dfrac{a^2-3a}{a^2-49} = \dfrac{a-7}{(a+3)(a-3)} \times \dfrac{a(a-3)}{(a+7)(a-7)}$

$= \dfrac{a}{(a+3)(a+7)}$

4.(C) $\dfrac{a^2-b^2}{2a^2+ab-b^2} = \dfrac{(a+b)(a-b)}{(a+b)(2a-b)} = \dfrac{a-b}{2a-b}$

5.(D) $\dfrac{4x^2-49y^2}{2x^2+5xy-7y^2} = \dfrac{(2x+7y)(2x-7y)}{(2x+7y)(x-y)}$

$= \dfrac{2x-7y}{x-y}$

6.(A) $\dfrac{324-4y^2}{2y+18} = \dfrac{(18-2y)(18+2y)}{2y+18}$

$= 18-2y = -2(y-9)$

7.(A) $\dfrac{(2x-y)^2-(x-2y)^2}{5x^2-5y^2} =$

$\dfrac{[(2x-y)+(x-2y)][(2x-y)-(x-2y)]}{5(x+y)(x-y)}$

$= \dfrac{(3x-3y)(x+y)}{5(x+y)(x-y)} = \dfrac{3}{5}$

8.(C) $\dfrac{a^2-1}{a^3+2a^2-a-2} = \dfrac{(a+1)(a-1)}{a^2(a+2)-1(a+2)}$

$= \dfrac{(a+1)(a-1)}{(a^2-1)(a+2)}$

$= \dfrac{1}{a+2}$

9.(D) Substitute $x = yc$ and $z = wc$ into the given expression

$\dfrac{3y^2c^2-ywc^2+w^2c^2}{3y^2-yw+w^2} = \dfrac{c^2(3y^2-yw+w^2)}{3y^2-yw+w^2} = c^2$

10.(C) Amount spent in the first b days $= by$ naira

Amount left unspent $= (x-by)$ naira

Number of days left $= a-b$

Amount left per day $= \dfrac{x-by}{a-b}$

11.(B) Let $a^{1/3} = x$ and $b^{1/3} = y$

Given expression $= \dfrac{x^3-y^3}{x-y} = \dfrac{(x-y)(x^2+xy+y^2)}{x-y}$

$= x^2+xy+y^2$

$= a^{2/3}+a^{1/3}b^{1/3}+b^{2/3}$

12.(D) $\dfrac{x+2}{x-3x-10} = \dfrac{x+2}{(x+2)(x-5)} = \dfrac{1}{x-5}$

This is undefined when $x-5=0$, or $x=5$

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13.(A) $\frac{a^2 + 15a + 50}{a - 5} = \frac{(a + 10)(a + 5)}{a - 5}$

This is undefined for $a - 5 = 0$. or $a = 5$

14.(C) $\frac{5}{a^2 - ab - 6b^2} = \frac{5}{(a - 3b)(a + 2b)}$

This is undefined when $a - 3b = 0$ or $a + 2b = 0$

i.e. when $b = a/3$ or $-a/2$

15.(A) $\frac{(x - 5)(x - 2)}{(x + 2)(x - 1)}$ is undefined for $x = 1$ or -2

16.(D) $\frac{x^3 - y^3 - z^{1/2}}{x + y - z} = \frac{(-3)^3 - (2)^3 - (4)^{1/2}}{-3 + 2 - 4}$

$= \frac{-27 - 8 - 2}{-3 + 2 - 4} = \frac{37}{5}$

17.(C) $\frac{1}{1 - y} + \frac{2}{1 + y} = \frac{1 + y + 2(1 - y)}{(1 - y)(1 + y)}$

$= \frac{3 - y}{1 - y^2}$

18.(B) $\left(\frac{3}{a} - \frac{15}{2b}\right) \div \frac{6}{ab} = \left(\frac{6b - 15a}{2ab}\right) \div \frac{6}{ab}$

$= \frac{6b - 15a}{2ab} \times \frac{ab}{6} = \frac{2b - 5a}{4}$

19.(D) $\frac{5}{6m} - \frac{3}{4m} = \frac{10 - 9}{12m} = \frac{1}{12m}$

20.(C) $\frac{3}{x + 2y} - \frac{2}{x - 3y} = \frac{3(x - 3y) - 2(x + 2y)}{(x + 2y)(x - 3y)}$

$= \frac{x - 13y}{(x + 2y)(x - 3y)}$

21.(A) $\frac{5}{m - n} - \frac{4}{n - m} = \frac{5}{m - n} + \frac{4}{m - n} = \frac{9}{m - n}$

22.(D) $\frac{1}{4x - 2y} - \frac{1}{y - 2x} = \frac{1}{2(2x - y)} + \frac{1}{2x - y}$

$= \frac{3}{2(2x - y)} = \frac{3}{4x - 2y}$

23.(A) $\frac{2a}{a^2 - b^2} + \frac{1}{b - a} = \frac{2a}{(a - b)(a + b)} - \frac{1}{a - b}$

$= \frac{2a - (a + b)}{(a - b)(a + b)} = \frac{a - b}{(a - b)(a + b)}$

$= \frac{1}{a + b}$

24.(B) $\frac{x - y}{x + y} - \frac{x + y}{x - y} = \frac{(x - y)^2 - (x + y)^2}{(x + y)(x - y)}$

$= \frac{(x - y + x + y)(x - y - x - y)}{(x + y)(x - y)}$

$= \frac{(2x)(-2y)}{x^2 - y^2} = \frac{-4xy}{x^2 - y^2}$

25.(C) $\frac{a^2 + b^2 + ab}{a + b} - \frac{a^2 + b^2 - ab}{a - b}$

$= \frac{(a^2 + b^2 + ab)(a - b) - (a^2 + b^2 - ab)(a + b)}{a^2 - b^2}$

$= \frac{(a^3 - a^2b + ab^2 - b^3 + a^2b - ab^2) - (a^3 + a^2b + ab^2 + b^3 - a^2b - ab^2)}{a^2 - b^2}$

$= \frac{-2b^3}{a^2 - b^2} = \frac{2b^3}{b^2 - a^2}$

26.(D) $\frac{5}{y + 1} - \frac{3}{1 - y} - \frac{7y - 1}{y^2 - 1} = \frac{5(y - 1) + 3(y + 1) - (7y - 1)}{(y + 1)(y - 1)}$

$= \frac{5y - 5 + 3y + 3 - 7y + 1}{(y + 1)(y - 1)}$

$= \frac{y - 1}{(y + 1)(y - 1)} = \frac{1}{y + 1}$

Hence $P = 1$

27.(A) $m^2 - n^2 = (m + n)(m - n)$

$= \left[\frac{2y}{1 - y} + \frac{1 + y}{1 - y}\right]\left[\frac{2y}{1 - y} - \frac{1 + y}{1 - y}\right]$

$= \left[\frac{3y + 1}{1 - y}\right]\left[\frac{y - 1}{1 - y}\right]$

$= \frac{3y + 1}{y - 1}$

28.(C) $\left[1 + \frac{y - 1}{\frac{1}{y + 1}}\right]\left[y + 2\right] = [1 + (y + 1)(y - 1)][y + 2]$

$= (1 + y^2 - 1)(y + 2) = y^2(y + 2)$

29.(B) $F = \frac{y}{y - 3} + \frac{y}{y + 4}$

When $y = -2$, $F = \frac{-2}{-5} + \frac{-2}{2} = \frac{2}{5} - 1 = -\frac{3}{5}$

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30.(D) $\frac{1}{a-2} + \frac{1}{a+2} + \frac{2a}{a^2-4} = \frac{(a+2)+(a-2)+2a}{a^2-4}$

$= \frac{4a}{a^2-4}$

31.(B) $\frac{p}{q} = \frac{1}{q} \div \frac{1}{p} = \frac{x+y}{x^2-2xy+y^2} \times \frac{x-y}{x^2+2xy+y^2}$

$= \frac{x^2-y^2}{(x-y)^2(x+y)^2} = \frac{x^2-y^2}{[(x-y)(x+y)]^2}$

$= \frac{x^2-y^2}{(x^2-y^2)^2} = \frac{1}{x^2-y^2}$

32.(C) Multiply numerator and denominator by $(a+1)^{1/2}$ to give

$\frac{a(a+1)^0 - (a+1)^1}{(a+1)^1} = \frac{a-a-1}{a+1} = \frac{-1}{a+1}$

33.(A) $\left[\frac{1}{a^{-1}} \div \frac{1}{b^{-1}}\right]^{-1} = (a \div b)^{-1} = \left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

34.(B) $\left(\frac{1}{x} - \frac{1}{y}\right) \div \left(\frac{x}{y} - \frac{y}{x}\right) = \frac{y-x}{xy} \div \frac{x^2-y^2}{xy}$

$= \frac{y-x}{xy} \times \frac{xy}{(x-y)(x+y)} = -\frac{1}{x+y}$

35.(D) $6m - n = 2(m+2n) = 2m + 4n$

$4m = 5n$, or $\frac{m}{n} = \frac{5}{4}$

36.(C) $\frac{y}{y-2} - \frac{y+2}{y+3} = \frac{y(y+3) - (y-2)(y+2)}{(y-2)(y+3)}$

$= \frac{y^2+3y-y^2+4}{(y-2)(y+3)} = \frac{3y+4}{(y-2)(y+3)}$

Hence $P = 3y + 4$

37.(D) $1 + \frac{2}{1 - \frac{5}{5+\frac{2}{y}}}$. $1 + \frac{2}{1 - \frac{5}{\frac{5y+2}{y}}}$

$= 1 + \frac{2}{1 - \frac{5y}{5y+2}} = 1 + \frac{2}{\frac{2}{5y+2}} = 1 + 5y + 2$

$= 5y + 3$

38.(A) $\frac{x}{3} + \frac{1}{2} = \frac{3}{4} + \frac{x}{4}$

$\frac{x}{3} - \frac{x}{4} = \frac{3}{4} - \frac{1}{2}$

$\frac{x}{12} = \frac{1}{4}$, or $x = 3$

39.(B) $\frac{4}{x} + \frac{1}{5x} = 3$

$\frac{20+1}{5x} = 3$, $\frac{21}{5x} = 3$

$x = \frac{21}{15} = 1\frac{2}{5}$

40.(C) $\frac{y+1}{2} = y - 3 + \frac{5}{y}$

Multiply through by $2y$:

$y(y+1) = 2y(y-3) + 10$

$y^2 + y = 2y^2 - 6y + 10$

$y^2 - 7y + 10 = 0$

$(y-5)(y-2) = 0$, $y = 5$ or 2

41.(C) $\frac{1}{z+1} - \frac{1}{z+3} = \frac{1}{4}$

$\frac{z+3-z-1}{(z+1)(z+3)} = \frac{1}{4}$

$\frac{2}{z^2+4z+3} = \frac{1}{4}$

$z^2 + 4z + 3 = 8$, or $z^2 + 4z - 5 = 0$

$(z+5)(z-1) = 0$, $z = -5$ or 1

42.(B) $\frac{\frac{2}{y}}{\frac{1}{a^2} + \frac{1}{b^2}} = k$

$\frac{\frac{2}{y}}{\frac{a^2+b^2}{a^2b^2}} = k$; $\frac{2}{y} \times \frac{a^2b^2}{a^2+b^2} = k$

$yk(a^2 + b^2) = 2a^2b^2$

$y = \frac{2a^2b^2}{k(a^2+b^2)}$

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43.(D) $\frac{3x+y}{3y-x} = \frac{2}{5}$

$15x + 5y = 6y - 2x$, or $17x = y$

$\frac{x+3y}{y-3x} = \frac{x+3(17x)}{17x-3x} = \frac{52x}{14x} = \frac{26}{7}$

44.(A) $f(y) = \frac{1}{y-1} + \frac{y-1}{y^2-1}$

$= \frac{1}{y-1} + \frac{y-1}{(y+1)(y-1)} = \frac{1}{y-1} + \frac{1}{y+1}$

$f(1-y) = \frac{1}{(1-y)-1} + \frac{1}{(1-y)+1}$

$= -\frac{1}{y} + \frac{1}{2-y} = -\frac{1}{y} - \frac{1}{y-2}$

45.(B) $\frac{2}{2x-1} - \frac{5}{3} = \frac{1}{x+2}$

$\frac{6-5(2x-1)}{3(2x-1)} = \frac{1}{x+2}$

$\frac{11-10x}{6x-3} = \frac{1}{x+2}$

$(11-10x)(x+2) = 6x-3$

$11x + 22 - 10x^2 - 20x = 6x - 3$

$10x^2 + 15x - 25 = 0$

$(2x+5)(5x-5) = 0$

$x = -5/2$ or 1

46.(D) $\text{LHS} = a\left[\frac{(m+1)(m+2)-(m-1)(m-2)}{m^2-4}\right]$

$= a\left[\frac{m^2+3m+2-m^2+3m-2}{m^2-4}\right]$

$= a\left[\frac{6m}{m^2-4}\right]$

The equation is thus

$\frac{a(6m)}{m^2-4} = 6m$, or $a = m^2 - 4$

47.(C) $\left(y+\frac{1}{y}\right)^2 = y^2 + \frac{1}{y^2} + 2$

Hence $y^2 + \frac{1}{y^2} = \left(y+\frac{1}{y}\right)^2 - 2$

$= 4^2 - 2 = 14.$

48.(A) $\frac{1}{x-1} + \frac{2}{x+1} = \frac{3}{x}$

Multiply through by $(x-1)(x+1)x$

$x(x+1) + 2x(x-1) = 3(x-1)(x+1)$

$x^2 + x + 2x^2 - 2x = 3x^2 - 3$

$-x = -3$, or $x = 3$.

49.(C) $\text{RHS} = \frac{1}{2}\left[\frac{x+2+9x}{x^2+2x}\right] = \frac{5x+1}{x^2+2x}$

Hence $\frac{Q}{x^2+2x} = \frac{5x+1}{x^2+2x}$, or

$Q = 5x + 1$

50.(A) $y^2 + y - 2 = (y+2)(y-1)$

Let $\frac{3}{y^2+y-2} = \frac{P}{y-1} + \frac{Q}{y+2}$

$3 = P(y+2) + Q(y-1)$

When $y = 1$, $3P = 3$ and $P = 1$

When $y = -2$, $-3Q = 3$ and $Q = -1$

Hence $\frac{3}{y^2+y-2} = \frac{1}{y-1} - \frac{1}{y+2}$

51.(C) $z - 3 = M(z+2) + N(1-z)$

When $z = 1$, $3M = -2$, and $M = -2/3$.

52.(B) $\frac{5y-12}{(y-2)(y-3)} = \frac{P}{y-2} + \frac{Q}{y-3}$

$5y - 12 = P(y-3) + Q(y-2)$

When $y = 3$, $Q = 15 - 12 = 3$

When $y = 2$, $-P = -2$, or $P = 2$

Hence $\frac{5y-12}{(y-2)(y-3)} = \frac{2}{y-2} + \frac{3}{y-3}$.

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53. (D) $5 + 2x = P(x + 1) + Q(x - 2)$

When $x = 2$, $3P = 5 + 2(2) = 9$, or $P = 3$

54. (B) Replace x with $x + 3$ to give

$$f(x + 3) = \frac{(x + 3) - 3}{11} + \frac{11}{(x + 3)^2 - 9}$$

$$= \frac{x}{11} + \frac{11}{(x + 3 + 3)(x + 3 - 3)}$$

$$= \frac{x}{11} + \frac{11}{x(x + 6)}.$$

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11 Inequalities

11.1 Definitions of Symbols

- The following symbols are used instead of the equality symbol to denote inequalities:

  $<$ means *less than*, e.g. $2 < 4$ means "2 is less than 4".

  $\leq$ means *less than or equal to*. If $x \leq 10$, then x can be 10, 9, 2, –8,...

  $>$ means *greater than*, e.g. $5 > 3$ means "5 is greater than 3".

  $\geq$ means *greater than or equal to*. If $x \geq 4$, then x can be 4, 5, 9, 20,...

- If, for any set of real numbers a, b, and c, $a < b$ and $b < c$, then $a < c$, e.g. since $3 < 7$ and $7 < 12$, it follows that $3 < 12$.

- The expression $a < b < c$ means that "$a < b$ and $b < c$", i.e. b lies between a and c. Similarly, the expression $a > b > c$ means that "$a > b$ and $b > c$", e.g. $3 < 7 < 12$, or $12 > 7 > 3$.

11.2 Addition Rule

- The direction of the inequality remains unchanged if the same quantity is added to or subtracted from each side of the inequality; i.e. for any real numbers x, y and z:

  (i) if $x < y$, then $x + z < y + z$

  (ii) if $x > y$, then $x + z > y + z$

EXAMPLE 11-1

Find the values of x for which

(a) $x - 3 > 5$

(b) $x + 7 < 2x + 4$

Solutions

(a) Add 3 to both sides of the inequality:

$x - 3 + 3 > 5 + 3$, or $x > 8$

(b) Subtract x from both sides:

$7 < 2x + 4 - x$

Subtract 4 from both sides:

$3 < x$, or $x > 3$

DRILL 1

Solve the following inequalities:

(a) $y + 7 < 10$ (b) $2x < x + 5$

(c) $x + 2 > 2x - 6$ (d) $3y - 7 > 2y + 1$

(e) $a - 5 > 5$ (f) $8m - 6 < 7m$

11.3 Multiplication Rule

- The direction of the inequality remains *unchanged* if each side of the inequality is multiplied or divided by the same *positive* number. For all positive values of a and b,

  (i) if $x < y$, then $ax < ay$ and $x/b < y/b$

  (ii) if $x > y$, then $ax > ay$ and $x/b > y/b$.

- The inequality is *reversed* if each side of the inequality is multiplied or divided by the same *negative* number. For all positive values of a:

  (i) If $x < y$, then $-ax > -ay$ and $-x/b > -y/b$

  (ii) If $x > y$, then $-ax < -ay$ and $-x/b < -y/b$.

EXAMPLE 11-2

Solve the following inequalities:

(a) $2x + 5(x - 3) > x + 9$

(b) $\frac{2}{x} > 3$, for $x > 0$.

Solutions

(a) Expand the bracket : $2x + 5x - 15 > x + 9$

Add $15 - x$ to both sides : $2x + 5x - x > 9 + 15$

or $6x > 24$

Divide both sides by 6 : $x > 4$

(b) Multiply both sides by x : $2 > 3x$

Divide both sides by 3 : $\frac{2}{3} > x$, i.e. $x < \frac{2}{3}$.

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EXAMPLE 11-3

Solve the inequalities:

(a) $4 - 3x \leq 10$

(b) $\frac{1}{x+1} > -\frac{1}{3}$, if $x < -1$

(c) $x - 3 \leq -2x + 5 \leq x + 11$

Solutions

(a) Subtract 4 from both sides : $-3x \leq 6$

Divide both sides by -3 and reverse the inequality sign : $x \geq -2$

(b) Since $x < -1$, $x + 1$ must be negative. Multiply both sides by $3(x + 1)$ and reverse inequality sign:

$3 < -(x + 1)$, or $3 < -x - 1$

Hence $3 + 1 < -x$, or $4 < -x$.

Multiply both sides by -1 and reverse inequality sign:

$-4 > x$, i.e. $x < -4$

(c) This means $x - 3 \leq -2x + 5$ and $-2x + 5 \geq x + 11$

$3x \leq 8$ and $-6 \leq 3x$

$x \leq 8/3$ and $x \geq -2$, or $-2 \leq x \leq 8/3$.

DRILL 2

Solve the following inequalities:

(a) $7 - 3(x - 1) < 13$

(b) $12 \geq 3(2y - 3)$

(c) $\frac{x}{3} + 2 > x - 2$

(d) $\frac{x}{4} - \frac{3x}{5} \geq 3 - \frac{x}{2}$

11.4 Second-Degree Inequalities

For any real numbers or factors p and q:

(i) $p \times q > 0$ implies that p and q are either both positive or both negative, i.e. either $p > 0$ and $q > 0$, or $p < 0$ and $q < 0$.

(ii) $p \times q < 0$ implies that p and q have opposite signs, i.e. either $p > 0$ and $q < 0$, or $p < 0$ and $q > 0$.

- Second-degree inequalities are solved by factorizing and performing the operations outlined above on each of the factors.

EXAMPLE 11-4

Solve the inequalities:

(a) $x^2 < 1$

(b) $x^2 - 2x - 3 \geq 0$

Solutions

(a) If $x^2 < 1$ then $x^2 - 1 < 0$

Factorize: $(x + 1)(x - 1) < 0$

Either (i) $x + 1 < 0$ and $x - 1 > 0$,

or (ii) $x + 1 > 0$ and $x - 1 < 0$.

(i) $x < -1$ and $x > 1$ (which is **not** possible since x cannot satisfy both conditions at the same time);

(ii) $x > -1$ and $x < 1$, or $-1 < x < 1$

(b) $(x - 3)(x + 1) \geq 0$

Either $(x - 3) \geq 0$ and $(x + 1) \geq 0$.

i.e. $x \geq 3$ and $x \geq -1$

Both are true only when $x \geq 3$

or $(x - 3) \leq 0$ and $(x + 1) \leq 0$

i.e. $x \leq 3$ and $x \leq -1$

Both are true only when $x \leq -1$

Hence the solution is $x \geq 3$ or $x \leq -1$

DRILL 3

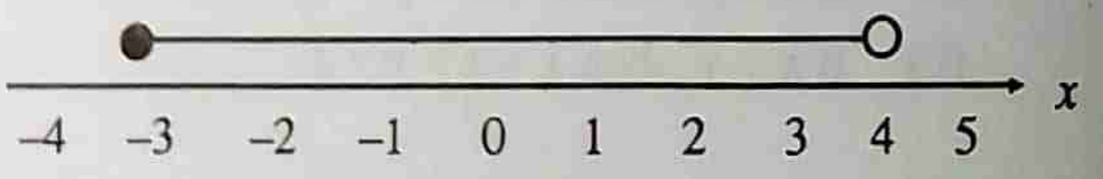
Illustrate the following inequalities graphically:

(a) $\frac{-5}{2} < x \leq 3$ (b) $x < -3; x \geq 2$

(c) $(x - 4)(x + 1) > 0$ (d) $(2x + 3)(x - 1) \leq 0$

11.5 Graphing of Inequalities

Graphically, an inequality involving only one variable (e.g. x) is represented as points on a line, e.g. the number line diagram illustrated below indicates all real numbers greater than or equal to -3 but less than 4, i.e. $-3 \leq x < 4$.



The solid circle indicates that the number -3 is included in the range of x while the open circle excludes the number 4.

- An inequality involving two variables (e.g. x and y) is represented graphically as whole regions in a (2–dimensional) plane, see Example 11-6.

EXAMPLE 11-5

Illustrate the following inequalities graphically

(a) $2 \geq x > -1$

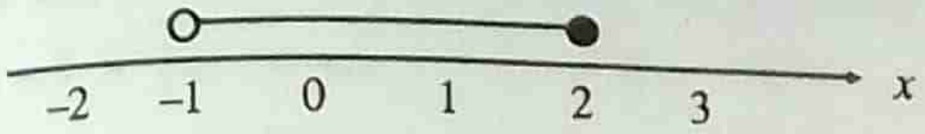
(b) $x^2 - 1 < 0$

(c) $x^2 - 2x - 3 \geq 0$

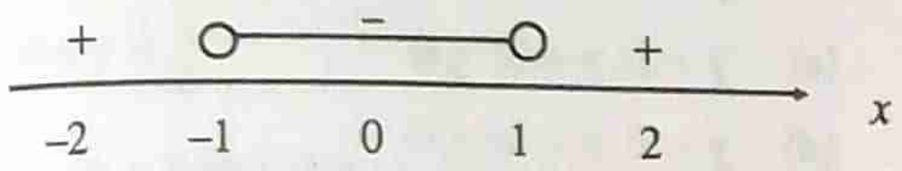
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Solutions

(a) The range of x covers all numbers between -1 and 2, and it also includes 2.



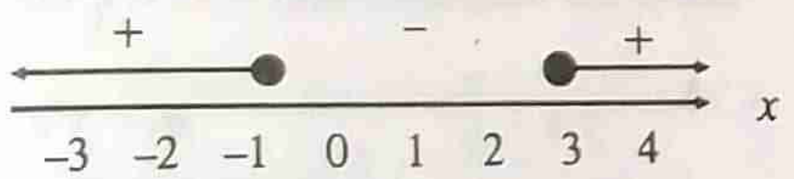
(b) The solution to this inequality, as shown in Example 11- 4(a) is $-1 < x < 1$,i.e. all numbers lying between -1 and 1 (Note that -1 and 1 are excluded).



This means that $x^2 - 1$ is negative only in the section indicated. This can be checked by testing a number in each section, e.g.

If $x = 0$, $x^2 - 1 = -1$ (which is *negative*)
If $x = -2$, $x^2 - 1 = 3$ (which is *positive*)
If $x = 2$, $x^2 - 1 = 3$ (which is *positive*)

(c) The solution to this inequality is $x \leq -1$ or $x \geq 3$ (Example 11-4(b)). This is illustrated below.



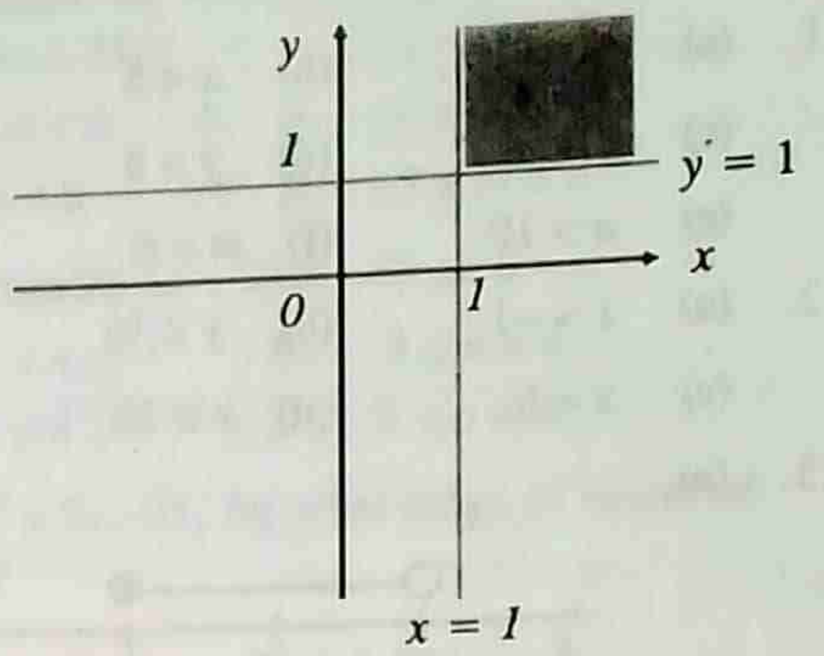
[As a check, the candidate should test a number in each section e.g. $x = -2$, 0 and 4].

EXAMPLE 11-6

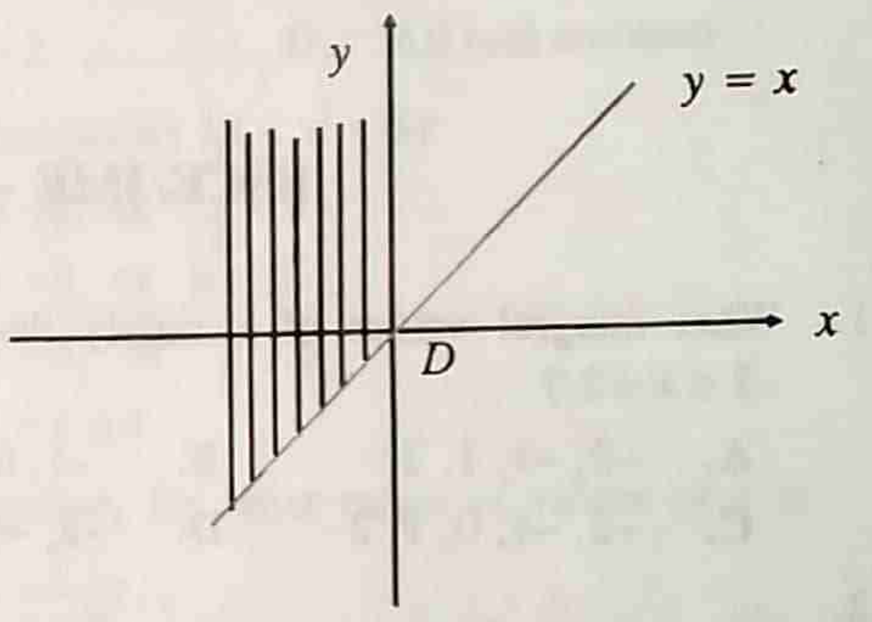
Graph the following inequalities.
(a) $x > 1, y > 1$
(b) $x \leq 0, y > x$

Solutions

(a) All points lying to the right of the line $x = 1$ satisfy the inequality $x > 1$ while all points lying above the line $y = 1$ satisfy $y > 1$. The region which is common to both (shaded region in the figure) satisfies the inequality $x > 1$, $y > 1$.

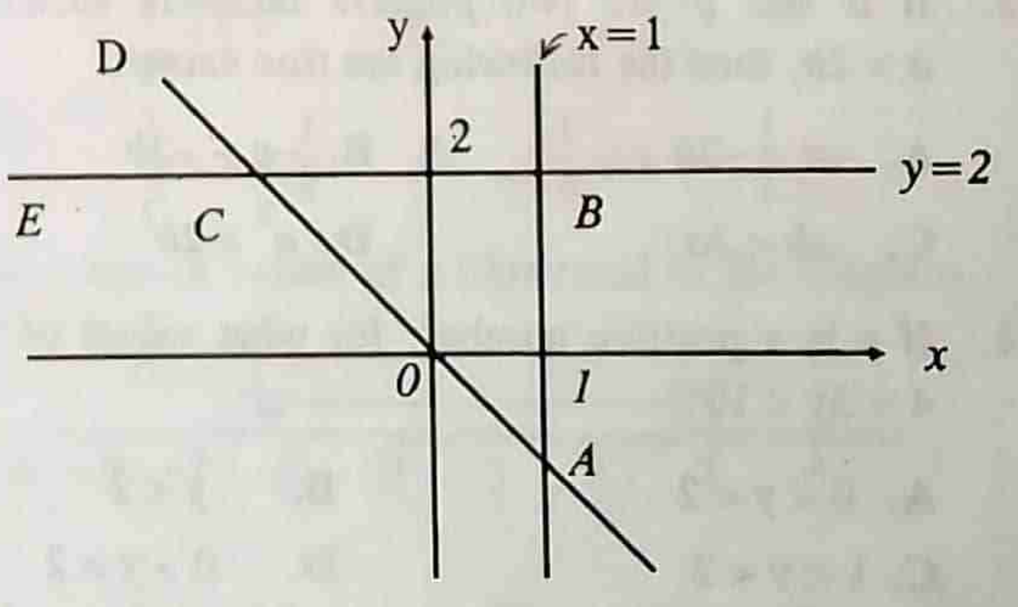


(b) Points lying on the y-axis (line $x = 0$) and to the left of the y-axis satisfy $x \leq 0$ while points lying above the line $y = x$ satisfy $y > x$. Thus the common region (shaded region) satisfies the inequality $x \leq 0$, $y > x$.



Note:Dashed boundary lines are not included in the desired region while solid lines are included.

DRILL 4

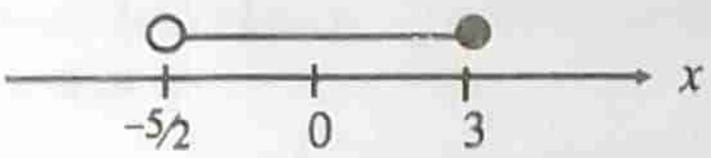


Write inequalities describing the following regions in the diagram above: All points lying
(a) within triangle ABC
(b) on or above line CD and above BC
(c) above line EC and on or below CD
(d) below lines EC and AC
(e) to the right of AB but above DA

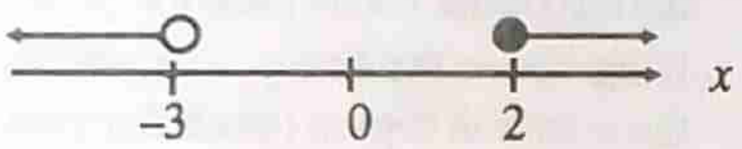
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ANSWERS TO DRILLS

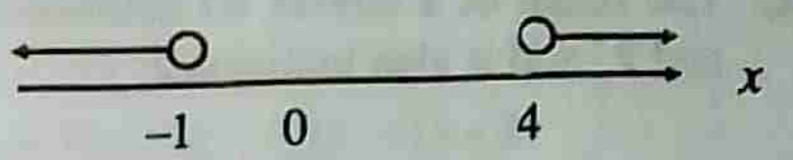
1. (a) $y < 3$ (b) $x < 5$
   (c) $x < 8$ (d) $y > 8$
   (e) $a > 10$ (f) $m < 6$
2. (a) $x > -1$ (b) $y \leq 7/2$
   (c) $x < 6$ (d) $x \geq 20$
3. (a)



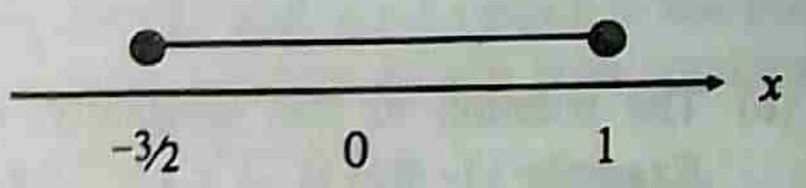
(b)



(c)



(d)



4. (a) $y + 3x > 0, x < 1, y < 2$
   (b) $y + 3x \geq 0, y > 2$
   (c) $y > 2, y + 3x \leqslant 0$
   (d) $y < 2, y + 3x < 0$
   (e) $x > 1, y + 3x > 0$

SSCE/JME –TYPE OBJECTIVE QUESTIONS

1. What integral values of x satisfy the inequality $-2 \leq x < 2$?

   A. $-2, -1, 1, 2$ **B.** $-1, 0, 1$
   C. $-2, -1, 0, 1, 2$ **D.** $-2, -1, 0, 1$

2. What is the least possible value of $\frac{9}{1 + 2y^2}$ if $0 \leq y \leq 2$?

   A. 9 **B.** 0 **C.** 1 **D.** 2

3. If a and b are two *positive* numbers such that $a > 2b$, then the following are true **except**

   A. $-a < -2b$ **B.** $-a > -2b$
   C. $-b < 2a$ **D.** $a^2 > 2b^2$

4. If y is a positive number, for what values of y is $4 + 3y < 10$?

   A. $0 < y < 2$ **B.** $y < 2$
   C. $1 < y < 2$ **D.** $0 > y > 2$

5. Solve the inequality $3x + 3 > 9$

   A. $x > 2$ **B.** $x > 3$
   C. $x > 4$ **D.** $x > 6$

6. Solve the following inequality $\frac{1}{3}(2y - 1) < 5$

   A. $y < -5$ **B.** $y < -6$
   C. $y < 7$ **D.** $y < 8$

7. Solve the inequalitiy $3a - 8 \geq 5a$

   A. $a \leq 4$ **B.** $a \geq 1$
   C. $a \leq -4$ **D.** $a \leq -1$

8. Solve the inequality $3(2y + 1) \geq \frac{1}{3}(2y - 9)$

   A. $y \geq -\frac{9}{8}$ **B.** $y \geq -\frac{1}{8}$
   C. $y \leq \frac{3}{8}$ **D.** $y \geq \frac{5}{8}$

9. Solve the inequality $\frac{5 - 3a}{2} \geq \frac{2a + 3}{5}$

   A. $a \leq -1$ **B.** $a \geq -1$
   C. $a \geq 1$ **D.** $a \leq 1$

10. Find the values of y satisfying the inequality $\frac{y}{2} - \frac{1}{3} < \frac{2y}{5} + \frac{1}{6}$

    A. $y < -5$ **B.** $y < 5$
    C. $y \geq 5$ **D.** $y > \frac{15}{2}$

11. Find all real numbers n for which $\frac{1}{3}(n + 1) - 1 > \frac{1}{5}(n + 4)$.

    A. $n < -1$ **B.** $n < 11$ **C.** $n > 11$ **D.** $n > 6$

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12. For what range of values of y is $y - 1 > 4(y + 2)$?

A. $y < -3$ B. $y > -3$

C. $2 < y < 3$ D. $-3 < y < -2$

13. The real numbers m which satisfy the inequality $\frac{3}{5}m + 3 > \frac{2}{3}m - 4$ are

A. $(m : m < 7)$ B. $(m : m > 7)$

C. $(m : m > 105)$ D. $(m : m < 105)$

14. For what range of values of y is $\frac{1}{y} > 2$?

A. $y < \frac{1}{2}$ B. $y < 0$ or $y > \frac{1}{2}$

C. $0 < y < \frac{1}{2}$ D. $1 < y < 2$

15. If $f(y) = 2y + 4$ and $g(y) = 6y + 7$ where $g(y) > 0$, for what range of values of y is $\frac{f(y)}{g(y)} < 1$?

A. $y > -12$ B. $y > -\frac{3}{4}$

C. $y > -\frac{4}{3}$ D. $y > -\frac{7}{6}$

16. If $(x - 3) < \frac{x}{3}$, then

A. $x > -9$ B. $x < 3$

C. $x < 4\frac{1}{2}$ D. $x < 9$

17. If p, q and r are positive, the range of values of x which satisfies the inequality $\frac{x}{p} + \frac{x}{q} + \frac{x}{r} > 1$ is

A. $x > \frac{pqr}{pq + qr + pr}$ B. $x > pqr$

C. $x > \frac{1}{p} + \frac{1}{q} + \frac{1}{r}$ D. $x > \frac{1}{pqr}$

18. What values of y satisfy the inequality $0 < \frac{y + 3}{y - 1} \leq 2$?

A. $-3 < y \leq 5$ B. $y < -3$ and $y \geq 5$

C. $y > -3$ D. $y \geq 5$

19. The integral values of z which satisfy the inequality $-1 < 2z - 5 \leq 5$ are

A. 2, 5 B. 2, 3, 4

C. 3, 4, 5 D. 2, 3, 4, 5

20. The integral values of y which satisfy the inequality $-1 < 5 - 2y \leq 7$ are

A. $-1, 0, 1, 2$ B. $0, 1, 2, 3$

C. $-1, 0, 1, 2, 3$ D. $-1, 0, 2, 3$

21. Find all values of m satisfying the inequality $-11 \leq 4 - 3m \leq 28$

A. $-5 < m \leq 8$ B. $-8 \leq m \leq 5$

C. $5 \leq m \leq 8$ D. $-5 \leq m \leq 8$

22. Solve the inequality $(a - 3)(a - 4) \leq 0$

A. $3 < a < 4$ B. $3 \leq a < 4$

C. $3 \leq a \leq 4$ D. $3 < a \leq 4$

23. If $f(y) = 2y^2 + 9y - 35$, for what range of values of y is $f(y) < 0$?

A. $-7 < y < \frac{5}{2}$ B. $-7 \leq y < 5$

C. $-5 \leq y < 7$ D. $-\frac{7}{2} < y \leq 5$

24. Find the real numbers n satisfying $n(\frac{3}{n} + n) > 2$

A. $n < 1$ B. $n > -1$

C. $n > 2$ D. All real numbers

25. Solve the inequality $18 < x^2 - 3x$

A. $x < -3$ or $x < 6$

B $x > -3$ or $x > 6$

C. $x < -3$ or $x > 6$

D. $-3 < x < 6$

26. If y is negative, for what range of values of y is $\frac{y + 1}{3} > \frac{1}{y + 3}$?

A. $3 < y < 4$ B. $-3 < y < 0$

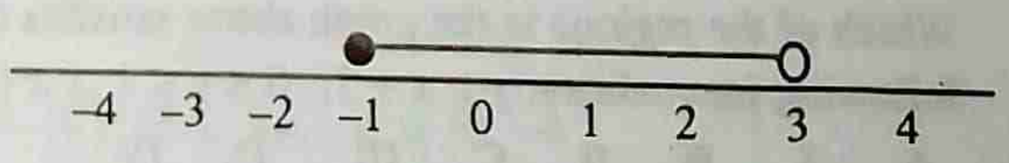
C. $-2 < y < -1$ D. $-4 < y < -3$

27. Solve the inequality $12y^2 < y + 1$

A. $-\frac{1}{4} < y < \frac{1}{3}$ B. $\frac{1}{4} < y < \frac{1}{3}$

C. $-\frac{1}{3} < y < \frac{1}{4}$ D. $-\frac{1}{4} < y < -\frac{1}{3}$

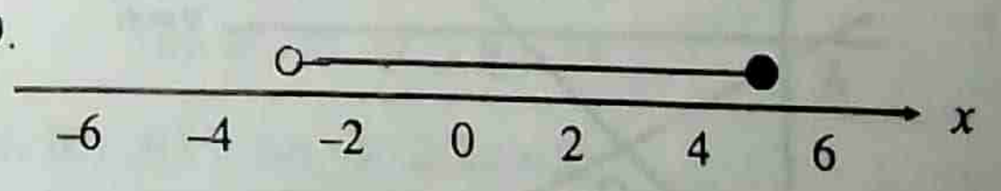
28. The range of values of x illustrated in the diagram below is



A. $-1 < x \leq 3$ B. $-1 \leq x < 3$

C. $-1 < x < 3$ D. $-1 \leq x \leq 3$

29.



The range of values of x illustrated in the diagram above is

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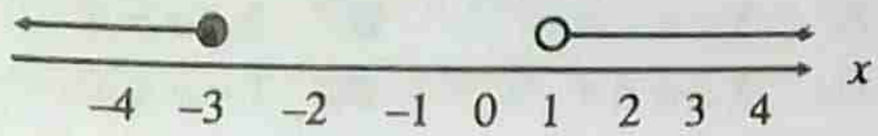
A. $(x : -3 < x \leq 5)$

B. $(x : -3 \leq x \leq 5)$

C. $(x : -4 < x \leq 4)$

D. $(x : -3 < x < 5)$

30.



The range of values of x illustrated in the diagram above is

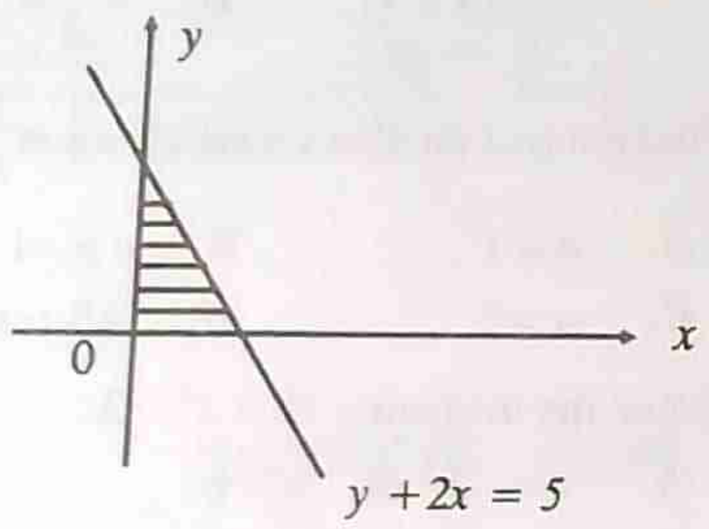
A. $-3 \leq + x \leq 1$

B. $-3 < x < 1$

C. $x \leq -3 \; ; x \geq 1$

D. $x \leq -3, x > 1$

31.



The shaded region indicated in the graph above is

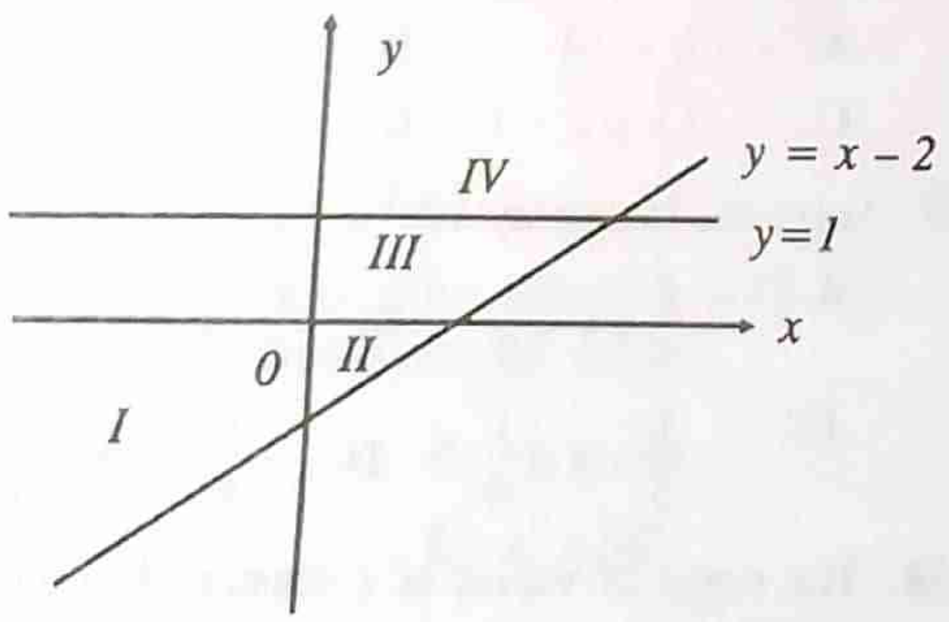
A. $y + 2x \leq 5$

B. $y + 2x \leq 5, x \geq 0$

C. $y + 2x \leq 5, x \geq 0, \; y \geq 0$

D. $y + 2x \geq 5 \; , x \geq 0 \; , y \geq 0$

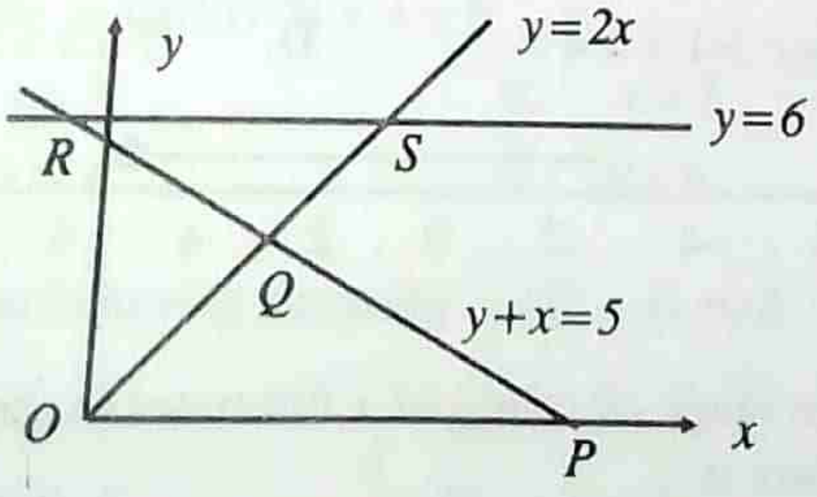
32.



Which of the regions in the graph above satisfies the following inequalities: $y \geq x + 2, \; 0 \leq y \leq 1 \; , x \geq 0$?

A. I B. II C. III D. IV

33.



In the graph above, which of the following solution sets is represented by the triangle *OPQ*?

A. $y \geq 2x \; , y \leq 0 \; , y + x \geq 5$

B. $y \leq 2x \; , y \geq 0 \; , y + x \leq 5$

C. $y \geq 2x \; , y \geq 0, y + x \leq 5$

D. $y \leq 2x \; , y \geq 0 \; , y + x \geq 5$

34. Referring to the graph in question 33, which of the following solution sets is represented by the triangle *QRS*?

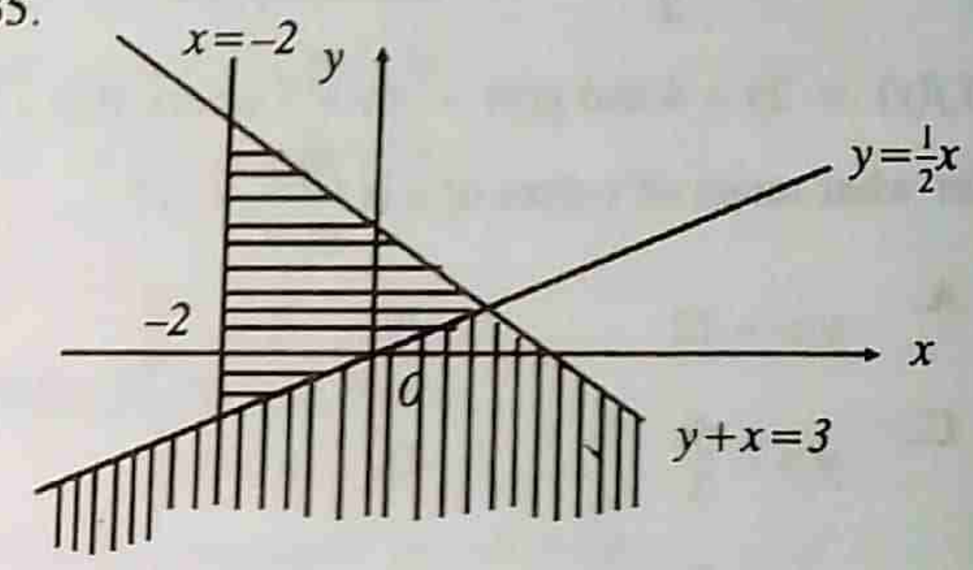
A. $y \geq 2x \; , y \leq 6 \; , y + x \geq 5$

B. $y \geq 2x, y \leq 6, y + x \leq 5$

C. $y \leq 2x, y \leq 6, y + x \geq 5$

D. $y \leq 2x, y \leq 6, y + x \leq 5$

35.



The area shaded with horizontal lines in the diagram above is the solution set of

A. $y + x \leq 3, \; 2y - x \geq 0, \; x + 2 \geq 0$

B. $y \; + x \geq 3, \; 2y - x \geq 0, \; x + 2 \leq 0$

C. $y + x \leq 3, \; 2y - x \leq 0, \; x + 2 \geq 0$

D. $y + x \leq 3, \; 2y - x \geq 0, \; x + 2 \leq 0$

36. Referring to the diagram in question 35, the area shaded with vertical lines is the solution set of

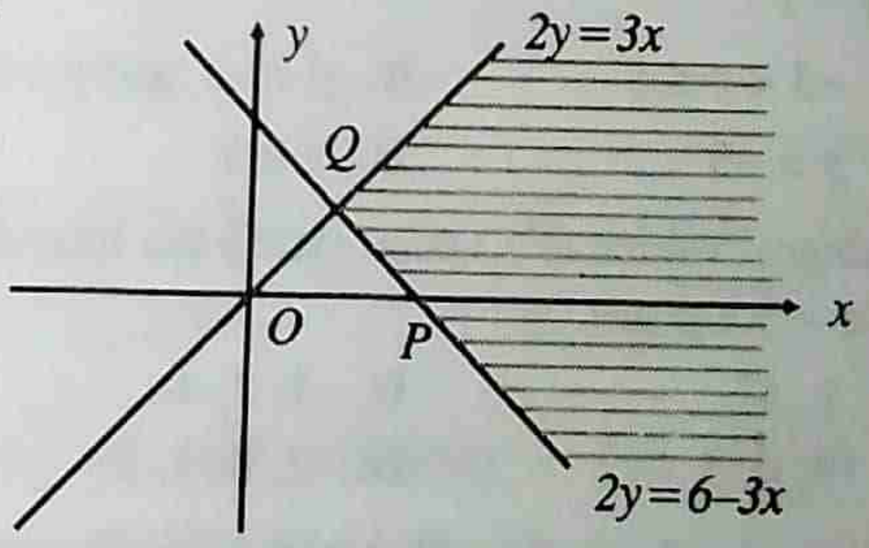
A. $y + x \leq 3, \; 2y - x \leq 0, \; x + 2 \geq 0$

B. $y + x \leq 3, \; 2y - x \geq 0, \; x + 2 \leq 0$

C. $y + x \geq 3, \; 2y - x \leq 0$

D. $y + x \leq 3, \; 2y - x \leq 0$

37.



Which of the following inequalities identify the shaded region in the diagram above?

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I. $2y - 3x \geq 0$ II. $3x + 2y \leq 6$

III. $2y - 3x \leq 0$ IV. $3x + 2y \geq 6$

A. I and II **B.** III and IV

C. I and IV **D.** II and III

38. The triangle OPQ in the diagram in question 37 is represented by the inequalities

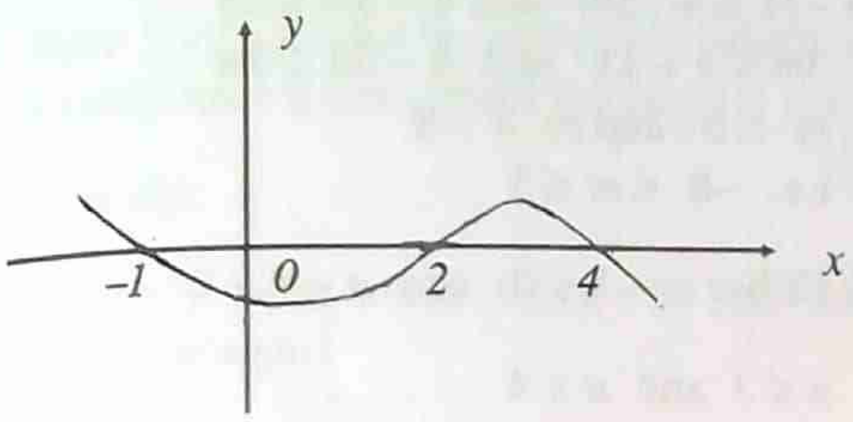
A. $2y - 3x \geq 0,\ 3x + 2y \leq 6,\ y \geq 0$

B. $2y - 3x \leq 0,\ 3x + 2y \leq 6,\ y \geq 0$

C. $2y - 3x \geq 0,\ 3x + 2y \geq 6,\ x \geq 0$

D. $2y - 3x \leq 0,\ 3x + 2y \leq 6,\ x \geq 0$

39.



In the graph of $y = f(x)$ sketched above, the range of values of x for which $f(x) > 0$ are

A. $x \leq -1\ ,\ 2 \leq x \leq 4$

B. $x < -1,\ 2 < x < 4$

C. $-1 \leq x \leq 2,\ 2 \leq x \leq 4$

D. $x < -1,\ x > 4$

40. Referring to the diagram in question 39, the range of values of x for which $f(x) \leq 0$ are

A. $-1 \leq x \leq 4$

B. $x \geq -1\ ,\ x \geq 4$

C. $2 \geq x \geq -1,\ x \geq 4$

D. $-1 < x < -2,\ x > 4$

ANSWER KEY

1.	**D**	8.	**A**	15.	**D**	22.	**C**	29.	**A**	36.	**D**
2.	**C**	9.	**D**	16.	**C**	23.	**A**	30.	**D**	37.	**B**
3.	**B**	10.	**B**	17.	**A**	24.	**D**	31.	**C**	38.	**B**
4.	**A**	11.	**C**	18.	**D**	25.	**C**	32.	**C**	39.	**B**
5.	**A**	12.	**A**	19.	**C**	26.	**D**	33.	**B**	40.	**C**
6.	**D**	13.	**D**	20.	**A**	27.	**A**	34.	**A**		
7.	**C**	14.	**C**	21.	**B**	28.	**B**	35.	**A**		

SOLUTIONS TO OBJECTIVE QUESTIONS

1. **(D)** These are integers between –2 and 2, and including –2.

2. **(C)** The function has its least value when the denominator is maximum. This is true for the maximum value of y in the specified range, i.e. $y = 2$ and the function is then $\frac{9}{1+8} = 1$

3. **(B)** If both sides of the inequality are multiplied by –1, the inequality must be *reversed*. Note also that since a and b are both positive, (C) must be true.

4. **(A)** $4 + 3y < 10,\ 3y < 6,$ or $y < 2$
The question also states that $y > 0$. Hence $0 < y < 2$

5. **(A)** $3x > 6,$ or $x > 2$

6. **(D)** $2y - 1 < 15,\ 2y < 16,$ or $y < 8$

7. **(C)** $-8 \geq 2a,$ or $a \leq -4$

8. **(A)** $9(2y + 1) \geq 2y - 9$

$18y + 9 \geq 2y - 9$

$16y \geq -18,$ or $y \geq -9/8$

9. **(D)** $5(5 - 3a) \geq 2(2a + 3)$

$25 - 15a \geq 4a + 6$

$19 \geq 19a,$ or $a \leq 1$

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10. **(B)** Multiply through by 30, which is the LCM of the denominators:

$15y - 10 < 12y + 5$
$3y < 15$, or $y < 5$

11. **(C)** Multiply through by 15:

$5(n + 1) - 15 > 3(n + 4)$

$5n - 10 > 3n + 12$

$2n > 22$, or $n > 11$

12. **(A)** $y - 1 > 4y + 8$

$-9 > 3y$, or $y < -3$

13. **(D)** Multiply through by 15:

$9m + 45 > 10m - 60$

$105 > m$, or $m < 105$

14. **(C)** This is a somewhat tricky question, which, if answered hastily, may lead to the wrong choice of option (A). However, it should also be noted that y must be positive, i.e. $y > 0$ and $y < \frac{1}{2}$.

15. **(D)** $\frac{2y + 4}{6y + 7} < 1$

$2y + 4 < 6y + 7$

$4y > -3$, or $y > -\frac{3}{4}$.

However, since we are also told that $g(y) > 0$, $6y + 7 > 0$ and $y > -7/6$. The range of y which satisfies both conditions is $y > -7/6$.

16. **(C)** Multiply through by 3:

$3(x - 3) < x$

$2x - 9 < 0$, $x < 4\frac{1}{2}$

17. **(A)** Multiply through by pqr:

$xqr + xpr + xpq > pqr$

$x(pq + qr + pr) > pqr$

$x > \frac{pqr}{pq + qr + pr}$

18. **(D)** $0 < y + 3$ and $y + 3 \leq 2(y - 1)$

$-3 < y$ and $5 \leq y$

These two conditions are both satisfied for $y \geq 5$.

19. **(C)** $-1 < 2z - 5$ and $2z - 5 \leq 5$

$z > 2$ and $z \leq 5$, $\Rightarrow$ $2 < x \leq 5$

i.e. $z = 3, 4$ or 5.

20. **(A)** $-1 < 5 - 2y$ and $5 - 2y \leq 7$

$2y < 6$ and $2y \geq -2$

$y < 3$ and $y \geq -1$, or $-1 \leq y < 3$

i.e. $y = -1, 0, 1, 2$

21. **(B)** $-11 \leq 4 - 3m$ and $4 - 3m \leq 28$
$3m \leq 4 + 11$ and $4 - 28 \leq 3m$
$m \leq 5$ and $m \geq -8$
i.e. $-8 \leq m \leq 5$

22. **(C)** **Either** $a - 3 \leq 0$ and $a - 4 \geq 0$

$a \leq 3$ and $a \geq 4$

(No value of a satisfies both conditions)

or $(a - 3) \geq 0$ and $(a - 4) \leq 0$

$a \geq 3$ and $a \leq 4$, i.e. $3 \leq a \leq 4$

23. **(A)** $2y^2 + 9y - 35 < 0$

$(2y - 5)(y + 7) < 0$

Either $2y - 5 < 0$ and $y + 7 > 0$

$y < \frac{5}{2}$ and $y > -7$, i.e. $-7 < y < \frac{5}{2}$

or $2y - 5 > 0$ and $y + 7 < 0$

$y > \frac{5}{2}$ and $y < -7$ (not feasible)

24. **(D)** Expand to give $3 + n^2 > 2$, or $n^2 > -1$

This is true for all real values of n, for which n^2 is positive.

25. **(C)** $x^2 - 3x - 18 > 0$

$(x - 6)(x + 3) > 0$

Either $(x - 6) > 0$ and $x + 3 > 0$
$x > 6$ and $x > -3$, i.e. $x > 6$
or $(x - 6) < 0$ and $(x + 3) < 0$
$x < 6$ and $x < -3$, i.e. $x < -3$

Hence $x < -3$ or $x > 6$.

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26.(D) It is necessary to consider two cases

(i) $0 > y > -3$ for which $y + 3$ is positive, and

(ii) $y < -3$ for which $y + 3$ is negative.

Case (i):

Multiply both sides by $y + 3$ and *retain* inequality sign:

$(y + 1)(y + 3) > 3$, or $y^2 + 4y > 0$

i.e. $y(y + 4) > 0$.

Since $y < 0$, $y + 4 < 0$ or $y < -4$, which is not feasible since it lies outside the range $0 > y > -3$.

Case (ii):

Multiply both sides by $y + 3$ and *reverse* inequality sign.

$(y + 1)(y + 3) < 3$, or $y^2 + 4y < 0$

$y(y + 4) < 0$

Since $y < 0$, $y + 4 > 0$, or $y > -4$

Hence the solution is $y < -3$ and $y > -4$, i.e.

$-4 < y < -3$

27.(A) $12y^2 - y - 1 < 0$

$(4y + 1)(3y - 1) < 0$

Either $(4y + 1) < 0$ and $(3y - 1) > 0$

i.e. $y < -\frac{1}{4}$ and $y > \frac{1}{3}$ (not feasible)

or $4y + 1 > 0$ and $3y - 1 < 0$

$y > -\frac{1}{4}$ and $y < \frac{1}{3}$, i.e.

$-\frac{1}{4} < y < \frac{1}{3}$

28.(B) This means all numbers between -1 and 3, including -1 but excluding 3.

29.(A) All numbers between -3 and 5, including 5 but excluding 3.

30.(D) All numbers less than and including -3, and all numbers greater than but excluding 1.

31.(C) These are points common to the regions

(i) on or below the line $y + 2x = 5$, i.e. $y + 2x \le 5$,

(ii) on or above the x-axis, i.e. $y \ge 0$, and

(iii) on or to the right of the y-axis, i.e. $x \ge 0$.

32.(C) Since all the regions lie above the line $y = x + 2$, the inequality $y \ge x + 2$ applies to all of them. Also $x \ge 0$ for regions II, III and IV. However, $0 \le y \le 1$ applies only to region III.

33.(B) These are points lying

(i) below or on $y + x = 5$, i.e. $y + x \le 5$

(ii) above or on the x-axis, i.e. $y \ge 0$, and

(iii) on or below the line $y = 2x$, i.e. $y \le 2x$

34.(A) These are points lying

(i) above or on $y + x = 5$, i.e $y + x \ge 5$

(ii) on or below the line $y = 6$, i.e. $y \le 6$, and

(iii) above or on the line $y = 2x$, i.e. $y \ge 2x$

35.(A) These are points lying

(i) on or below the line $y + x = 3$, i.e $y + x \le 3$,

(ii) on or above the line $y = \frac{1}{2}x$, i.e. $y \ge \frac{1}{2}x$, or $2y - x \ge 0$, and

(iii) on or to the right of the line $x = -2$, i.e. $x \ge -2$, or $x + 2 \ge 0$.

36. (D)These are points lying

(i) on or below $y + x = 3$, i.e. $y + x \le 3$, and

(ii) on or below $y = \frac{1}{2}x$, i.e. $y \le \frac{1}{2}x$, or $2y - x \le 0$.

37. (B)These are points lying

(i) on or below $2y = 3x$, i.e. $2y \le 3x$, or $2y - 3x \le 0$ and

(ii) on or above the line $2y = 6 - 3x$, i.e. $2y \ge 6 - 3x$, or $3x + 2y \ge 6$.

38.(B) These are points lying

(i) on or below $2y = 3x$, i.e. $2y \le 3x$ or $2y - 3x \le 0$,

(ii) on or below $2y + 3x = 6$, i.e $2y + 3x \le 6$

(iii) on or above the x–axis, i.e. $y \ge 0$.

39.(B) $f(x) > 0$ in the regions lying above the x-axis, i.e for values of x less than -1, and those lying between 2 and 4.

40.(C) $f(x) \le 0$ for the points lying on or below the x–axis, i.e. for values of x lying between -1 and 2, including -1 and 2; and for x greater than or equal to 4.

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12 Sequences and Series

12.1 Definitions

- A **sequence** is an ordered list of terms in which each succeeding term can be obtained from the previous term by a simple rule or logic, e.g.

(i) $-20, -15, -10, -5, 0, 5, 10, \ldots$
(succeeding term = previous term +5).

(ii) $-16, -4, -1, -{}^1/_4, -{}^1/_{16}, \ldots$
(succeeding term = previous term x ${}^1/_4$).

(iii) $-16, 4, -1, {}^1/_4, -{}^1/_{16}, \ldots$
(succeeding term = previous term x $-{}^1/_4$).

- A **series** is the summation of all the terms of a given sequence.
- When there is a limit to the sequence, the summation gives a **finite series**.
- If there is no limit to the sequence, the summation results in an **infinite series.**
- Finite series always have a finite sum, but it is not always possible to obtain the sum of an infinite series.

12.2 Arithmetic Progression

- A sequence in which there is a **common difference**, d, between consecutive terms is called an arithmetic sequence or **arithmetic progression (A.P.)**.
- If the first term is a, then the terms of the sequence are $a,\ a + d,\ a + 2d, \ldots, a + (n-1)d$.
- From above, the ***n*th term**, t_n , of an A.P. is

$$t_n = a + (n-1)d$$

- Example (i) under section (12.1) is an A.P. with $a = -20$ and $d = 5$. The 7th term in that sequence is $t_7 = -20 + (7-1) \times 5 = -20 + 6 \times 5 = 10$.

- The sum , S_n , of the first n terms of an A.P. is given by

$S_n = a + (a+d) + (a+2d) + \ldots + [a + (n-1)d]$

This is simplified to give:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

or

$$S_n = \frac{n}{2}(a + t_n)$$

EXAMPLE 12 - 1

The second and sixth terms of an A.P. are 10 and 34. Find the (a) common difference (b) first term. (c) *n*th term, (d) 100th term, and (e) sum of the first 100 terms.

Solution

Let a = first term

(a) Second term is

$$a + d = 10 \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

and the sixth term is

$$a + 5d = 34 \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

(ii) – (i): $4d = 24$, or $d = 6$

(b) From eq. (i) above, $a = 10 - d = 10 - 6 = 4$

(c) *n*th term $= a + (n-1)d$
$= 4 + (n-1) \times 6 = 6n - 2$

(d) 100th term $= a + 99d = 4 + (99 \times 6)$
$= 598$

(e) $S_n = \frac{n}{2}(a + t_n)$

$S_{100} = {}^{100}/_2(4 + 598) = 30{,}100$

DRILL 1

Find the 10th term and the sum of the first 10 terms in the following sequences

(a) $-10, -5, 0, 5, \ldots$
(b) $1, 2, 3, 4, \ldots$
(c) ${}^1/_4, {}^3/_4, {}^5/_4, {}^7/_4, \ldots$
(d) $10, 8, 6, 4, \ldots$

12.3 Geometric Progression

- A **geometric progression (G.P.)** is a sequence in which there is *a* **common ratio, *r*,** between any two consecutive terms, which is constant. In other words, each term is the product of the preceding term and the common ratio.
- If the first term is a, then the terms of the entire sequence are: $a,\ ar,\ ar^2,\ ar^3, \ldots ar^{n-1}$. The ***n*th** term of a G.P. is given by

$$t_n = ar^{n-1}$$

- Examples (ii) and (iii) under section (12.1) are geometric progressions. For (ii): $a = -16$ and $r = {}^1/_4$.
- The sum, S_n, of the first n terms of a G.P. is given by $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$, which simplifies to

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

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or $$S_n = \frac{a(r^n - 1)}{r - 1}$$

- The first version is particularly suitable for finding S_n for $r < 1$ and the second version for $r > 1$, although either equation may be used.

EXAMPLE 12 – 2

The third and fifth terms of geometric progression are 27 and 243. Find the (a) common ratio, (b) first term, and (c) sum of the first six terms.

Solutions

(a) 1st term ..... $= a$
3rd term is..... $ar^2 = 27$ .....(i)
5th term is..... $ar^4 = 243$ .....(ii)
(ii) ÷ (i): $r^2 = 243 \div 27 = 9$
$r = \pm 3$

(b) From (i), $a = 27/r^2 = 27/9 = 3$.
The progression is either 3, 9, 27, 81, 243, ...
or 3, –9, 27, –81, 243, ...

(c) $S_n = \frac{a(r^n - 1)}{r - 1}$

For $r = 3$, $S_6 = \frac{3(3^6 - 1)}{3 - 1} = 1{,}092$

For $r = -3$, $S_6 = \frac{3[1 - (-3)^6]}{1 - (-3)} = \frac{3(1 - 3^6)}{1 + 3} = -546$

EXAMPLE 12 – 3

Find (a) the number of terms in the G.P.
$5 + 15 + 45 + ... + 1215$
(b) the sum of the G.P.

Solutions

(a) 1st term, $a = 5$; 2nd term, $ar = 15$
$r = 15/a = 3$;
Last term = nth term = $t_n = ar^{n-1}$
$1215 = 5(3)^{n-1}$
$243 = 3^{n-1}$
$3^5 = 3^{n-1}$
$n - 1 = 5$, or $n = 6$

DRILL 2

Find the common ratio, sixth term and the sum of the first six terms of the following sequences:
(a) –10, 20, –40, ...
(b) $^1/_3, ^1/_6, ^1/_{12}$, ...
(c) $1, 3a, 9a^2$, ...
(d) 2, 6, 18, ...

- If $|r| < 1$, $r^n \to 0$ as $n \to \infty$. The sum of an **infinite** G.P. series is thus given by

$$S_\infty = \frac{a(1 - r^\infty)}{1 - r}, \text{ or}$$

$$S_\infty = \frac{a}{1 - r}$$

EXAMPLE 12 – 4

Find the sum of the infinite series
$\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \cdots$

Solution

This is an infinite G.P. with $a = ^1/_2$ and $r = ^1/_3$.
Sum to infinity, $S_\infty = \frac{a}{1 - r}$
$= \frac{^1/_2}{1 - ^1/_3}$
$= ^3/_4$.

DRILL 3

Find the sum of the following infinite G.P.
(a) $1 - ^1/_2 + ^1/_4 - ...$
(b) $0.8 + 0.08 + 0.008 + ...$
(c) $1 + x + x^2 + x^3 + ...$, for $|x| < 1$
(d) $1 - y^2 + y^4 - y^6 + ...$, for $|y| < 1$

ANSWERS TO DRILLS

1. (a) 35, 125 (b) 10, 55
(c) $\frac{19}{4}$, 25 (d) –8, 10

2. (a) –2, 320, 210
(b) $\frac{1}{2}, \frac{1}{96}, \frac{21}{32}$
(c) $3a, 243a^5, \frac{729a^6 - 1}{3a - 1}$
(d) 3, 486, 728

3. (a) $\frac{2}{3}$ (b) $\frac{8}{9}$
(c) $\frac{1}{1 - x}$ (d) $\frac{1}{1 + y^2}$

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SSCE/JME–TYPE OBJECTIVE QUESTIONS

1. The 8th term of the A.P. –4, –2, 0 .... is
   A. 12 **B.** 10 **C.** –10 **D.** 8

2. The first term of an A.P. is –1 and the common difference is 0.5. The 6th term is
   A. 0.5 **B.** 1.0 **C.** 1.5 **D.** 2.5

3. The first term of an A.P. is equal to one–half the common difference, d. The 8th term of the A.P. is
   A. $4d$ **B.** $6d$ **C.** $13d/2$ **D.** $15d/2$

4. The 23rd term of the sequence 4, 7, 10, ... is
   A. 67 **B.** 70 **C.** 73 **D.** 92

5. The nth term of the A.P. 10, 3, –4, ... is
   A. $17-7n$ **B.** $18+7n$ **C.** $18-7n$ **D.** $16-7n$

6. The 11th term of the A.P. 15, 9, 3, –3,... is
   A. 75 **B.** 51 **C.** –45 **D.** –51

7. The 1st and the 11th terms of an A.P. are –5 and 20 respectively. The common difference is
   A. $2^1/_2$ **B.** 3 **C.** $3^1/_2$ **D.** 4

8. The 5th and 10th terms of an A.P. are 0 and 10 respectively. the 20th term is
   A. 20 **B.** 30 **C.** 36 **D.** 40

9. The 2nd and 9th terms of an A.P. are 391 and 328 respectively. the 7th term is
   A. 400 **B.** 373 **C.** 355 **D.** 346

10. Find the values p and q in the arithmetic progression $-12, p, q, 18, ...$
    A. 1, 11 **B.** –1, 8 **C.** –2, 8 **D.** 0, 9

11. The first winner in a lottery gets N2,000 while subsequent winners get N50 less than the preceding winner. The winner in the 20th position gets
    A. N950 **B.** N1,000 **C.** N1,050 **D.** N1,100

12. The first term of an A.P. is 19. If the difference between the 3rd and 6th terms is 18, the 10th term is
    A. 67 **B.** 73 **C.** 79 **D.** 85

13. The heights of 10 pupils form an arithmetic sequence. If the shortest pupil is 1.1 m and the tallest is 1.55 m, the common difference is
    A. 5.0 cm **B.** 5.5 cm **C.** 50.0 cm **D.** 55.0 cm

14. The sum of the 1st and 2nd terms of an A.P. is 8 and the 10th term is 38. The sum of the 7th and 8th terms is
    A. 30 **B.** 48 **C.** 56 **D.** 64

15. The first term of an A.P. is –11 and the 5th term is 1. The sum of the first term eight terms is
    A. –4 **B.** 0 **C.** 4 **D.** 12

16. The first term of an A.P. is 6 and the fifth term is 18. Find the number of terms in the series having a sum of 162.
    A. 21 **B.** 15 **C.** 12 **D.** 9

17. The sum of the first 21 terms of the progression –18, –15, –12,.... is
    A. 441 **B.** 336 **C.** 270 **D.** 252

18. The 4th term of an A.P. is 6. If the sum of the 8th and 9th terms is –72, the common difference is
    A. $-9^1/_3$ **B.** –8 **C.** $-6^2/_3$ **D.** 4

19. The sum of the first eight terms of the A.P. $\log x, \log x^2, \log x^3, ...$ is
    A. $\log x^8$ **B.** $\log x^9$ **C.** $\log x^{36}$ **D.** $\log x^{72}$

20. The 10th and 15th terms of an A.P. are –5 and $-7^1/_2$ respectively. The sum of the first 20 terms is
    A. 60 **B.** –105 **C.** $-52^1/_2$ **D.** –20

21. A G.P. has a common ratio of 3. If the difference between the 1st term and 5th term is 160, the 5th term is
    A. 120 **B.** 160 **C.** 162 **D.** 180

22. The 2nd and 5th terms of a G.P. are $^2/_3$ and $^1/_{12}$ respectively.The 1st term is
    A. $^1/_2$ **B.** $1^1/_3$ **C.** $1^2/_3$ **D.** $2^1/_3$

23. The seventh term of the G.P $^{16}/_9, -^8/_3, 4, ...$ is
    A. $-^9/_2$ **B.** 16 **C.** $-^{32}/_3$ **D.** $^{81}/_4$

24. If $^4/_3, m, 1, n$ form a G.P., the product of m and n is
    A. 1 **B.** $^3/_4$ **C.** $^9/_{16}$ **D.** $^4/_3$

25. The values of p and q in the G.P. $10, p, q, -^5/_4$ are, respectively
    A. $5, -^5/_2$ **B.** $-5, ^5/_2$ **C.** $^5/_2, -5$ **D.** $-^5/_2, 5$

26. The seventh term in the progression 27, 81, 243,... is
    A. 3^5 **B.** 3^7 **C.** 3^9 **D.** 3^{11}

27. A G.P. has 6 and 24 as its 2nd and 4th terms respectively. The sum of the first four terms is
    A. 40 **B.** 45 **C.** 60 **D.** 66

28. The sum of the first five terms of the G.P. 3, –6, 12, is
    A. 93 **B.** 31 **C.** –33 **D.** 33

29. The 1st and 4th terms of a G.P. are –3 and 24 respectively. The sum of the first four terms is
    A. 15 **B.** –15 **C.** 45 **D.** –45

30. The common ratio in the G.P. log 2, log 4, log 16, ... is

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A. 2 B. 3 C. 4 D. log 2

31. The sum of the first two terms of a G.P. is p and the sum of the last two terms is q. If the G.P. has n terms, the common ratio is

A $\frac{p}{q}$ B $\frac{q}{p}$

C. $\left[\frac{p}{q}\right]^{\frac{1}{n-2}}$ D $\left[\frac{q}{p}\right]^{\frac{1}{n-2}}$

32. The sum to infinity of the progression $1-x+x^2-x^3+\ldots$ is

A. $\frac{1}{1+x}$ B. $\frac{1}{1-x}$ C. $\frac{1}{x-1}$ D. $\frac{1}{x}$

33. Find the sum to infinity of the sequence $3, 2, {}^4/_3, {}^8/_9, \ldots$

A. 18 B. 10 C. 9 D. 8

34. Find the sum to infinity of the series $1 + {}^7/_{10} + ({}^7/_{10})^2 + ({}^7/_{10})^3 + \ldots$

A. $2{}^7/_{10}$ B. $3{}^1/_3$ C. 5 D. 7

35. If the nth term of a sequence is given by $2 \times 3^{n-2}$, the first three terms of the sequence are

A. $3/2, 0, 6$ B. $3/2, 2, 2/3$
C. $2/3, 2, 6$ D. $2/3, 1, 3/2$

36. The nth term of a sequence is $(-1)^{n-1} \cdot 2^{n-2}$ The sum of the first three terms of the sequence is

A. $4{}^1/_2$ B. $3{}^1/_2$ C. $1{}^1/_2$ D. $-3{}^1/_2$

37. If the nth term of a sequence is given by $(-2)^{1-n}$, the fourth term of the sequence is

A. ${}^1/_{16}$ B. -8 C. ${}^1/_8$ D. $-{}^1/_8$

38. The nth term of the sequence $-1, {}^3/_4, -{}^3/_5, {}^1/_2, \ldots$ is given by

A. $\frac{-3}{n+2}$ B. $(-1)^{n+1} \cdot \frac{3}{n+2}$ C. $\frac{n}{1-2n}$ D. $(-1)^n \cdot \frac{3}{n+2}$

39. The nth term of the sequence 6, 10, 15, 21,... is

A. $\frac{n(n+2)}{2}$ B. $\frac{(n+2)(n+3)}{2}$
C. $(n+1)(n+2)$ D. $2n(n+2)$

40. The sum of the first n terms of a sequence is given by $S_n = n^2$. the nth term of the sequence is

A. $2n-1$ B. $2n+1$ C. $1-2n$ D. $2n+3$

ANSWER KEY

1. B	8. B	15. A	22. B	29. A	36. C
2. C	9. D	16. D	23. D	30. A	37. D
3. D	10. C	17. D	24. A	31. D	38. D
4. B	11. C	18. A	25. B	32. A	39. B
5. A	12. B	19. C	26. C	33. C	40. A
6. C	13. A	20. B	27. B	34. B	
7. A	14. C	21. C	28. D	35. C	

SOLUTIONS TO OBJECTIVE QUESTIONS

1. (B) Common difference,
$d = -2 - (-4) = 2$, $a = -4$
$t_8 = a + 7d = (-4) + 7(2) = 10$

2. (C) $a = -1$, $d = 0.5$
$t_6 = a + 5d = (-1) + 5(0.5) = 1.5$

3. (D) $a = d/2$
$t_8 = a + 7d = d/2 + 7d = 15d/2$

4. (B) $a = 4$, $d = 7 - 4 = 3$
$t_{23} = a + 22d = 4 + 22(3) = 70$

5. (A) $a = 10$, $d = 3 - 10 = -7$

nth term $= t_n = a + (n-1)d$
$= 10 + (n-1)(-7)$
$= 17 - 7n$

6. (C) $a = 15$, $d = 9 - 15 = -6$
$t_{11} = a + 10d = 15 + 10(-6) = -45$

7. (A) $a = -5$ .....(i)
$a + 10d = 20$ .....(ii)
Substituting for a in (ii) gives
$-5 + 10d = 20$, or $d = 2.5$

8 (B) $t_5 = a + 4d = 0$.....(i)
$t_{10} = a + 9d = 10$.....(ii)
(ii) – (i): $5d = 10$, $d = 2$
From (i): $a = -4d = -4(2) = -8$
20th term $= a + 19d = -8 + 19(2) = 30$

9. (D) $t_2 = a + d = 391$ ..... (i)
$t_9 = a + 8d = 328$ .....(ii)
(ii) – (i): $7d = -63$, or $d = -9$
From (i) : $a = 391 - d = 391 - (-9) = 400$

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7th term = $a + 6d = 400 + 6(-9) = 346$

10. (C) Common difference
$= p - (-12) = q - p = 18 - q$
Hence $p + 12 = q - p \Rightarrow 2p - q = -12$....(i)
and $p + 12 = 18 - q \Rightarrow p + q = 6$.....(ii)
(i) + (ii) gives $3p = -6$, or $p = -2$
and from (ii): $q = 6 - p = 6 - (-2) = 8$

11. (C) This is an arithmetic sequence of the form
2,000, 1,950, 1,900, 1,850, ...
Hence $a = 2{,}000$ and $d = -50$
20th term = $a + 19d = 2000 + 19(-50)$
= N1,050

12. (B) $a = 19$
$t_3 = a + 2d = 19 + 2d$
$t_6 = a + 5d = 19 + 5d$
$t_6 - t_3 = (19 - 5d) - (19 + 2d) = 3d = 18$,
or $d = 6$
10th term = $a + 9d = 19 + a(6) = 73$

13. (A) The arithmetic sequence is of the form
1.1, . . . , 1.55
Hence $a = 1.1$ and $t_{10} = a + 9d = 1.55$
$1.1 + 9d = 1.55$, or $9d = 0.45$ and
$d = 0.05$ m, or 5.0 cm

14. (C) $t_1 + t_2 = a + (a + d) = 2a + d$
Hence $2a + d = 8$ .....(i)
$t_{10} = a + 9d = 38$. ..... (ii)
(ii) x 2: $2a + 18d = 76$ .....(iii)
(iii) – (i): $17d = 68$, or $d = 4$
From (i): $a = (8 - d)/2 = (8 - 4)/2 = 2$
$t_7 = a + 6d = 2 + 6(4) = 26$
$t_8 = a + 7d = 2 + 7(4) = 30$
$t_7 + t_8 = 26 + 30 = 56$

15. (A) $a = -11$ ..... (i)
$a + 4d = 1$ ..... (ii)
From (i) and (ii): $4d = 12$, or $d = 3$
$S_n = {}^n/_2 [2a + (n - 1)d]$
$S_8 = {}^8/_2 [2(-11) + 7(3)]$
$= 4(-22 + 21) = -4$

16. (D) $a = 6$ ..... (i)
$a + 4d = 18$ ..... (ii)
From (i) and (ii): $4d = 12$, $d = 3$
$S_n = n/2 [2(6) + (n - 1)(3)] = 162$
$12n + 3n^2 - 3n = 324$
$3n^2 + 9n - 324 = 0$
$(3n + 36)(n - 9) = 0$
$n = 9$ [Note that n *cannot* be negative]

17. (D) $a = -18$, $d = -15 - (-18) = 3$
$S_{21} = {}^{21}/_2[2(-18) + 20(3)] = 252$

18. (A) $a + 3d = 6$ .....(i)
8th term = $a + 7d$
9th term = $a + 8d$
$(a + 7d) + (a + 8d) = -72$, or
$2a + 15d = -72$ .....(ii)
Multiply (i) by 2 to give
$2a + 6d = 12$ .....(iii)
(ii) – (iii): $9d = -84$, or $d = -9^1/_3$

19. (C) The series can be written as
$\log x + 2\log x + 3\log x + ...$
Common difference,
$d = 2\log x - \log x = \log x$
$a = \log x$
$S_8 = {}^n/2 [2a + (n - 1)d]$
$= {}^8/_2 [2\log x + 7\log x]$
$= 36\log x = \log x^{36}$

20. (B) 10th term = $a + 9d = -5$ ..... (i)
15th term = $a + 14d = -{}^{15}/_2$..... (ii)
(ii) – (i): $5d = -{}^5/_2$, or $d = -{}^1/_2$
From (i): $a = -5 - 9d = -5 - 9(-{}^1/_2) = -1/2$
$S_{20} = {}^{20}/_2 [2(-{}^1/_2) + 19(-{}^1/_2)] = -105$

21. (C) 1st term = a, common ratio, $r = 3$
5th term = $ar^4 = a(3)^4 = 81a$
5th term – 1st term = $81a - a = 80a = 160$
$a = 2$
5th term = $2(3)^4 = 162$

22. (B) $t_2 = ar = {}^2/_3$ ..... (i)
$t_5 = ar^4 = {}^1/_{12}$ ..... (ii)
(ii) ÷ (i) $r^3 = {}^1/_{12} \times 3/_2$, or $r = {}^1/_2$
From (i), 1st term, $a = \frac{2}{3r} = \frac{2}{3({}^1/_2)} = \frac{4}{3}$

23. (D) 1st term $a = {}^{16}/_9$
$r = \frac{-8/3}{16/9} = -\frac{3}{2}$
Seventh term = $ar^6 = \frac{16}{9}(-{}^3/_2)^6 = \frac{81}{4}$

24. (A) Common ratio, $r = {}^1/_m = {}^n/_1$
[from 2nd, 3rd and 4th terms]
Hence $mn = 1$

25. (B) Common ratio = $\frac{p}{10} = \frac{q}{p} = \frac{-5}{4q}$
$p^2 = 10q$ ..... (i)
$-5p = 4q^2$ .....(ii)
From (ii), $p = -4q^2/5$, which on substituting into (i) gives
$(-4q^2/5)^2 = 10q$
$\frac{16}{25} q^4 = 10q$
$q^3 = \frac{250}{10} = \frac{125}{8}$, or $q = \frac{5}{2}$
From (ii) $p = \frac{-4(25/4)}{5} = -5$

26. (C) $a = 27$, $r = 81/27 = 3$
Seventh term = $ar^6 = 27(3^6) = 3^3(3^6) = 3^9$

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27. (B) $t_2 = ar = 6$ ..... (i)
$t_4 = ar^3 = 24$ ..... (ii)
(ii) ÷ (i): $r^2 = 24/6 = 4$, or $r = 2$
From (i): $a = 6/2 = 3$

Sum of first n terms, $S_n = \frac{a(r^n - 1)}{r - 1}$

Sum of first 4 terms, $S_4 = \frac{3(2^4 - 1)}{2 - 1} = 45$

28. (D) $a = 3, r = -6/3 = -2$
$S_5 = \frac{a(r^5 - 1)}{r - 1} = \frac{3[(-2)^5 - 1]}{-2 - 1} = 33$

29. (A) $a = -3$
$t_4 = ar^3 = -3r^3 = 24$
$r^3 = -8$, or $r = -2$

Sum of first 4 terms, $S_4 = \frac{-3[(-2)^4 - 1]}{-2 - 1} = 15$

30. (A) The G.P can be written as
log 2, 2log 2, 4log 2, ...
Common ratio $= \frac{2\log 2}{\log 2} = 2$

31. (D) 1st term = a; 2nd term = ar
$(n-1)$th term = ar^{n-2}; nth term = ar^{n-1}
$a + ar = p$, or
$a(1 + r) = p$ ..... (i)
$ar^{n-2} + ar^{n-1} = q$, or
$ar^{n-2}(1 + r) = q$ ..... (ii)
(ii) ÷ (i): $r^{n-2} = q/p$

$$r = \left[\frac{q}{p}\right]^{\frac{1}{n-2}}$$

32. (A) This is a G.P. with $a = 1$ and $r = -x$
$S_\infty = \frac{a}{1 - r} = \frac{1}{1 + x}$

33. (C) This is a geometric sequence with $a = 3$ and $r = {}^2/_3$.
$S_\infty = \frac{a}{1 - r} = \frac{3}{1 - {}^2/_3} = 9$

34. (B) $a = 1, r = 7/10$
$S_\infty = \frac{1}{1 - {}^7/_{10}} = \frac{10}{3} = 3{}^1/_3$

35. (C) nth term = $2(3)^{n-2}$
1st term = $2(3)^{-1} = 2/3$ ..... ($n = 1$)
2nd term = $2(3)^0 = 2$ ..... ($n = 2$)
3rd term = $2(3)^1 = 6$ ..... ($n = 3$)

36. (C) nth term = $(-1)^{n-1} \cdot 2^{n-2}$
1st term = $(-1)^0 . 2^{-1} = {}^1/_2$
2nd term = $(-1)^1 . 2^0 = -1$
3rd term = $(-1)^2 . 2^1 = 2$
Sum of first three terms = ${}^1/_2 - 1 + 2 = 1{}^1/_2$

37. (D) nth term = $(-2)^{1-n}$
4th term ($n = 4$) is $(-2)^{-3} = -{}^1/_8$

38. (D) This is neither an A.P. nor a G.P
The quickest approach to solving this type of problem is to try out some of the given terms in each of the options provided.
Option (A) is ruled out because its 2nd term ($n = 2$) is negative.
Option (B) is ruled out because its 1st term ($n = 1$) is positive.
Option (C) is ruled out because its 2nd term is not equal to ${}^3/_4$.

39. (B) Option (A) is ruled out because its 1st term ($n = 1$) is not equal to 6.
All the four terms provided are consistent with option (B)

40. (A) Sum of first n terms = $S_n = n^2$
Sum of first $(n - 1)$ terms = $(n - 1)^2$
nth term = $S_n - S_{n-1} = n^2 - (n - 1)^2 = 2n - 1$.

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13 Lines, Triangles and Polygons

13.1 Intersecting and Parallel Lines

- **Angles** are formed at the point of intersection of two lines.
- Angles are defined according to their sizes as follows:
  (i) **acute** angle (less than 90^0)
  (ii) **right** angle (90^0)
  (iii) **obtuse** angle (between 90^0 and 180^0)
  (iv) **reflex** angle (between 180^0 and 360^0)

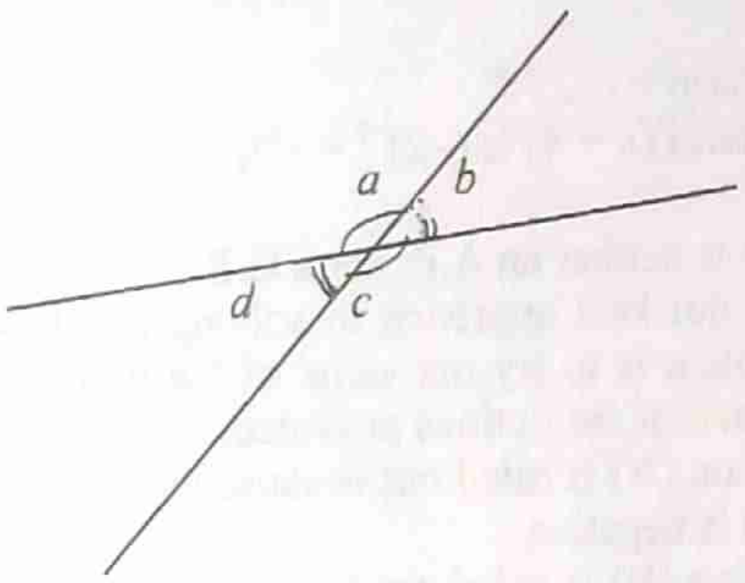


Fig. 13.1

- Referring to Fig 13.1:
  (i) *angles at a point* add up to 360^0
  $a + b + c + d = 360^0$
  (ii) *angles on a straight line* up to 180^0
  $a + b = 180^0 = c + d$
  (iii) *vertically opposite* angles are equal
  $a = c$ and $b = d$

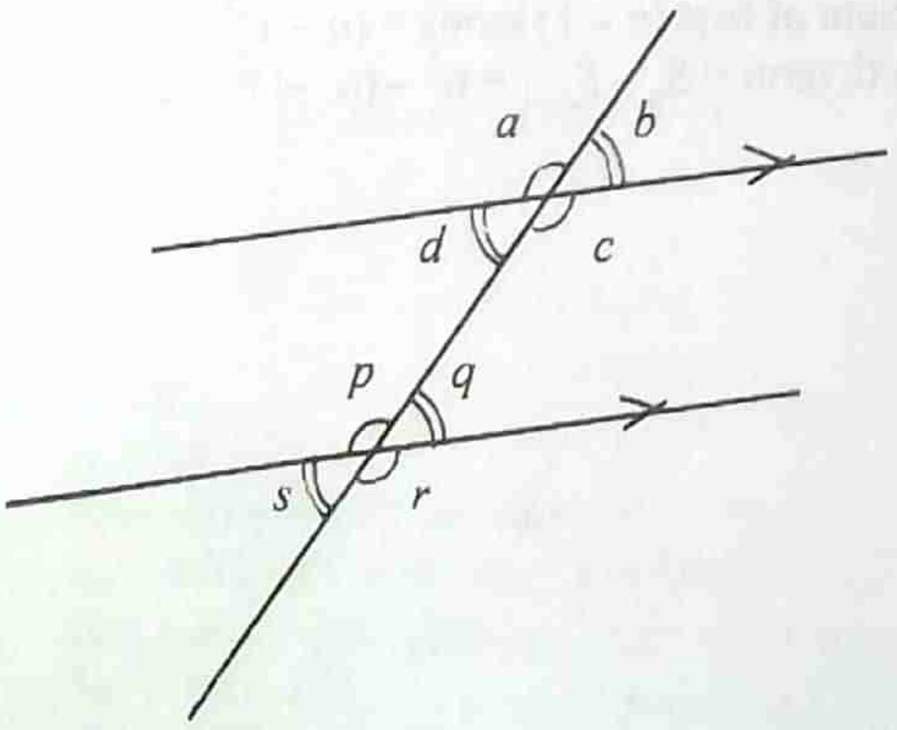


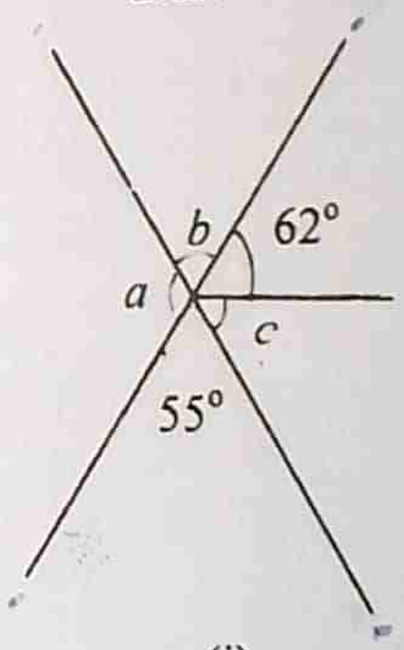
Fig. 13.2

- For two parallel lines intersected by a third line (*transversal*) (Fig. 13.2):
  (i) *alternate angles* are equal
  $c = p$ and $d = q$
  (ii) *corresponding angles* are equal
  $a = p$ and $c = r$
  (iii) two *interior angles* on the same side of the transversal add up to 180^0
  $d + p = 180^0 = c + q$

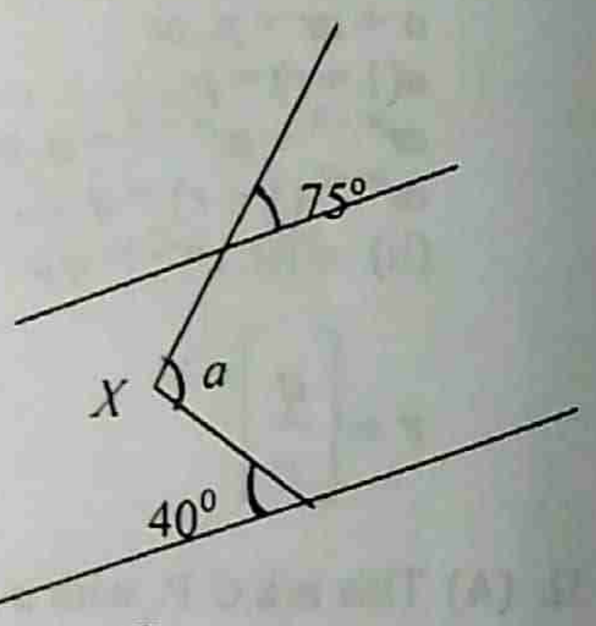
- Two angles x and y **complementary** if their sum is 90^0, i.e. if $x + y = 90^0$
- Two angles x and y are **supplementary** if their sum is 180^0 i.e. if $x + y = 180^0$

EXAMPLE 13 - 1

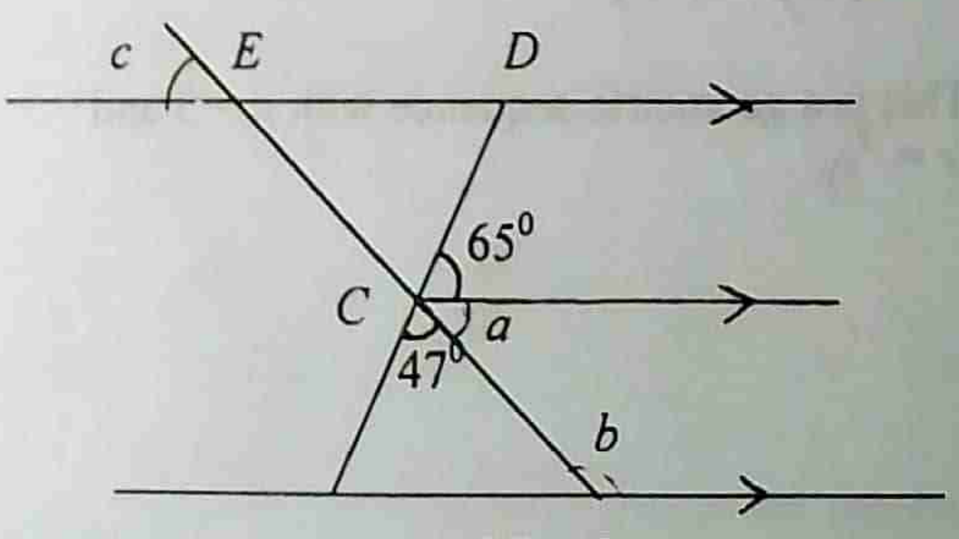
Determine the angles a, b, c in the following figures:



(i)



(ii)



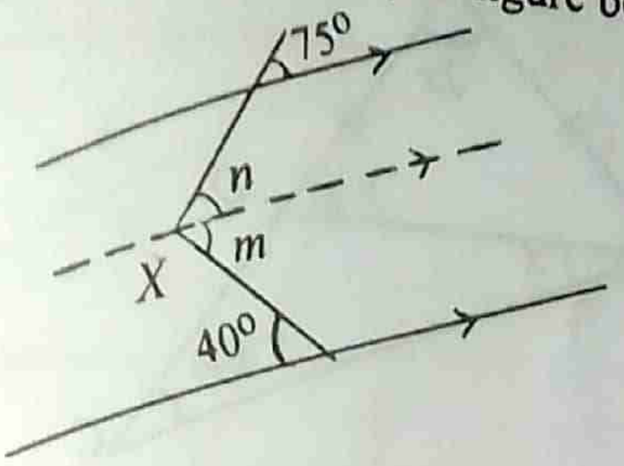
A (iii) B

Solutions

(i) $55^0 + c + 62^0 = 180^0$ (*angles on a straight line*)
$c = 180^0 - 62^0 - 55^0 = 63^0$
$b = 55^0$ (*vertically opposite angles*)
$a + b = 180^0$
$a = 180^0 - b = 180^0 - 55^0 = \mathbf{125^0}$

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(ii) Draw a line through X, parallel to the two given parallel lines (see figure below)

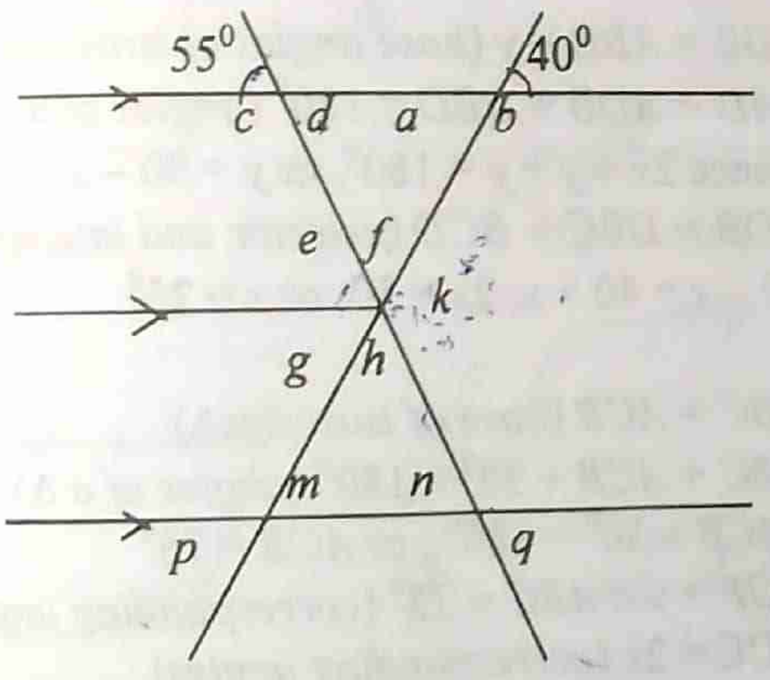


$m = 40^0$ (*alternate angles*);
$n = 75^0$ (*corresponding angles*)
$a = m + n = 40^0 + 75^0 = \mathbf{115^0}$

(iii) $65^0 + a + 47^0 = 180^0$ (*angles on a straight line*)
$a = 180^0 - 65^0 - 47^0 = 68^0$
$a + b = 180^0$ (*interior angles*)
$b = 180^0 - a = 180^0 - 68^0 = \mathbf{112^0}$
angle $DEC = a$ (*corresponding*)
$c = DEC$ (*vertically opposite*)
Hence $c = a = 68^0$

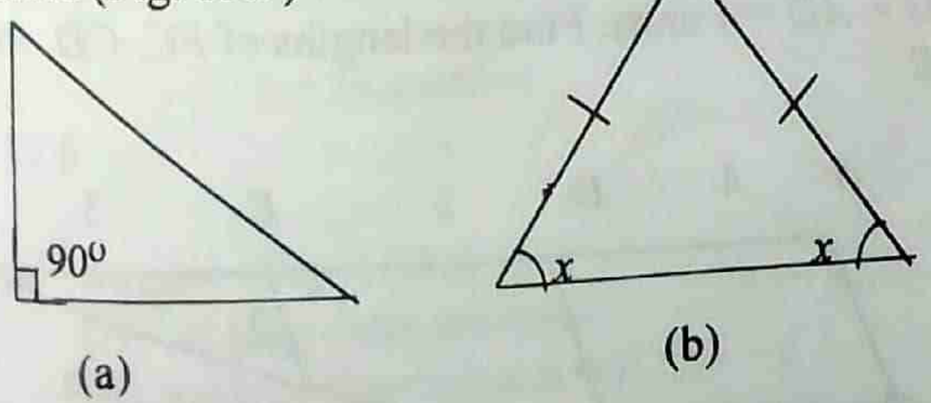
DRILL 1

Find the values of the angles $a, b, c, d, e, f, g, h, k, m, n, p, q$ in the figure below

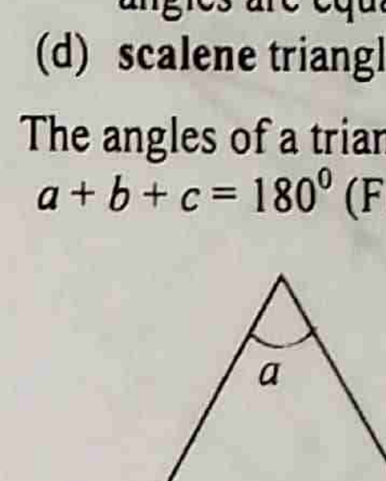


13.2 Triangles

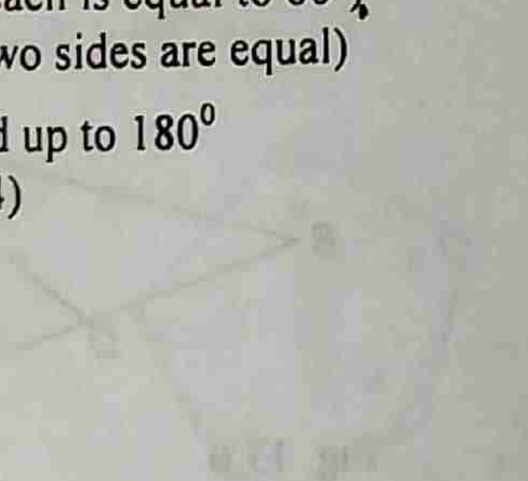
- A **triangle** is a figure bounded by three straight lines.
- Triangles are classified according to the following rules (Fig. 13.3)



(a) (b)



(c)



$p \neq q \neq r$
(d)

Fig. 13.3

- (a) **right-angled** triangle (one of the angles is 90^0)
  (b) **isosceles** triangle (two of the sides are equal; two of the angles facing the equal sides are equal
  (c) **equilateral** triangle (all sides are equal, all angles are equal and each is equal to 60^0)
  (d) **scalene** triangle (no two sides are equal)
- The angles of a triangle add up to 180^0
  $a + b + c = 180^0$ (Fig. 13.4)

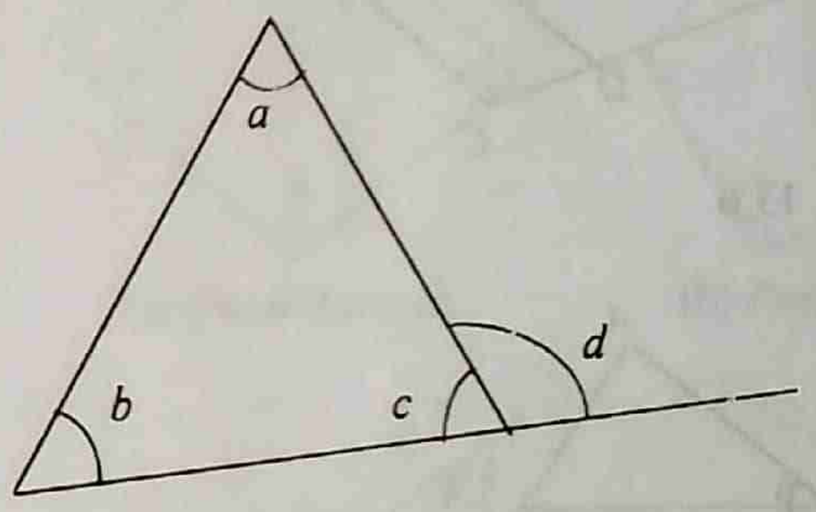


Fig. 13.4

- The exterior angle of a triangle is equal to the sum of the two interior opposite angles. In Fig. 13.4:
  $d = a + b$
- Two triangles are **congruent** if any of the following sets of properties of one are identical to those of the other:
  (i) two sides and included angle (*SAS*)
  (ii) two angles and one side (*AAS*)
  (iii) three sides (*SSS*)
- Two triangles are **similar** if they have the same shape but different sizes. This is true if all the angles of one are equal to those of the other. The ratio of corresponding sides are equal (Fig. 13.5), e.g. ΔABC and ΔPQR are similar if $\hat{A} = P$, $B = Q$ and $C = R$. Also
  $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

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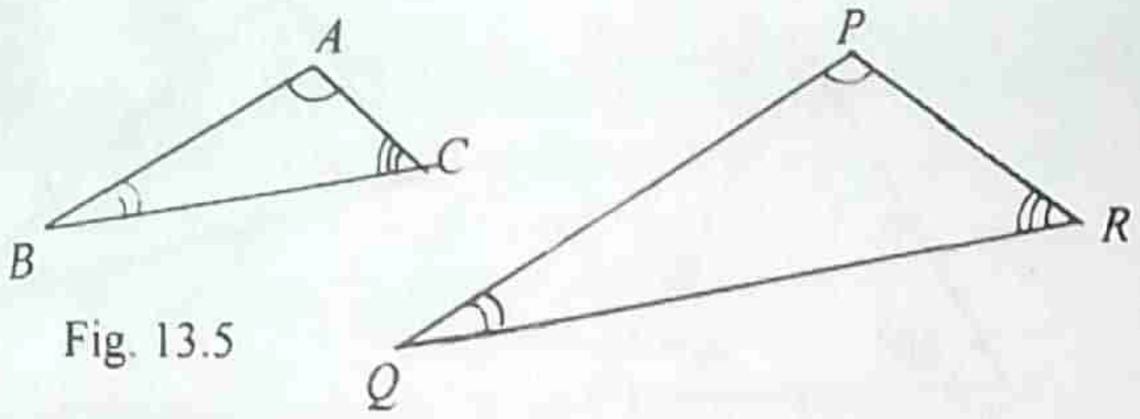


Fig. 13.5

- A line connecting two sides of a triangle, and is parallel to the third side, divides the two sides in the same ratio. In Fig. 13.6,

$$\frac{BD}{DC} = \frac{BE}{EA}$$

Also, since triangles ABC and EBD are similar,

$$\frac{BE}{AB} = \frac{BD}{BC} = \frac{DE}{AC}$$

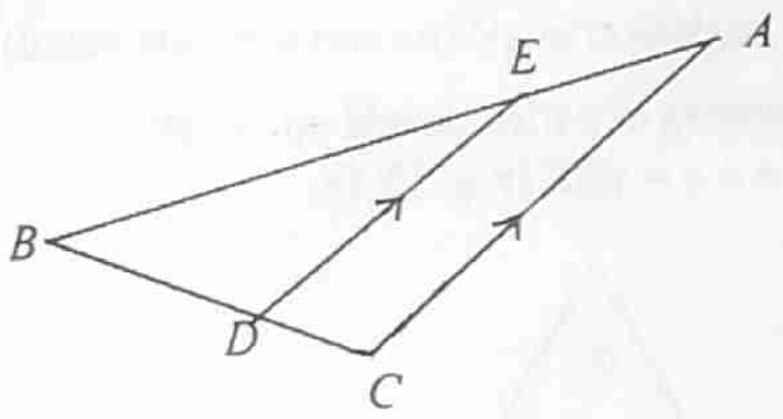


Fig. 13.6

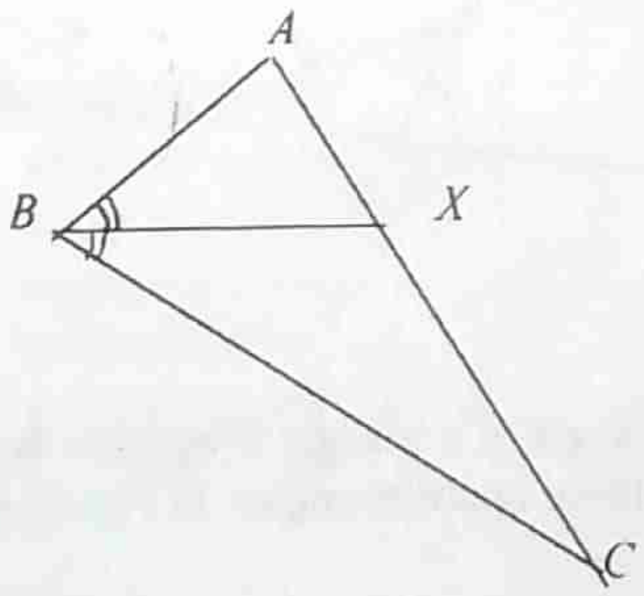


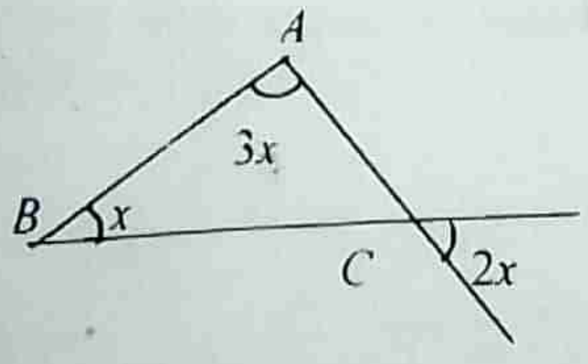
Fig. 13.7

If a line bisects one angle of a triangle, it divides the side opposite the angle in the ratio of the sides containing the angle. In Fig. 13.8:

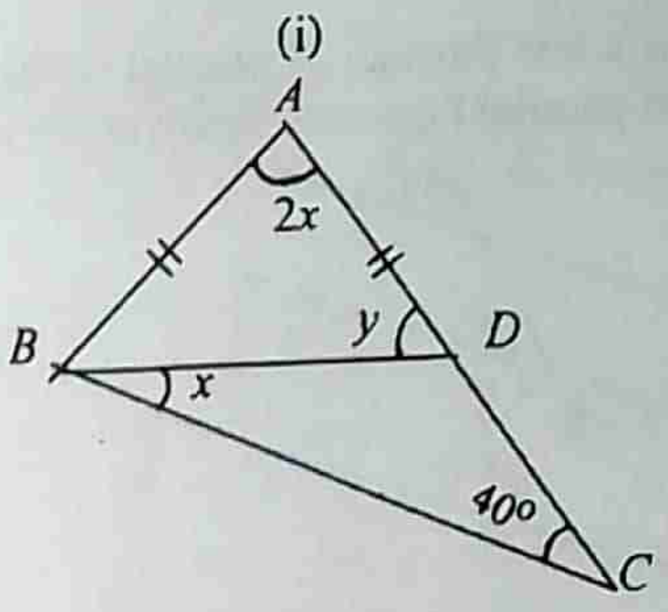
$$\frac{CX}{XA} = \frac{BC}{BA}$$

EXAMPLE 13.2

Determine the values of x and y in the following figures.

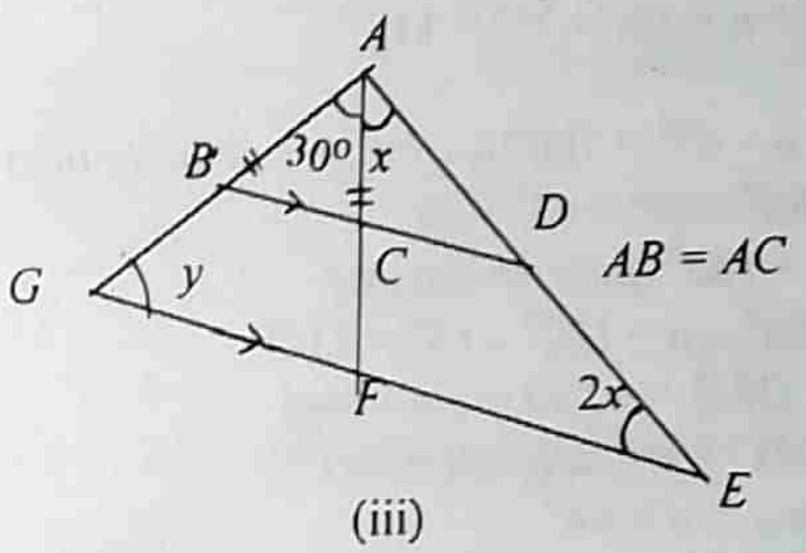


(i)



$AB = AD$

(ii)



(iii)

Solutions

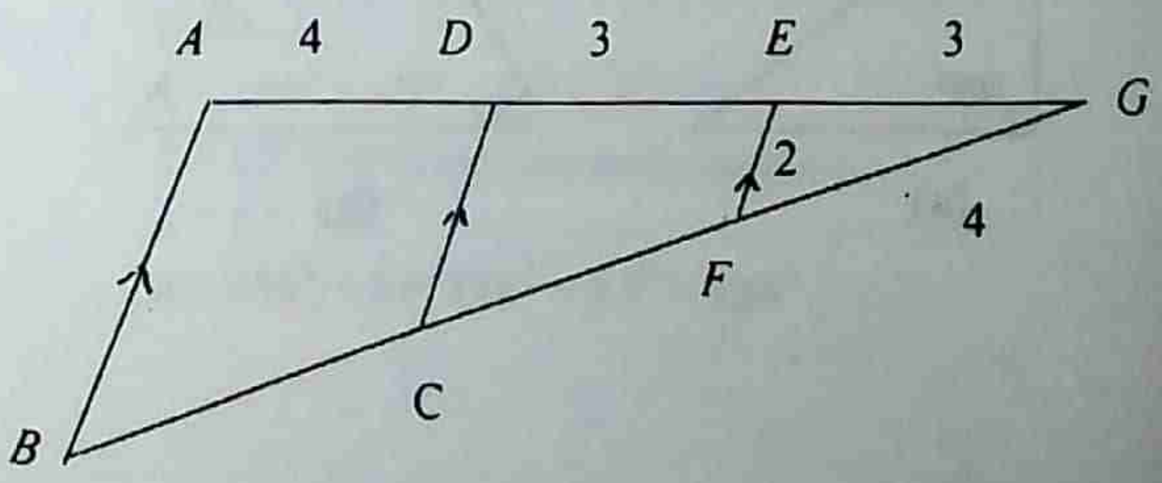
(i) $ACB = 2x$ (*vertically opposite*)
$B\hat{A}C + ABC + ACB = 180^0$ (*angles of a* Δ)
$3x + x + 2x = 180^0$
$6x = 180^0$ or $x = \mathbf{30^0}$

(ii) $ADB = ABD = y$ (*base angles of isosceles* Δ)
$BAD + ADB + ABD = 180^0$ (*angles of a* Δ)
Hence $2x + y + y = 180^0$, or $y = 90 - x$
$ADB = DBC + BCD$ (*exterior and interior angles*)
$90 - x = 40 + x$, $2x = 50$, or $x = \mathbf{25^0}$

(iii) $ABC = ACB$ (*base of isosceles* Δ)
$ABC + ACB + 30^0 = 180^0$ (*angles of a* Δ)
$2ACB + 30^0 = 180^0$, or $ACB = 75^0$
$BGF = y = ABC = 75^0$ (*corresponding angles*)
$ADC = 2x$ (*corresponding angles*)
$ACB = CAD + ADC$ (*exterior and interior angles*)
$75^0 = x + 2x = 3x$, or $x = \mathbf{25^0}$

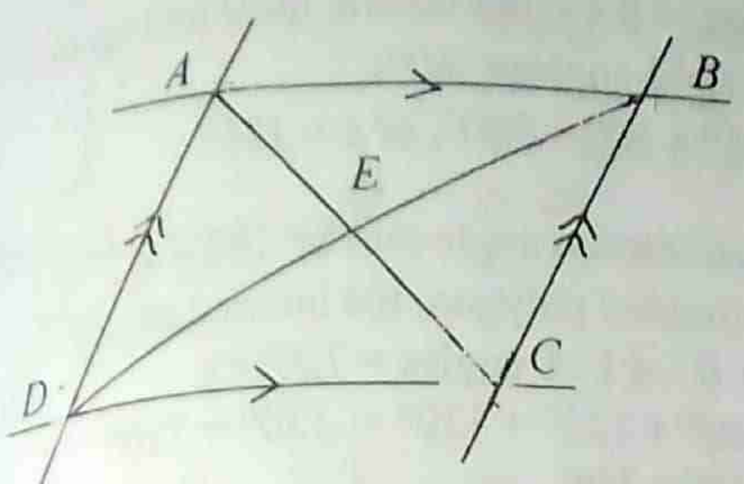
DRILL 2

In the figure below, $AB//DC//EF$, $DE = FG = 3$ units, $FG = AD = 4$ units. Find the lengths of FC, CD, BC and AB



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DRILL 3



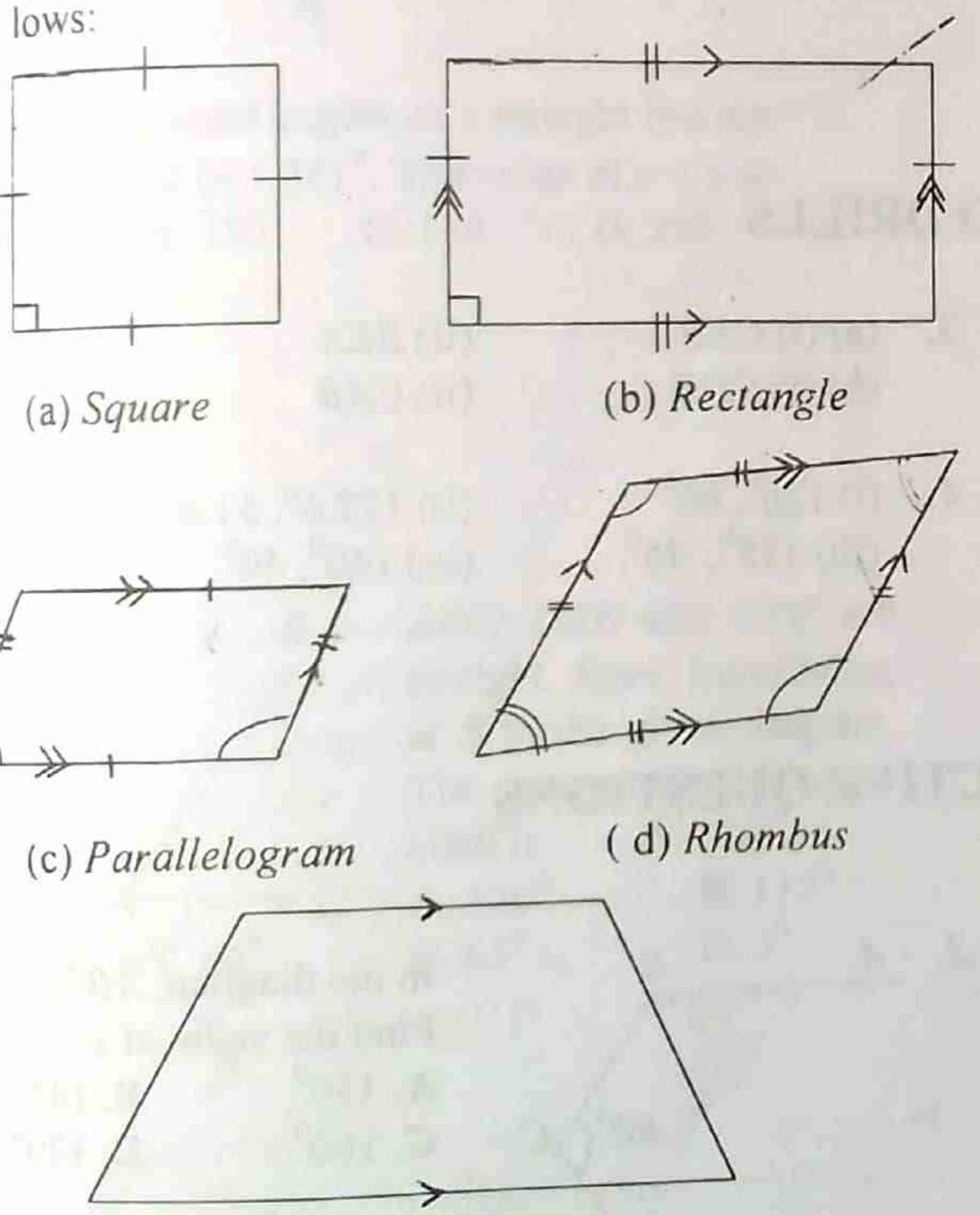
In the figure above, $AB//DC$ and $AD//BC$. Also $AB = DE$ and $AD = BC$. Which triangle is

(a) similar to (i) AED (ii) DEC ?

(b) congruent to (i) ABD (ii) ACD ?

13.3 Polygons

- A **polygon** is a plane figure which is bounded by many straight sides.
- A *regular polygon* has all sides equal and all angles equal.
- A **quadrilateral** is a four-sided figure, or a polygon with four sides. Quadrilaterals are classified as follows:



(a) *Square* (b) *Rectangle*

(c) *Parallelogram* (d) *Rhombus*

(e) *Trapezium*

Fig. 13.8

(a) **Square** (Fig. 13.8a)
- (i) all four sides are equal
- (ii) each of the angles is 90^0
- (iii) the diagonals are equal and they bisect each other.

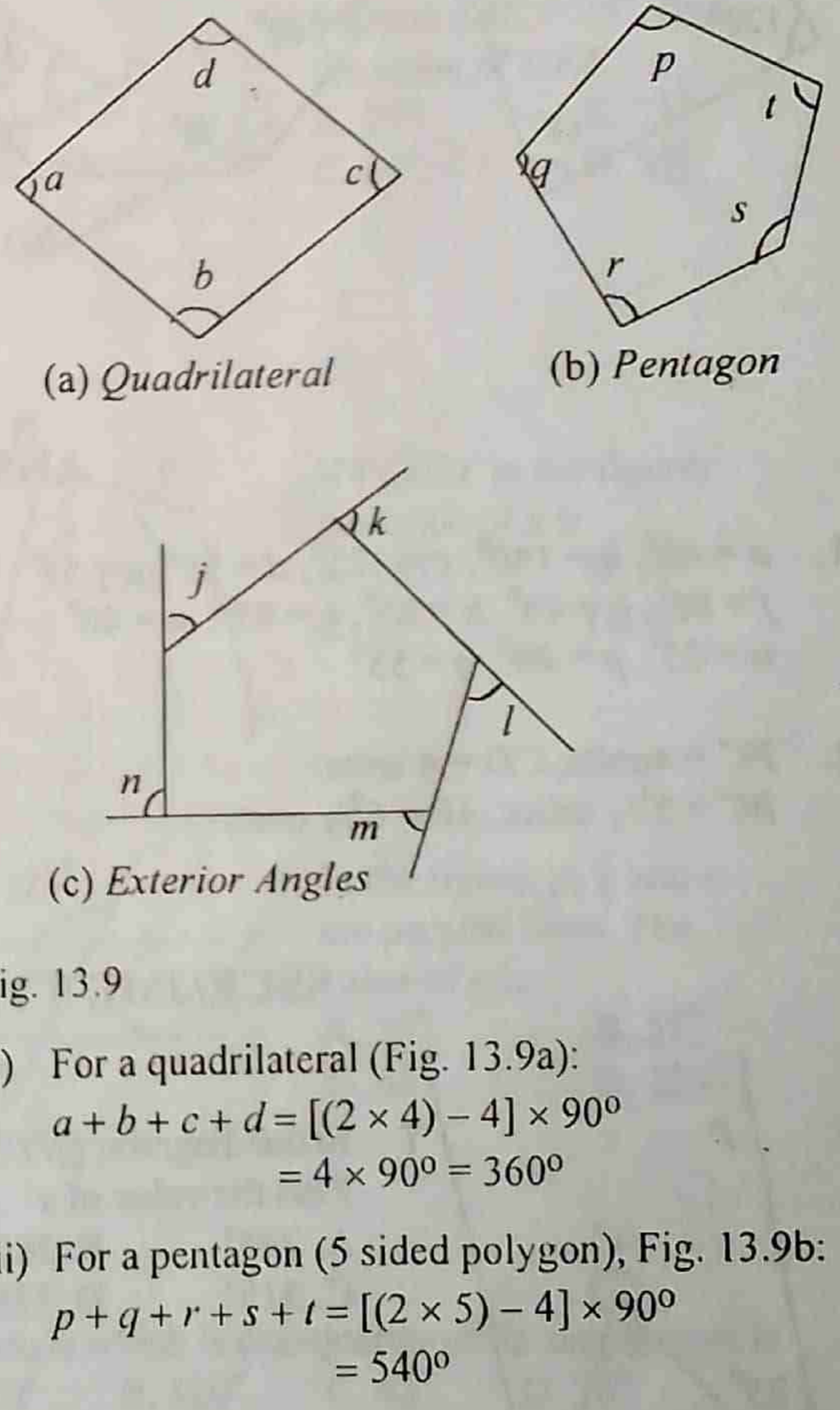
(b) **Rectangle** (Fig. 13.8b)
- (i) opposite sides are equal and parallel
- (ii) each of the angles is 90^0
- (iii) the diagonals are equal and they bisect each other.

(c) **Parallelogram** (Fig. 13.8c)
- (i) opposite sides are equal and parallel
- (ii) opposite angles are equal
- (iii) the diagonals bisect each other

(d) **Rhombus** (Fig. 13.8d)
This is a special type of parallelogram with all four sides equal

(e) **Trapezium** (Fig. 13.8e)
One pair of opposite sides are parallel while the other pair are not.

- The interior angles of an n-sided polygon add up to $2n - 4$ right angles, or $(2n - 4) \times 90^0$, e.g.



(a) *Quadrilateral* (b) *Pentagon*

(c) *Exterior Angles*

Fig. 13.9

(i) For a quadrilateral (Fig. 13.9a):

$$a + b + c + d = [(2 \times 4) - 4] \times 90^o$$
$$= 4 \times 90^o = 360^o$$

(ii) For a pentagon (5 sided polygon), Fig. 13.9b:

$$p + q + r + s + t = [(2 \times 5) - 4] \times 90^o$$
$$= 540^o$$

The exterior angles of a polygon add up to 4 right angles or 360^0 (Fig. 13.9c):

$j + k + l + m + n = 360^0$

Note that this is true regardless of the number of sides in the polygon.

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EXAMPLE 13-3

Determine the value of each interior angle in a 10-sided polygon.

Solution

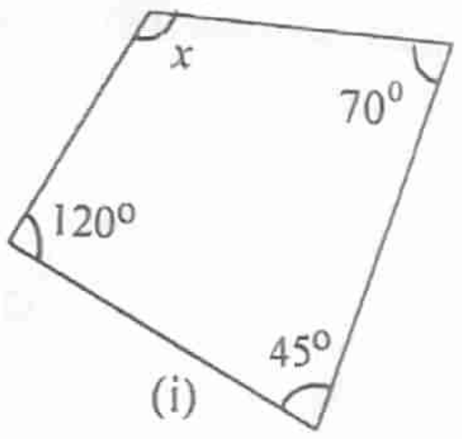
Let a = each interior angle. Since there are 10 such angles:

$10a = [(2 \times 10) - 4] \times 90^0 \quad = 16 \times 90^0$

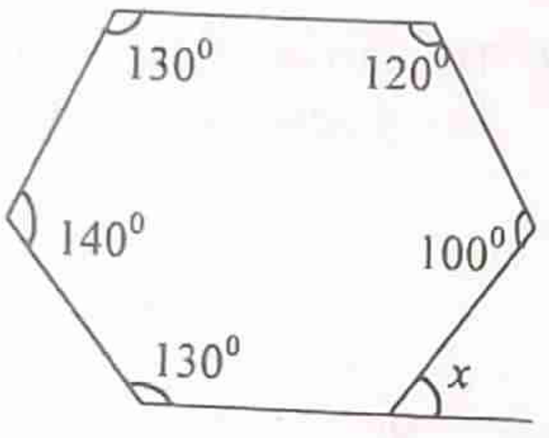
$a = 16 \times 90/10 = \mathbf{144^0}$

EXAMPLE 13-4

Determine the value of x in the two figures shown below:



(i)



(ii)

Solutions

(i) Interior angles of the quadrilateral must add up to 2 $\times$ 4 – 4, or 4 right angles (360°),
$x + 120^0 + 45^0 + 70^0 = 360^0$, or $x = \mathbf{125^0}$

(ii) The unmarked interior angle mut be $180 - x$.
Since this is 6-sided polygon, the interior angles add up to 2 x 6 – 4 8 rt. angles = 720° i.e.
$180^0 - x + 100^0 + 120^0 + 130^0 + 130^0 = 720^0$
$x = 800^0 - 720^0 = \mathbf{80^0}$.

DRILL 4

(a) Determine the magnitude of each interior and each exterior angle in a regular polygon with n sides for
(i) $n = 6$ (ii) $n = 7$
(iii) $n = 8$ (iv) $n = 9$.

ANSWERS TO DRILLS

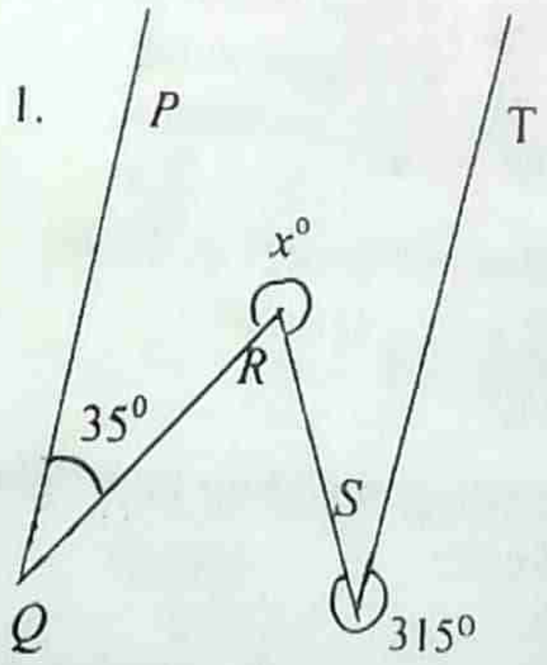
1. $a = 40^0$, $b = 140^0$, $c = 125^0$, $d = 55^0$, $e = 55^0$
$f = 85^0$, $g = 40^0$, $h = 85^0$, $k = 95^0$, $m = 40^0$
$n = 55^0$, $p = 40^0$, $q = 55^0$

2. $FC = 4$units, $CD = 4$ units
$BC = 5^1/_3$ units, $AB = 6^2/_3$ units

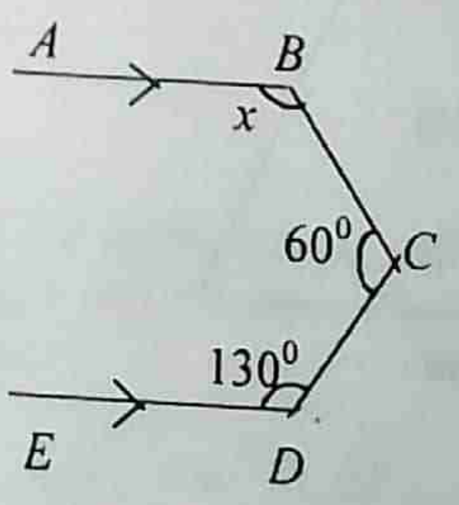
3. (a) (i) CEB (ii) BEA
(b) (i) CDB (ii) CAB

4. (i) $120^0, 60^0$ (ii) $128.6^0, 51.4^0$
(iii) $135^0, 45^0$ (iv) $140^0, 40^0$

SSCE/JME-TYPE OBJECTIVE QUESTIONS

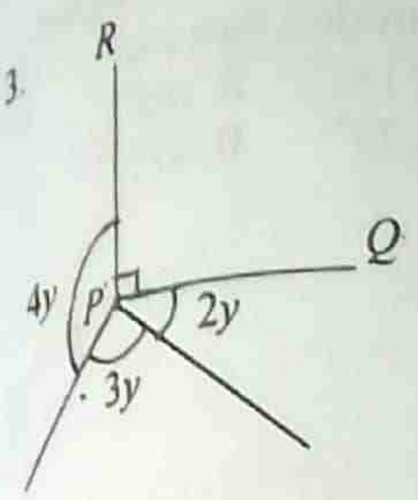
1. 

In the diagram, $QP//ST$
Find the value of x
A. 245° **B.** 280°
C. 315° **D.** 325°

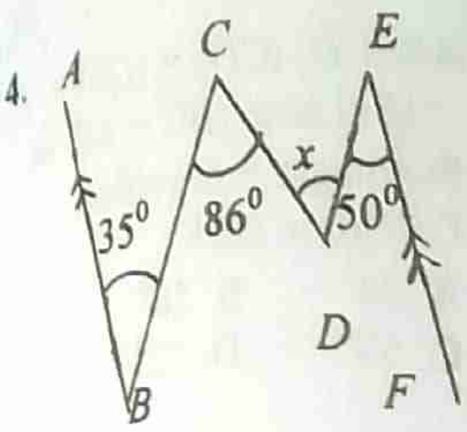
2. 

In the diagram, $AB//ED$
Find the value of x
A. 130^0 **B.** 145^0
C. 160^0 **D.** 170^0

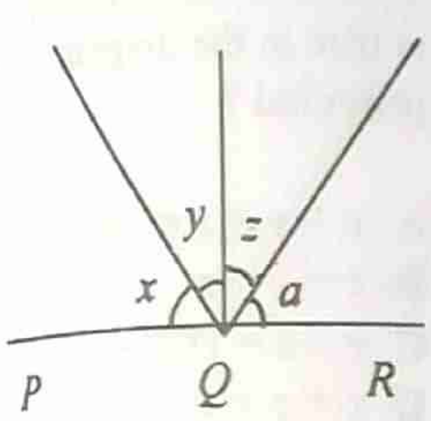
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3. 

If PQ is perpendicular to PR in the given diagram, the value of y is
A. 30^0 B. 22.5^0
C. 15^0 D. 7.5^0

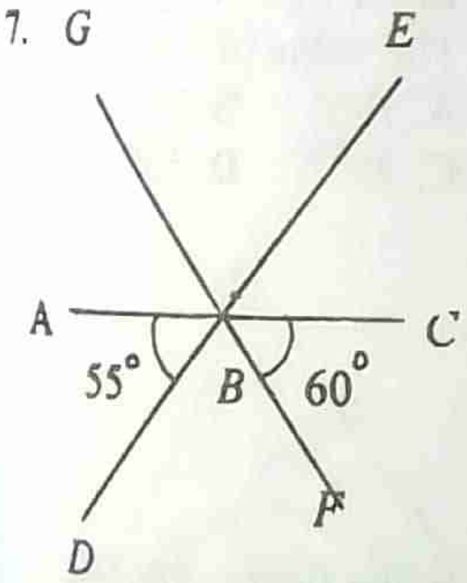
4. 

In the diagram, $AB//EF$ Calculate the value of x
A. 136^0 B. 121^0
C. 101^0 D. 86^0

5. 

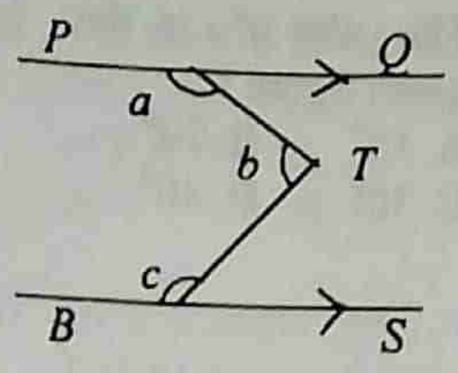
In the diagram, PQR is a straight line and $a = {}^1/_2 (x + y + z)$ The value of a is
A. 45^0 B. 60^0
C. 90^0 D. 120^0

6. The two adjacent angles on a straight line are $(x - 60^0)$ and $(y + 30)^0$. The value of $x + y$ is
A. 90 B. 150 C. 180 D. 210

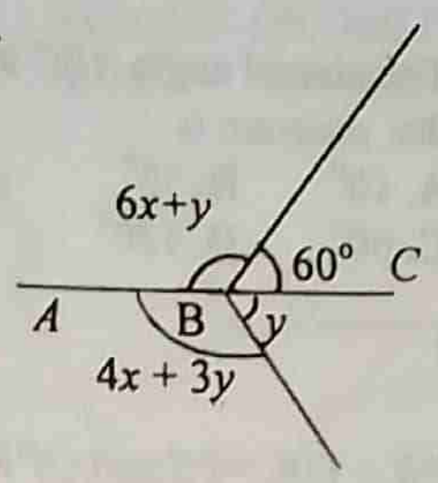
7. 

ABC, DBE and GBF are straight lines intersecting at B in the given diagram. The magnitude of angle GBE is
A. 120^0 B. 115^0
C. 65^0 D. 5^0

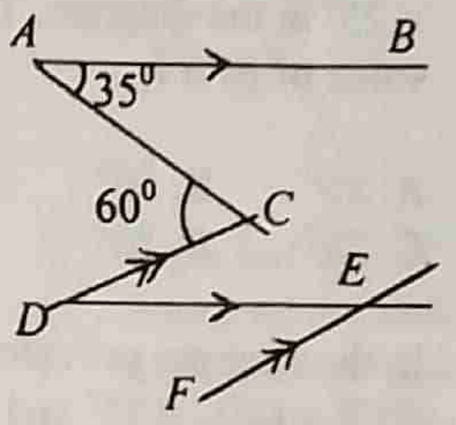
8. When two straight lines intersect
I. adjacent angles are equal
II. adjacent angles are supplementary
III. vertically opposite angles are equal
IV. vertically opposite angles are supplementary
Which of the above statements are true?
A. I and III only B. II and IV only
C. I and IV only D. II and III only

9. 

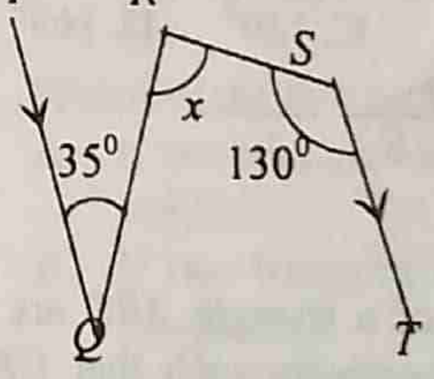
If $PQ//RS$ is the given figure, the value of $a + b + c$ is
A. 360^0 B. 270^0
C. 200^0 D. 180^0

10. 

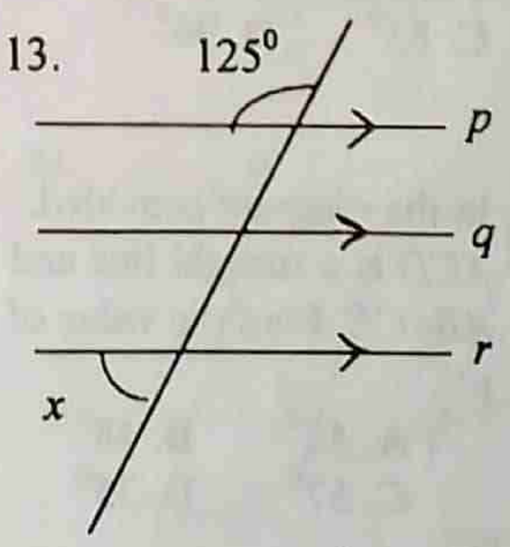
ABC is a straight line in the given figure. The value of x and y are, respectively,
A. 30^0, 15^0 B. 15, 30^0
C. 30^0, 45^0 D. 15, 45^0

11. 

In the diagram, $AB//DE$ and $DC//FE$. If $ACD = 60°$ and $BAC = 35°$, find the value of DEF.
A. $25°$ B. $35°$
C. $60°$ D. $95°$

12. 

If $PQ//ST$ in the figure, the value of x is
A. 165^0 B. 95^0
C. 85^0 D. 50^0

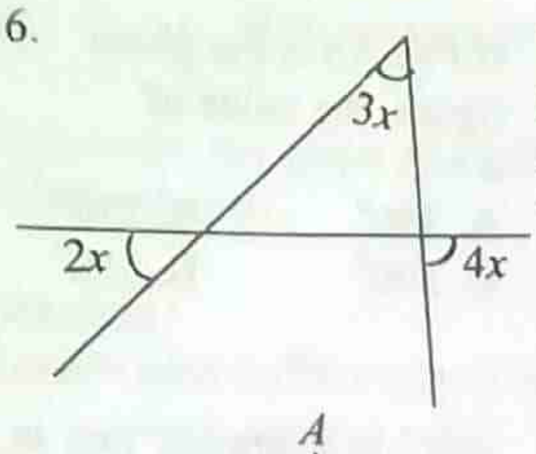
13. 

In the figure, p, q and r are parallel lines. The value of x is
A. 35^0 B. 55^0
C. 65^0 D. 125^0

14. The angle which is one-quarter of its supplement is
A. 210^0 B. 120^0 C. 45^0 D. 36^0

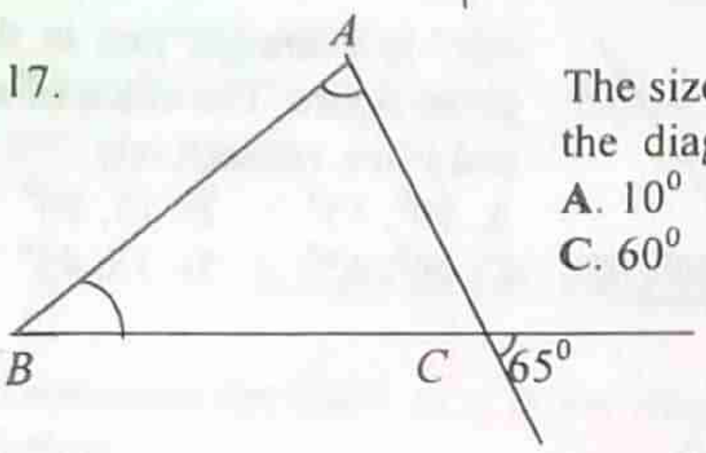
15. The angle which is one third of its complement is
A. $22{}^1/_2{}^0$ B. 30^0 C. 45^0 D. 75^0

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16. 

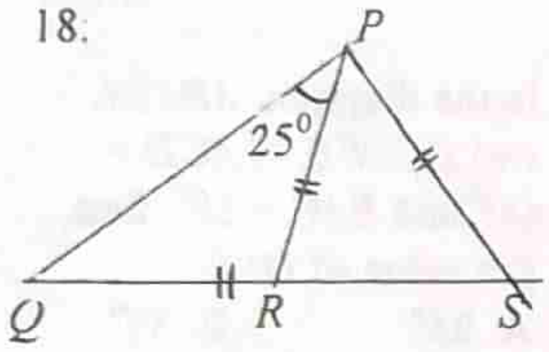
The value of x in the given figure is

A. 15^0 **B**. 20^0
C. 30^0 **D**. 40^0

17. 

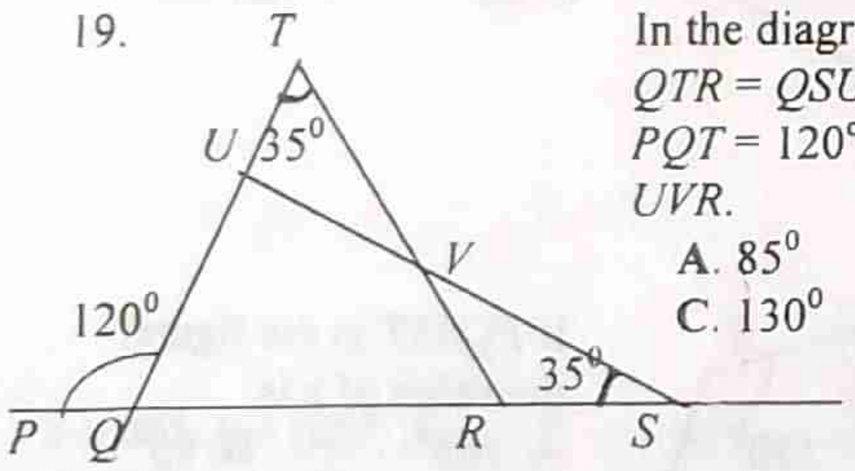
The size of angle ABC in the diagram is

A. 10^0 **B**. 55^0
C. 60^0 **D**. 120^0

18. 

$PR = QR = PS$ and $QPR = 25^\circ$ in the diagram. The value of RPS is

A. 25° **B**. 50°
C. 75° **D**. 80°

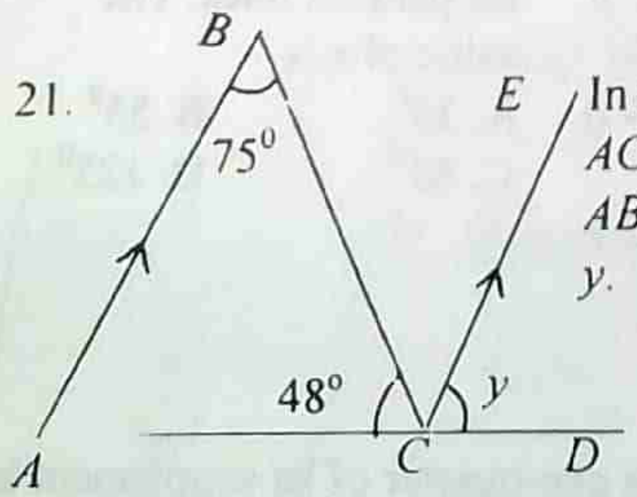
19. 

In the diagram provided $QTR = QSU = 35^\circ$ and $PQT = 120^\circ$. Calculate UVR.

A. 85^0 **B**. 120^0
C. 130^0 **D**. 165^0

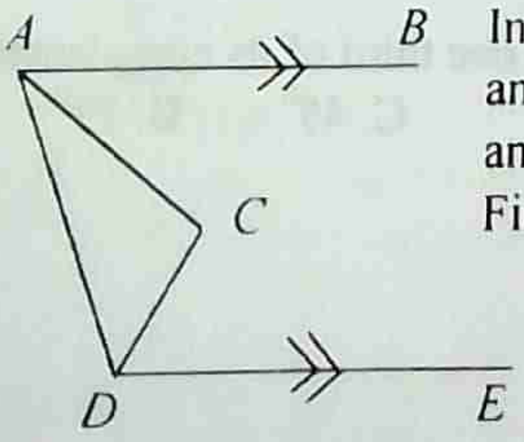
20. The sides AB and AC of a triangle ABC are produced to D and E respectively such that $DBC = 125^\circ$ and $BCE = 95^\circ$. Find the value of BAC

A. 40^0 **B**. 55^0 **C**. 85^0 **D**. 90^0

21. 

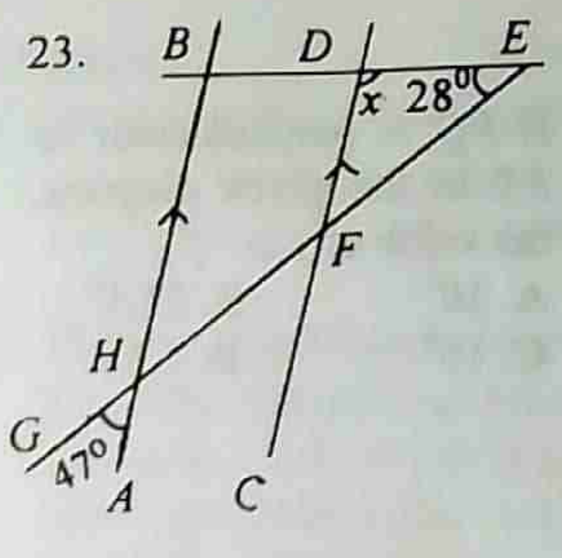
In the diagram provided, ACD is a straight line and $AB//CE$. Find the value of y.

A. 42^0 **B**. 48^0
C. 57^0 **D**. 75^0

22. 

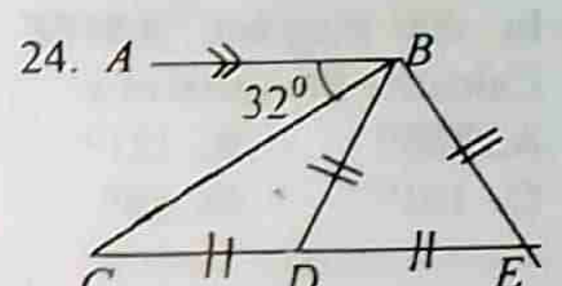
In the diagram, $AB//CD$ and the bisectors of BAD and ADE meet at C. Find ACD.

A. 120^0 **B**. 90^0
C. 60^0 **D**. 45^0

23. 

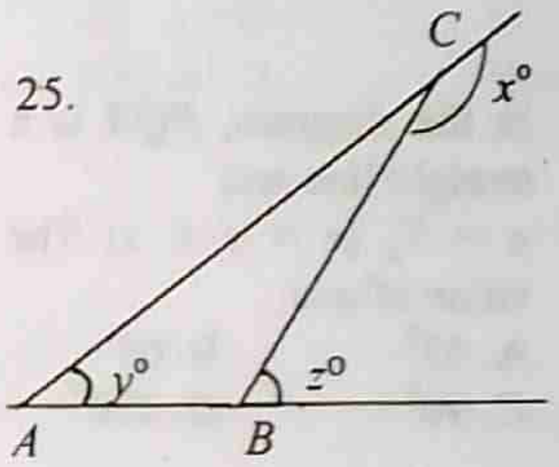
If $AB//CD$ in the diagram provided, then x is

A. 133° *B*. 105°
C. 75° **D**. 47°

24. 

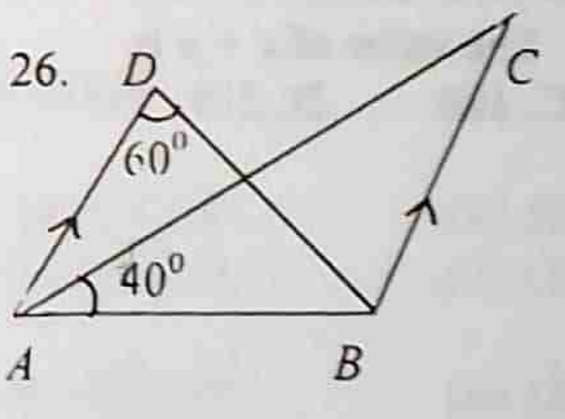
$AB//CD$, $|CD| = |DB| = |BE|$ and $ABC = 32^\circ$ in the diagram provided. Calculate DBE

A. 64^0 **B**. 58^0
C. 52^0 **D**. 32^0

25. 

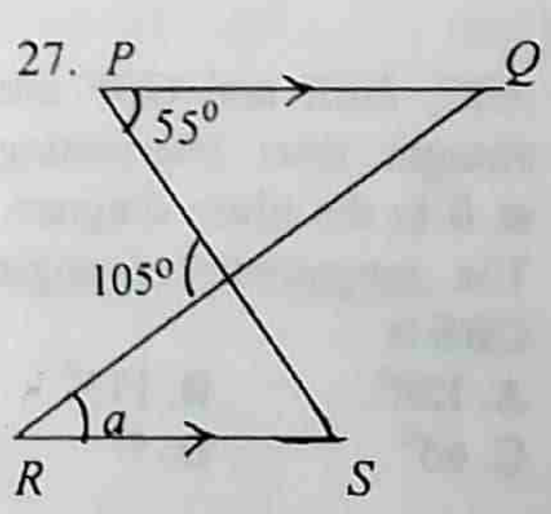
Which of the following is true in the diagram provided ?

A. $x + y + z = 180^0$
B. $x = y + z$
C. $x - z = 180^0 - y$
D. $x + z = y + 180^0$

26. 

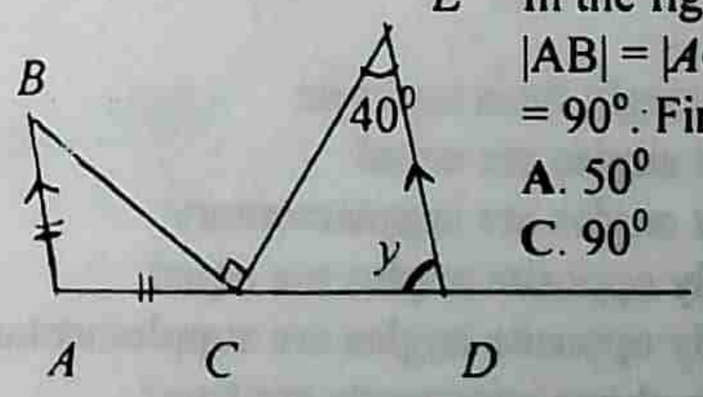
In the figure, $AD//BC$ and DB bisects ABC. Find ACB

A. 20^0 **B**. 30^0
C. 40^0 **D**. 60^0

27. 

In the figure, $PQ//RS$. The value of a is

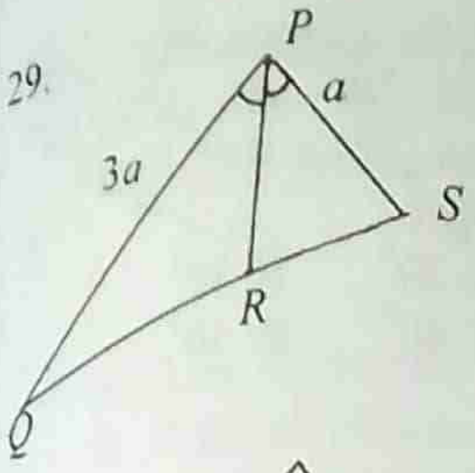
A. 105^0 **B**. 75^0
C. 55^0 **D**. 50^0

28. 

In the figure, $AB//DE$, $|AB| = |AC|$ and $BCE = 90^\circ$. Find y

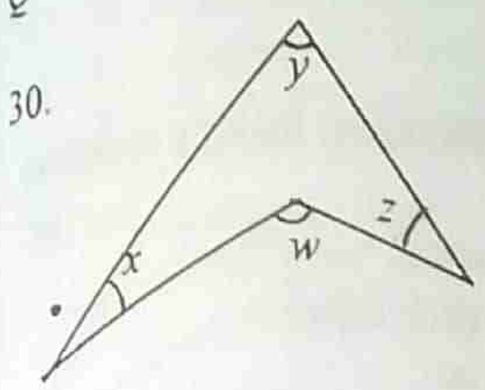
A. 50^0 **B**. 80^0
C. 90^0 **D**. 100^0

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29. 

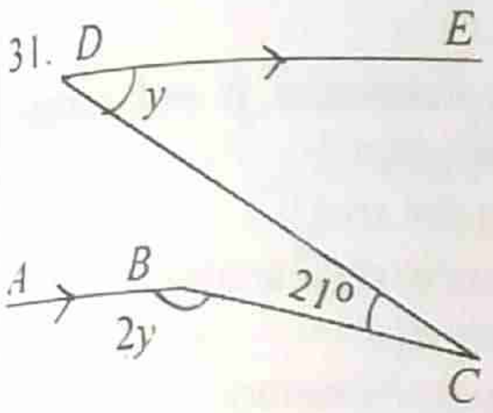
In the figure, PR bisects QPS, $|PS| = a$ and $|PQ| = 3a$. Find the ratio $SR : SQ$.

A. 1 : 2 **B**. 1 : 3
C 1 : 4 **D**. 2 : 5

30. 

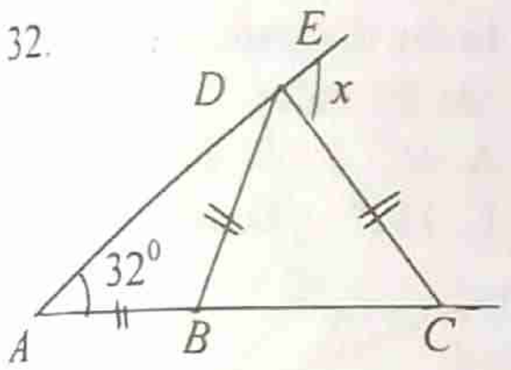
The relationship between the angles indicated in the figure is

A. $w = 180^\circ - (x + y + z)$
B. $w = x + y + z$
C. $w = x - y + z$
D. $w = x + 2y + z$

31. 

In the figure, $AB//DE$ and $BCD = 21^\circ$. The value of y is

A 21^0 **B**. 42^0
C. 63^0 **D**. 67^0

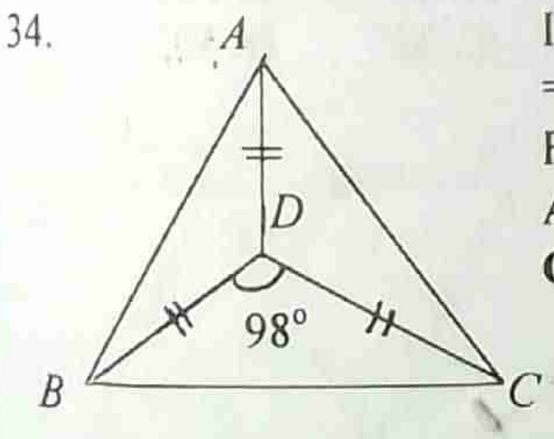
32. 

If ABC and ADE are straight lines and $|AB| = |BD| = |CD|$ in the diagram, Find the value of x.

A. 32^0 **B**. 64^0
C. 86^0 **D**. 96^0

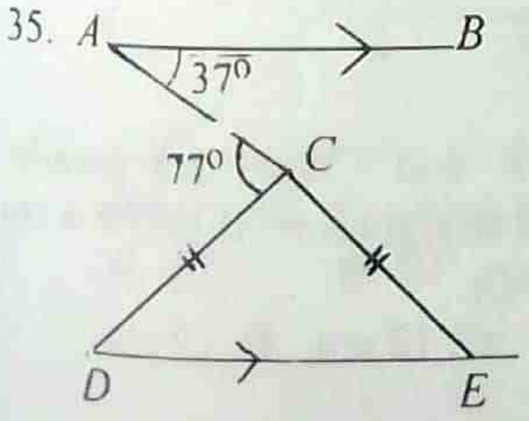
33. In a triangle ABC, the bisector of BAC meets BC at D. The line AB is produced to E and the bisector of EBC meets AD produced at F. If BAC is 52^0 and $BDF = 70^0$, Calculate BFD.

A. 37^0 **B**. 42^0 **C**. 68^0 **D**. 75^0

34. 

In the figure, $|AD| = |BD| = |CD|$ and $BDC = 98^0$. Find BAC.

A. 45° **B**. 49°
C. 82° **D**. 98°

35. 

In the diagram, $AB//DE$, $BAC = 37^\circ$, $ACD = 77^\circ$ and $|CD| = |CE|$. Find DCE.

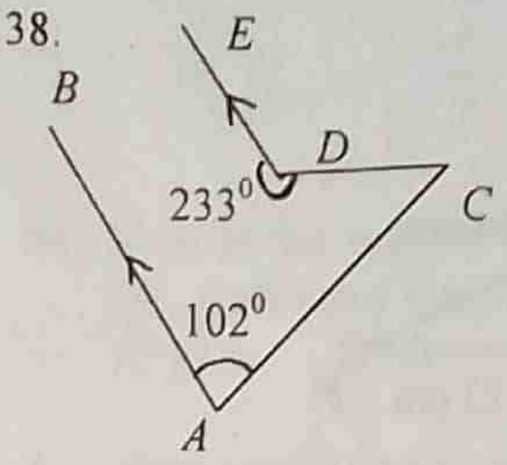
A. 66^0 **B**. 74^0
C. 100^0 **D**. 103^0

36. The lengths of the three sides of an isosceles triangle are $y + 3$, $2y + 3$ and $2y - 3$ respectively. The value of y is

A. 0 **B**. 1 **C**. 3 **D**. 6

37. In a triangle ABC, $BAC = 48^\circ$ and $ABC = 100^\circ$. Calculate the acute angle between the internal bisectors of ABC and BCA.

A. 16^0 **B**. 48^0 **C**. 66^0 **D**. 82^0

38. 

In the diagram $AB//DE$, reflex $EDC = 233^\circ$ and $BAC = 102^\circ$ Find DCA

A. 25° **B**. 50°
C. 53° **D**. 78°

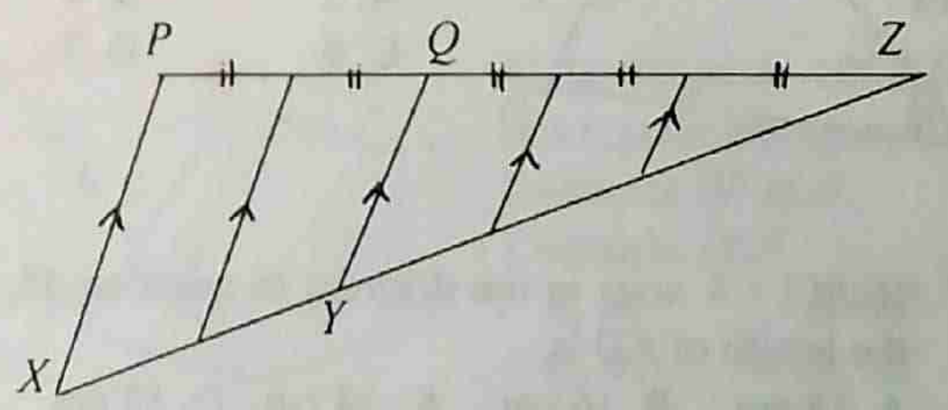
39. A triangle has two of its angles specified as 45^0 and 45^0. Which of the following is (are) true ? It is

I. an equilateral triangle
II. an isosceles triangle
III. a right-angled triangle

A. I only **B**. II only
C. I and III only **D**. II and III only

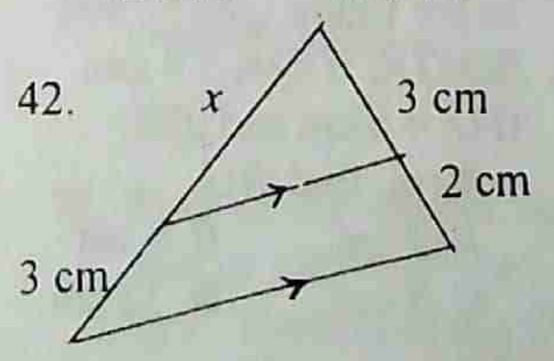
40. Which of the following statements is **false** ?

A. If two triangles are congruent, then they are also similar.
B. If two triangles are similar, then they are also congruent.
C. An equiangular triangle is also equilateral
D. An equilateral triangle is also equiangular.

41. 

The ratio $|XY| : |XZ|$ in the figure above is

A. 2 : 3 **B**. 3 : 2 **C**. 2 : 5 **D**. 5 : 3

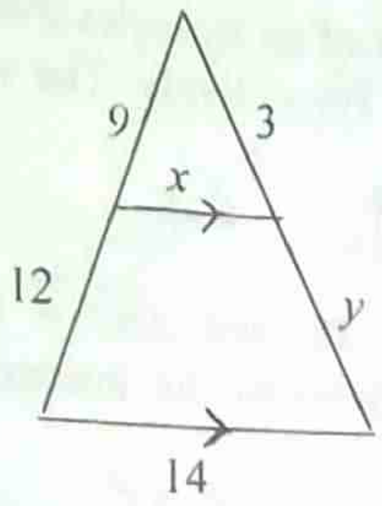
42. 

The value of x in the given figure is

A. 4.0 cm
B. 4.5 cm
C. 5.0 cm
D. 6.0 cm

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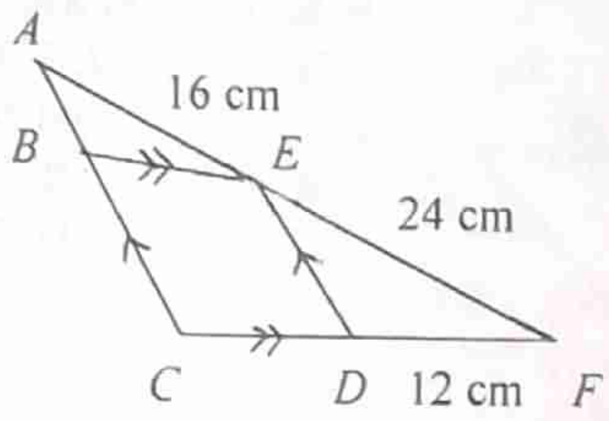
43.



The values of x and y in the figure provided are, respectively

A. 7 and 10
B. 4 and 6
C. 6 and 7
D. 6 and 4

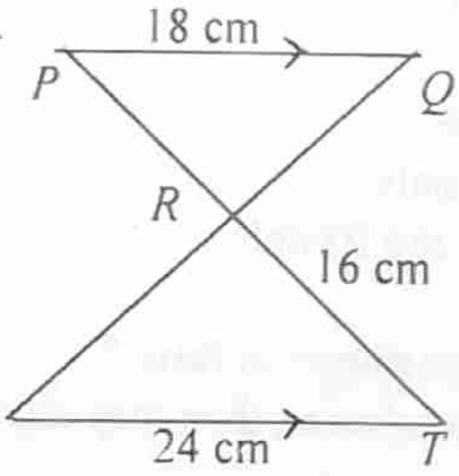
44.



In the figure provided, $CB//DE$ and $CD//BE$; $|EF| = 24$ cm, $|AE| =$ 16 cm and $|DF| = 12$ cm. Find $|BE|$.

A. 4 cm **B**. 6 cm **C**. 8 cm **D**. 12 cm

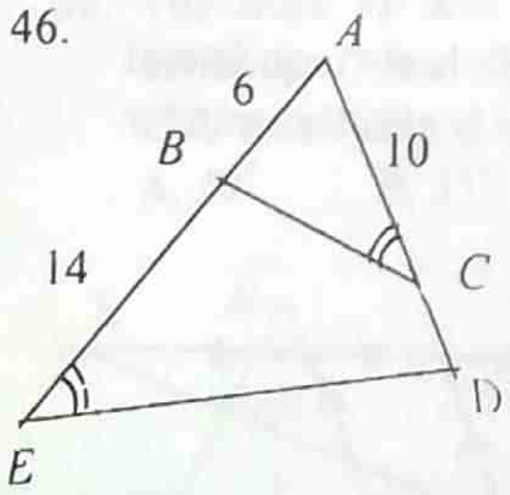
45.



In the figure, $PQ//ST$; $|ST| = 24$ cm, $|RT|$ 16 cm and $PQ = 18$ cm. Calculate $|PR|$

A. 12 cm **B**. 15 cm
C. 16 cm **D**. 18 cm

46.



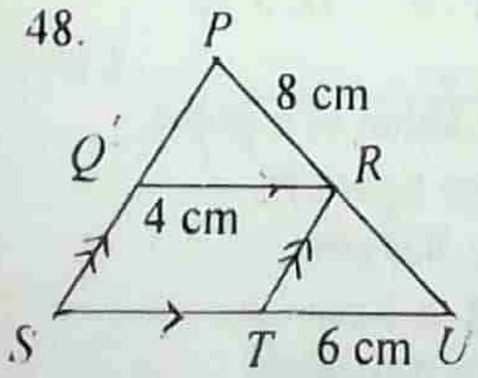
In the figure, $BCA = BED$, $|AB| = 6$ units, $|AC| = 10$ units and $|BE| = 14$ units. Find $|CD|$

A. 2 **B**. 4
C. 6 **D**. 7

47. If $|BC| = 8$ units in the diagram in question 46, then the length of ED is

A. 18 cm **B**. 16 cm **C**. 14 cm **D**. 12 cm

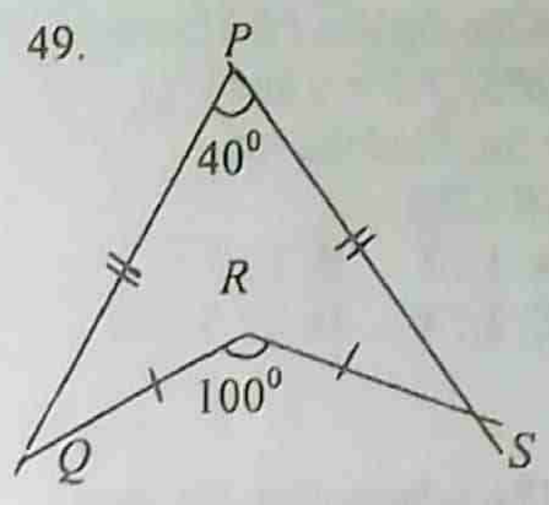
48.



In the figure $QR//ST$ and $SQ//TR$. If $|QR| = 4$ cm, $|TU| = 6$ cm and $|PR| = 8$ cm, find $|RU|$

A. 4 cm **B**. 6 cm
C. 8 cm **D**. 12 cm

49.



In the diagram, $|PQ| = |PS|$ and $|QR| = |RS|$, $QPS = 40°$ and $QRS = 100°$. Find PQR

A. 20^0 **B**. 30^0
C. 40^0 **D**. 50^0

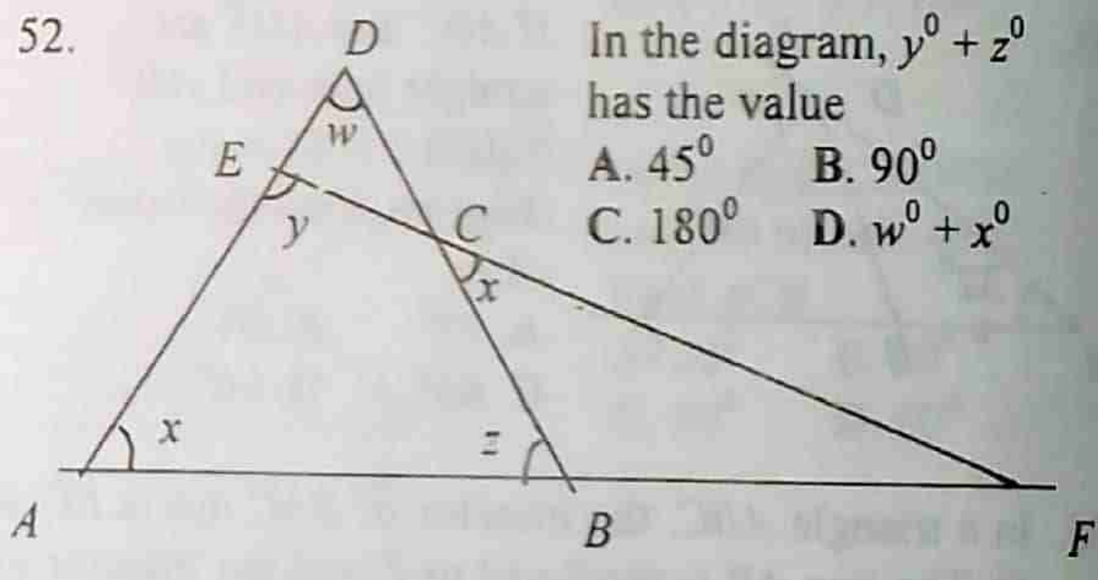
50. Which of the following statements about a rhombus is not necessarily true ?

A. Opposite angles are equal
B. The diagonals bisect each other
C. The diagonals are equal
D. The diagonals bisect the angles through which they pass.

51. Which of the following statements is **not** necessarily true about a parallelogram ?

A. Each diagonal bisects the area
B. The diagonals intersect at right angles
C. Opposite angles are equal
D. Adjacent angles are supplementary.

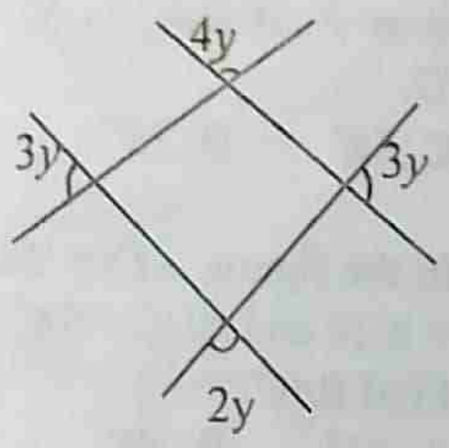
52.



In the diagram, $y^0 + z^0$ has the value

A. 45^0 **B**. 90^0
C. 180^0 **D**. $w^0 + x^0$

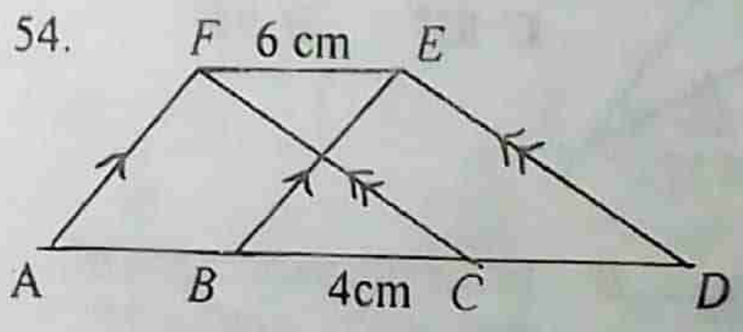
53.



In the diagram, y has the value

A. 15° **B**. 30°
C. 36° **D**. 45°

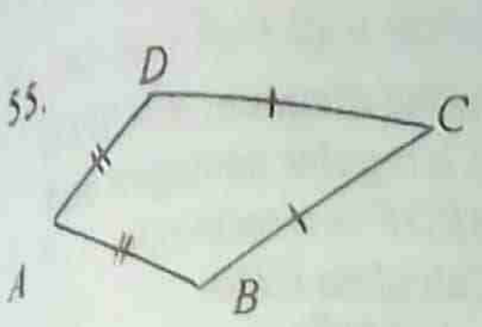
54.



In the diagram, $ABEF$ and $CDEF$ are parallelograms and $ABCD$ is a straight line. If $|BC| = 4$ cm and $|FE| = 6$ cm, find $|AD|$.

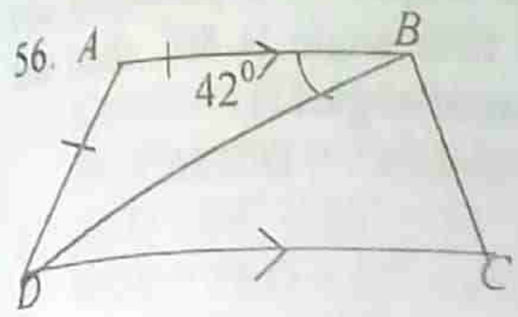
A. 18 cm **B**. 16 cm **C**. 12 cm **D**. 10 cm

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55. 

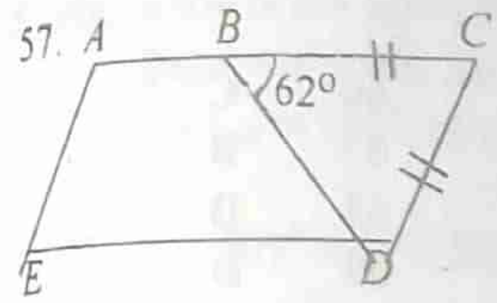
In the diagram, $|AB| = |AD|$ and $|BC| = |DC|$. Which of the following statements is **not** necessarily true?

A. Line AC bisects angle BAD
B. Line AC bisects angle BCD
C. Angle ADC equals angle ABC
D. Lines AC and CD bisect each other

56. 

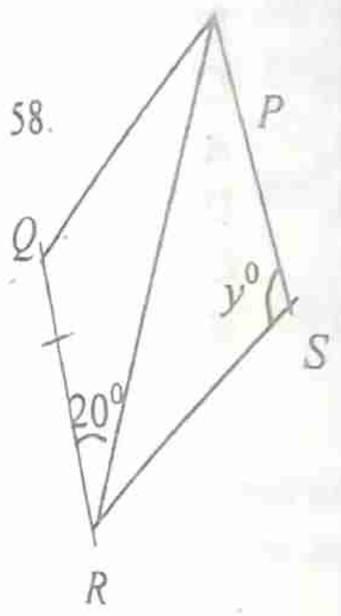
In the figure, $ABCD$ is a trapezium with $AB//DC$ $AB = AD$, $ABC = 42°$ and $BAD = 2BCD$. What is DBC?

A. 90° B. 84°
C. 64° D. 48°

57. 

$ACDE$ is a parallelogram in the diagram provided and $BC = CD$, $DBC = 62°$. The size of angle CAE is

A. 124° B. 120°
C. 118° D. 136°

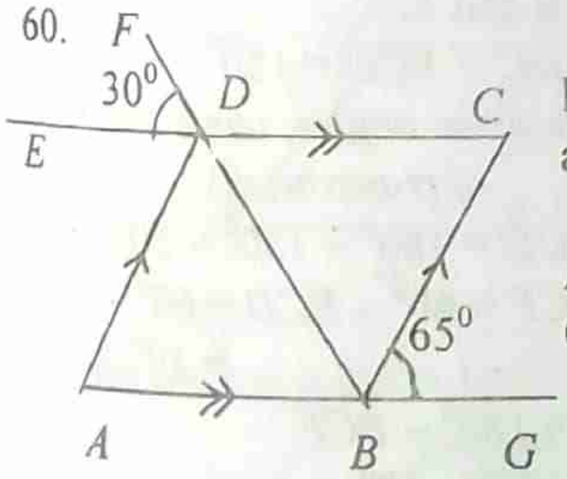
58. 

$PQRS$ is rhombus and $QRP = 20°$ in the given diagram. What is the value of y ?

A. 110^0 B. 130^0
C. 140^0 D. 160^0

59. The angles of a quadrilateral are $4x$, $x + 10$, $2x - 30$ and $3x + 20$. The value of x is

A. 28^0 B. 36^0 C. 40^0 D. 63^0

60. 

In the diagram, $ABCD$ is a parallelogram. Find ADB

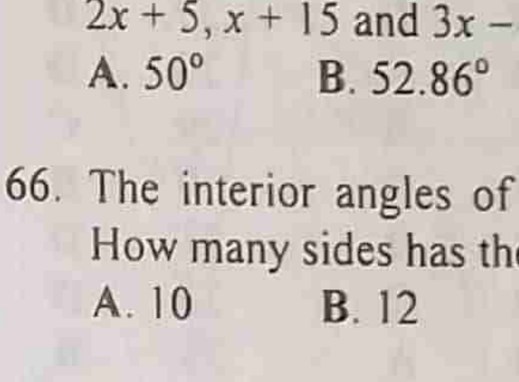
A. 30^0 B. 65^0
C. 85^0 D. 95^0

61. If the angles of a pentagon are y, $2y$, $y + 10$, $y - 10$ and $y + 60$, the value of y is

A. 40° B. 60° C. 75° D. 80°

62. The ratio of the interior angle to the exterior angle in a regular polygon is 4 : 1. The number of sides in the polygon is

A. 10 B. 8 C. 6 D. 5

63. 

The exterior angles of a polygon are marked as shown in the diagram. The The value of y is

A. 15^0 B. 20^0
C. 24^0 D. 30^0

64. Each of the exterior angles of a regular polygon is 24°. How many sides has the polygon ?

A. 15 B. 12 C. 10 D. 8

65. The exterior angles of a quadrilateral are given as x, $2x + 5$, $x + 15$ and $3x - 10$. The value of x is

A. 50° B. 52.86° C. 60° D. 61.43°

66. The interior angles of a polygon add up to 1980^0. How many sides has the polygon ?

A. 10 B. 12 C. 13 D. 15

67 Each interior angle of an eight-sided polygon is equal to

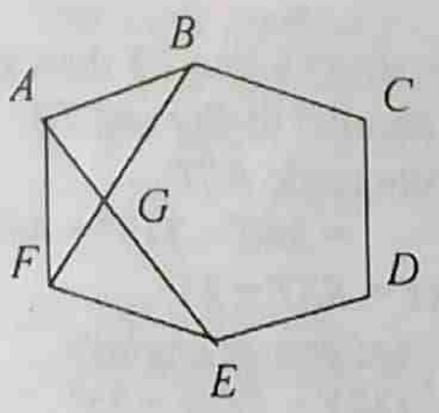
A. 140^0 B. 135^0 C. 130^0 D. 120^0

68. The interior angles of a pentagon are 130°, 118°, 80°, 78° and x. The value of x is

A. 75^0 B. 108^0 C. 120^0 D. 134^0

69 The size of each interior angle of a regular polygon with n sides is 160^0. The value of n is

A. 12 B. 14 C. 16 D. 18

70. 

In the diagram, $ABCDEF$ is a regular hexagon, AE intersects BF at G. Calculate EGF

A. 90^0 B. 60^0
C. 45^0 D. 30^0

71. Which of the following is **not** an exterior angle of a ***regular*** polygon ?

A. 24° B. 21° C. 18° D. 15°

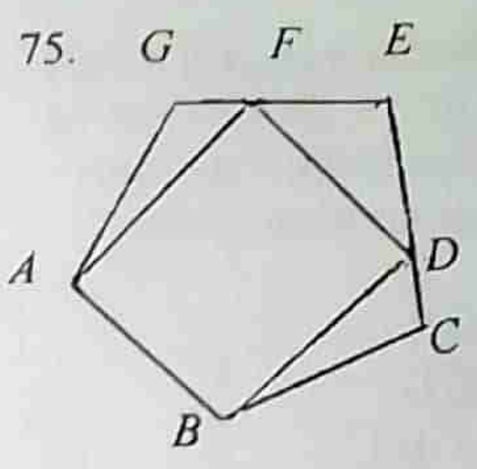
72. Four of the interior angles of a pentagon are specified as $(80 - y)^0$, $(80 + y)^0$, $(120 - 2y)^0$ and $(120 + 2y)^0$. The size of the fifth interior angle must be

A. 120^0 B. 130^0 C. 140^0 D. 144^0

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73. A regular polygon with $(2m + 1)$ sides has each interior angle equal to 144^0. The value of m is
A. $4\frac{1}{2}$ **B.** 5 **C.** 8 **D.** 10

74. In a regular pentagon $ABCDE$, AC intersects BD at F. Calculate CBD.
A. 108° **B.** 72° **C.** 60° **D.** 36°

75. 

In the diagram, $ABCDEG$ is a regular pentagon and $ABDF$ is a rectangle. Calculate CDB.
A. 108^0 **B.** 90^0
C. 54^0 **D.** 18^0

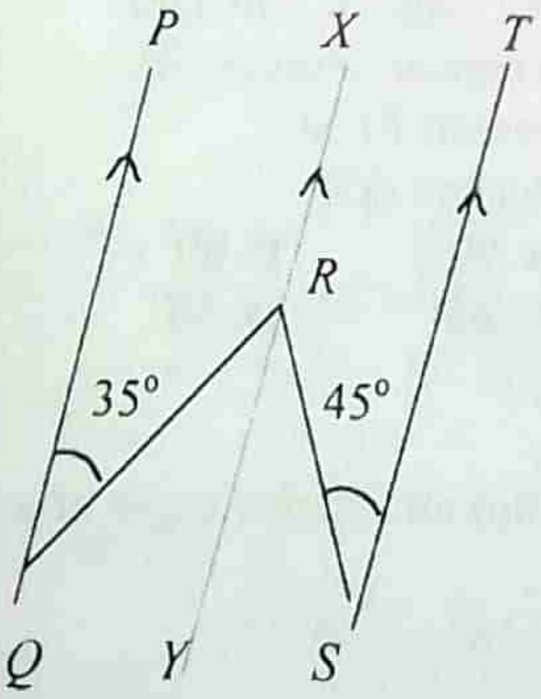
76 Four of the five interior angles of a pentagon are equal. If the size of the fifth angle is 80^0, then the size of each of the four equal angles is
A. 100^0 **B.** 115^0 **C.** 120^0 **D.** 136^0

ANSWER KEY

1.	B	14.	D	27.	D	40.	B	53.	B	66.	C
2.	D	15.	A	28.	D	41.	C	54.	B	67.	B
3.	A	16.	B	29.	C	42.	B	55.	D	68.	D
4.	C	17.	C	30.	B	43.	D	56.	A	69.	D
5.	B	18.	D	31.	D	44.	C	57.	A	70.	B
6.	D	19.	C	32.	D	45.	A	58.	C	71.	B
7.	C	20.	A	33.	B	46.	A	59.	B	72.	C
8.	D	21.	C	34.	B	47.	B	60.	C	73.	A
9.	A	22.	B	35.	C	48.	D	61.	D	74.	D
10.	B	23.	B	36.	D	49.	B	62.	A	75.	C
11.	A	24.	C	37.	C	50.	C	63.	B	76.	B
12.	C	25.	D	38.	A	51.	B	64.	A		
13.	B	26.	A	39.	D	52.	C	65.	A		

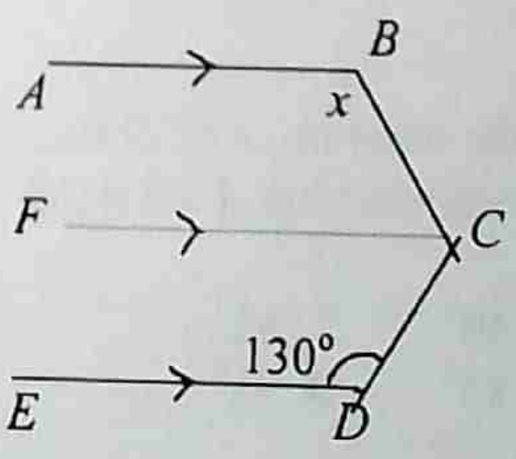
SOLUTIONS TO OBJECTIVE QUESTIONS

1. **(B)**



Construct a line XY through R, parallel to PQ and TS.
Acute angle RST
$= 360^\circ - 315^\circ = 45^\circ$
$SRY = RST = 45^0$
(*alternate angles*)
$QRY = PQR = 35^\circ$
(*alternate angles*)
$x = 360^\circ - (SRY + QRY)$
$= 360^\circ - (45^\circ + 35^\circ)$
$= 280^\circ$

2. **(D)**

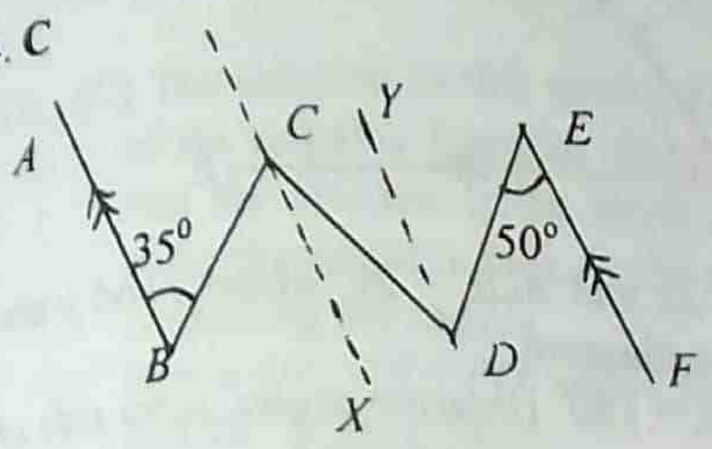


Construct a line CF through C, parallel to AB and ED.
$EDC + FCD = 180^0$
(*interior angles, same side of transversal*)
$FCD = 180^0 - 130^0 = 50^\circ$
$BCF = 60^0 - FCD = 60^\circ - 50^\circ$
$= 10^0$
$x = 180^0 - BCF$
$= 180^0 - 10^0$
$= 170^\circ$

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3. **(A)** $4y + 3y + 2y + 90^\circ = 360^\circ$ (*angles at a point*)
$9y = 270^\circ$, or $y = 30^\circ$.

4. **C**



Construct CX and DY such that $CX//DY//AB//EF$
$BCX = ABC = 35^0$ (*alternate angles*)
$\therefore$ $XCD = 86^0 - 35^0 = 51^0 = CDY$ (*alternate*)
$EDY = FED = 50^0$ (*alternate*)
$x = CDY + EDY = 101^\circ$.

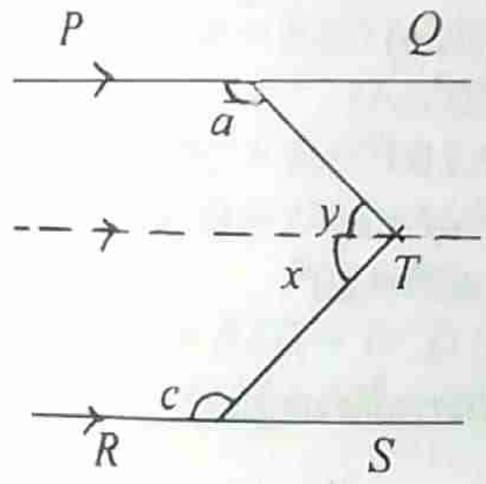
5. **(B)** $x + y + z + a = 180^0$ (*angles on a straight line*)
But $x + y + z = 2a$ (*given*)
$\therefore$ $2a + a = 180^\circ$, or $a = 60^\circ$.

6. **(D)** $(x - 60)^0 + (y + 30)^0 = 180^0$
(*angles on a straight line*)
$x + y - 30 = 180$
$x + y = 210$

7. **(C)** $DBF + 60^0 + 55^0 = 180^0$
(*angles on a straight line*)
$DBF = 180^0 - 115^0 = 65^0 = GBE$
(*vertically opposite angles*)

8. **(D)** Adjacent angles are *supplementary*. Vertically opposite angles are *equal*.

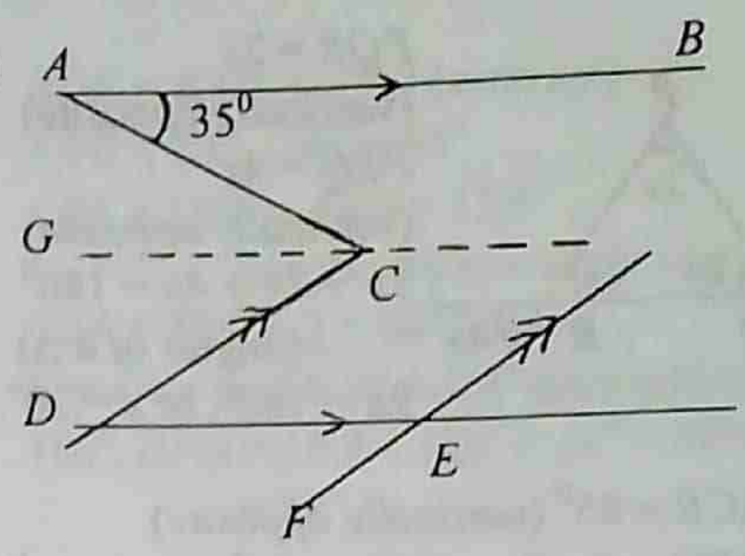
9. **(A)**



Draw a line through T parallel to PQ and RS. Then
$y = 180^\circ - c$ and
$y = 180^\circ - a$
$b = x + y = 360^\circ - a - c$,
$a + b + c = 360^\circ$

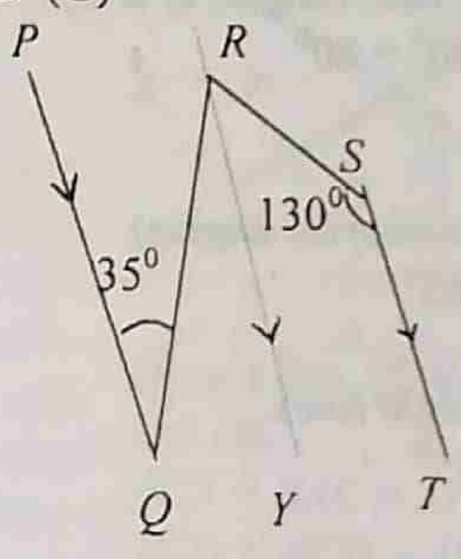
10. **(B)** $6x + y + 60^0 = 180^0$
(*angles on a straight line*)
$\therefore$ $6x + y = 120^0$ ..... (i)
$4x + 3y + y = 180^0$
(*angles on a straight line*)
$\therefore$ $x + y = 45^0$ .....(ii)
(i) – (ii): $5x = 75^0$, or $x = 15^0$
From (ii): $y = 45^0 - 15^0 = 30^0$

11. **(A)**



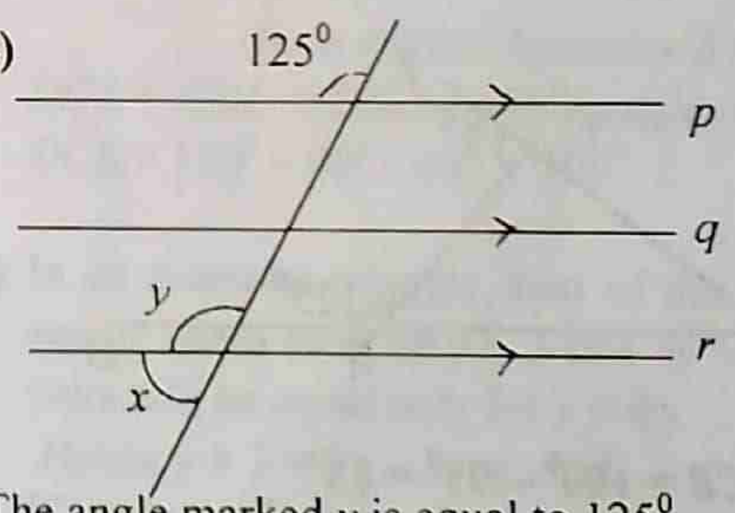
Construct a line GC through C, parallel to AB and DE.
$ACG = BAC = 35^\circ$ (*alternate angles*)
$GCD = 60^\circ - 35^\circ = 25^\circ$
$CDE = GCD = 25^\circ$ (*alternate angles*)
$DEF = CDE = 25^\circ$ (*alternate angles*)

12. **(C)**



Construct a line RY through R, parallel to PQ and ST
$QRY = PQR = 35^\circ$
(*alternate angles*)
$SRY = 180^\circ - RST$
$= 180^0 - 130^0 = 50^\circ$
(*interior angles*)
$x = QRY + SRY$
$= 35^0 + 50^0 = 80^0$

13. **(B)**



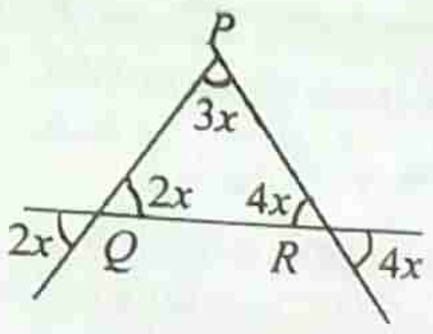
The angle marked y is equal to 125^0
(*corresponding angles*)
$x + y = 180^0$ (*angles on a straight line*)
$x = 180^0 - y = 180^0 - 125^0 = 55^0$.

14. **(D)** Let the angle $= x$
Its supplement $= 180^0 - x$
Hence $x = {}^1/_4\,(180^0 - x)$
$5x = 180^0$, or $x = 36^0$

15. **(A)** Let the angle $= y$
Its complement $= 90^0 - y$
Hence $y = {}^1/_3\,(90^0 - y)$
$4y = 90^0$, or $y = 22{}^1/_2{}^0$

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16. **(B)**



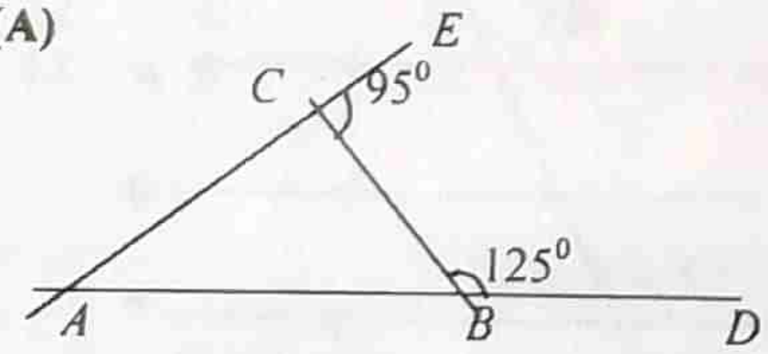
$PQR = 2x$ (*vertically opposite*)
$PRQ = 4x$ (*vertically opposite*)
$3x + 2x + 4x = 180^0$ (*angles of a* Δ)
$9x = 180°$, or $x = 20°$.

17. **(C)** $ACB = 65^0$ (*vertically opposite*)
$ABC + BAC + ACB = 180^0$ (*angles of a* Δ)
$ABC = 180^0 - 55^0 - 65^0 = 60^0$

18. **(D)** $PQR = QPR = 25°$
(*base angles, isosceles* Δ)
$PRS = PQR + QPR = 50°$ (*exterior angles*)
$PSR = PRS = 50°$
(*base angles, isosceles* Δ)
$RPS + PRS + PSR = 180^0$ (*angles of a* Δ)
$RPS = 180^0 - 50^0 - 50^0 = 80^0$

19. **(C)** $PQT = QTR + QRT$
(*exterior and two interior angles*)
$QRT = 120° - 35° = 85°$
$SRT + QRT = 180^0$
(*angles on a straight line*)
$SRT = 180^0 - 85^0 = 95^0$
$UVR = SRT + VSR$
(*exterior and two interior angles*)
$= 95^0 + 35^0 = 130^0$

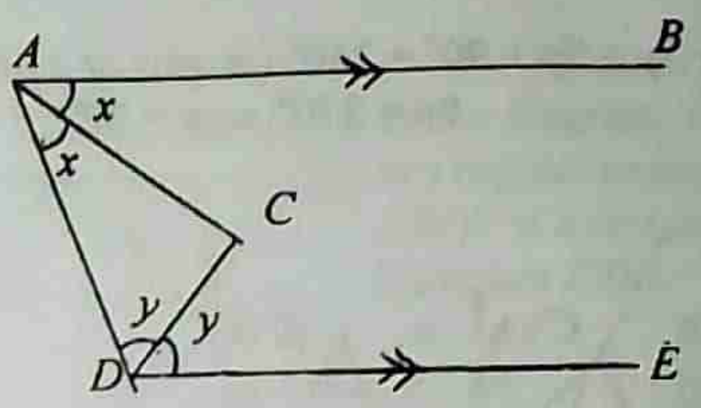
20. **(A)**



$ACB = 180^0 - 95^0 = 85^0$
(*angles on a straight line*)
$DBC = BAC + ACB$ (*exterior*)
$BAC = DBC - ACB = 125^0 - 85^0$
$= 40^0$

21. **(C)** $BCE = ABC = 75^0$ (*alternate angles*)
$ACB + BCE + y = 180^0$
(*angles on a straight Line*)
$y = 180^0 - 48^0 - 75^0 = 57^0$

22. **(B)**



Let $BAD = 2x$ and $ADE = 2y$, where x and y are shown in the diagram.
$BAD + ADE = 180^0$ (*interior angles, same side of transversal*)
$2x + 2y = 180^0$, or $x + y = 90°$
$ACD + CAD + ADC = 180°$ (*angles of a* Δ)
$ACD = 180^0 - (x + y) = 180^0 - 90^0 = 90^0$

23. **(B)**. $HFC = GHA = 47^0$ (*corresponding angles*)
$DFE = HFC = 47^0$
(*vertically opposite angles*)
$x + DFE + DEF = 180°$ (*angles of a* Δ)
$x = 180^0 - 28^0 - 47^0 = 105^0$

24. **(C)** $BCD = ABC = 32^0$ (*alternate angles*)
$DBC = BCD = 32^0$ (*base angles, isosceles* Δ)
$BDE = BCD + DBC = 64^0$ (*exterior*)
$BED = BDE = 64^0$ (*base angles, isosceles* Δ)
$DBE + BED + BDE = 180°$ (*angles of a* Δ)
$DBE = 180^0 - 64^0 - 64^0 = 52°$

25. **(D)** $ACB = 180° - x$ (*angles on a straight line*)
$z = y + ACB$
(*exterior and two interior angles*)
$z = y + 180° - x$, or
$x + z = y + 180°$

26. **(A)** $DBC = ADB = 60^0$ (*alternate angles*)
$ABD = DBC = 60^0$ (*given*)
$ABC = 60^0 + 60^0 = 120^0$
$ACB + ABC + CAB = 180^0$
(*angles of a* Δ)
$ACB = 180^0 - 120^0 - 40^0 = 20^0$

27. **(D)** $PSR = QPS = 55^0$ (*alternate angles*)
$105^0 = a + PSR$
(*exterior and two interior angles*)
$a = 105^0 - PSR = 105^0 - 55^0 = 50^0$

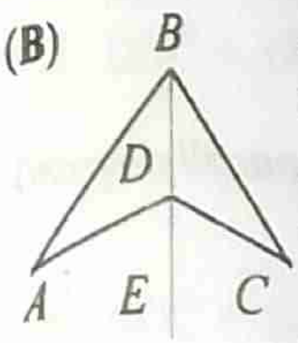
28. **(D)** $BAC + y = 180°$
(*interior angles, same side of transversal*)
$\therefore BAC = 180^0 - y$
$ABC = ACB$ (*base angles, isosceles* Δ)
$BAC + ABC + ACB = 180°$ (*angles of a* Δ)
$\therefore ACB = ABC = y/2$
$ECD = 180° - 40° - y$ (*angles of a* Δ)

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$ACB + BCE + ECD = 180^0$
(*angles on a straight line*)
$y/2 + 90^0 + 140^0 - y = 180^0$, or $y = 100^0$

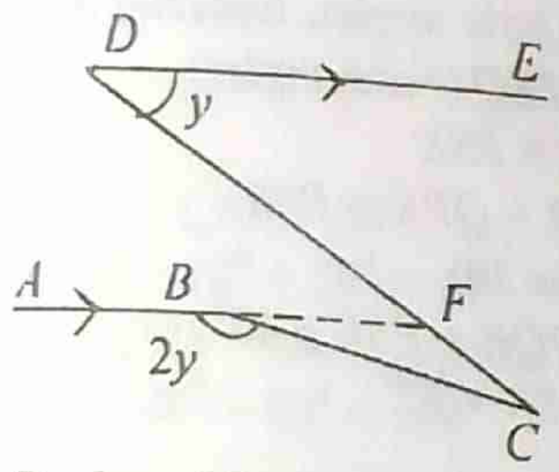
29. **(C)** The solution to this problem requires the use of the **angular bisector theorem** which states that *the bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angles.*
Hence $QR : RS = PQ : PS = 3 : 1$
and $SR : SQ = 1 : 4$

30. **(B)**



Join B to D and produce to E
$CDE = DCB + CBD$ (*exterior*)
$ADE = DAB + ABD$ (*exterior*)
$w = ADE + CDE$
$= DAB + ABD + CBD + DCB$
Noting that $ABD + CBD = y$,
$DAB = x$ and $DCB = z$,
$w = x + y + z$

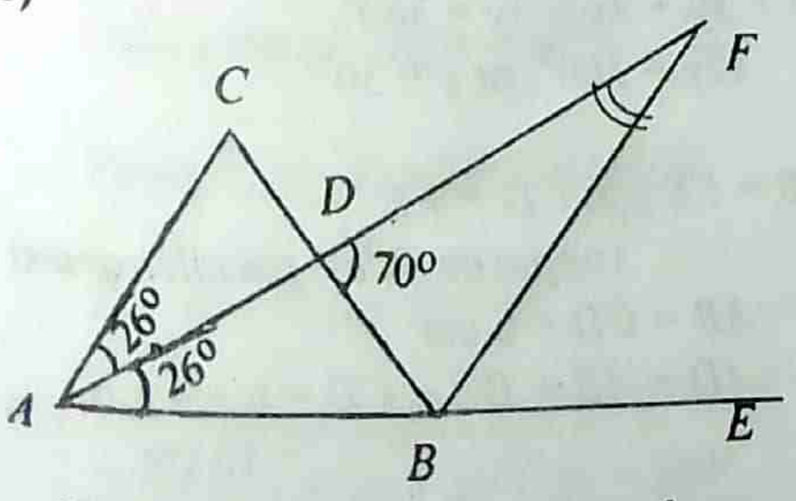
31. **(D)**



Produce AB to meet CD at F.
CBF = $180^0 - 2y$ (*angles on a straight line*)
BFD = y (*alternate angles*)
= CBF + BCD (*exterior*)
Hence $y = 180^0 - 2y + 21^0$
$3y = 201^0$, or $y = 67^0$.

32. **(D)** $ADB = BAD = 32^0$
(*base angles, isosceles* Δ)
$DBC = BAD + ADB = 64^0$ (*exterior*)
$BCD = DBC = 64^0$
(*base angles, isosceles* Δ)
$x = BAD + BCD$ (*exterior angle*)
$= 32^0 + 64^0 = 96^0$

33. **(B)**

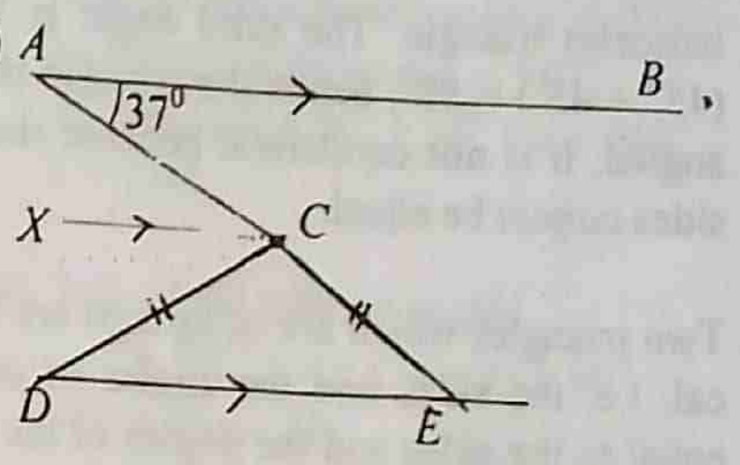


Sketch the diagram as shown above.

$BDF = BAD + ABD$ (*exterior*)
$\therefore$ $ABD = 70^0 - 26^0 = 44^0$
$CBE = 180^0 - ABD = 136^0$
(*angles on a straight line*)
$CBF = 136^0 \div 2 = 68^0$ (*given*)
Since the angles of ΔABF must add up to 180^0, $BFD = 180^0 - (26^0 + 44^0 + 68^0)$
$= 42^0$

34. **(B)** Let $DAB = DBA = x$ and $DAC = DCA = y$
$BDC = DBA + DAB + DAC + DCA$
(*See problem 30*)
$\therefore$ $BDC = 2(x + y) = 98^0$
$BAC = DAB + DAC = x + y = 49^0$

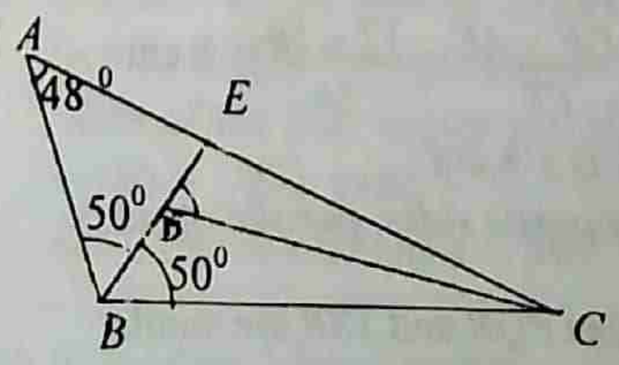
35. **(C)**



Draw a line XC through C, parallel to AB and DE.
$ACX = BAC = 37^0$ (*alternate*)
$\lambda CD = ACD - ACX = 77^0 - 37^0 = 40^0$
$CDE = XCD = 40^0$ (*alternate*)
$DEC = CDE = 40^0$
(*base angles, isosceles* Δ)
$DCE + CDE + DEC = 180^0$ (*angles of a* Δ)
$DCE = 180^0 - 40^0 - 40^0 = 100^0$

36: **(D)** In an *isosceles* triangle, two of the sides are equal. These must be $y + 3$ and $2y - 3$ (Other pairs will be equal only for $y = 0$).
Hence $y + 3 = 2y - 3$, or $y = 6$
[Check: the lengths are $y + 3 = 6 + 3 = 9$; $2y + 3 = 15$ and $2y - 3 = 9$]

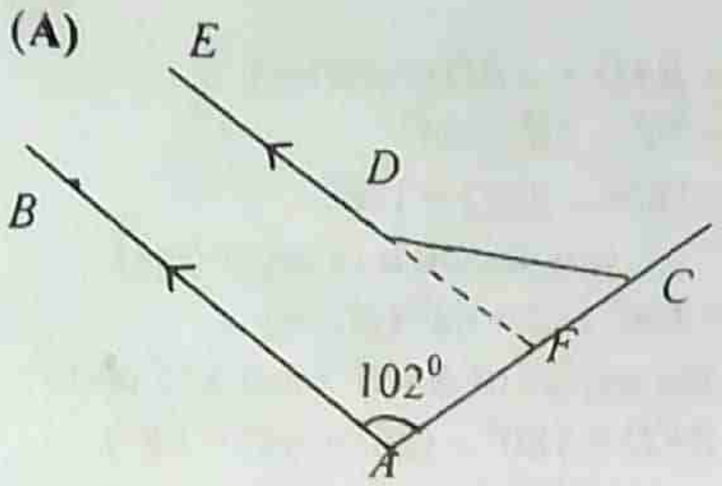
37. **(C)**



Referring to the diagram, BD and CD are the bisectors of ABC and BCA respectively.
$BCE + ABC + BAC = 180^0$ (*angles of a* Δ)
$BCE = 180^0 - 100^0 - 48^0 = 32^0$
$DCB = 32^0/2 = 16^0$
$EDC = DBC + DCB$ (*exterior*)
$= 50^0 + 16^0 = 66^0$

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38. **(A)**



Project ED to meet AC at F
$DFC = BAF = 102^0$ (*corresponding*)
$CDF = 233^0 - 180^0 = 53^0$
$DCA + CDF + DFC = 180^0$ (*angles of a* Δ)
$DCA = 180^0 - 102^0 - 53^0 = 25^0$

39. **(D)** Since two of the angles are equal, it is an isosceles triangle. The third angle is $180^0 - (45^0 + 45^0) = 90^0$, hence the triangle is right-angled. It is **not** equilateral because the three sides *cannot* be equal.

40. **(B)** Two triangles which are congruent are identical, i.e. the sides and the angles of one are equal to the sides and the angles of the other. Two triangles, though similar (i.e. having similar shapes) may be of different sizes. When all the angles of a triangle are equal (*equiangular*), all the sides must also be equal (*equilateral*), and vice-versa.

41. **(C)** $XY : XZ = PQ : PZ = 2 : 5$

42. **(B)** $\frac{x}{3} = \frac{3}{2}$, or $x = {}^9/_2 = 4.5$ cm

43. **(D)** $\frac{x}{14} = \frac{9}{12+9} = \frac{9}{21}$, or $x = 6$

$\frac{y}{3} = \frac{12}{9}$, or $y = 4$

44. **(C)** $\frac{CD}{DF} = \frac{AE}{EF}$

$CD = \frac{DF \times AE}{EF} = \frac{12 \times 16}{24} = 8$ cm

$BE = CD = 8$ cm
(*opposite sides, parallelogram*)

45. **(A)** Triangles PQR and TSR are similar

$\frac{PR}{PQ} = \frac{TR}{TS} = \frac{16}{24} = \frac{2}{3}$

$PR = PQ \times {}^2/_3 = 18 \times {}^2/_3 = 12$ cm

46. **(A)** $BCA = BED$ (*given*)
BAC is common to triangles ABC and ADE
Hence angle ABC = angle ADE, and
triangle ABC is similar to triangle ADE

$\frac{AD}{AE} = \frac{AB}{AC} = \frac{6}{10} = \frac{3}{5}$

$AD = 20 \times 3/5 = 12$
$CD = AD - AC = 12 - 10 = 2$

47. **(B)** $\frac{ED}{AE} = \frac{CB}{AC} = \frac{8}{10} = \frac{4}{5}$

$ED = AE \times 4/5 = 20 \times 4/5 = 16$ cm
[Note that the ratio of the sides of ΔABC is 6 : 8 : 10, which is also the ratio of the sides of ΔADE, i.e 12 : 16 : 20]

48. **(D)** $ST = QR = 4$ cm
(*opposite sides, parallelogram*)

$\frac{RU}{PR} = \frac{TU}{ST} = \frac{6}{4} = \frac{3}{2}$

$RU = PR \times 3/2 = 8 \times 3/2 = 12$ cm

49. **(B)** If Q is joined to S, then $RQS = RSQ$
(*base angles, isosceles* Δ)
and $PQS = PSQ$ (*base angles*)
Hence $PQR = PSR$
$QRS = PQR + QPR + PSR$
(*See problem 30*)
i.e. $100^0 = PQR + PSR + 40^0$
or $PQR = PSR = 30^0$

50. **(C)** The diagonals are equal only for a *square*

51. **(B)** The angle between the diagonals depends in the relative lengths of the sides of the parallelogram.

52. **(C)** $BCE = 180^0 - BCF$ (*angles on a str. line*)
$= 180^0 - x$
In the quadrilateral $ABCE$,
$x + z + (180^0 - x) + y = 360^0$
or $y + z = 180^0$

53. **(B)** Each of the interior angles is vertically opposite (and hence equal) to the marked exterior angles. The sum of all the interior angles is 360^0. Hence
$4y + 3y + 2y + 3y = 360^0$
$12y = 360^0$, or $y = 30^0$

54. **(B)** $AB = FE$ and $CD = FE$
(*opposite sides, parallelogram*)
$\therefore$ $AB = CD = 6$ cm
$AD = AB + BC + CD = 6 + 4 + 6$
$= 16$ cm

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55. **(D)** If A is joined to C, the first three statements follow from the similarity of Δs ADC and ABC. Also, AC bisects DB but BD does *not* bisect AC.

56. **(A)** $BDA = ABD = 42^\circ$
(*base angles, isosceles* Δ)
$BAD = 180^\circ - BDA - ABD$
(*angles of a* Δ)
$= 180^\circ - 42^\circ - 42^\circ = 96^\circ$
$BCD = {}^1/_2\,(BAD) = 48^\circ$ (*given*)
$BDC = ABD = 42^\circ$ (*alternate*)
$DBC = 180^\circ - BDC - BCD$
(*angles of a* Δ)
$= 180^0 - 42^0 - 48^0 = 90^0$

57. **(A)** $BDC = DBC = 62^0$
(*base angles, isosceles* Δ)
$BCD = 180^0 - BDC - DBC$ (*angles of a* Δ)
$= 180^0 - 62^0 - 62^0 = 56^0$
$CAE = 180^0 - BCD$
$= 180^\circ - 56^\circ = 124^\circ$.

58. **(C)** $PRS = QRP = 20^0$
$QRS = 20^0 + 20^0 = 40^0$
$y + QRS = 180^0$ (*interior angles*)
$y = 180^0 - 40^0 = 140^0$

59. **(B)** The angles of a quadrilateral add up to 360^0.
$4x + x + 10 + 2x - 30 + 3x + 20 = 360^0$,
$10x = 360^0$, or $x = 36^0$

60. **(C)** $ABD = EDF = 30^\circ$ (*corresponding*)
$DAB = GBC = 65^\circ$ (*corresponding*)
$ADB + ABD + DAB = 180^\circ$
$ADB = 180^0 - 30^0 - 65^0 = 85^0$

61. **(D)** The angles of a pentagon ($n = 5$) must add up to $2n - 4 = 2(5) - 4 = 6$ right angles, or 540^0.
$y + 2y + y + 10^\circ + y - 10^\circ + y + 60^0 = 540^0$
$6y = 480^0$, or $y = 80^0$

62. **(A)** Each interior angle $= \dfrac{2n-4}{n} \times 90$

Each exterior angle $= \dfrac{360^0}{n}$

Hence $\dfrac{(2n-4) \times 90}{n} = 4\left(\dfrac{360^0}{n}\right)$

$2n - 4 = 16$, or $n = 10$

63. **(B)** The exterior angles add up to 360^0
$\therefore\ 4y + 4y + 2y + 3y + 5y = 360^0$
$18y = 360^0$, or $y = 20^0$

64. **(A)** $24n = 360^0$
$n = 360/24 = 15$

65. **(A)** $x + (2x + 5) + (x + 15) + (3x - 10) = 360^0$
$7x = 350^0$, or $x = 50^0$

66. **(C)** Sum of the interior angles
$= (2n - 4) \times 90^0 = 1980^0$
$2n - 4 = 22$, or $n = 13$

67. **(B)** Sum of the interior angles = $(2 \times 8) - 4$ right angles = 12 right angles = 1080^0
Each interior angle = $1080^0/8 = 135^0$

68. **(D)** Sum of the interior angles = $2(5) - 4$ right angles = 6 right angles = 540^0.
$130^0 + 118^0 + 80^0 + 78^0 + x = 540^0$
$x = 134^0$

69. **(D)** Sum of the interior angles
$= 160^\circ \times n = (2n - 4) \times 90^\circ$
$16n = 18n - 36$
$2n = 36$, or $n = 18$

70. **(B)** Each interior angle of a reg. hexagon ($n = 6$)
$= \dfrac{(2n-4) \times 90^\circ}{n} = \dfrac{(12-4) \times 90^\circ}{6} = 120^\circ$
Since the hexagon is regular, all sides are equal $\Rightarrow AF = FE$ and $FAE = GEF$
But $AFE = 120^\circ$ and $AFE + FAE + CEF = 180^\circ$
Hence $FAE = GEF = 30^\circ = AFG$
$EFG = 120^0 - 30^0 = 90^0$
$EGF = 180^0 - 90^0 - 30^0 = 60^0$

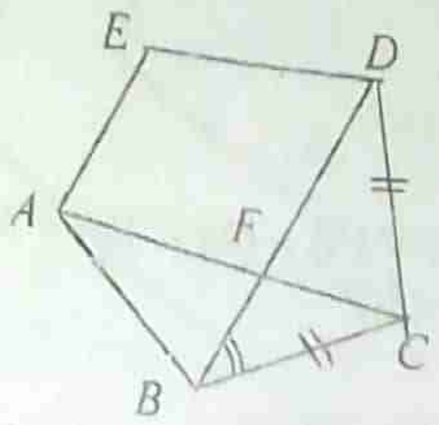
71. **(B)** The exterior angles of a regular polygon are equal and add up to 360^0. Hence each exterior angle must be a factor of 360^0. All, except 21^0, are factors of 360^0

72. **(C)** The sum of the interior angles of a pentagon ($n = 5$) is $2(5) - 4$ right angles = 540°.
Sum of the four angles specified = $(80 - y)^0 + (80 + y)^0 + (120 - 2y)^0 + (120 + 2y)^0 = 400^0$. Hence fifth angle = $540^0 - 400^0 = 140^0$

73. **(A)** Sum of the interior angles
$= [2(2m + 1) - 4]$ right angles
$= (4m - 2) \times 90^0$
Each interior angle = 144^0
Hence $(2m + 1)(144^\circ) = (4m - 2) \times 90^0$
$3.2m + 1.6 = 4m - 2$, or
$m = 4.5$

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74. **(D)**



Each interior angle = $540^0/5 = 108^0$
Hence $BCD = 108^0$
Since the pentagon is regular, $BC = CD$ and $CBD = BDC$
(*base angles, isosceles* Δ),
Also $BCD + CBD + BDC = 180^0$
(*angles of a* Δ)
$\Rightarrow CBD = (180^0 - 108^0)/2 = 36^0$

75. **(C)** Each interior angle of a regular pentagon $= 108^0$ (*see problem 74*)
Hence $ABC = BCD = 108^0$
$ABD = 90^0$ (*angle of a rectangle*)
$CBD = 108^0 - 90^0 = 18^0$
$CDB = 180^0 - CBD - BCD$ (*angles of a* Δ)
$= 180^0 - 18^0 - 108^0 = 54^0$

76. **(B)** Let each of the equal angles $= x$
Hence $4x + 80^0 = 540^0$
$4x = 460^0$, or $x = 115^0$.

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14 Circles

14.1 Definitions

- The following terms are used in relation to a circle (Fig. 14.1):

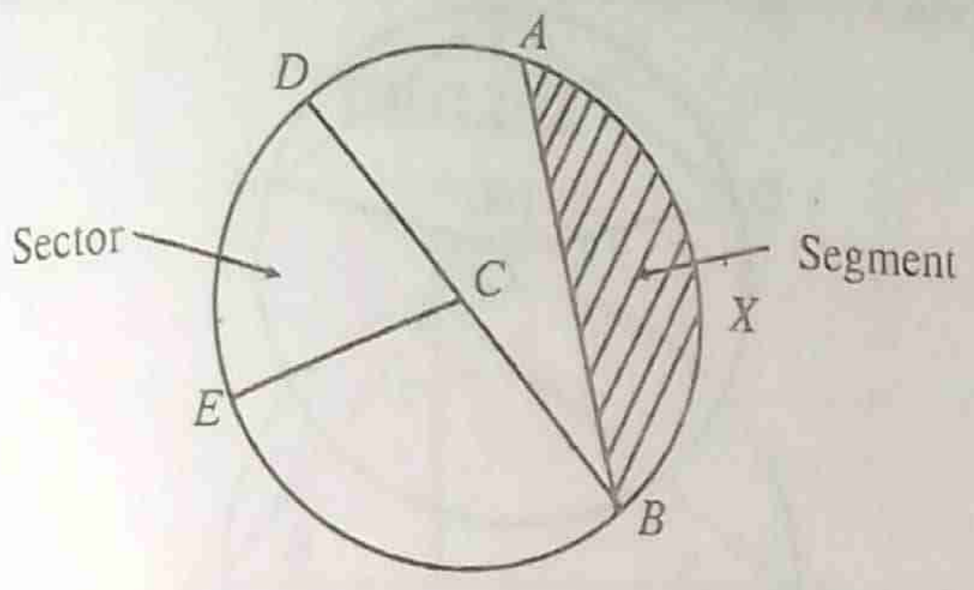


Fig. 14.1

(i) *Diameter:* a line passing through the centre C and joining two points on the circle, e.g. BD.

(ii) *Radius:* a line joining the centre to a point on the circle, e.g. CB, CD or CE.

(iii) *Chord:* a line, other than the diameter, joining two points on a circle, e.g. AB.

(iv) *Circumference:* the boundary or perimeter of a circle.

(v) *Arc:* part of the circumference, e.g. DE.

(vi) *Segment:* part of the area of a circle bounded by a chord and an arc, e.g. ABX.

(vii) *Sector:* part of the area of a circle bounded by two radii and an arc, e.g. CDE.

14.2 Theorems on Circles

1. The line which joins the centre of a circle to the midpoint of a chord is perpendicular to the chord (Fig. 14.2). If $AB = DB$, CD is perpendicular to AB.

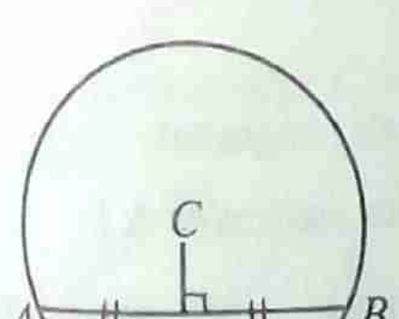


Fig. 14.2

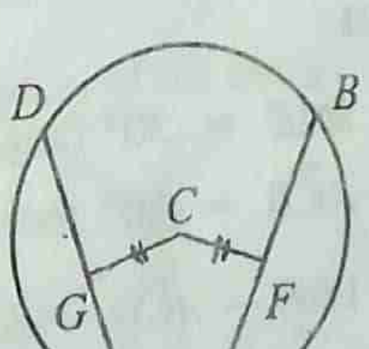


Fig. 14.3

2. Equal chords of a circle (or of equal circles) are equidistant from the centre of the circle (Fig. 14.3): If $AB = DE$, then $CF = CG$.

†3. The angle subtended by an arc at the centre of a circle is twice that subtended at any point in the remaining part of the circumference (Fig. 14.4): $A\hat{C}B = 2(A\hat{D}B)$.

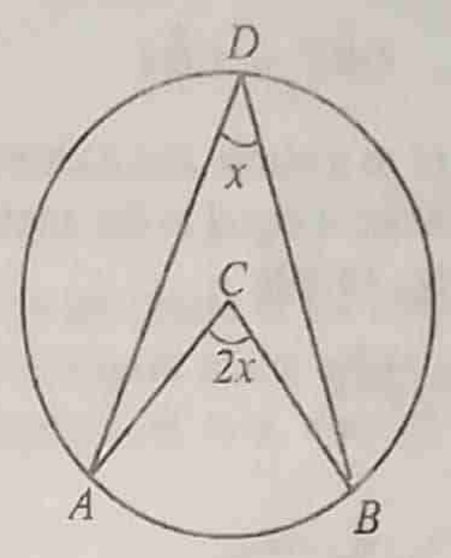


Fig. 14.4

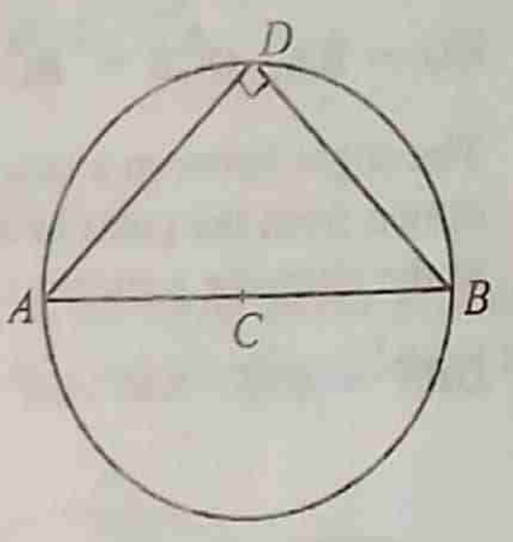


Fig. 14.5

4. The angle in a semicircle is a right angle (Fig. 14.5): If AB is a diameter, then $A\hat{D}B = 90°$.

†5. Angles in the same segment are equal (Fig. 14.6): $B\hat{A}D = B\hat{E}D$.

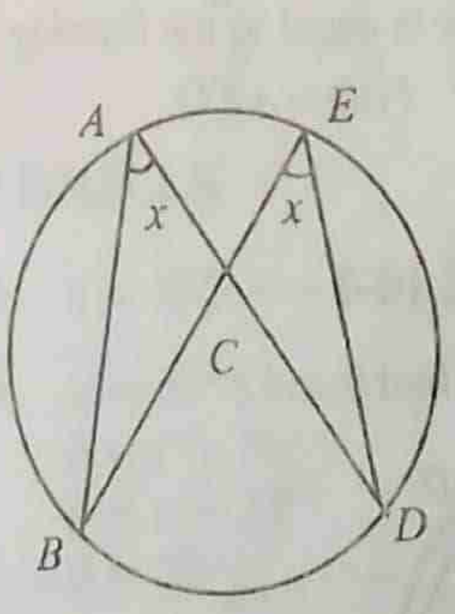


Fig. 14.6

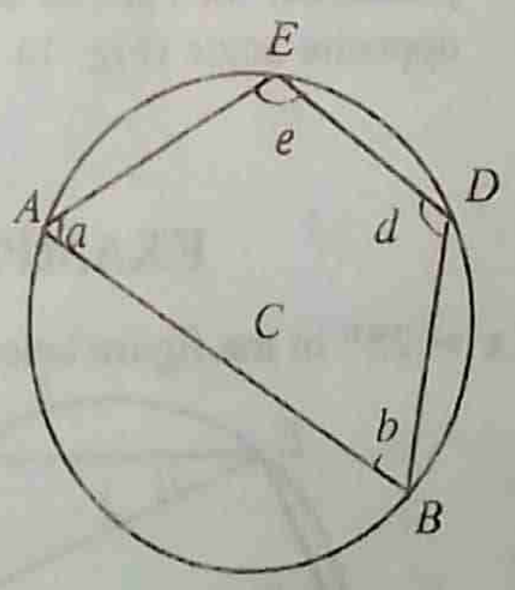


Fig. 14.7

6. Angles in opposite segments are supplementary, i.e. they add up to 180° (Fig. 14.7):
$a + d = 180° = b + e$
$ABDE$ is also called a **cyclic quadrilateral.**

7. A tangent to a circle is perpendicular to the radius at the point of contact (Fig. 14.8): CD is perpendicular to AB.

†Proof required in SSCE

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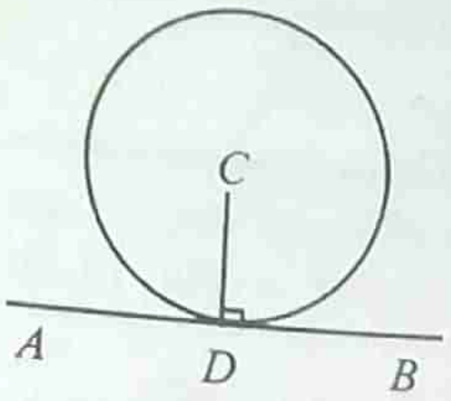


Fig. 14.8

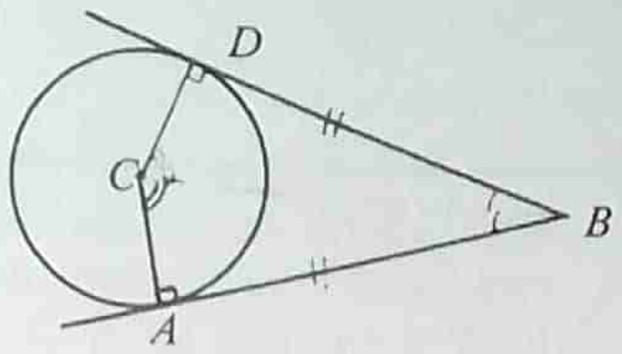


Fig. 14.9

8. Two tangents drawn to a circle from an external point are equal in length and subtend equal angles at the centre (Fig. 14.9):

BD = BA, $D\hat{C}B = B\hat{C}A$, $D\hat{B}C = C\hat{B}A$

9. The angle between a tangent to a circle and a chord drawn from the point of contact is equal to the angle in the alternate segment (Fig. 14.10):

$DB\hat{E} = B\hat{F}E$ and $A\hat{B}F = B\hat{E}F$

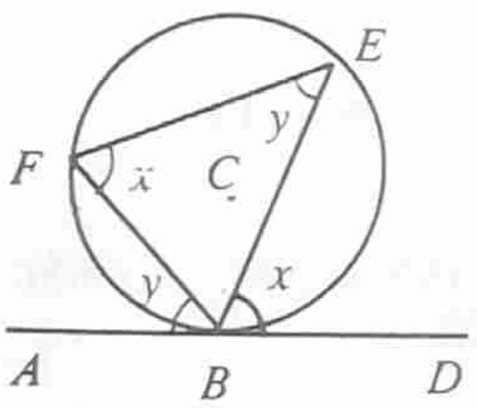


Fig. 14.10

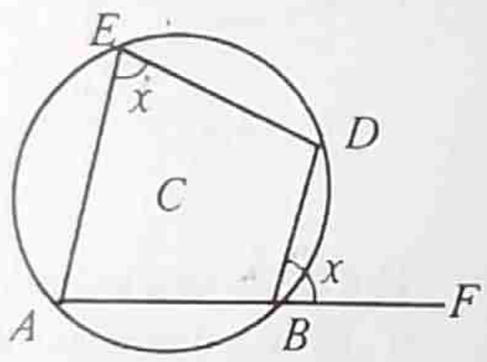
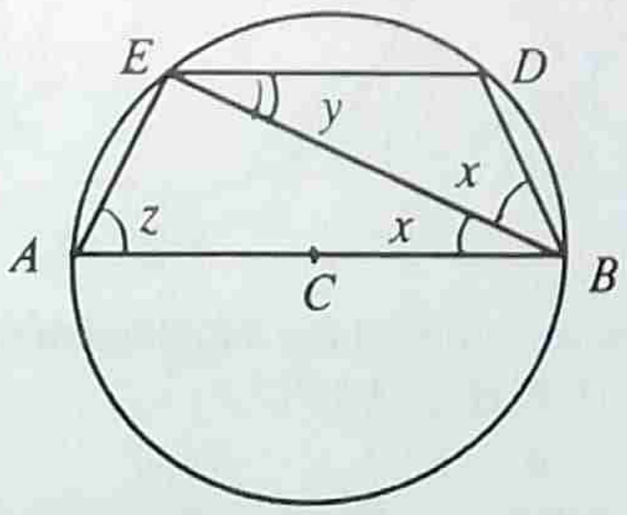


Fig. 14.11

10. If one of the sides of a cyclic quadrilateral is produced, the exterior angle is equal to the interior, opposite angle (Fig. 14.11) $F\hat{B}D = A\hat{E}D$

EXAMPLE 14-1

If $x = 25°$ in the figure below, find y and z



Solution

$C\hat{B}D = 2x = 2(25°) = 50°$

$A\hat{E}D = 180° - C\hat{B}D$ *(opp. angles cyclic quad.)*

$= 180° - 50° = 130°$

$A\hat{E}B = 90°$ *(angle in a semicircle)*

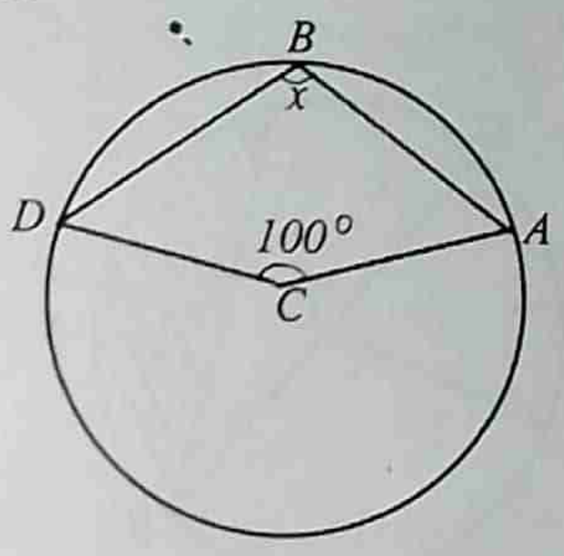
$y = A\hat{E}D - A\hat{E}B = 130° - 90° = 40°$

In ΔAEB, $x + z + A\hat{E}B = 180°$

$z = 180° - 90° - 25° = 65°$

EXAMPLE 14-2

Find the value of x in the figure below if C is the centre of the circle.



Solution

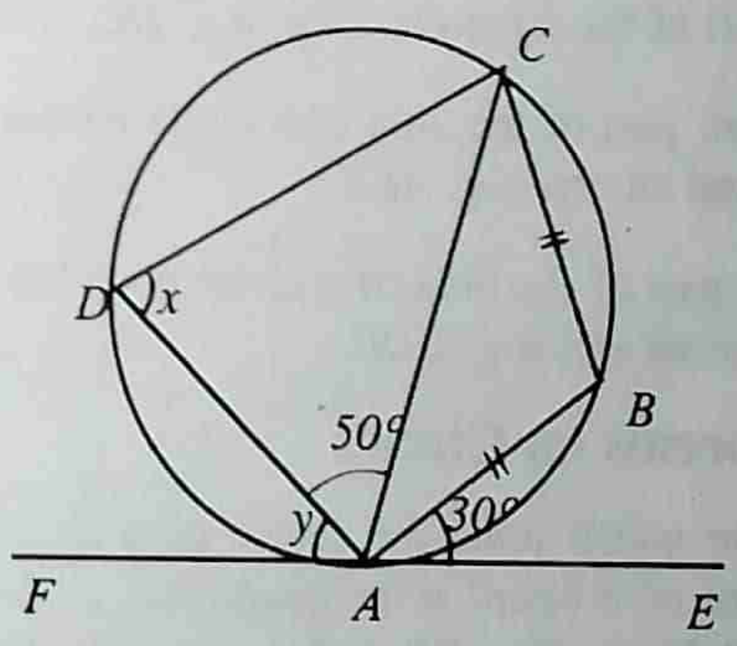
Reflex $D\hat{C}A$ at C = 360° – 100° = 260°.

This is equal to $2x$ *(angle at centre = twice angle at circumference)*

$2x = 260°$, or $x = 130°$

EXAMPLE 14-3

Find the values of x and y in the figure below if FAE is a tangent to the circle at A, and $AB = BC$.



Solution

$A\hat{C}B = B\hat{A}E = 30°$ (angles in alt. segment)

$B\hat{A}C = A\hat{C}B = 30°$ *(base angles isosceles Δ)*

$A\hat{B}C = 180° - A\hat{C}B - B\hat{A}C = 180° - 30° - 30° = 120°$

$x = A\hat{D}C = 180° - A\hat{B}C$ *(opp. angles, cyclic quad.)*

$= 180° - 120° = 60°$

$D\hat{C}A = 180° - 50° - x = 180° - 50° - 60° = 70°$

$y = D\hat{C}A = 70°$ *(angles in alt. segment).*

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DRILL 1

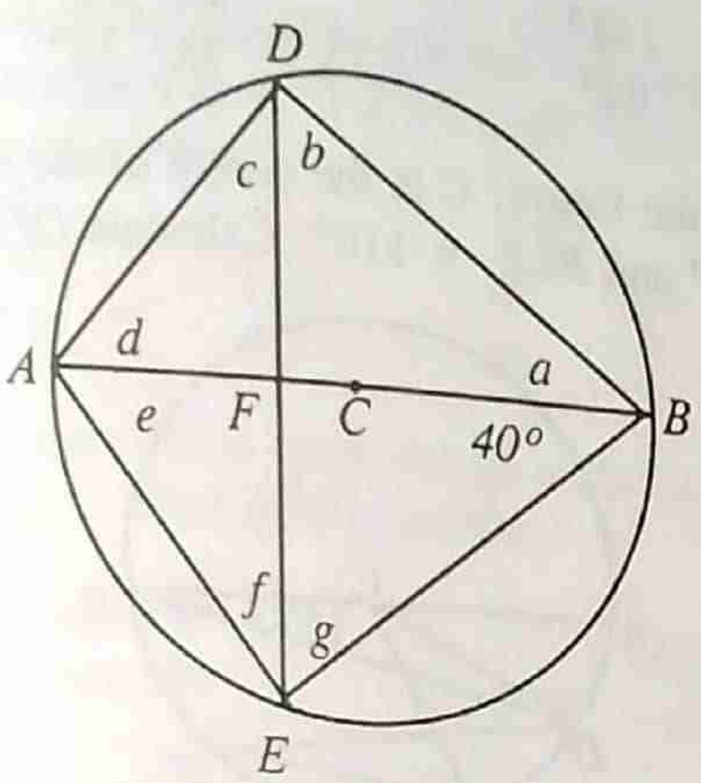
Given a circle of radius 5cm,

(a) how far is a chord of length 6 cm from the centre of the circle?

(b) what is the length of a chord which is 2cm from the centre of the circle?

(c) what is the maximum length of a chord of the circle?

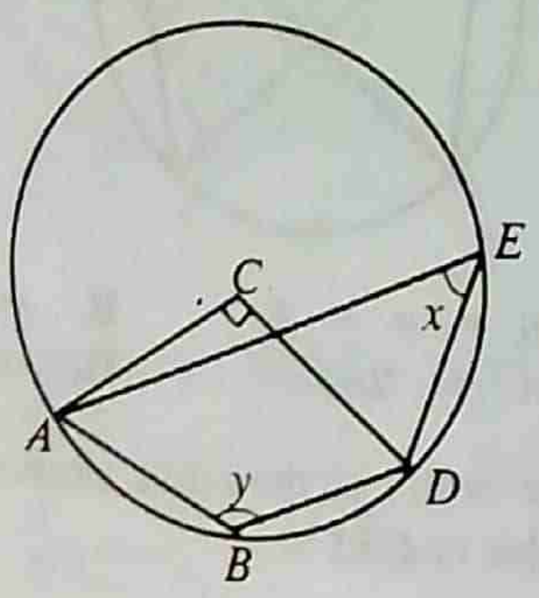
DRILL 2



If C is the centre of the circle in the figure above, $|AD| = |AE|$ and $A\hat{B}E = 40^{\circ}$, find the values of a, b, c, d, e, f and g.

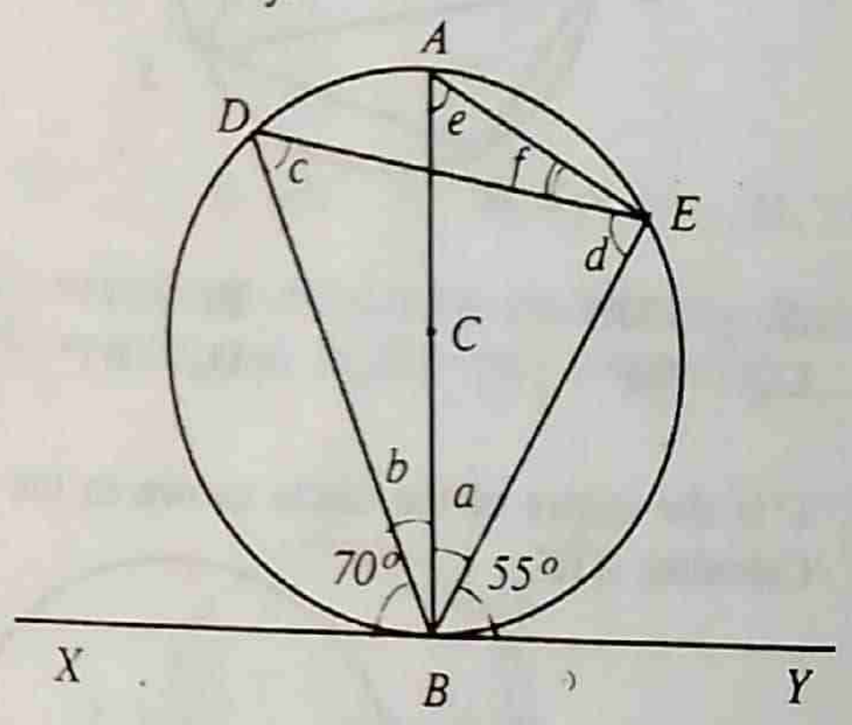
DRILL 3

Find the values of x and y in the figure below if C is the centre of the circle, and $A\hat{C}D = 90^{\circ}$



DRILL 4

C is the centre of the circle in the figure below and ***XBY*** is a tangent to the circle at B. Find the magnitude of the angles a, b, c, d, e and f.

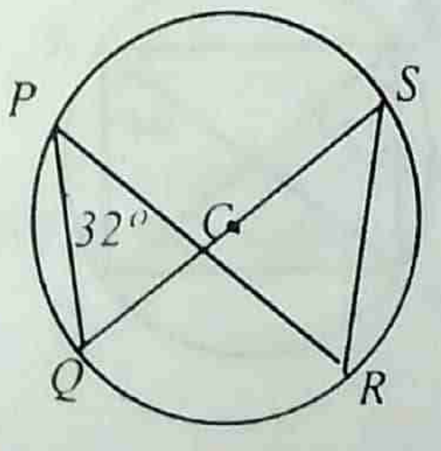


ANSWERS TO DRILLS

1. (a) 4 cm (b) 9.17 cm
   (c) 10 cm (the diameter of the circle)
2. $a = c = f = 40^{\circ}$
   $b = d = e = g = 50^{\circ}$
3. $x = 45^{\circ}$, $y = 135^{\circ}$
4. $a = 35^{\circ}$
   $b = f = 20^{\circ}$
   $c = e = 55^{\circ}$
   $d = 70^{\circ}$

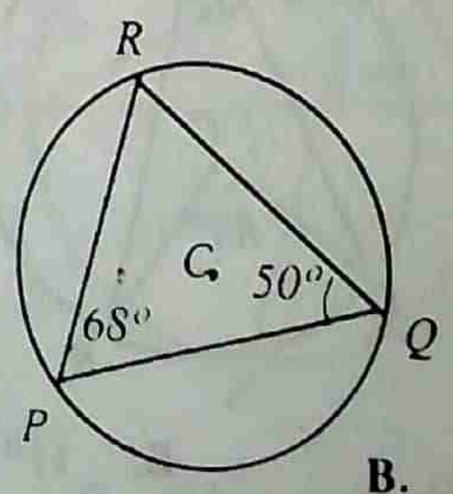
SSCE/JME – TYPE OBJECTIVE QUESTIONS

1. In the diagram, C is the centre of the circle. What is $S\hat{Q}R$?



A. 148° B. 122°
C. 58° D. 32°

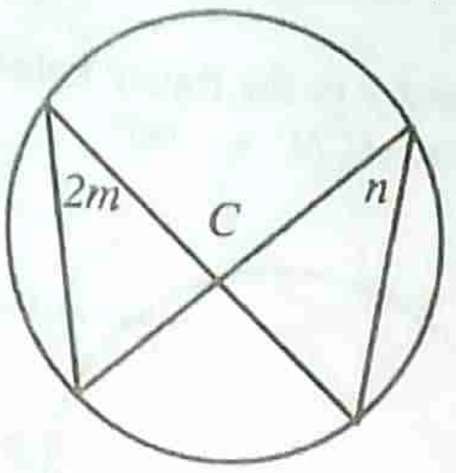
2. In the diagram, C is the centre of the circle. Calculate $P\hat{C}Q$



A. 62° B. 100°
C. 118° D. 124°

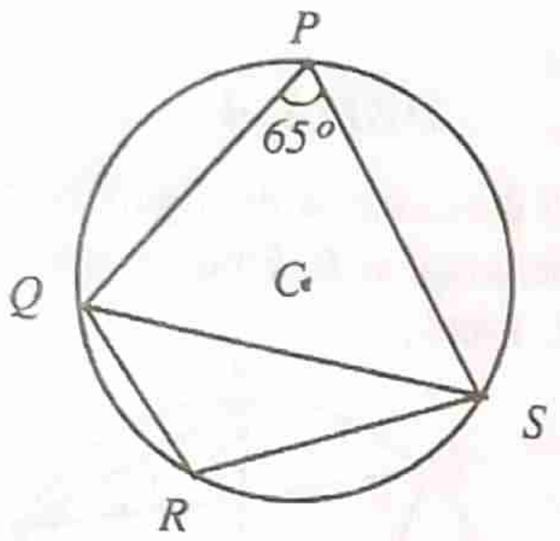
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3. Referring to the diagram, express m in terms of n



A. $m = n + 2$
B. $m = n - 2$
C. $m = 2n$
D. $m = n/2$

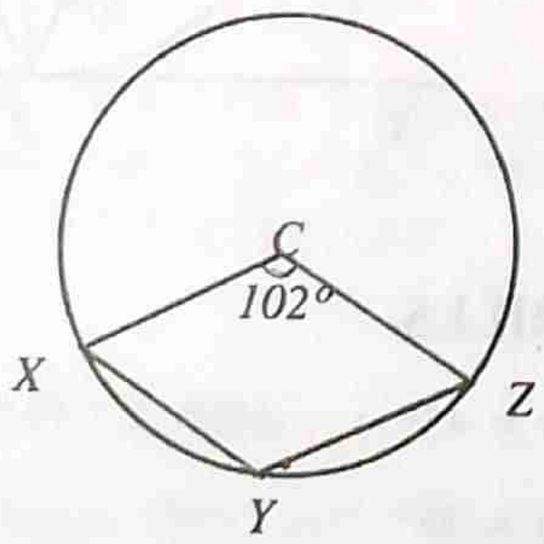
4. C is the centre of the circle shown in the diagram. The value of $\hat{QRS}$ is



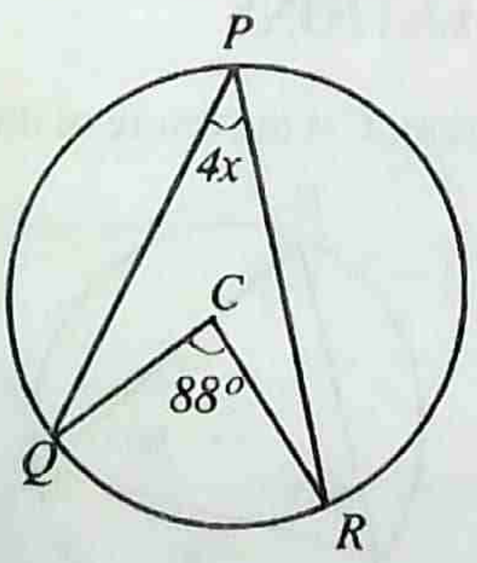
A. 130°
B. 115°
C. 90°
D. 65°

5. C is the centre of the circle shown in the diagram. Calculate $\hat{XYZ}$

A. 129°
B. 102°
C. 98°
D. 78°

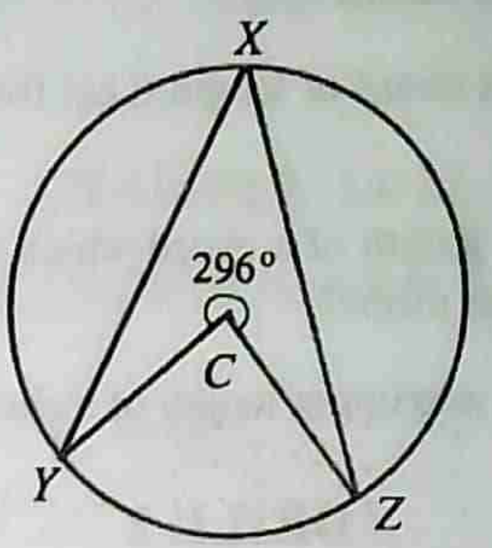


6. If C is the centre of the circle shown in the diagram, the value of x is



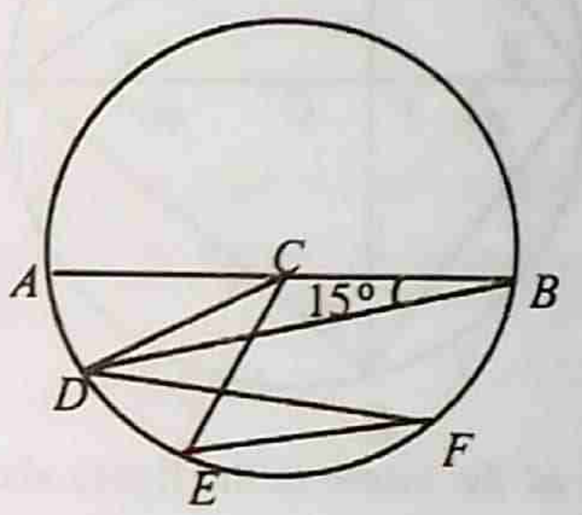
A. $5\frac{1}{2}°$
B. 11°
C. 22°
D. 44°

7. C is the centre of the circle shown in the figure, and reflex angle $\hat{YCZ} = 296°$. Find $\hat{YXZ}$.



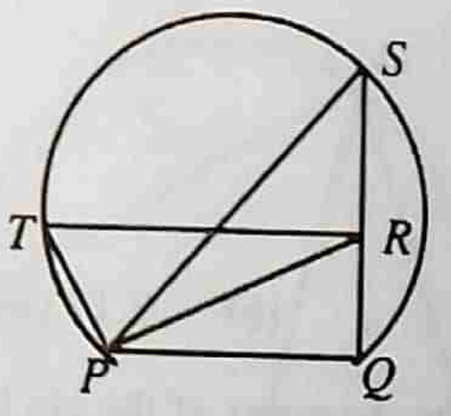
A. 148°
B. 104°
C. 64°
D. 32°

8. In the figure, C is the centre of the circle, $\hat{ABD} = 15°$ and $\hat{BCE} = 110°$. Calculate $\hat{DFE}$



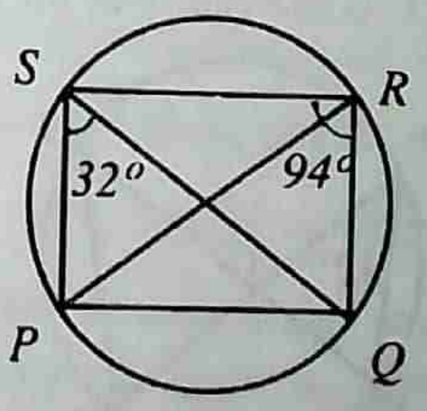
A. 55°
B. 30°
C. 20°
D. 15°

9. $PQST$ is the major arc of a circle (see diagram). Which angle is equal to $\hat{PTQ}$?



A. $\hat{PSQ}$
B. $\hat{PRQ}$
C. $\hat{SQT}$
D. $\hat{RTQ}$

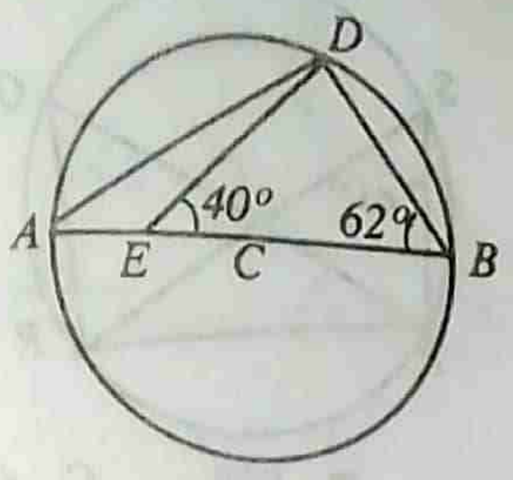
10. In the figure, $PQRS$ is a cyclic quadrilateral, $\hat{SRQ} = 94°$ and $\hat{PSQ} = 32°$. Find $\hat{SQP}$.



A. 62°
B. 58°
C. 47°
D. 32°

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11. AB is a diameter of the circle shown in the diagram. If $A\hat{B}D = 62°$ and $D\hat{E}B = 40°$, find $A\hat{D}E$.



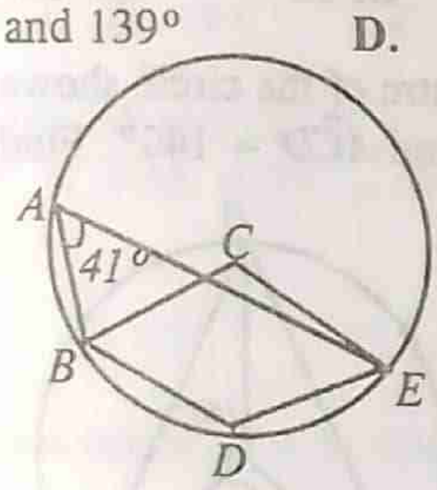
A. 10° B. 12°
C. 20° D. 22°

12. $PQRS$ is a cyclic quadrilateral and the diagonals PR and QS intersect at T. If $S\hat{P}R = 47°$ and $P\hat{T}Q = 98°$, calculate $P\hat{R}Q$.

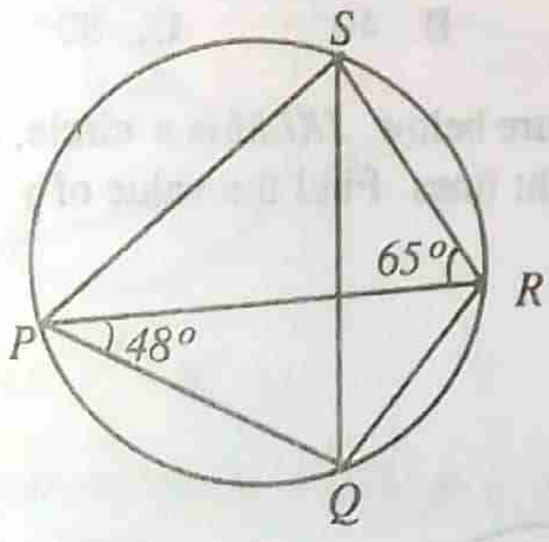
A. 145° B. 94°
C. 51° D. 47°

13. C is the centre of the circle shown in the diagram and $B\hat{A}E = 41°$. Find $B\hat{C}E$ and $B\hat{D}E$.

A. 41° and 49° B. 82° and 41°
C. 41° and 139° D. 82° and 139°



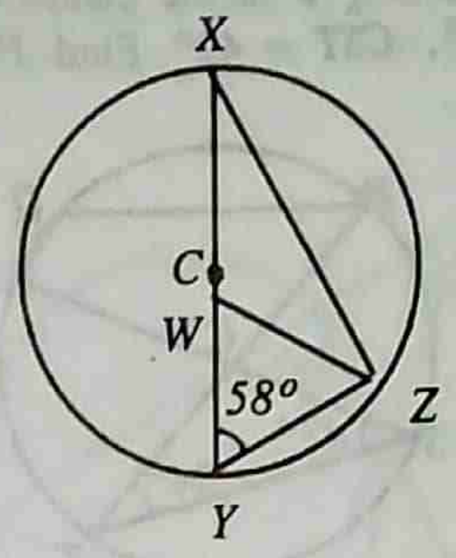
14. $PQRS$ is a cyclic quadrilateral (see diagram). If $|SR| = |QR|$, $S\hat{R}P = 65°$ and $R\hat{P}Q = 48°$, find $P\hat{R}Q$



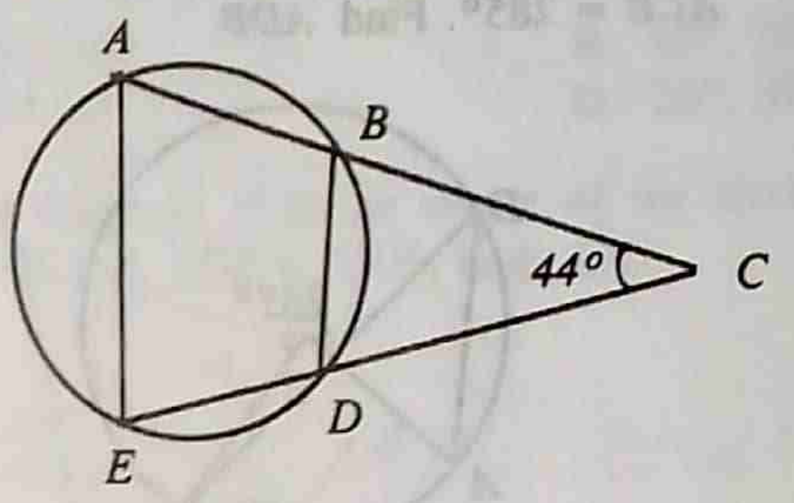
A. 19° B. 25°
C. 48° D. 65°

15. In the diagram, C is the centre of the circle, $|YW| = |YZ|$ and $X\hat{Y}Z = 58°$. Calculate $X\hat{Z}W$.

A. 61° B. 58°
C. 32° D. 29°

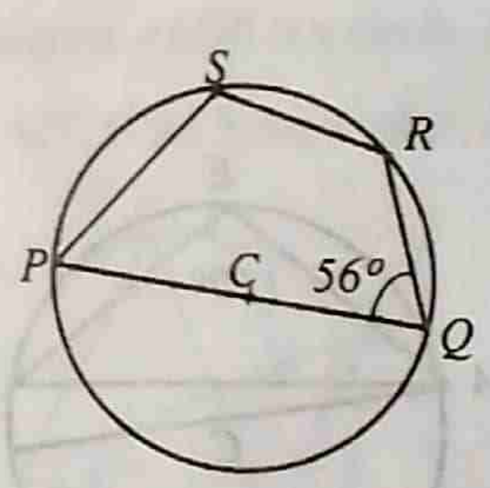


16. In the diagram below, $|CD| = |CB|$ and $B\hat{C}D = 44°$. Find $A\hat{E}D$.



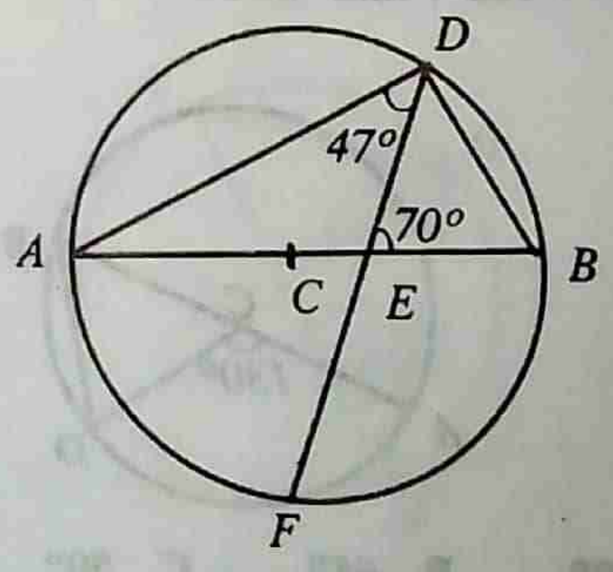
A. 68° B. 46° C. 44° D. 22°

17. PQ is a diameter of the circle shown in the diagram. $|PS| = |SR|$ and $P\hat{Q}R = 56°$. Find $S\hat{P}Q$.



A. 28° B. 62° C. 84° D. 118°

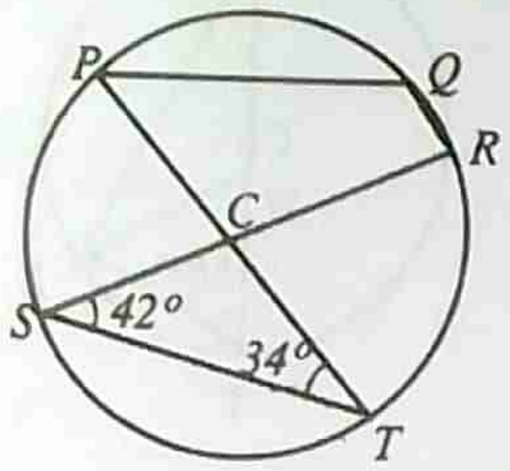
18. ACB is a diameter of the circle shown in the figure. If $A\hat{D}E = 47°$ and $B\hat{E}D = 70°$, calculate $D\hat{B}E$.



A. 23° B. 43° C. 47° D. 67°

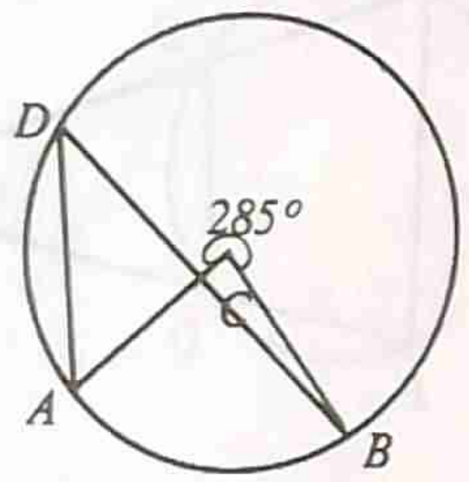
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19. In the diagram, C is the centre of the circle and $C\hat{T}S = 34°$, $C\hat{S}T = 42°$. Find $P\hat{Q}R$.



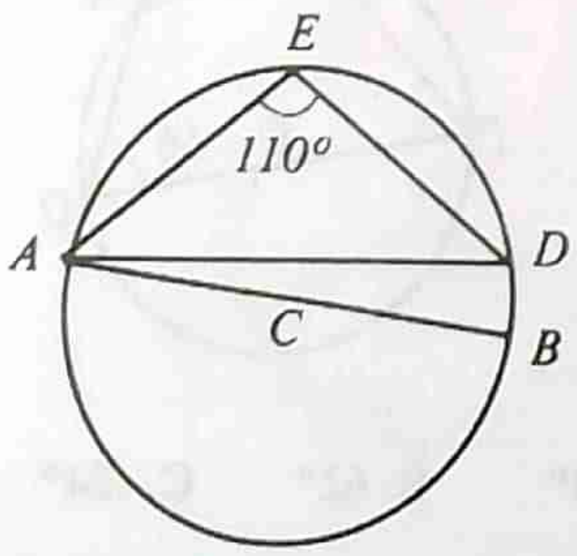
A. 76° **B.** 104° **C.** 128° **D.** 256°

20. In the figure, C is the centre of the circle and reflex $A\hat{C}B = 285°$. Find $A\hat{D}B$.



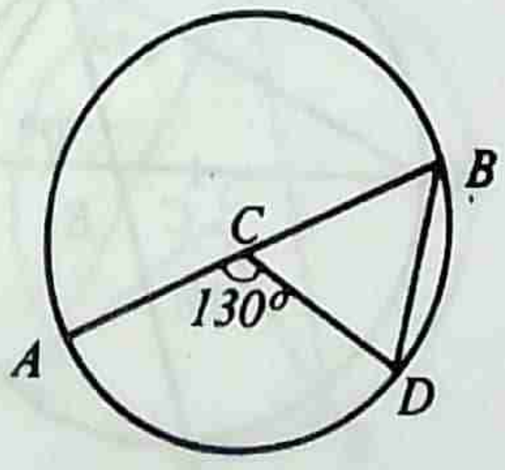
A. 37½° **B.** 55° **C.** 75° **D.** 142½°

21. ACB is the diameter of the circle shown in the figure. If $A\hat{E}D = 110°$, find $D\hat{A}B$.



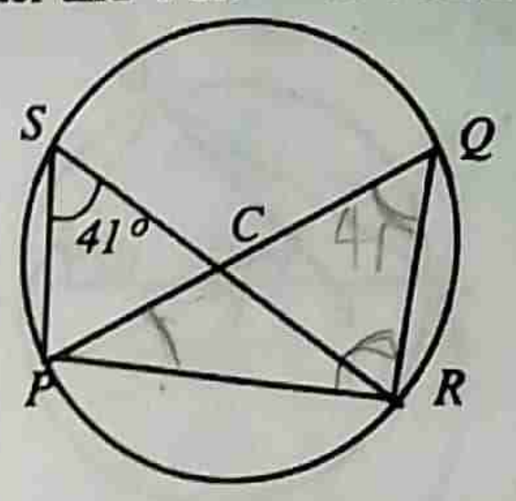
A. 110° **B.** 70° **C.** 35° **D.** 20°

22. If C is the centre of the circle in the diagram and $A\hat{C}D = 130°$, find $C\hat{D}B$.



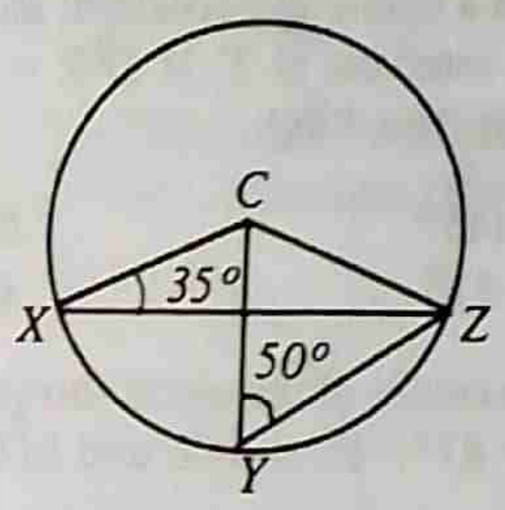
A. 90° **B.** 65° **C.** 50° **D.** 25°

23. In the diagram, C is the centre of the circle, PCQ is a diameter and $P\hat{S}R = 41°$. Find $R\hat{P}Q$.



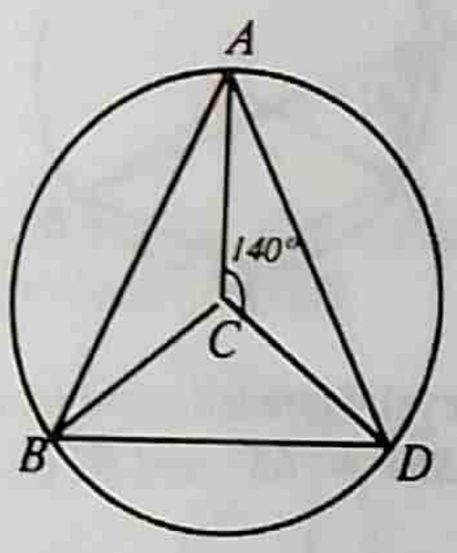
A. 139° **B.** 98° **C.** 49° **D.** 41°

24. C is the centre of the circle XYZ shown in the figure, $C\hat{X}Z = 35°$ and $C\hat{Y}Z = 50°$. Find $X\hat{C}Y$.



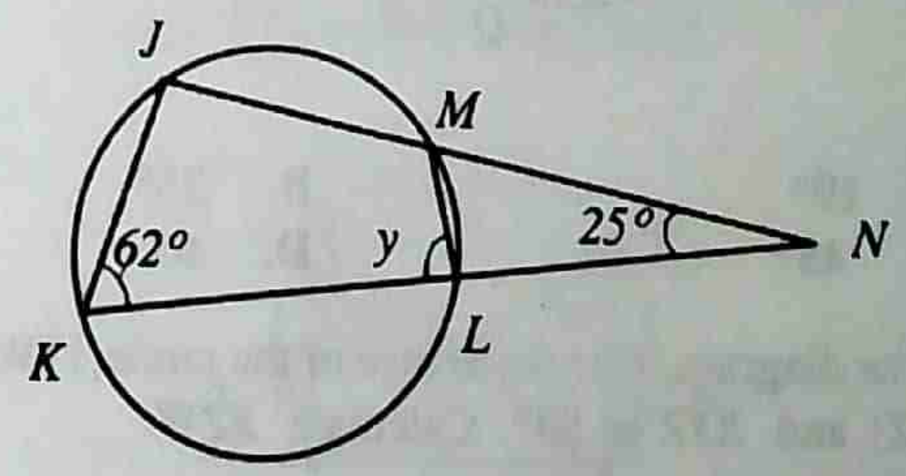
A. 15° **B.** 30° **C.** 55° **D.** 85°

25. C is the centre of the circle shown in the figure and $AB = AD$ and $A\hat{C}D = 140°$. Find $B\hat{C}D$.



A. 20° **B.** 40° **C.** 80° **D.** 140°

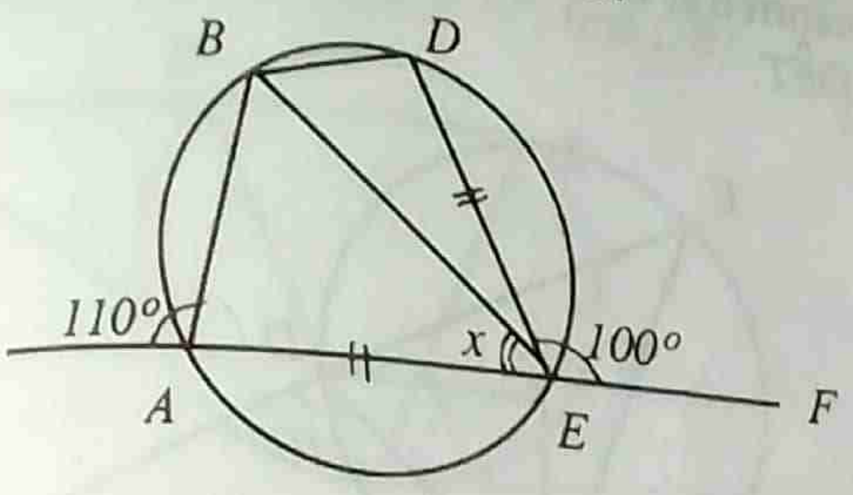
26. In the figure below, $JKLM$ is a circle, JMN and KLN are straight lines. Find the value of y.



A. 37° **B.** 87° **C.** 93° **D.** 118°

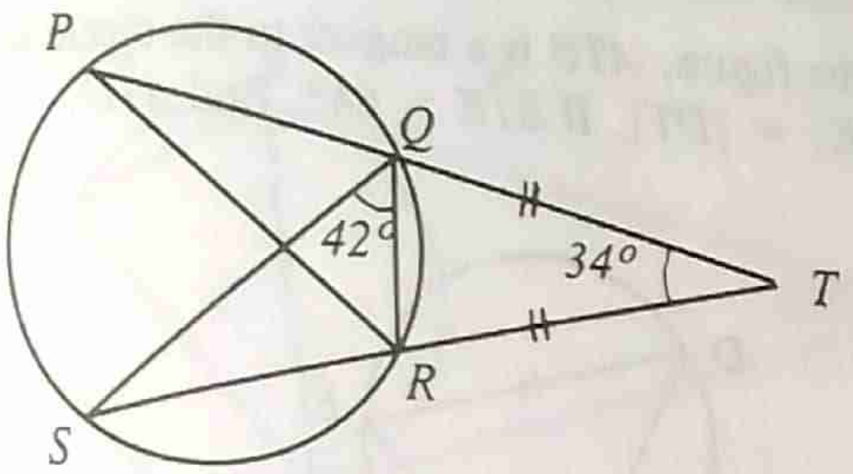
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27. $ABDE$ is a circle and chord AE = chord DE (see diagram). Calculate the value of x.



A. 60° B. 50° C. 30° D. 20°

28. In the figure below, PQT and SRT are straight lines and $|QT| = |RT|$. Find $Q\hat{P}R$.

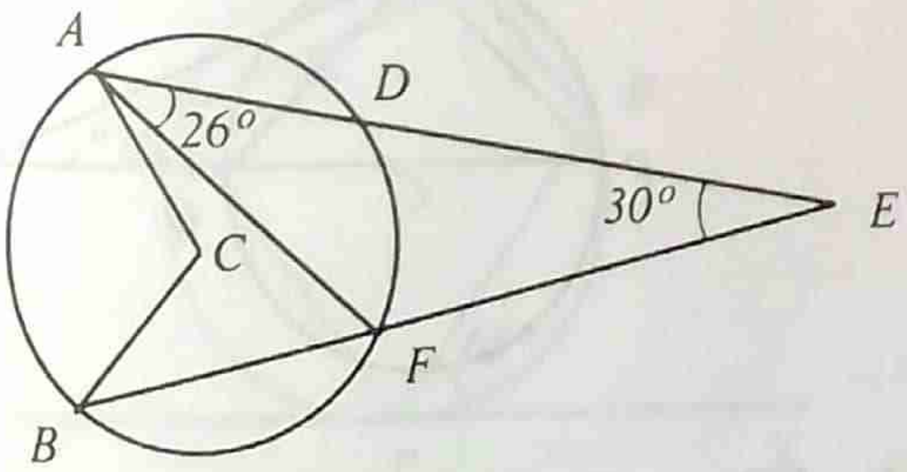


A. 107° B. 76° C. 42° D. 31°

29. Two chords AB and DE of a circle when produced meet at F, If $F\hat{A}E = 25°$ and $A\hat{F}D = 32°$, What is $D\hat{B}F$?

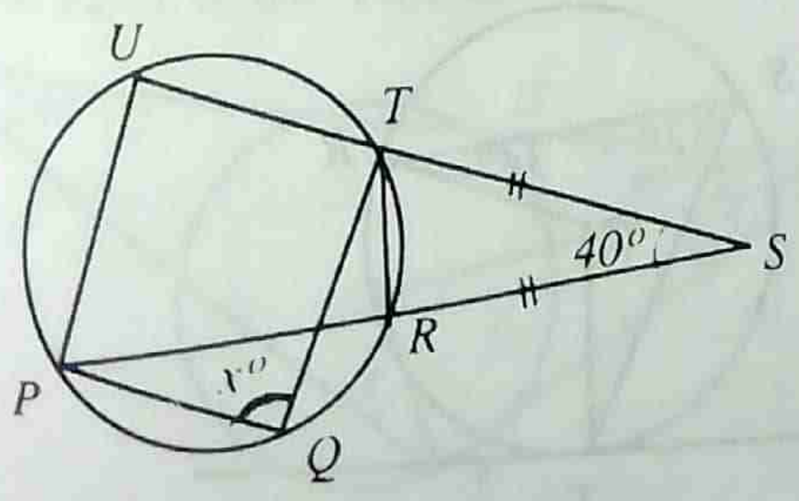
A. 7° B. 57° C. 116° D. 123°

30. In the figure below, C is the centre of the circle. Find $A\hat{C}B$.



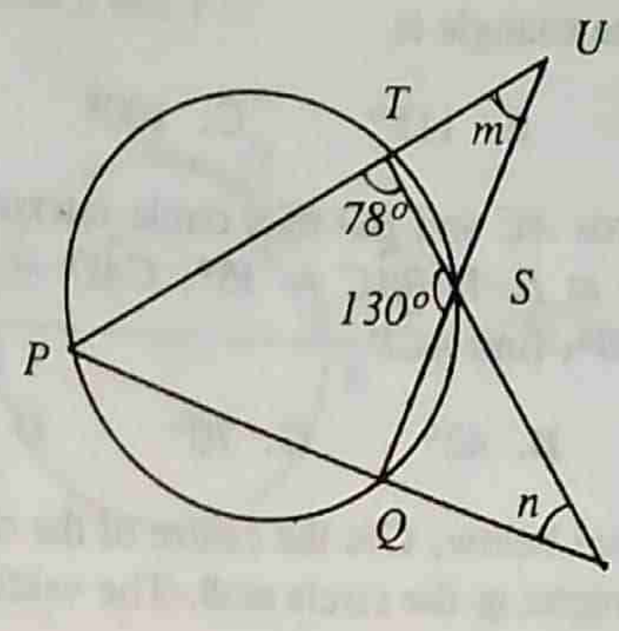
A. 56° B. 112° C. 124° D. 150°

31. In diagram below, $PQTU$ is a circle and $|SR| = |ST|$. Find x.



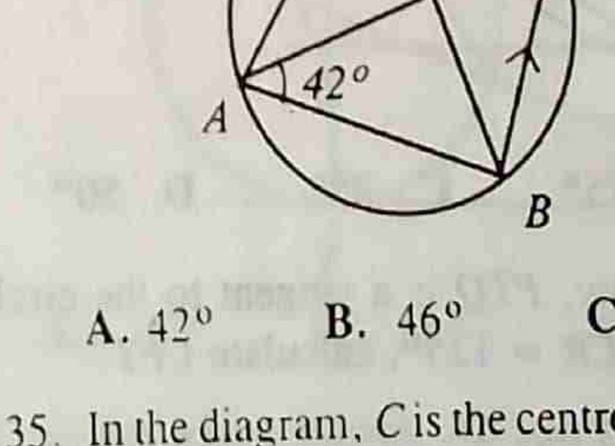
A. 40° B. 70° C. 110° D. 140°

32. In the diagram, $PQST$ is a cyclic quadrilateral. If $P\hat{T}S = 78°$ and $Q\hat{S}T = 130°$, find $(m.n)$.



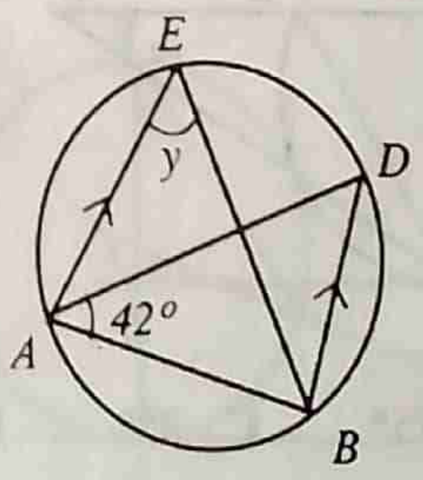
A. 28°, 52° B. 42°, 28°
C. 39°, 65° D. 28°, 39°

33. In the diagram, C is the centre of the circle, If $AB||DE$ and $C\hat{D}E = 130°$, find $A\hat{B}C$.



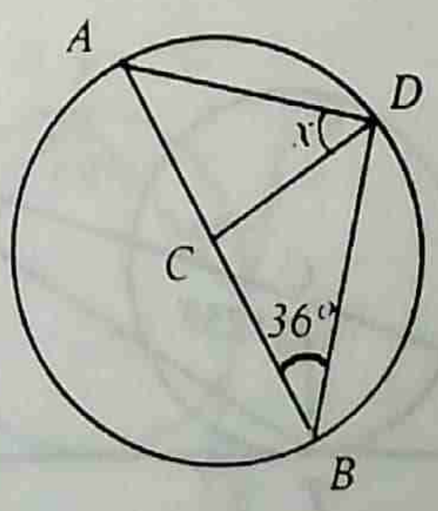
A. 65° B. 50° C. 40° D. 32½°

34. In the diagram $ABDE$ is a circle, $AE||DB$ and $B\hat{A}D = 42°$. If BE bisects $A\hat{B}D$, find y.



A. 42° B. 46° C. 48° D. 84°

35. In the diagram, C is the centre of the circle. The value of x is



A. 36° B. 64° C. 54° D. 72°

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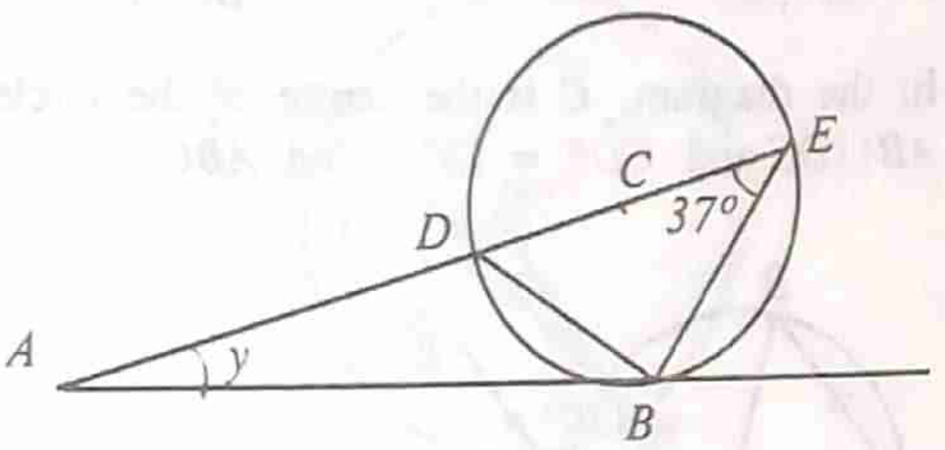
36. All the vertices of an isosceles triangle lie on a circle and each of the base angles of the triangle is $65°$. The angle subtended at the centre of the circle by the base of the triangle is

A. $130°$ **B.** $115°$ **C.** $100°$ **D.** $65°$

37. Two chords AC and BD of a circle intersect inside the circle at E. If $B\hat{A}C = 35°$, $C\hat{A}D = 42°$ and $A\hat{B}D = 70°$, find $A\hat{C}B$.

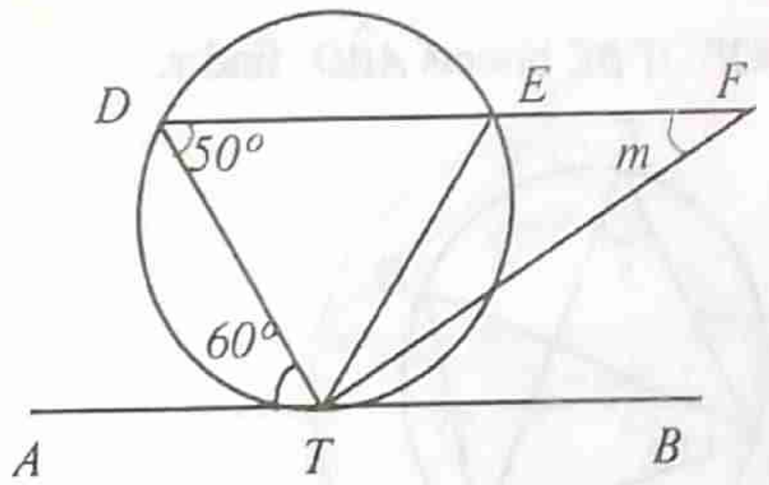
A. $33°$ **B.** $42°$ **C.** $70°$ **D.** $77°$

38. In the figure below, C is the centre of the circle and AB is a tangent to the circle at B. The value of y is



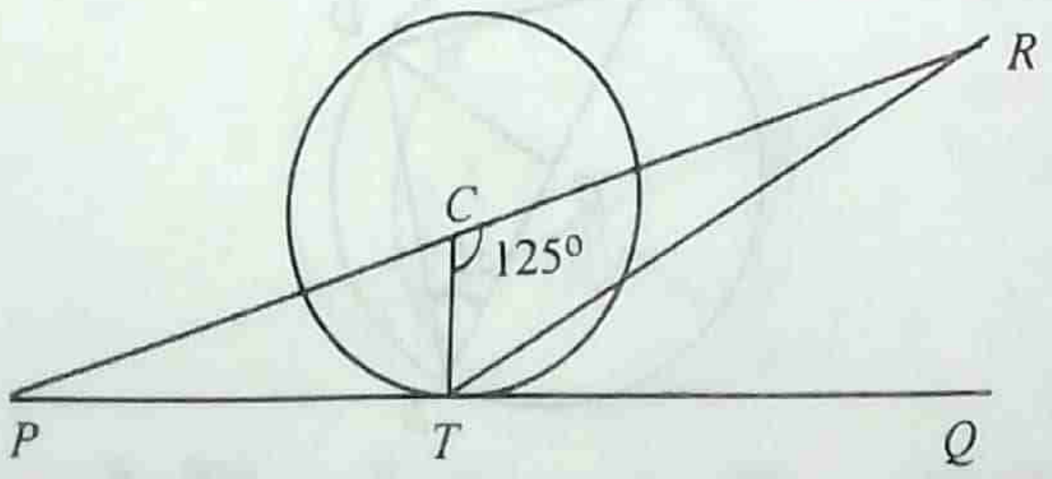
A. $53°$ **B.** $37°$ **C.** $23°$ **D.** $16°$

39. ATB is a tangent to the circle DTE at T (see diagram). If TF bisects $B\hat{T}E$, the value of m is



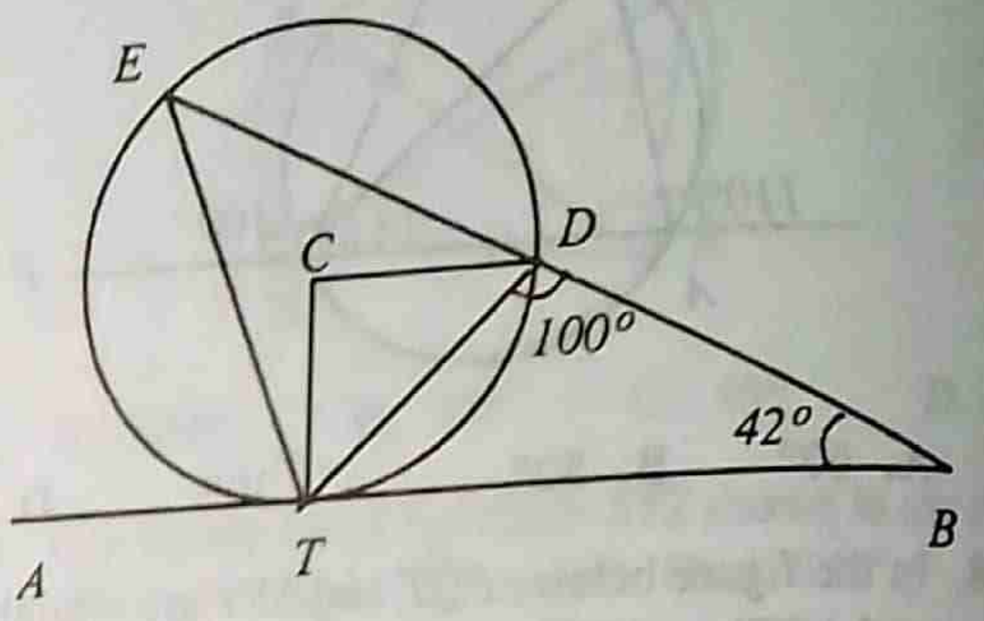
A. $10°$ **B.** $25°$ **C.** $35°$ **D.** $50°$

40. In the figure below, PTQ is a tangent to the circle centred at C. If $T\hat{C}R = 125°$, calculate $C\hat{P}T$.



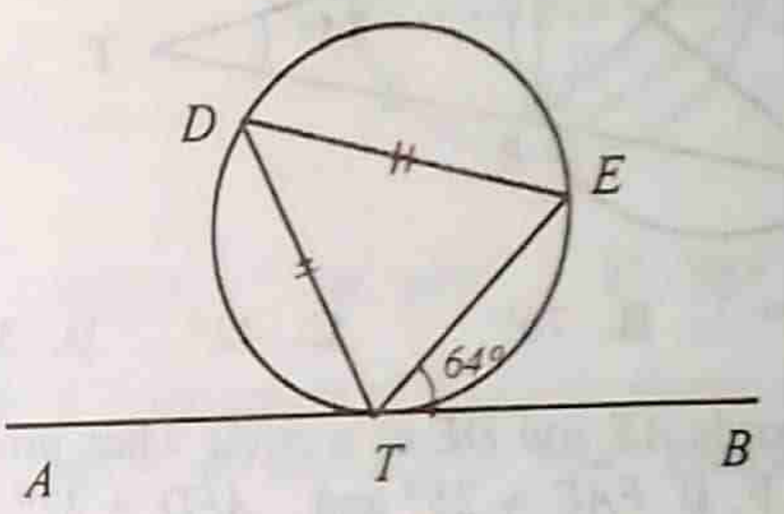
A. $25°$ **B.** $35°$ **C.** $45°$ **D.** $55°$

41. In the diagram below, ATB is a tangent to the circle centred at C. If $B\hat{D}T = 100°$ and $D\hat{B}T = 42°$, find $D\hat{E}T$.



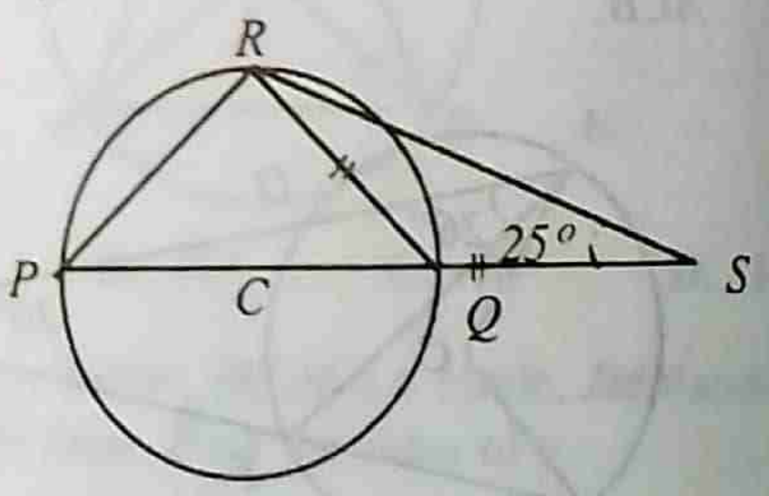
A. $50°$ **B.** $48°$ **C.** $42°$ **D.** $38°$

42. In the figure, ATB is a tangent to the circle at T and $|DE| = |DT|$. If $B\hat{T}E = 64°$, find $A\hat{T}D$.



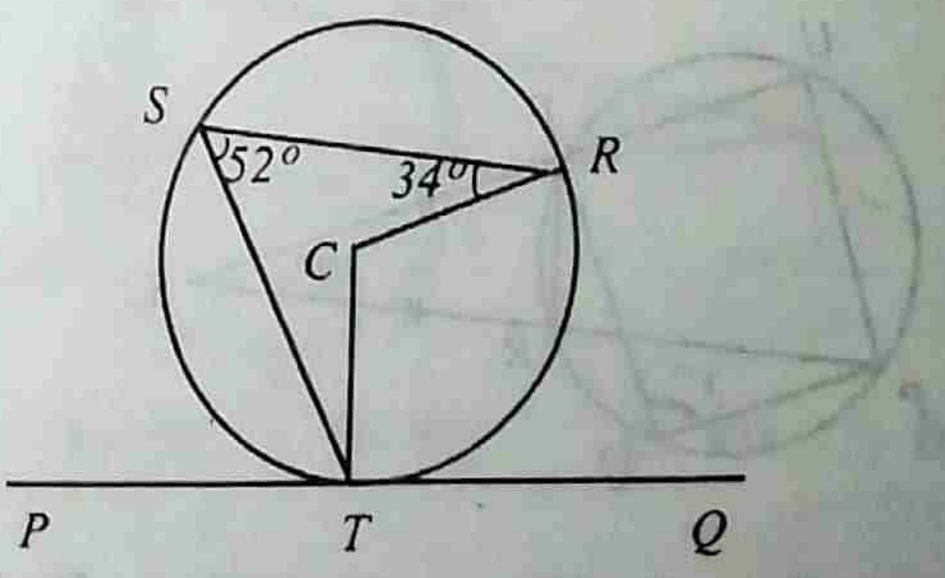
A. $32°$ **B.** $42°$ **C.** $58°$ **D.** $64°$

43. In the diagram below, C is the centre of the circle and $|QS| = |QR|$. If $R\hat{S}Q = 25°$, find $R\hat{P}C$.



A. $65°$ **B.** $50°$ **C.** $40°$ **D.** $25°$

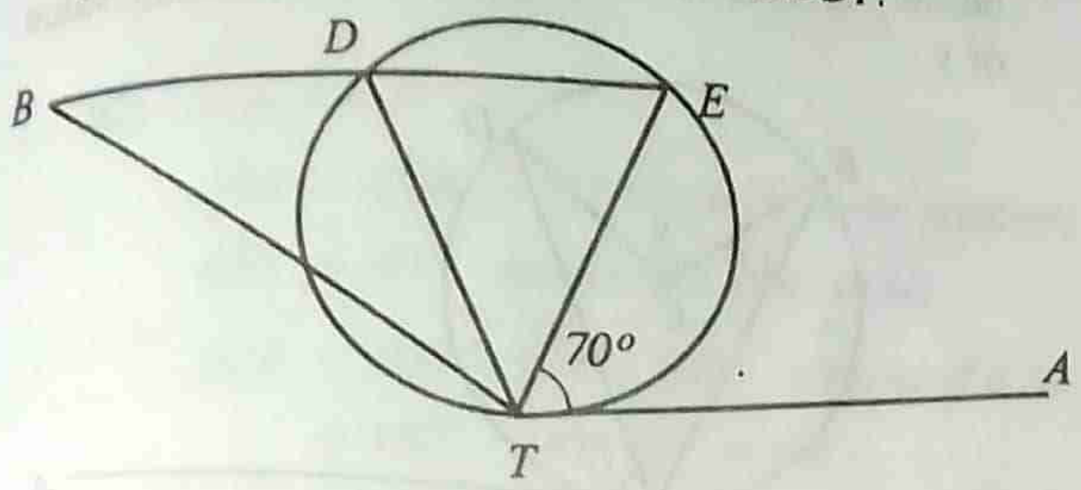
44. PTQ is a tangent to the circle with centre C (see diagram below). Find $S\hat{T}P$.



A. $72°$ **B.** $68°$ **C.** $56°$ **D.** $52°$

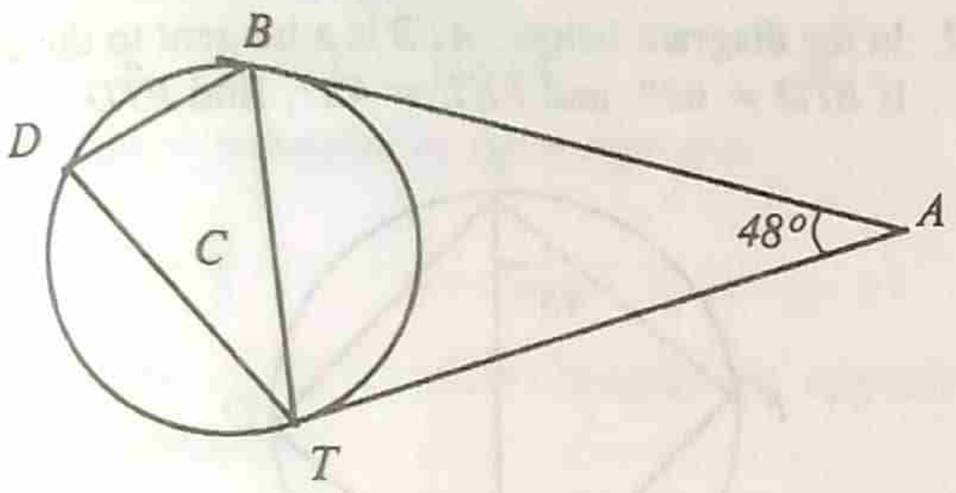
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45. In the figure below, AT is a tangent to the circle at T. If $|BD| = |DT|$ and $A\hat{T}E = 70°$, find $D\hat{B}T$.



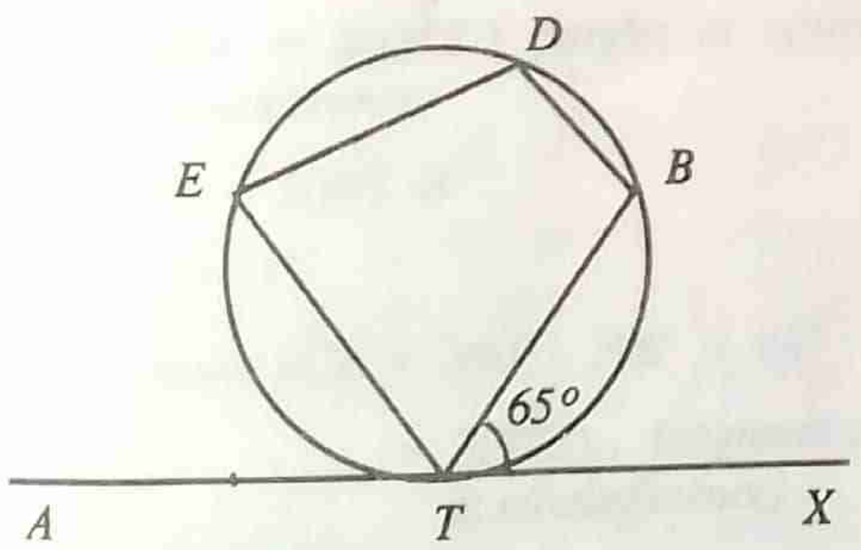
A. 20° B. 35° C. 50° D. 70°

46 In the diagram below, AT and AB are tangents to the circle centred at C. If $B\hat{A}T = 48°$, find $B\hat{D}T$



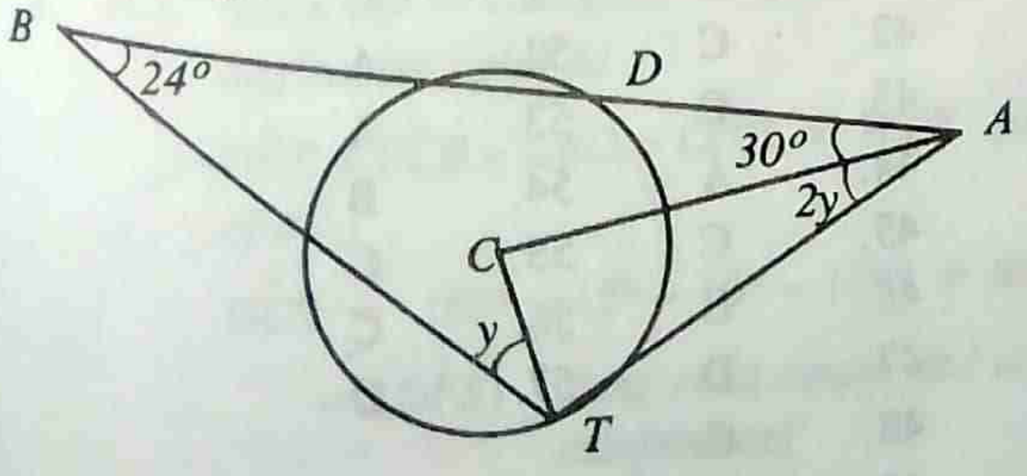
A. 42° B. 48° C. 66° D. 72°

47. In the diagram below, $BDET$ is a cyclic quadrilateral. ATX is a tangent to the circle at T, $B\hat{T}X = 65°$ and $|TB| = |TE|$. Find $B\hat{D}E$.



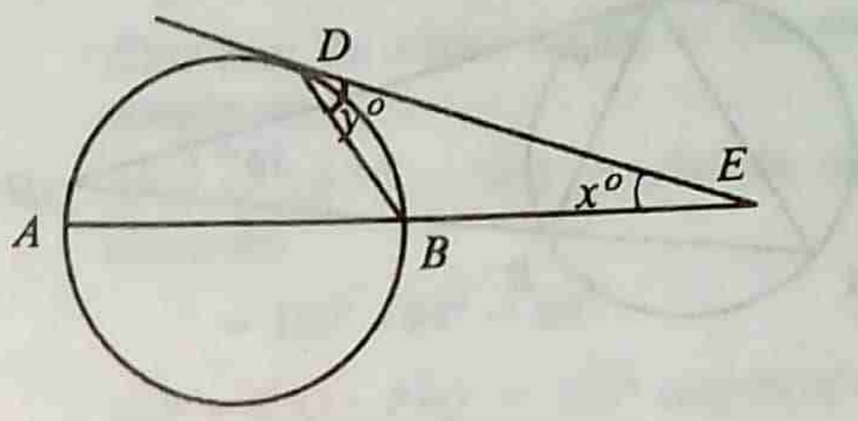
A. 65° B. 100° C. 115° D. 130°

48. AT is a tangent to the circle, centred at C as shown in the figure below. Find $C\hat{A}T$



A. 12° B. 15° C. 24° D. 30°

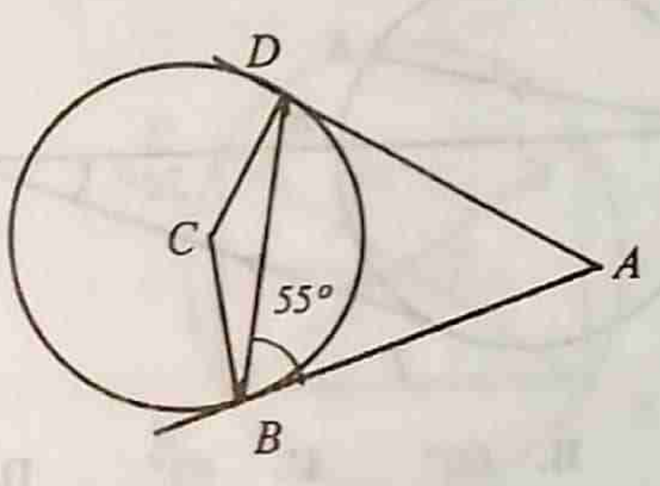
49. In the figure below AB is a diameter of the circle and ED is a tangent to the circle. The correct relationship between x and y is



A. $x + y = 90°$ B. $x = y$

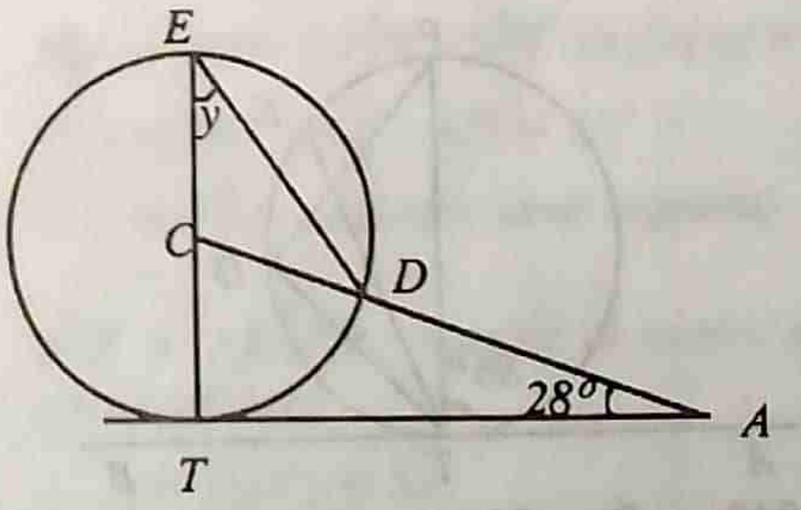
C. $x + 2y = 180°$ D. $x + 2y = 90°$

50. AD and AB are tangents to the circle centred at C. If $A\hat{B}D = 55°$, find $B\hat{C}D$.



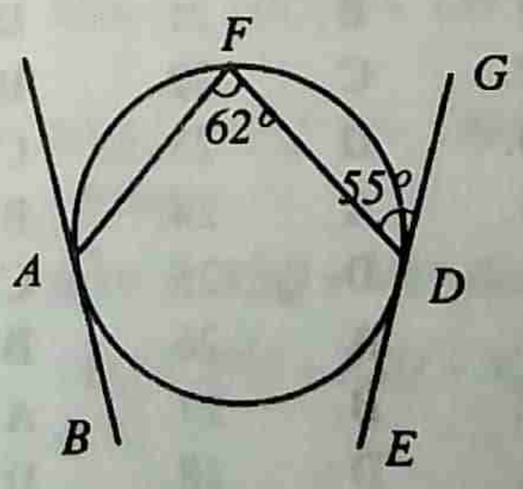
A. 125° B. 110° C. 90° D. 55°

51. In the diagram below, C is the centre of the circle and AT is a tangent. The value of y is



A. 62° B. 56° C. 31° D. 28°

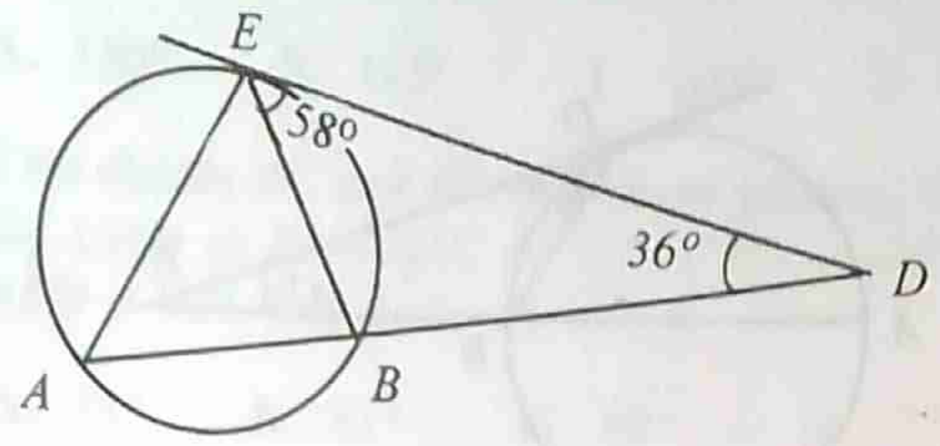
52. In the figure shown, AB and DE are tangents to the circle. If $F\hat{D}G = 55°$ and $A\hat{F}D = 62°$, find $F\hat{A}B$.



A. 117° B. 124° C. 125° D. 152°

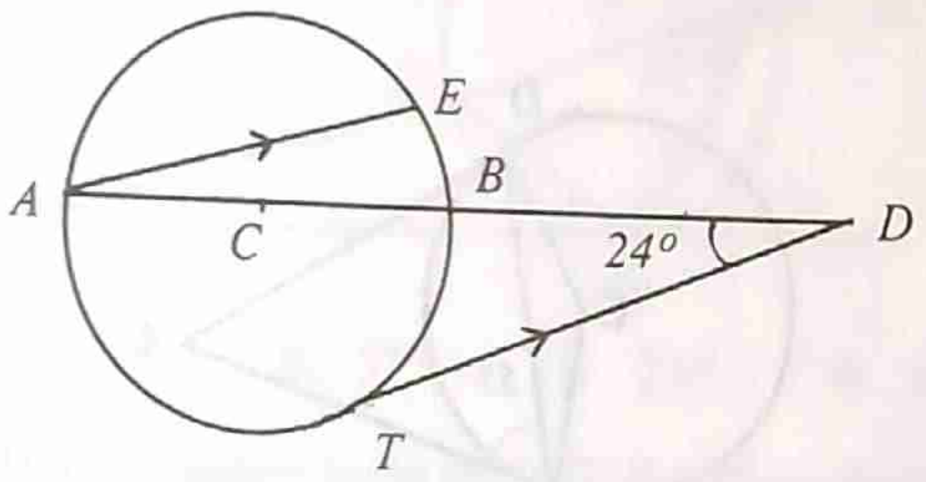
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53. In the diagram below, DE is a tangent to the circle at E. If $E\hat{D}B = 36°$ and $B\hat{E}D = 58°$, find $A\hat{E}B$.



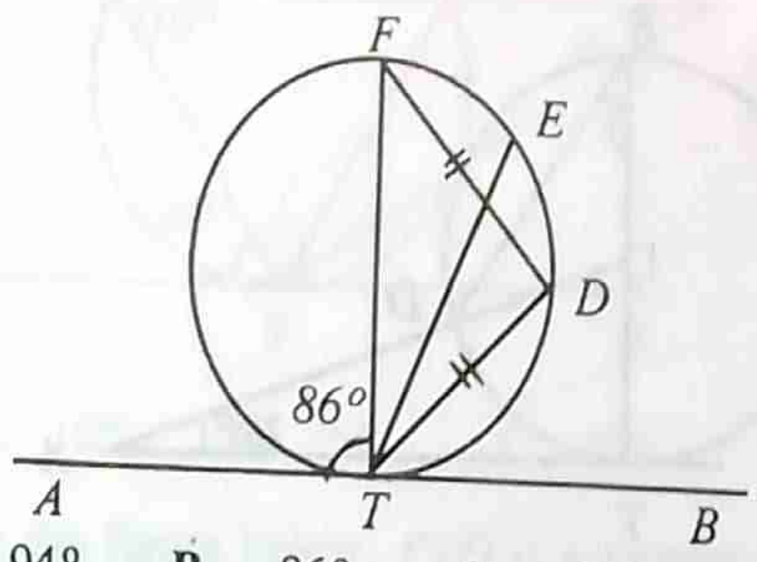
A. 28° **B.** 36° **C.** 58° **D.** 86°

54. In the figure below, C is the centre of the circle, $AE \| TD$ and $B\hat{D}T = 24°$. Find $E\hat{B}C$.



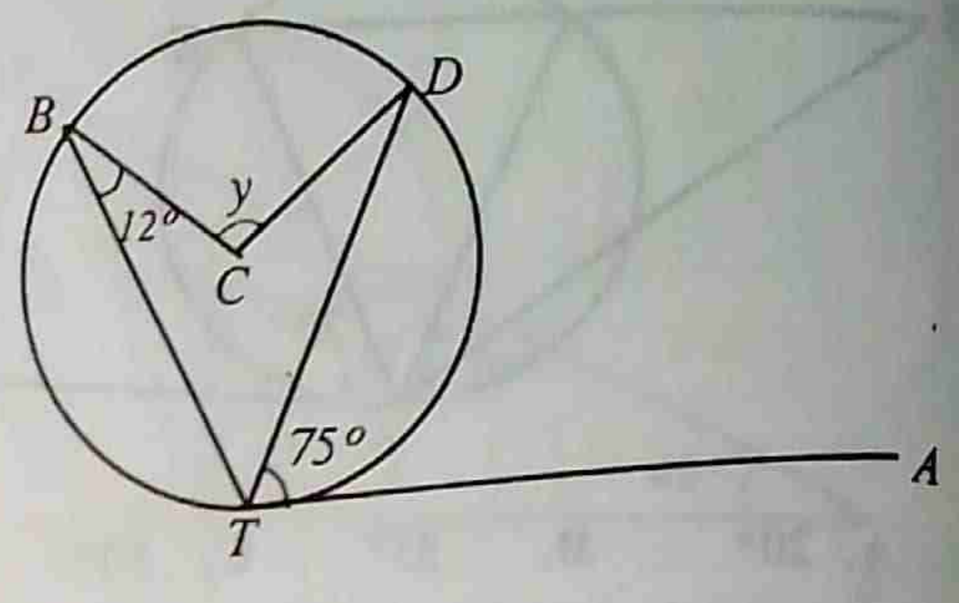
A. 90° **B.** 66° **C.** 48° **D.** 24°

55. In the diagram below, ATB is a tangent to the circle at T. If $|DT| = |DF|$ and $A\hat{T}F = 86°$, calculate $T\hat{E}D$.



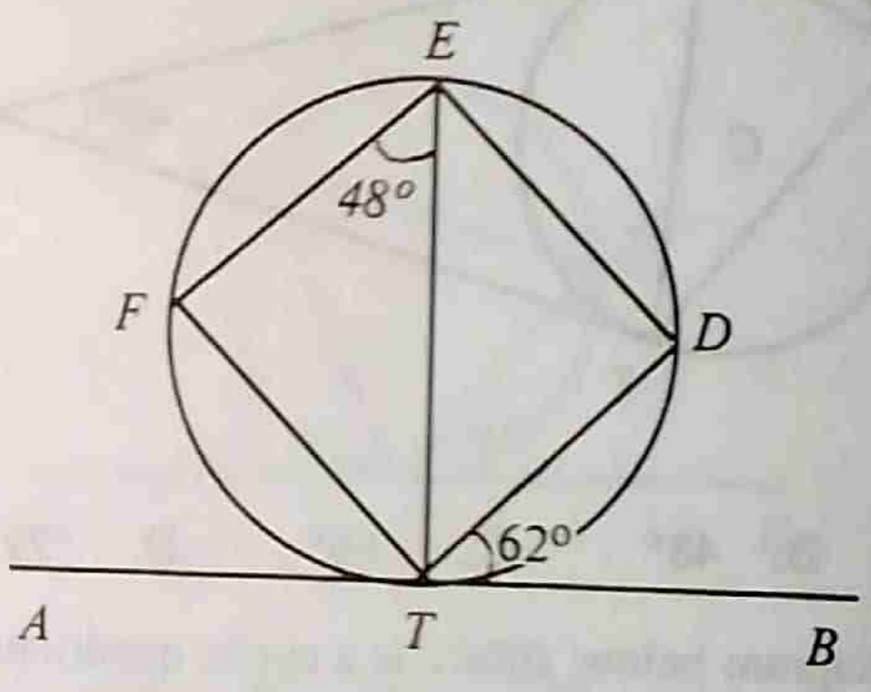
A. 94° **B.** 86° **C.** 47° **D.** 43°

56. AT is a tangent to the circle centred at C (see diagram below). If $A\hat{T}D = 75°$ and $C\hat{B}T = 12°$, find the value of y.



A. 24° **B.** 27° **C.** 54° **D.** 75°

57. In the diagram below, ATB is a tangent to the circle. If $B\hat{T}D = 62°$ and $F\hat{E}T = 48°$, find $F\hat{T}D$.



A. 124° **B.** 110° **C.** 86° **D.** 70°

ANSWER KEY

1.	C	11.	B	21.	D	31.	C	41.	D	51.	C
2.	D	12.	C	22.	B	32.	A	42.	C	52.	A
3.	D	13.	D	23.	C	33.	B	43.	C	53.	A
4.	B	14.	A	24.	B	34.	B	44.	A	54.	B
5.	A	15.	D	25.	C	35.	C	45.	C	55.	C
6.	B	16.	A	26.	B	36.	C	46.	C	56.	C
7.	D	17.	B	27.	A	37.	A	47.	D	57.	D
8.	C	18.	D	28.	D	38.	D	48.	C		
9.	A	19.	C	29.	D	39.	C	49.	D		
10.	A	20.	A	30.	B	40.	B	50.	B		

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SOLUTIONS TO OBJECTIVE QUESTIONS

1. (C) Join Q to R

$Q\hat{S}R = Q\hat{P}R = 32^\circ$ *(angles in same segment)*

$Q\hat{R}S = 90^\circ$ *(angle in a semicircle)*

$S\hat{Q}R = 180^\circ - Q\hat{S}R - Q\hat{R}S$ *(angles of a Δ)*

$= 180^\circ - 32^\circ - 90^\circ = 58^\circ$

2. (D) Join C to P and to Q

$P\hat{R}Q = 180^\circ - 68^\circ - 50^\circ = 62^\circ$

$P\hat{C}Q = 2(P\hat{R}Q)$ *(angles subtended at centre and at circumference)*

$= 2(62^\circ) = 124^\circ$

3. (D) $2m = n$ *(angles in same segment)*

$m = \frac{n}{2}$

4. (B) $Q\hat{R}S + Q\hat{P}S = 180^\circ$ *(angles in opposite segments)*

$Q\hat{R}S = 180^\circ - 65^\circ = 115^\circ$

5. (A) Reflex $X\hat{C}Z = 360^\circ - 102^\circ = 258^\circ$

$= 2(X\hat{Y}Z)$ . . . *(angles subtended at centre and at circumference)*

$X\hat{Y}Z = 258^\circ/2 = 129^\circ$

6. (B) $Q\hat{C}R = 2(Q\hat{P}R)$ *(angles at centre and at circumference)*

$88^\circ = 2(4x)$, or

$x = 11^\circ$

7. (D) Acute $Y\hat{C}Z = 360^\circ - 296^\circ = 64^\circ$

$= 2(Y\hat{X}Z)$... *(angles at centre and at circumference)*

$64^\circ = 2(Y\hat{X}Z)$

$Y\hat{X}Z = 32^\circ$

8. (C) Since $CD = CB$, $C\hat{D}B = C\hat{B}D = 15^\circ$ *(base angles, isosceles Δ)*

$D\hat{C}E + E\hat{C}B + C\hat{B}D + C\hat{D}B = 180^\circ$ *(angles of a Δ)*

$D\hat{C}E = 180^\circ - 15^\circ - 15^\circ - 110^\circ = 40^\circ$

$= 2(D\hat{F}E)$ ... *(angles at centre and at circumference)*

$D\hat{F}E = 40^\circ/2 = 20^\circ$

9. (A) $P\hat{T}Q = P\hat{S}Q$ *(angles in same segment).*

(This may be easier to see if the circle is completed).

10. (A) $S\hat{P}Q = 180^\circ - S\hat{R}Q$ *(angles in opposite segments)*

$= 180^\circ - 94^\circ = 86^\circ$

$S\hat{Q}P + S\hat{P}Q + P\hat{S}Q = 180^\circ$ *(angles of a Δ)*

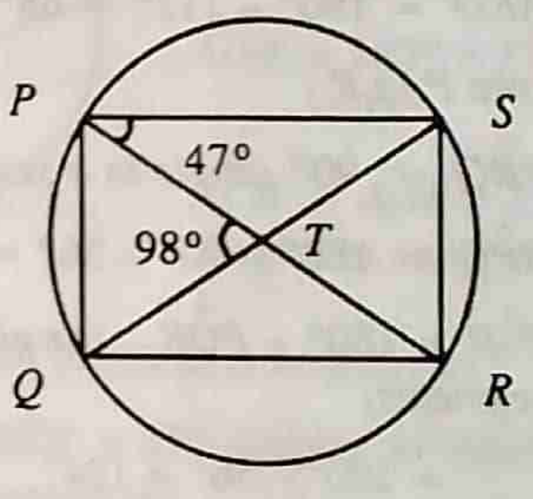
$S\hat{Q}P = 180^\circ - 86^\circ - 32^\circ = 62^\circ$

11. (B) $A\hat{D}B = 90^\circ$ *(angle in a semicircle)*

$B\hat{D}E = 180^\circ - 40^\circ - 62^\circ = 78^\circ$ (angles of a Δ)

$A\hat{D}E = A\hat{D}B - B\hat{D}E = 90^\circ - 78^\circ = 12^\circ$

12. (C)



Referring to the figure:

$P\hat{T}S = 180^\circ - 98^\circ = 82^\circ$

$P\hat{S}T = 180^\circ - P\hat{T}S - S\hat{P}T$ *(angles of a Δ)*

$= 180^\circ - 82^\circ - 47^\circ = 51^\circ$

$= P\hat{R}Q$ *(angles in same segment)*

13. (D) $B\hat{C}E = 2(B\hat{A}E)$ ...*(angles at centre and at circumference)*

$= 2(41^\circ) = 82^\circ$

Reflex $B\hat{C}E = 360^\circ - 82^\circ = 278^\circ$

$= 2(B\hat{D}E)$ ...*(angles at centre and at circumference)*

$B\hat{D}E = 278^\circ/2 = 139^\circ$

14. (A) $R\hat{S}Q = R\hat{P}Q = 48^\circ$ *(angles in same segment)*

$S\hat{Q}R = R\hat{S}Q = 48^\circ$ *(base angles isosceles Δ)*

$P\hat{R}Q + P\hat{R}S + R\hat{S}Q + S\hat{Q}R = 180^\circ$ *(angles of a Δ)*

$P\hat{R}Q = 180^\circ - 65^\circ - 48^\circ - 48^\circ = 19^\circ$

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15. **(D)** $\hat{YZW} = \hat{YWZ}$ *(base angles, isosceles Δ)*

$\hat{YZW} + \hat{YWZ} = 180° - 58° = 122°$ *(angles of a Δ)*

$\therefore \hat{YZW} = 122°/2 = 61°$

$\hat{XZY} = 90°$ *(angle in a semicircle)*

$\hat{XZW} = \hat{XZY} - \hat{YZW} = 90° - 61° = 29°$

16. **(A)** $\hat{CBD} = \hat{CDB}$ *(base angles, isosceles Δ)*

$\hat{CBD} + \hat{CDB} = 180° - 44° = 136°$ *(angles of a Δ)*

$\therefore \hat{CBD} = 136°/2 = 68°$

$\hat{ABD} = 180° - \hat{CBD} = 180° - 68° = 112°$

$\hat{AED} + \hat{ABD} = 180°$ *(angles in opposite segments)*

$\hat{AED} = 180° - 112° = 68°$

17. **(B)** Join P to R

$\hat{PRQ} = 90°$ *(angle in a semicircle)*

$\hat{RPQ} = 180° - 90° - 56° = 34°$

$\hat{PSR} = 180° - \hat{PQR}$ *(angles in opposite segments)*

$= 180° - 56° = 124°$

$\hat{SPR} = \hat{SRP} = (180° - 124°) \div 2 = 28°$

$\hat{SPQ} = \hat{SPR} + \hat{RPQ} = 28° + 34° = 62°$

18. **(D)** $\hat{ADB} = 90°$ *(angle in a semicircle)*

$\hat{EDB} = \hat{ADB} - \hat{ADE} = 90° - 47° = 43°$

$\hat{DBE} = 180° - \hat{DEB} - \hat{EDB}$ *(angles of a Δ)*

$= 180° - 70° - 43° = 67°$

19. **(C)** $\hat{PCS} = \hat{RCT} = 42° + 34° = 76°$ *(exterior angle and two opposite interior angles)*

$\hat{SCT} = 180° - 42° - 34° = 104°$ *(angles of a Δ)*

Reflex $\hat{PCR} = \hat{PCS} + \hat{SCT} + \hat{RCT}$

$= 76° + 104° + 76° = 256°$

$= 2(\hat{PQR})$ *... (angles at centre and at circumference)*

$\hat{PQR} = 256°/2 = 128°$

20. **(A)** Acute $\hat{ACB} = 360° - 285° = 75°$

$= 2(\hat{ADB})$ *... (angles at centre and at circumference)*

$\hat{ADB} = 75°/2 = 37\frac{1}{2}°$

21. **(D)** Join B to D

$\hat{ADB} = 90°$ *(angle in a semicircle)*

$\hat{ABD} = 180° - \hat{AED}$ *(angles in opposite segments)*

$= 180° - 110° = 70°$

$\hat{DAB} = 180° - \hat{ADB} - \hat{ABD}$ *(angles of a Δ)*

$= 180° - 90° - 70° = 20°$

22. **(B)** $\hat{CBD} = \frac{1}{2}(\hat{ACD})$ *... (angles at centre and at circumference)*

$= \frac{1}{2}(130°) = 65°$

Since $CB = CD$, $\hat{CDB} = \hat{CBD} = 65°$ *(base angles isosceles Δ)*

23. **(C)** $\hat{PQR} = \hat{PSR} = 41°$ *(angles in same segment)*

$\hat{PRQ} = 90°$ *(angle in a semicircle)*

$\hat{RPQ} = 180° - \hat{PQR} - \hat{PRQ}$ *(angles of a Δ)*

$= 180° - 41° - 90° = 49°$

24. **(B)** $\hat{CZX} = \hat{CXZ} = 35°$ *(base angles, isosceles Δ)*

$\hat{CYZ} = \hat{CZY} = 50°$ *(base angles, isosceles Δ)*

$\hat{XCZ} = 180° - \hat{CXZ} - \hat{CZX}$

$= 180° - 35° - 35° = 110°$

$\hat{YCZ} = 180° - \hat{CYZ} - \hat{CZY}$

$= 180° - 50° - 50° = 80°$

$\hat{XCY} = \hat{XCZ} - \hat{YCZ} = 110° - 80° = 30°$

25. **(C)** $\hat{CAD} = \hat{ADC} = \frac{1}{2}(180° - 140°) = 20°$ *(base angles, isosceles Δ)*

Since $AB = AD$, $BC = CD$ and AC is common, triangles CAB and CAD are congruent *(SSS)*, hence $\hat{CAB} = 20°$

$\hat{BAD} = \hat{CAB} + \hat{CAD} = 20° + 20° = 40°$

$\hat{BCD} = 2(\hat{BAD}) = 2(40°) = 80°$ *(angles at centre and at circumference)*

26. **(B)** $\hat{LMN} = \hat{JKL} = 62°$ *(ext. angle, cyclic quad)*

$y = \hat{LMN} + \hat{MNL} = 62° + 25° = 87°$

27. **(A)** Join A to D

$\hat{EAD} = \hat{EDA} = \frac{1}{2}(\hat{DEF}) = \frac{1}{2}(100°) = 50°$

$\hat{ABE} = \hat{EDA} = 50°$ *(angles in same segment)*

$\hat{BAD} = 180° - 110° - 50° = 20°$

$x = 180° - \hat{ABE} - \hat{BAD} - \hat{EAD}$

$= 180° - 50° - 20° - 50° = 60°$

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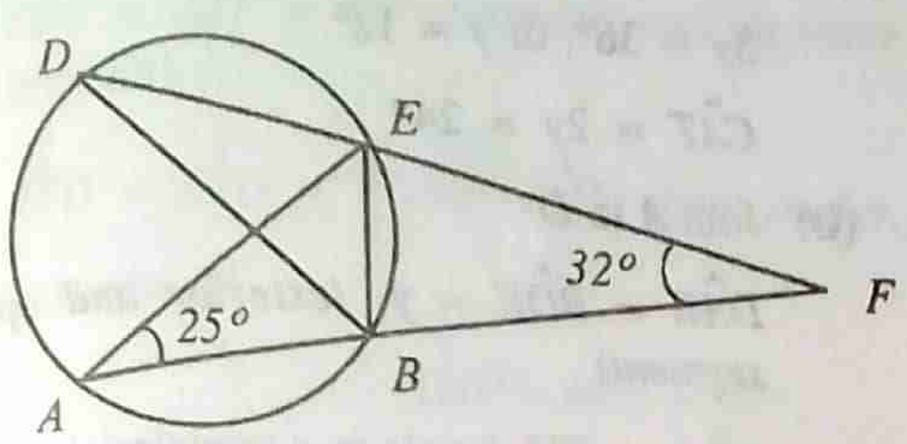
28. (D) $R\hat{Q}T = Q\hat{R}T = \frac{1}{2}(180° - 34°) = 73°$ *(base angles, isosceles Δ)*

$S\hat{Q}T = S\hat{Q}R + R\hat{Q}T = 42° + 73° = 115°$

$Q\hat{S}R = 180° - S\hat{Q}T - Q\hat{T}R$ *(angles of a Δ)*

$= 180° - 115° - 34° = 31°$

29. (D) Referring to the diagram below:



$B\hat{D}E = B\hat{A}E = 25°$ *(angles in same segment)*

$D\hat{B}F = 180° - B\hat{D}E - E\hat{F}B$ *(angles of a Δ)*

$= 180° - 25° - 32° = 123°$

30. (B) $A\hat{F}B = F\hat{A}D + F\hat{E}D = 26° + 30° = 56°$

$A\hat{C}B = 2(A\hat{F}B) = 2(56°) = 112°$

31. (C) $S\hat{R}T = R\hat{T}S = \frac{1}{2}(180° - 40°) = 70°$ *(base angles isosceles Δ)*

$P\hat{R}T = R\hat{T}S + R\hat{S}T = 70° + 40° = 110°$

$x = P\hat{R}T = 110°$ *(angles in same segment)*

32. (A) $S\hat{T}U = 180° - 78° = 102°$

$T\hat{S}U = 180° - 130° = 50°$

$m = 180° - S\hat{T}U - T\hat{S}U$

$= 180° - 102° - 50° = 28°$

$P\hat{Q}S = 180° - P\hat{T}S = 180° - 78° = 102°$

$Q\hat{S}R = 180° - 130° = 50°$

$P\hat{Q}S = Q\hat{S}R + n$

$102° = 50° + n$, or $n = 52°$

33. (B) $B\hat{A}C + C\hat{D}E = 180°$ *(interior angles, same side of transversal)*

$B\hat{A}C = 180° - 130° = 50°$

$A\hat{B}C = B\hat{A}C$ *(base angles, isosc. Δ, CA = CB)*

$\therefore \quad A\hat{B}C = 50°$

34. (B) $D\hat{B}E = y$ *(alternate angles)*

$E\hat{B}A = D\hat{B}E = y$ *(given)*

$A\hat{D}B = y$ *(angles in same segment)*

$B\hat{A}D + A\hat{D}B + D\hat{B}A = 180°$ *(angles of a Δ)*

$42° + y + 2y = 180°$

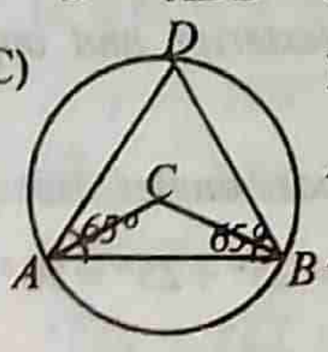
$3y = 138°$, or $y = 46°$

35. (C) $A\hat{D}B = 90°$ *(angle in a semicirle)*

Since $CD = CB$, $C\hat{D}B = C\hat{B}D = 36°$ *(base angles, isosceles Δ)*

$x = A\hat{D}B - C\hat{D}B = 90° - 36° = 54°$

36. (C)

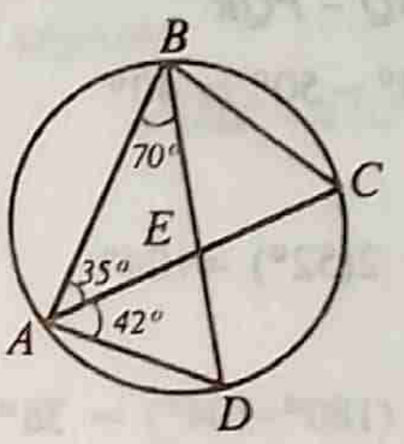


Referring to the diagram:

$A\hat{D}B = 180° - 65° - 65° = 50°$

$A\hat{C}B = 2(A\hat{D}B) = 2(50°) = 100°$

37. (A)



Referring to the figure:

$A\hat{D}B + B\hat{A}D + A\hat{B}D =$ $180°$ *(angles of a Δ)*

$A\hat{D}B = 180° - 77° - 70°$

$= 33°$

$A\hat{C}B = A\hat{D}B = 33°$

(angles in same segment)

38. (D) $E\hat{B}D = 90°$ *(angle in a semicircle)*

$D\hat{B}A = D\hat{E}B = 37°$ *(angles at exterior and opposite segment)*

$E\hat{B}A = E\hat{B}D + D\hat{B}A = 90° + 37° = 127°$

$y + D\hat{E}B + E\hat{B}A = 180°$

$y = 180° - 37° - 127° = 16°$

39. (C) $E\hat{T}B = T\hat{D}E = 50°$ *(exterior and opposite segment)*

$E\hat{T}F = \frac{1}{2}(E\hat{T}B) = 50°/2 = 25°$ *(given)*

$D\hat{E}T = A\hat{T}D = 60°$ *(exterior and opposite segment)*

$= E\hat{T}F + m$

$\therefore \quad m = DET - ETF = 60° - 25° = 35°$

40. (B) $P\hat{T}C = 90°$ *(tangent perpendicular to radius)*

$C\hat{P}T = T\hat{C}R - P\hat{T}C = 125° - 90° = 35°$

41. (D) $B\hat{T}D = 180° - 100° - 42° = 38°$

$C\hat{T}B = 90°$ *(tangent perpendicular to radius)*

$C\hat{T}D = C\hat{T}B - B\hat{T}D = 90° - 38° = 52°$

$C\hat{D}T = C\hat{T}D = 52°$ *(base angles, isosceles Δ, CT = CD)*

$T\hat{C}D = 180° - C\hat{T}D - C\hat{D}T$

$= 180° - 52° - 52° = 76°$

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$D\hat{E}T = \frac{1}{2}(T\hat{C}D) = \frac{1}{2}(76°) = 38°$ *(angles at centre and at circumference)*

42. (C) $E\hat{D}T = E\hat{T}B = 64°$ *(exterior and opposite segment)*

$D\hat{E}T = D\hat{T}E = \frac{1}{2}(180° - 64°) = 58°$

$A\hat{T}D = D\hat{E}T = 58°$ *(exterior and opposite segment)*

43. (C) $Q\hat{R}S = R\hat{S}Q = 25°$ *(base angles, isosceles Δ)*

$P\hat{Q}R = Q\hat{R}S + R\hat{S}Q = 25° + 25° = 50°$

$P\hat{R}Q = 90°$ *(angle in a semicircle)*

$R\hat{P}C = 180° - P\hat{R}Q - P\hat{Q}R$

$= 180° - 90° - 50° = 40°$

44. (A) Join T to R

$T\hat{C}R = 2(R\hat{S}T) = 2(52°) = 104°$

Since $CT = CR$,

$C\hat{R}T = C\hat{T}R = \frac{1}{2}(180° - 104°) = 38°$ *(base angles, isosceles Δ)*

$S\hat{T}P = S\hat{R}T = 34° + 38° = 72°$ *(exterior and opposite segment)*

45. (B) $E\hat{D}T = A\hat{T}E = 70°$ *(exterior and opposite segment)*

$B\hat{D}T = 180° - E\hat{D}T = 180° - 70° = 110°$

$D\hat{B}T = D\hat{T}B = \frac{1}{2}(180° - B\hat{D}T)$ *... (base angles, isosceles Δ)*

$= \frac{1}{2}(180° - 110°) = 35°$

46. (C) Join C to B and to T

$A\hat{B}C = A\hat{T}C = 90°$ *(tangents perpendicular to radius)*

$T\hat{C}B = 360° - B\hat{A}T - A\hat{B}C - A\hat{T}C$ *(angles of a quadrilateral)*

$= 360° - 48° - 90° - 90° = 132°$

$B\hat{D}T = \frac{1}{2}(T\hat{C}B) = \frac{1}{2}(132°) = 66°$

47. (D) Join E to B

$B\hat{E}T = B\hat{T}X = 65°$ *(exterior and opposite segment)*

$E\hat{B}T = B\hat{E}T = 65°$ *(base angles, isosceles Δ)*

$E\hat{T}B = 180° - B\hat{E}T - E\hat{B}T$

$= 180° - 65° - 65° = 50°$

$B\hat{D}E = 180° - E\hat{T}B$ *(angles in opposite segments)*

$= 180° - 50° = 130°$

48. (C) $A\hat{C}T = C\hat{T}B + T\hat{B}A + B\hat{A}C = y + 54°$

$C\hat{T}A = 90°$ *(tangent perpendicular to radius)*

$C\hat{A}T + A\hat{C}T + C\hat{T}A = 180°$ *(angles of a Δ)*

$2y + (y + 54°) + 90° = 180°$

$3y = 36°$, or $y = 12°$

$C\hat{A}T = 2y = 24°$

49. (D) Join A to D

$D\hat{A}B = B\hat{D}E = y$ *(exterior and opposite segment)*

$B\hat{D}A = 90°$ *(angle in a semicircle)*

$E\hat{D}A = B\hat{D}A + B\hat{D}E = 90° + y$

In Δ AED, $D\hat{A}B + B\hat{E}D + E\hat{D}A = 180°$

$y + x + 90° + y = 180°$

$x + 2y = 90°$

50. (B) $A\hat{D}B = A\hat{B}D = 55°$

$C\hat{D}A = C\hat{B}A = 90°$ *(tangents perpendicular to radius)*

$C\hat{B}D = C\hat{D}B = 90° - 55° = 35°$

$B\hat{C}D = 180° - C\hat{B}D - C\hat{D}B$

$= 180° - 35° - 35° = 110°$

51. (C) $T\hat{C}D = 2(C\hat{E}D) = 2y$ *(angles at centre and at circumference)*

$C\hat{T}A = 90°$

$T\hat{C}D + C\hat{T}A + T\hat{A}D = 180°$ *(angles of a Δ)*

$2y + 90° + 28° = 180°$

$2y = 62°$, or $y = 31°$

52. (A) Join A to D

$D\hat{A}B = A\hat{F}D = 62°$ *(exterior and opposite segment)*

$F\hat{A}D = F\hat{D}G = 55°$ *(exterior and opposite segment)*

$F\hat{A}B = F\hat{A}D + D\hat{A}B = 55° + 62° = 117°$

53. (A) $E\hat{A}B = D\hat{E}B = 58°$ *(exterior and opposite segment)*

In ΔAED, $A\hat{E}B + D\hat{E}B + E\hat{D}B + E\hat{A}B = 180°$

$\therefore A\hat{E}B = 180° - 58° - 36° - 58° = 28°$

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54. **(B)** Join E to B

$E\hat{A}B = B\hat{D}T = 24^\circ$ *(alternate angles)*

$A\hat{E}B = 90^\circ$ *(angle in a semicircle)*

$E\hat{B}C = 180^\circ - E\hat{A}B - A\hat{E}B$

$= 180^\circ - 24^\circ - 90^\circ = 66^\circ$

55. **(C)** Join E to D

$F\hat{D}T = A\hat{T}F = 86^\circ$ *(exterior and opposite segment)*

$T\hat{F}D = F\hat{T}D = \frac{1}{2}(180^\circ - F\hat{D}T)$ *(base angles, isosceles Δ)*

$= \frac{1}{2}(180^\circ - 86^\circ) = 47^\circ$

$T\hat{E}D = T\hat{F}D = 47^\circ$ *(angles in same segment)*

56. **(C)** Join B to D

$T\hat{B}D = A\hat{T}D = 75^\circ$ *(exterior and opposite segment)*

$C\hat{B}D = T\hat{B}D - C\hat{B}T = 75^\circ - 12^\circ = 63^\circ$

$C\hat{D}B = C\hat{B}D = 63^\circ$ *(base angles isosceles Δ)*

In ΔBCD, $y + C\hat{D}B + C\hat{B}D = 180^\circ$

$\therefore y = 180^\circ - 63^\circ - 63^\circ = 54^\circ$

57. **(D)** $T\hat{E}D = B\hat{T}D = 62^\circ$ *(exterior and opposite segment)*

$F\hat{E}D = F\hat{E}T + T\hat{E}D = 48^\circ + 62^\circ = 110^\circ$

$F\hat{T}D = 180^\circ - F\hat{E}D$ *(angles in opposite segment)*

$= 180^\circ - 110^\circ = 70^\circ$

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15 Trigonometry

15.1 Ratios of an Acute Angle

- In a right-angled triangle ABC (Fig. 15.1):

$BC = a$ = opposite side to angle θ
$AC = b$ = adjacent side to angle θ
$AB = c$ = hypotenuse

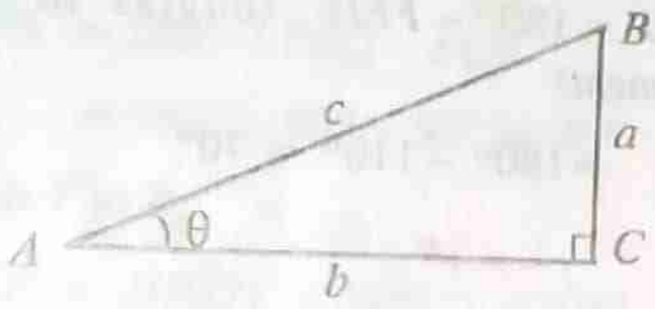


Fig. 15.1

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}; \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}; \quad \text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{c}{a}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{c}{b}; \quad \cot\theta = \frac{1}{\tan\theta} = \frac{b}{a}$$

- Angle ABC, which is equal to $90° - \theta$, is said to be *complementary* to θ. For this angle:

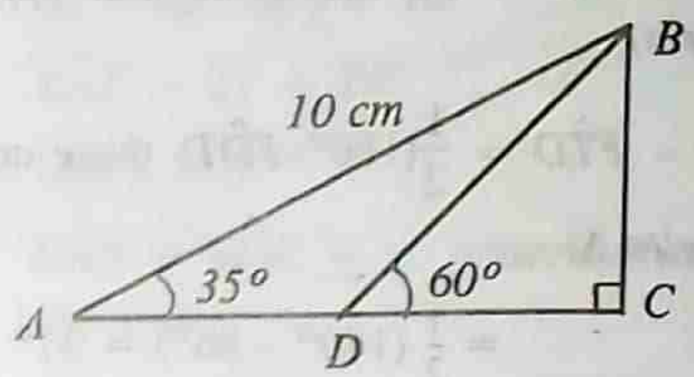
$\sin(90° - \theta) = b/c = \cos\theta$
$\cos(90° - \theta) = a/c = \sin\theta$
$\tan(90° - \theta) = b/a = \cot\theta$

- It is often helpful to remember the sine, cosine and tangent of the following special angles:

θ	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

EXAMPLE 15-1

In the given figure, calculate $|BD|$ and $|AD|$.



Solution

$BC/AB = \sin 35° \Rightarrow BC = AB \sin 35° = 10 \sin 35°$

From sine tables, $\sin 35° = 0.5736$.

Hence $BC = 10 \times 0.5736 = 5.736$ cm

$BC/BD = \sin 60° \Rightarrow BD = BC/\sin 60° = 5.736/0.866$
$= 6.62$ cm (Use log tables)

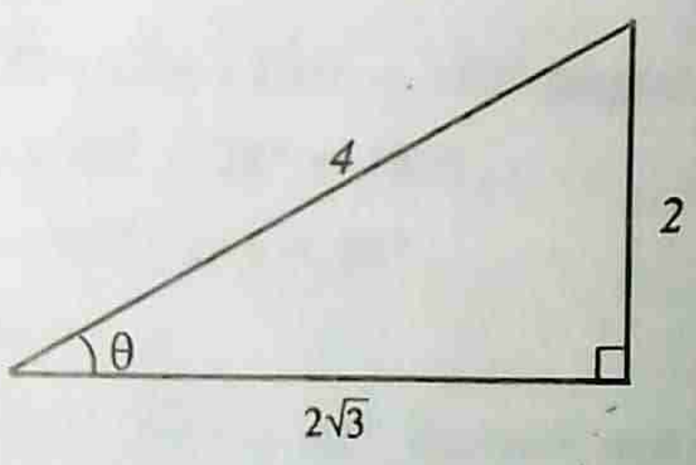
$BC/CD = \tan 60° = 1.732$

$CD = BC/1.732 = 5.736/1.732 = 3.31$ cm

$AC/AB = \cos 35° \Rightarrow AC = AB \cos 35° = 10(0.8192)$
$= 8.19$ cm

$AD = AC - CD = 8.19 - 3.31 = 4.88$ cm

DRILL 1



In the figure above, find (a) $\sin\theta$ (b) $\cos\theta$ (c) $\tan\theta$ (d) $\text{cosec}\,\theta$ (e) $\sec\theta$ and (f) $\cot\theta$

15.2 Angles Greater than 90° but Less than 360°

- A simple rule for obtaining the sine, cosine and tangent of angles greater than 90° is illustrated in Fig. 15.2 which is summarised as follows:

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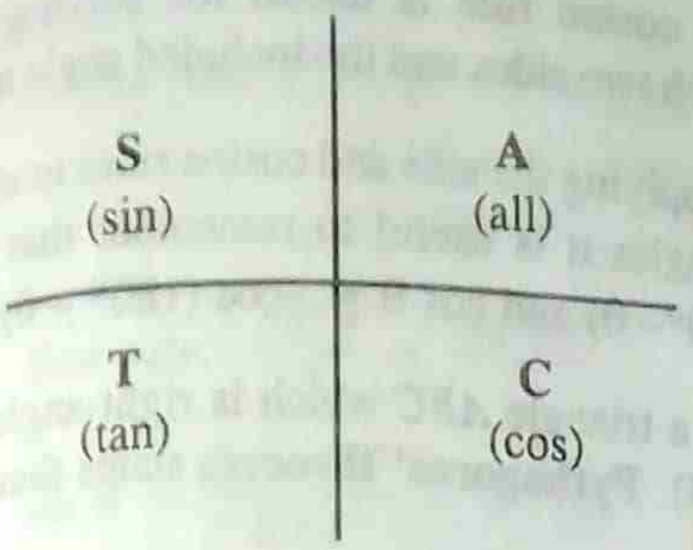


Fig. 15.2

1st Quadrant ($0° < \theta < 90°$) – **All** are positive

2nd Quadrant ($90° < \theta < 180°$) – Only **sine** is positive

3rd Quadrant ($180° < \theta < 270°$) – Only **tangent** is positive

4th Quadrant ($270° < \theta < 360°$) – Only **cosine** is positive

- *First Quadrant:* For any angle lying in the first quadrant ($0° < \theta < 90°$), sine, cosine and tangent are all positive, e.g. $\sin 30° = 0.5$, $\cos 30° = 0.866$ and $\tan 30° = 0.577$.

- *Second Quadrant:* The reference angle for the second quadrant ($90° < \theta < 180°$) is $180° - \theta$. Thus

  $\sin \theta = \sin (180° - \theta)$,

  e.g. $\sin 120° = \sin (180° - 120°) = \sin 60°$

  $\cos \theta = -\cos (180° - \theta)$,

  e.g. $\cos 120° = -\cos (180° - 120°) = -\cos 60°$

  $\tan \theta = -\tan (180° - \theta)$,

  e.g. $\tan 120° = -\tan (180° - 120°) = -\tan 60°$.

- *Third Quadrant:* The reference angle for the third quadrant ($180° < \theta < 270°$) is $\theta - 180°$. Thus

  $\sin \theta = -\sin (\theta - 180°)$

  e.g. $\sin 210° = -\sin (210° - 180°) = -\sin 30°$

  $\cos \theta = -\cos (\theta - 180°)$

  e.g. $\cos 210° = -\cos (210° - 180°) = -\cos 30°$

  $\tan \theta = \tan (\theta - 180°)$

  e.g. $\tan 210° = \tan (210° - 180°) = \tan 30°$

- *Fourth Quadrant:* The reference angle for the fourth quadrant is $360° - \theta$. Thus $\sin \theta = -\sin (360° - \theta)$,
  e.g. $\sin 300° = -\sin (360° - 300°) = -\sin 60°$

  $\cos \theta = \cos (360° - \theta)$

e.g. $\cos 300° = \cos (360° - 60°)$

$\tan \theta = -\tan (360° - \theta)$

e.g. $\tan 300° = -\tan (360° - 300°) = -\tan 60°$

EXAMPLE 15–2

Determine the values of the following, using tables where necessary.

(a) $\sin 135°$ (b) $\cos 250°$

(c) $\tan 320°$ (d) $\cos 180°$

Solutions

(a) $135°$ lies in the second quadrant.
$\sin 135° = \sin (180° - 135°) = \sin 45° = 0.707$

(b) $250°$ lies in the third quadrant
$\cos 250° = -\cos (250° - 180°) = -\cos 70° = -0.342$

(c) $320°$ lies in the fourth quadrant
$\tan 320° = -\tan (360° - 320°) = -\tan 40° = -0.839$

(d) $\cos 180° = -\cos (180° - 180°) = -\cos 0° = -1$

DRILL 2

Given that $\cos 60° = \frac{1}{2}$, $\sin 60° = \frac{\sqrt{3}}{2}$ and $\tan 60° = \sqrt{3}$, find the values of

(a) $\sin 120°$ (b) $\cos 120°$ (c) $\tan 240°$

(d) $\sin 240°$ (e) $\cos 300°$ (f) $\tan 300°$.

15.3 Angles Greater than 360° and Negative Angles

- The trigonometric ratios of an angle remain unchanged by the addition (or subtraction) of multiples of 360° to (or from) the angle, i.e.

  $\sin \theta = \sin (\theta + 360°) = \sin (\theta + 720°) = \ldots$

  $\sin \theta = \sin (\theta - 360°) = \sin (\theta - 720°) = \ldots$

  $\cos \theta = \cos (\theta + 360°) = \cos (\theta + 720°) = \ldots$

  $\cos \theta = \cos (\theta - 360°) = \cos (\theta - 720°) = \ldots$

  $\tan \theta = \tan (\theta + 360°) = \tan (\theta + 720°) = \ldots$

  $\tan \theta = \tan (\theta - 360°) = \tan (\theta - 720°) = \ldots$

- The trigonometric ratios of a negative angle, $-\theta$, are thus given by

  $\sin (-\theta) = \sin (360° - \theta) = -\sin \theta$

  $\cos (-\theta) = \cos (360° - \theta) = \cos \theta$

  $\tan (-\theta) = \tan (360° - \theta) = -\tan \theta$

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EXAMPLE 15–3

Find the values of the following, using tables where necessary:

(a) sin 420° (b) cos 690° (c) tan 600°

(d) sin (–30°) (e) cos (–210°) (f) tan (–120°)

Solutions

(a) $\sin 420° = \sin (420° - 360°) = \sin 60° = 0.866$

(b) $\cos 690° = \cos (690° - 360°) = \cos 330°$
$= \cos (360° - 330°) = \cos 30°$
$= 0.866$

(c) $\tan 600° = \tan (600° - 360°) = \tan 240°$
$= \tan (240° - 180°) = \tan 60° = 1.732$

(d) $\sin (-30°) = -\sin 30° = -0.5$

(e) $\cos (-210°) = \cos 210° = -\cos (210° - 180°)$
$= -\cos 30° = -0.866$

(f) $\tan (-120°) = -\tan 120° = -[(-\tan (180° - 120°)]$
$= \tan 60° = 1.732$

DRILL 3

Find the values of

(a) cos 540° (b) sin 540° (c) tan 480°

(d) tan (–40°) (e) sin (–150°) (f) cos (–60°)

15–4 Sine and Cosine Rules; Pythagoras' Theorem

- In a triangle ABC with side a facing angle A, side b facing angle B and side c facing angle C (Fig. 15.3), the **sine rule** is stated as

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

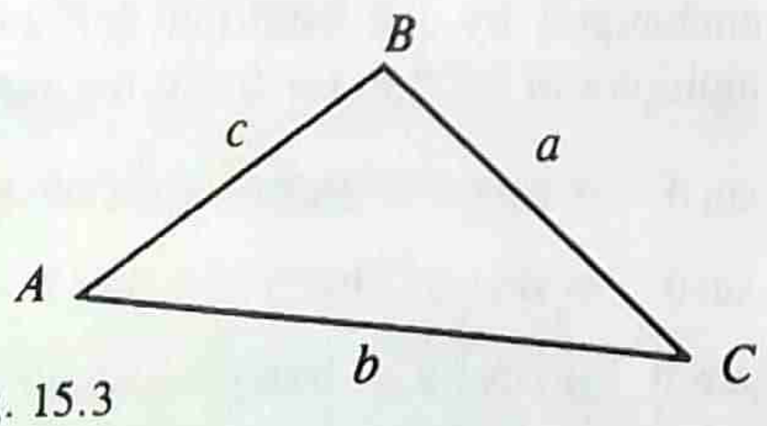


Fig. 15.3

- The sine rule is useful for solving triangles in which either two sides and an angle or two angles and a side are known.

- Referring to Fig. 15.3, the **cosine rule** states that

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

- The cosine rule is useful for solving triangles in which two sides and the included angle are specified.

- In applying the sine and cosine rules to obtuse-angled triangles it is useful to remember that $\sin \theta = \sin (180° - \theta)$ and $\cos \theta = -\cos (180° - \theta)$.

- For a triangle ABC which is right-angled at C (Fig. 15.4), **Pythagoras' theorem** states that

$$c^2 = a^2 + b^2$$

where c is the hypotenuse.

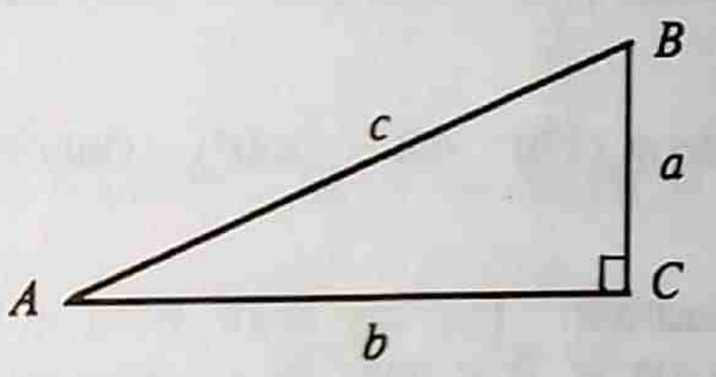
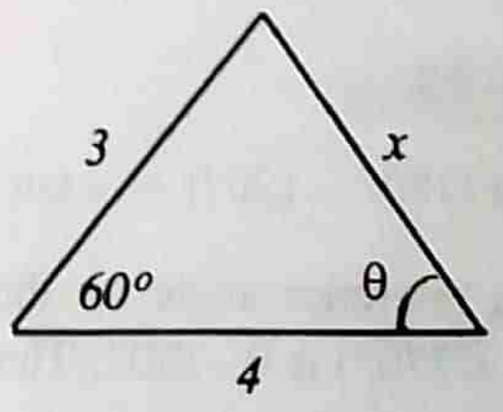


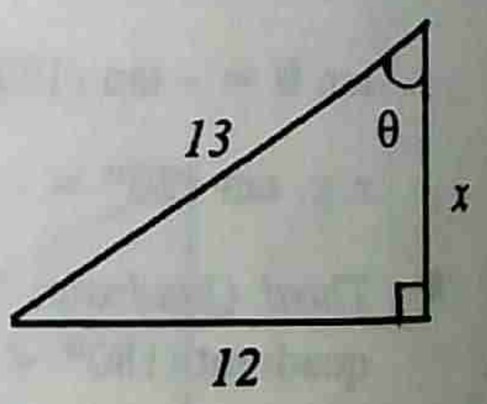
Fig. 15.4

EXAMPLE 15–4

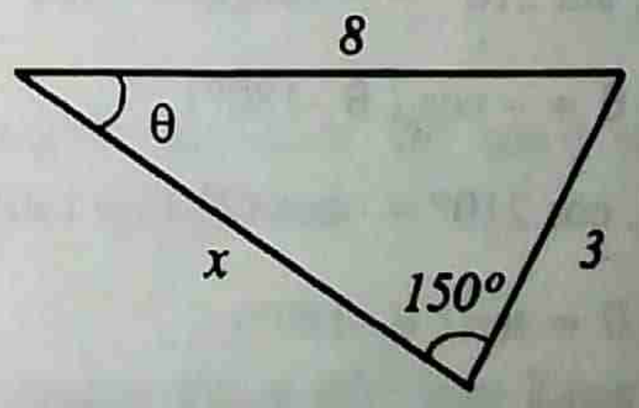
Determine the values of x and θ in each of the following figures:



(a)



(b)



(c)

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Solutions

(a) *Cosine rule:* $x^2 = 3^2 + 4^2 - 2(3)(4)\cos 60^o$

$= 16 + 9 - 24(0.500) = 13$

$x = \sqrt{13} = 3.61$

Sine rule: $\frac{\sin\theta}{3} = \frac{\sin 60^o}{x}$

$\sin\theta = \frac{3\sin 60^o}{3.61} = 0.7197$

$\theta = 46.0^o$

(b) *Pythagoras' Theorem:* $13^2 = 12^2 + x^2$

$x^2 = 169 - 144 = 25$, or $x = 5$

$\sin\theta = \frac{12}{13} = 0.923$

$\theta = 67.4^o$

(c) *Sine rule:* $\frac{\sin 150^o}{8} = \frac{\sin\theta}{3}$

$\sin\theta = \frac{3\sin 150^o}{8}$

$= 0.375 \sin 150^o$

But $\sin 150^o = \sin(180^o - 150^o) = \sin 30^o = 0.500$

Hence $\sin\theta = 0.375\,(0.500) = 0.1875$, or $\theta = \mathbf{10.8^o}$

Angle facing side $x = 180^o - (150^o + \theta)$

$= 180^o - 160.8^o = 19.2^o$.

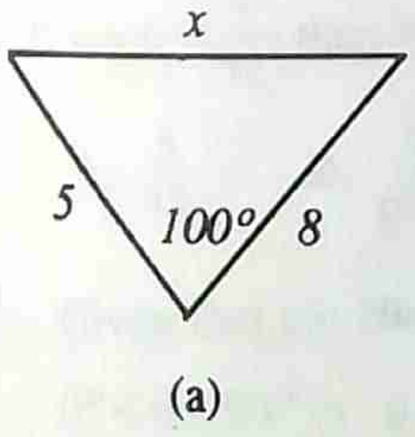
Sine rule: $\frac{x}{\sin 19.2^o} = \frac{8}{\sin 150^o}$

$x = 8 \sin 19.2^o / \sin 150^o = 8 \times 0.3289/0.500$

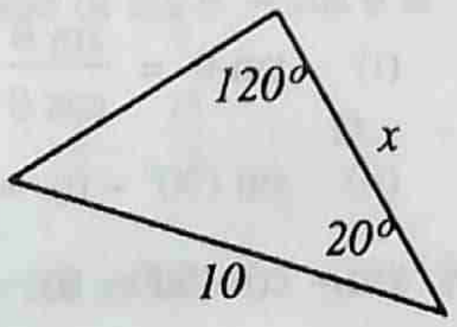
$= 5.26$

DRILL 4

Find the values of x in each of the following figures:



(a)



(b)

15.5 Graphs of Trigonometric Functions

- The procedure for drawing graphs of trigonometric functions is similar to that for algebraic functions.
- A table of values is constructed with selected values of the independent variable (θ) and the values of the dependent variable calculated from the given function.
- The independent variable is displayed on the abscissa and the dependent variable on the ordinate.
- Smooth curves are used to join points on the graph.

EXAMPLE 15–5

(a) Using an interval of 30°, draw the graph of $y = \cos 2\theta - \sin\theta$ for $0^o \le \theta \le 180^o$.

(b) Use your graph to determine:

(i) the minimum value of y and

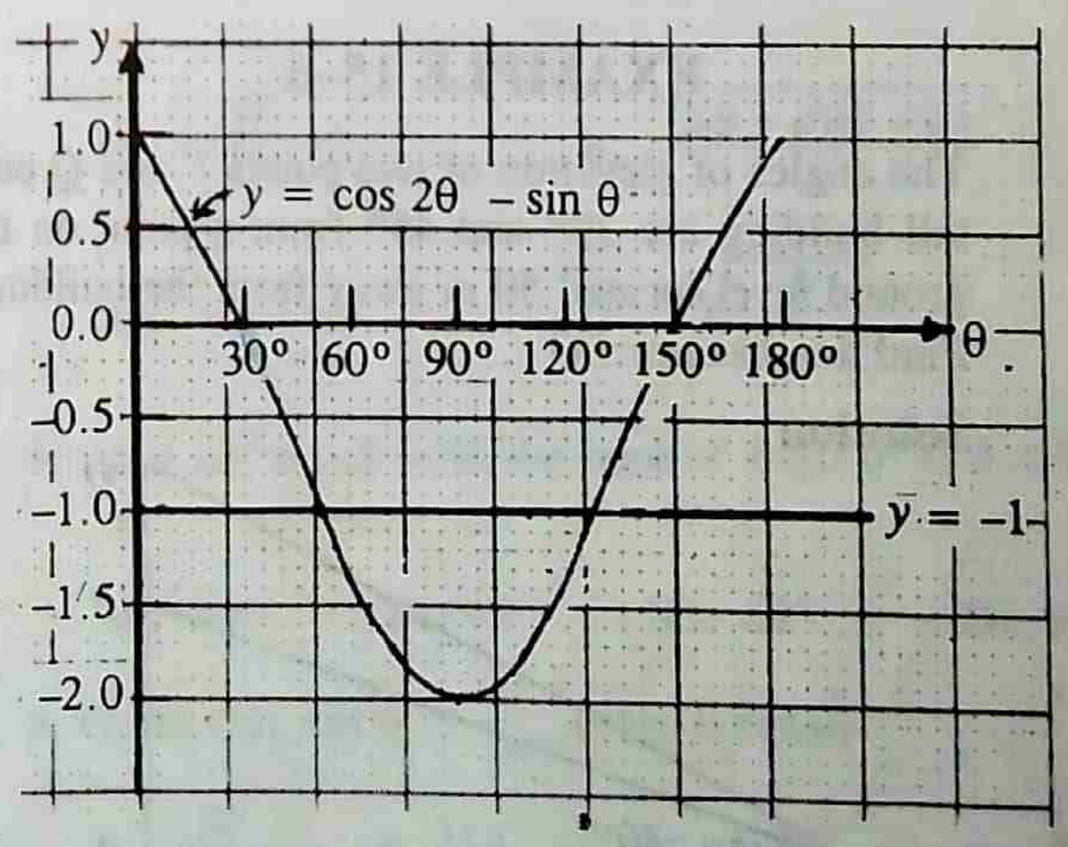
(ii) the values of θ for which $\cos 2\theta = \sin\theta$.

(iii) the solution to the equation $\cos 2\theta - \sin\theta = -1$.

Solutions

Table of Values

θ	$\cos 2\theta$	$\sin\theta$	$y = \cos 2\theta - \sin\theta$
0°	1.0	0.0	1.0
30°	0.5	0.5	0.0
60°	–0.5	0.87	–1.37
90°	–1.0	1.0	–2.0
120°	–0.5	0.87	–1.37
150°	0.5	0.5	0.0
180°	1.0	0.0	1.0



(a) See figure above for graph.

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(b) (i) Minimum value of y is –2 and this occurs at $\theta = 90°$.

(ii) If $\cos 2\theta = \sin \theta$, then $\cos 2\theta - \sin \theta = 0$. Hence the required values of θ are the intercepts of the curve with the θ–axis ($y = 0$). These are $\theta = 30°$ and $\theta = 150°$

(iii) The solution to the equation $\cos 2\theta - \sin \theta = -1$ is given by the points of intersection of $y = \cos 2\theta - \sin \theta$ and $y = -1$. These are read from the graph as $\theta = 52°$ or $128°$

DRILL 5

From the graph in Example 15–5, find

(a) the values of y when $\theta = 45°$, $100°$ and $160°$

(b) the values of θ for which $y = 0.5$ and -0.5.

15.6 Angles of Elevation and Depression

- If an observer O with his eye level at OA views an object P located **above** OA, the angle θ_E which OP makes with OA is called the **angle of elevation** (Fig. 15.5).

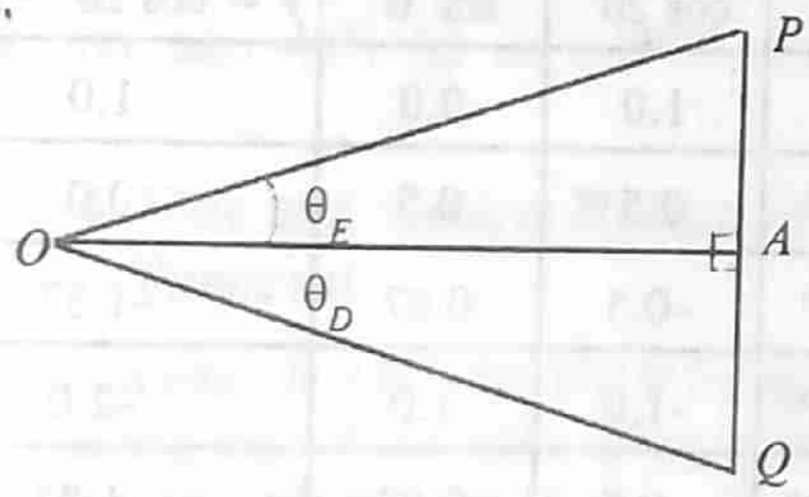


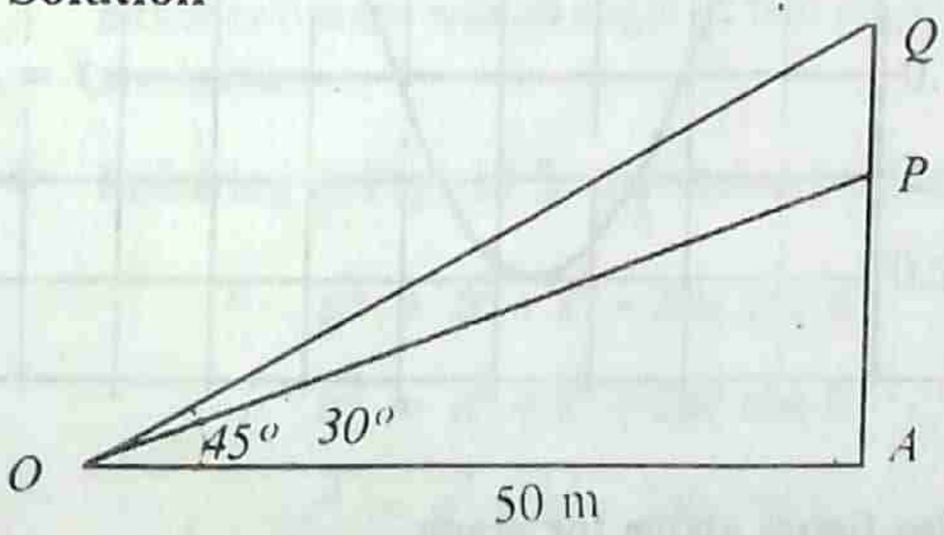
Fig. 15.5

- If the same observer views another object Q located **below** OA, the angle θ_D which OQ makes with OA is the **angle of depression.**

EXAMPLE 15–6

The angles of elevation of two points P and Q on a tall building are $30°$ and $45°$ from a point on the ground level located 50 m away from the building. Find the distance PQ.

Solution



The points P and Q are viewed from point O, on the same horizontal level as A (see diagram above).

From the figure,

$AP/50 = \tan 30°$

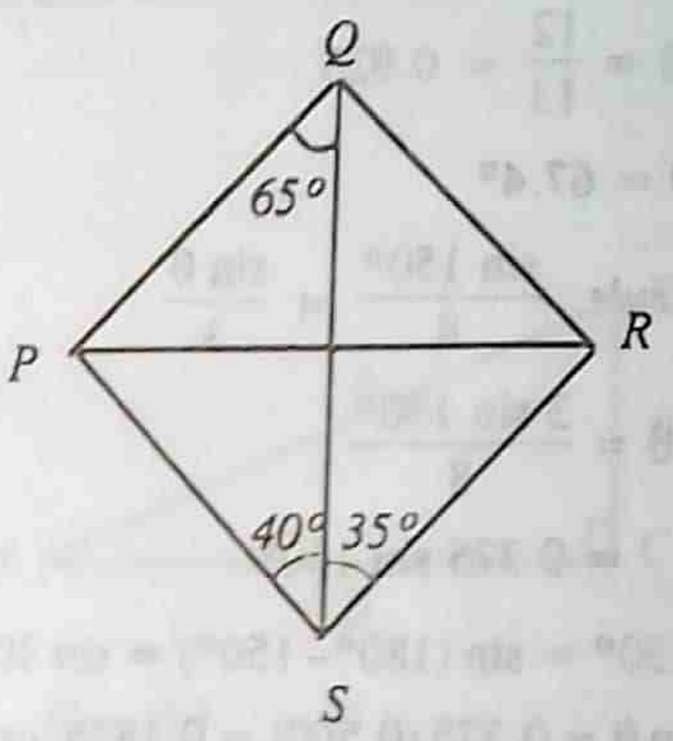
$AP = 50 \tan 30° = 50 \times 0.5774 = 28.87$ m

$AQ/50 = \tan 45° = 1$, or

$AQ = 50$ m

$PQ = AQ - AP = 50 - 28.87 = 21.13$ m

DRILL 6



In the figure above, PR is horizontal and QS is vertical. What is the angle of

(a) elevation of Q from P?

(b) depression of P from Q?

(c) elevation of R from S?

(d) depression of S from R?

(e) elevation of P from S?

(f) depression of S from P?

15.7 Trigonometric Identities

- The following trigonometric identities are often found useful in solving problems on trigonometry:

(i) $\tan \theta = \dfrac{\sin \theta}{\cos \theta}$

(ii) $\sin (90° - \theta) = \cos \theta$

(iii) $\cos (90° - \theta) = \sin \theta$

(iv) $\tan (90° - \theta) = \cot \theta$

(v) $\sin^2 \theta + \cos^2 \theta = 1$

(vi) $\sec^2 \theta = 1 + \tan^2 \theta$

(vii) $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$

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EXAMPLE 15–7

If $\sin\theta = 0.40$, find the values of

(a) $\cos\theta$ (b) $\tan\theta$ (c) $\sec^2\theta$ (d) $\text{cosec}^2\theta$

Solutions

(a) $\cos^2\theta = 1 - \sin^2\theta = 1 - 0.16 = 0.84.$
Hence $\cos\theta = 0.917$

(b) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{0.40}{0.917} = 0.436$

(c) $\sec^2\theta = 1 + \tan^2\theta = 1 + 0.19 = 1.19$

(d) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{0.436} = 2.29$

$\text{cosec}^2\theta = 1 + \cot^2\theta = 1 + 5.25 = 6.25$

[Alternatively, $\text{cosec}^2\theta = \dfrac{1}{\sin^2\theta} = \dfrac{1}{0.16} = 6.25$]

ANSWERS TO DRILLS

1. (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{3}}$
   (d) 2 (e) $\frac{2}{\sqrt{3}}$ (f) $\sqrt{3}$
2. (a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{2}$ (c) $\sqrt{3}$
   (d) $-\frac{\sqrt{3}}{2}$ (e) $\frac{1}{2}$ (f) $-\sqrt{3}$
3. (a) –1 (b) 0 (c) –1.732
   (d) –0.839 (e) –0.5 (f) 0.5
4. (a) 10.1 (b) 7.4
5. (a) For $\theta = 45°$, $y = -0.7$
   For $\theta = 100°$, $y = -1.9$
   For $\theta = 160°$, $y = 0.4$
   (b) For $y = 0.5$, $\theta = 18°$ and $162°$
   For $y = -0.5$, $\theta = 41°$ and $139°$
6. (a) 25° (b) 25° (c) 55°
   (d) 55° (e) 50° (f) 50°

SSCE/JME – TYPE OBJECTIVE QUESTIONS

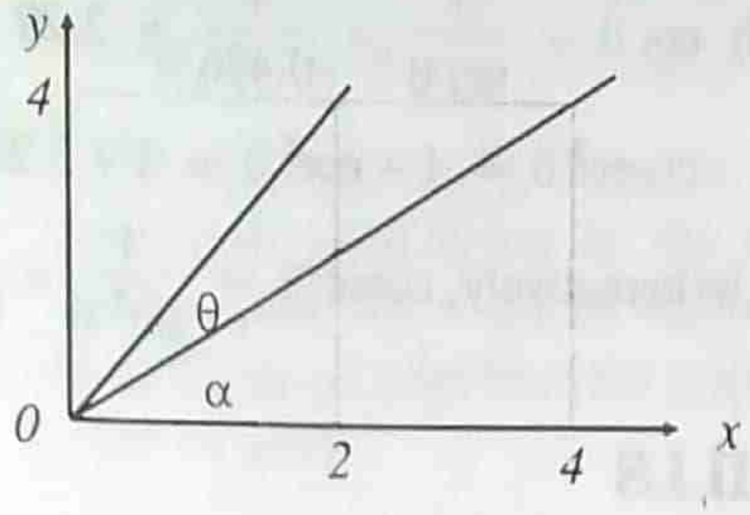
1. If $\cos\theta = \frac{4}{5}$, then for $0° < \theta < 90°$, $\tan\theta$ is equal to
   A. $\frac{5}{4}$ **B.** $\frac{4}{3}$ **C.** $\frac{3}{4}$ **D.** $\frac{5}{3}$
2. If $\cos\theta = \sin 50°$, then θ is
   A. 90° **B.** 50° **C.** 45° **D.** 40°
3. If $\tan\theta = \frac{5}{12}$, then the value of $\sin\theta - \cos\theta$ is
   A. $\frac{5}{13}$ **B.** $\frac{7}{13}$ **C.** $\frac{12}{13}$ **D.** $-\frac{7}{13}$
4. Given that $\sin\theta = \frac{4}{5}$, the value of $1 - \cos^2\theta$ for $0° < \theta < 90°$ is
   A. $\frac{16}{25}$ **B.** $\frac{9}{25}$ **C.** $\frac{9}{16}$ **D.** $\frac{2}{5}$
5. If $\sin\theta = x$, the value of $\tan\theta$ for $0° < \theta < 90°$ is
   A. $1 - x$ **B.** $\frac{x}{\sqrt{1-x^2}}$ **C.** $\frac{x}{x-1}$ **D.** $\frac{x}{\sqrt{x^2-1}}$
6. The value of $\frac{\sin 35°}{\cos 55°} + \frac{\cos 15°}{\sin 75°}$ is
   A. 2 **B.** 1 **C.** 0 **D.** –2
7. If $\sin\theta = \frac{a}{b}$, where θ is an acute angle, then $\cot\theta$ equals
   A. $\frac{\sqrt{b^2-a^2}}{b}$ **B.** $\frac{\sqrt{(b+a)(b-a)}}{a}$
   C. $\frac{a}{\sqrt{b^2-a^2}}$ **D.** $\sqrt{\frac{b}{b^2-a^2}}$
8. If $4\sin^2\theta - 3 = 0$, the value of θ for $0° < \theta < 90°$ is
   A. 30° **B.** 45° **C.** 60° **D.** 90°
9. Given that $\cot\theta = \frac{m}{n}$, $\text{cosec}\,\theta$ equals
   A. $\sqrt{\frac{m}{n}}$ **B.** $\frac{1}{n}(n^2+m^2)$ **C.** $\frac{1}{n}\sqrt{n^2+m^2}$ **D.** $\frac{n}{m}$

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10. The hypotenuse of a right-angled isosceles triangle is 8 units. The length of each of the other sides is

A. $4\sqrt{2}$ **B.** $\frac{4}{\sqrt{2}}$ **C.** 4 **D.** $4(\sqrt{2}-1)$

11.



The value of angle θ in the diagram shown above is

A. 17.5° **B.** 18.4° **C.** 20.6° **D.** 22.5°

12. If $\cos\theta = \frac{x}{y}$, then $1 + \tan^2\theta$ equals

A. $\frac{x^2}{y^2}$ **B.** $\frac{y^2-x^2}{y^2}$ **C.** $\frac{y^2-x^2}{y^2}$ **D.** $\frac{y^2}{x^2}$

13. Given that $\tan^2\theta + \sec^2\theta = 3$, the value of θ (for $0° < \theta < 90°$) is

A. 90° **B.** 60° **C.** 45° **D.** 30°

14. The expression $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$ can be simplified to

A. $\sin\theta$ **B.** $\frac{\sin\theta}{1-\cos\theta}$

C. $\frac{1+\cos\theta}{\sin\theta}$ **D.** $1-\sin\theta$

15. If $\sin\theta = \frac{1}{2}$ and $0° < \theta < 90°$, evaluate

$$\frac{\cos^2\theta}{\tan(90°-\theta)}$$

A. $\frac{\sqrt{3}}{4}$ **B.** $\frac{\sqrt{3}}{2}$ **C.** $\frac{4}{\sqrt{3}}$ **D.** $4\sqrt{3}$

16. For any value of θ, the value of

$$\frac{\sin^2\theta}{1-\cos\theta} + \frac{\sin^2\theta}{1+\cos\theta} \text{ is}$$

A. $\cos^2\theta$ **B.** $\sin^2\theta$ **C.** 1 **D.** 2

17. If $\tan\theta = \frac{2}{3}$, then $\sec^2\theta$ equals

A. $\frac{13}{4}$ **B.** $\frac{13}{9}$ **C.** $\frac{9}{13}$ **D.** $\frac{4}{13}$

18. Given that $\cos^2\theta - \sin^2\theta = \frac{1}{8}$, find the value of $\tan\theta$

A. $\sqrt{7}$ **B.** $3\sqrt{7}$ **C.** $\frac{\sqrt{7}}{3}$ **D.** $\frac{3}{\sqrt{7}}$

19. The value of $1 + \sec^2 60°$ is

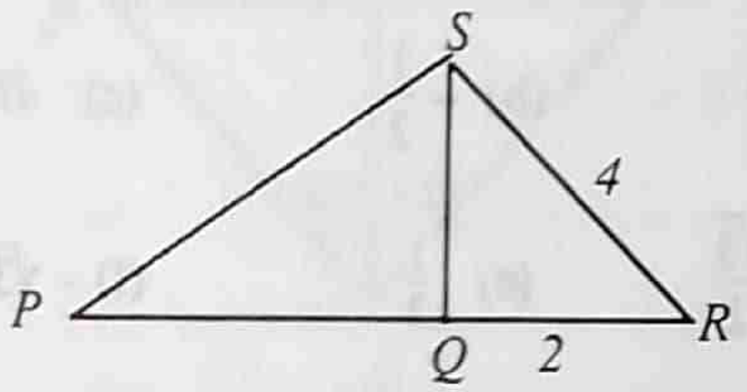
A. 2 **B.** $2\frac{1}{3}$ **C.** 4 **D.** 5

20. If $\cos\theta = \frac{\sqrt{3}}{2}$, evaluate $\frac{\cot\theta - \operatorname{cosec}\theta}{\sec\theta + \tan\theta}$

A. $\frac{3-2\sqrt{3}}{3}$ **B.** $\frac{\sqrt{3}-2}{3}$

C. $\frac{\sqrt{3}}{2} - 1$ **D.** $\frac{1}{2}$

21.



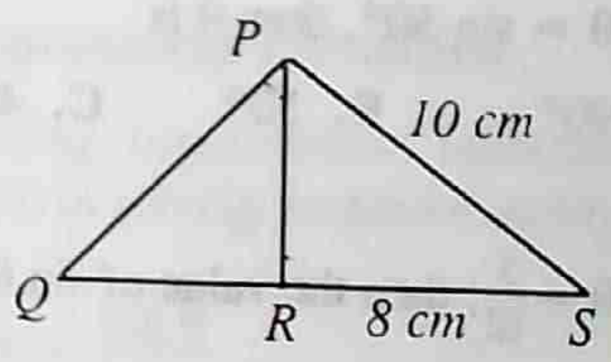
In the figure, SQ is perpendicular to PR and $|SQ| = |PQ|$. If $|SR| = 4$ and $|QR| = 2$, find $P\hat{S}R$.

A. 60° **B.** 75° **C.** 90° **D.** 120°

22. A ladder of length 5 m rests against a vertical wall and makes an angle of 30° with the wall. How far is the foot of the ladder from the wall?

A. $2\frac{1}{2}$ m **B.** $\frac{5\sqrt{3}}{2}$ m **C.** 5 m **D.** $5\sqrt{2}$ m

23.



In the figure, $Q\hat{P}S = P\hat{R}S = 90°$, $|PS| = 10$ cm and $|RS| = 8$ cm. Find $|QR|$.

A. 2 cm **B.** 4.5 cm **C.** 6 cm **D.** 12.5 cm

24.

In the triangle ABD, AC is the altitude from A to BD. If $|AD| = 15$ cm, $|AB| = 13$ cm and $|CD| = 9$ cm, find $|BD|$.

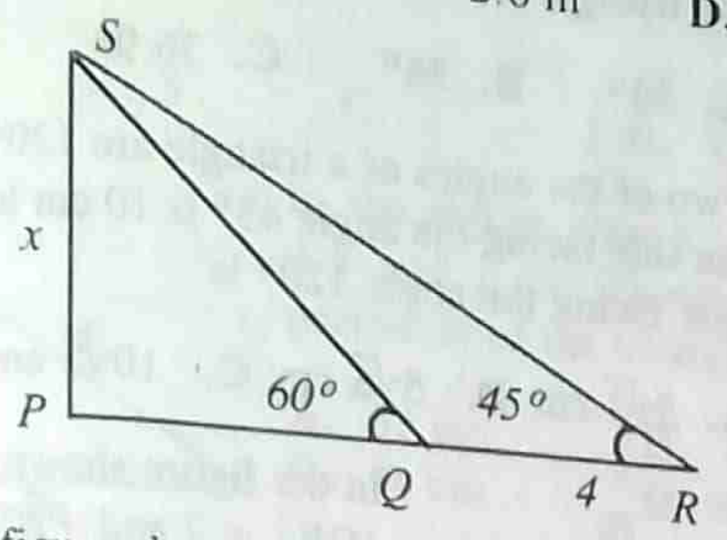
A. 12 cm **B.** 13 cm **C.** 14 cm **D.** 15 cm

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25. A ladder resting against a vertical wall makes an angle $\tan^{-1} 2.4$ with the ground. If the foot of the ladder is 1 m from the wall, the length of the ladder is

A. 1 m **B.** 2.4 m **C.** 2.6 m **D.** 5.2 m

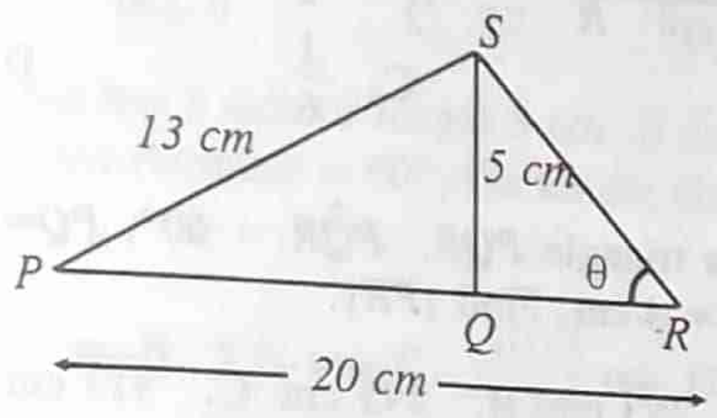
26.



In the figure shown find x if $|QR| = 4$.

A. $4\sqrt{3}$ **B.** $4(1 + \sqrt{3})$ **C.** $2(3 - \sqrt{3})$ **D.** $6 + 2\sqrt{3}$

27.



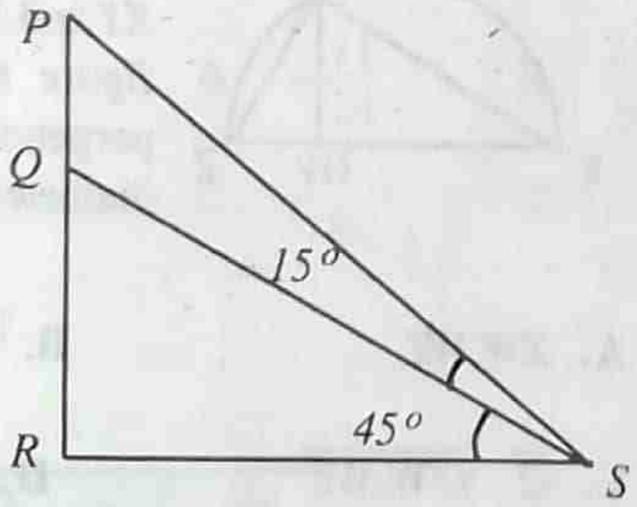
In the diagram, SQ is perpendicular to PR, $|PS| = 13$ cm and $|SQ| = 5$ cm. Find $\tan\theta$.

A. $\frac{1}{4}$ **B.** $\frac{5}{12}$ **C.** $\frac{12}{13}$ **D.** $\frac{5}{8}$

28. An isosceles triangle of sides 65 mm, 65 mm and 50 mm is inscribed in a circle. The radius of the circle is

A. $35\frac{5}{24}$ mm **B.** 60 mm **C.** 40 mm **D.** 35 mm

29.

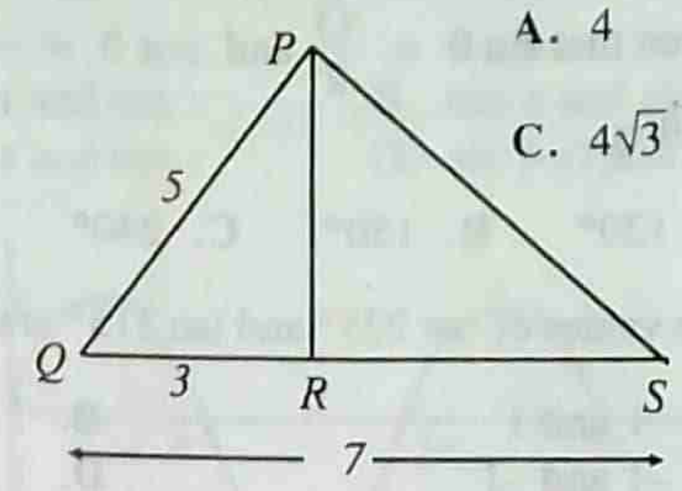


In the diagram provided, $|PR| = 12$ cm. The length of PQ is

A. $4\sqrt{3}$ cm **B.** $4(3 - \sqrt{3})$ cm

C. $12(\sqrt{3} - 1)$ cm **D.** 4 cm

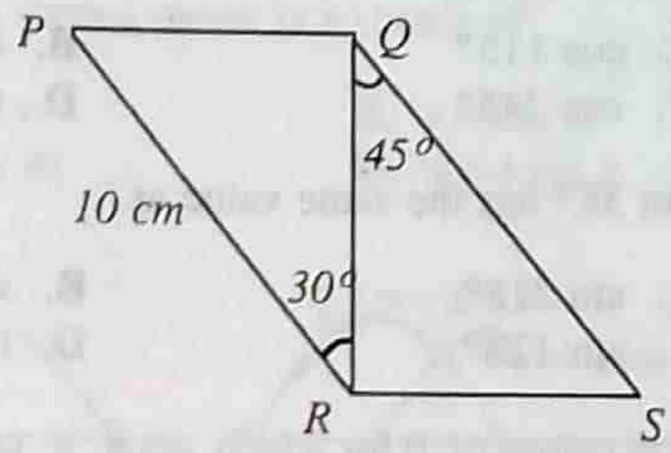
30. In the figure, PR is perpendicular to QS. If $|PQ| = 5$, $|QS| = 7$ and $|QR| = 3$, find $|PS|$.



A. 4 **B.** $4\sqrt{2}$

C. $4\sqrt{3}$ **D.** $\frac{5\sqrt{2}}{2}$

31.



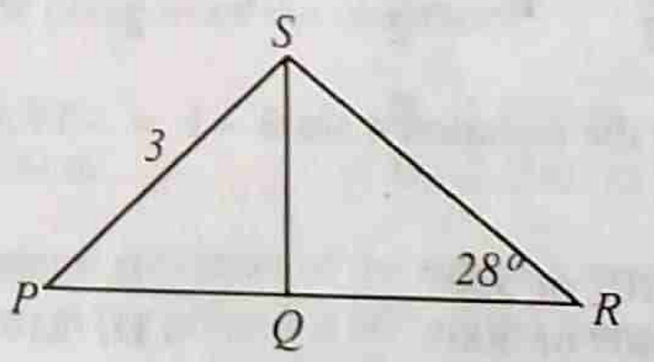
In the diagram shown above, $P\hat{Q}R = Q\hat{R}S = 90°$, $S\hat{Q}R = 45°$, $P\hat{R}Q = 30°$ and $|PR| = 10$ cm. Find $|QS|$.

A. $5\sqrt{6}$ cm **B.** $5\sqrt{3}$ cm **C.** $\frac{5\sqrt{3}}{2}$ cm **D.** $5\sqrt{2}$ cm

32. An inelastic string of length 1.0 m anchored to a point X has a stone tied to the end Y. How high does the stone rise when the string XY is inclined at 30° to the original vertical position?

A. $\frac{1}{2}$ m **B.** $\frac{\sqrt{3}}{2}$ m **C.** $\frac{2 - \sqrt{3}}{2}$ m **D.** $\frac{2}{\sqrt{3}}$ m

33.



In the diagram, $P\hat{S}R = S\hat{Q}R = 90°$, $S\hat{R}Q = 28°$ and $|PS| = 3$. Find $|QR|$.

A. $3\tan 28°\cos 28°$ **B.** $3\cot 28°\cos 62°$

C. $3\tan 28°\cos 62°$ **D.** $3\cot 28°\sin 62°$

34. If $\cos\theta > 0$ and $\sin\theta < 0$, then

A. $270° < \theta < 360°$ **B.** $180° < \theta < 270°$

C. $90° < \theta < 180°$ **D.** $0° < \theta < 90°$

35. The value of $2\cos 270° + \sin 270°$ is

A. 1 **B.** 0 **C.** –1 **D.** –2

36. If $\sin^{-1}(0.966) = 75°$, then $\sin^{-1}(-0.966)$ equals

A. 75° or 105° **B.** 105° or 255°

C. 195° or 285° **D.** 255° or 285°

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37. Given that $\sin\theta = \frac{\sqrt{3}}{2}$ and $\cos\theta = -\frac{1}{2}$, the value of θ is

A. 120° **B.** 150° **C.** 240° **D.** 300°

38. The values of tan 225° and tan 315° are respectively

A. –1 and 1 **B.** 1 and –1
C. –1 and –1 **D.** 1 and 1

39. Cos 65° is equal to

A. cos 115° **B.** sin 115°
C. cos 245° **D.** cos 295°

40. Sin 38° has the same value as

A. sin 218° **B.** sin (–38°)
C. sin 128° **D.** cos 308°

41. The values of θ for which $\sin\theta = \cos\theta$ are, within the range $0° < \theta < 360°$,

A. 45° and 135° **B.** 45° and 225°
C. 45° and 315° **D.** 135° and 225°

42. If $\cos\theta = -\frac{1}{2}$, the values of θ could be

A. 120° or 240° **B.** –60° or 120°
C. –60° or 240° **D.** 120° or 300°

43. Without using tables, evaluate sin (–1320°)

A. $\frac{1}{2}$ **B.** $-\frac{1}{2}$ **C.** $\frac{\sqrt{3}}{2}$ **D.** $-\frac{\sqrt{3}}{2}$

44. Solve the equation $4\sin\theta - 1 = -3$ for $0° < \theta < 360°$.

A. 210° or 330° **B.** 30° or 150°
C. 240° or 300° **D.** 210° or 300°

45. The value of sin (870°) is

A. $\frac{\sqrt{3}}{2}$ **B.** $-\frac{\sqrt{3}}{2}$ **C.** $\frac{1}{2}$ **D.** $-\frac{1}{2}$

46. The sine, cosine and tangent of 240° are, respectively

A. $\frac{\sqrt{3}}{2}$, $-\frac{1}{2}$ and $\sqrt{3}$ **B.** $-\frac{\sqrt{3}}{2}$, $-\frac{1}{2}$ and $\sqrt{3}$

C. $-\frac{\sqrt{3}}{2}$, $\frac{1}{2}$ and $-\sqrt{3}$ **D.** $\frac{\sqrt{3}}{2}$, $\frac{1}{2}$ and $\sqrt{3}$

47. Which of the following have the same value as sin 15°?

I. sin 165° II. sin 285°
III. sin (–15°) IV. sin (–195°)

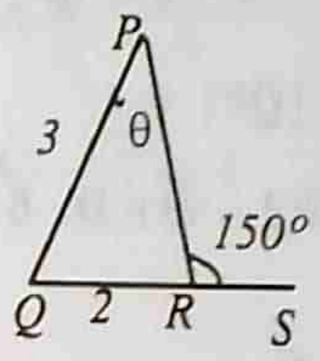
A. I and II only **B.** III and IV only
C. I and III only **D.** I and IV only

48. The two perpendicular sides of a right-angled triangle are 9 cm and 12 cm in length. The smallest angle of the triangle is

A. 30° **B.** 36° **C.** 36.9° **D.** 53.1°

49. Two of the angles of a triangle are 120° and 45°. If the side facing the angle 45° is 10 cm long, then the side facing the angle 120° is

A. $5\sqrt{3}$ cm **B.** $5\sqrt{6}$ cm **C.** $10\sqrt{2}$ cm **D.** 20 cm

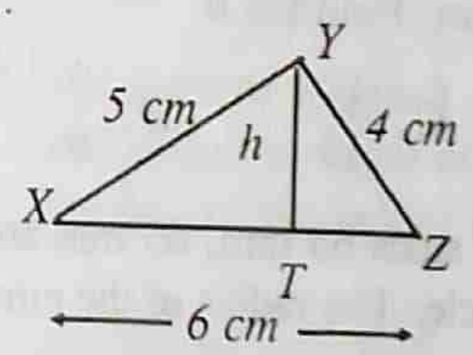
50. 

In the figure shown, $|PQ| = 3$, $|QR| = 2$ and $P\hat{R}S = 150°$. The value of sin θ is

A. $\frac{1}{3}$ **B.** $\frac{1}{2}$
C. $\frac{1}{6}$ **D.** $\frac{2}{3}$

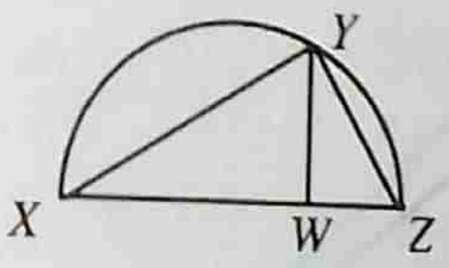
51. In a triangle PQR, $P\hat{Q}R = 60°$, $PQ = 3$ cm and $QR = 4$ cm. Find $|PR|$.

A. $2\sqrt{3}$ cm **B.** $2\sqrt{5}$ cm **C.** $\sqrt{13}$ cm **D.** 5 cm

52. 

The value of h in the diagram provided is

A. $\frac{5}{4}$ cm **B.** $\frac{4}{5}$ cm
C. $\frac{5\sqrt{7}}{2}$ cm **D.** $\frac{5\sqrt{7}}{4}$ cm

53. 

XYZ is a semicircle where XY and YZ are chords in the figure shown. YW is the perpendicular from Y to the diameter XZ. YW is equal to

A. $XW.WZ$ **B.** $\sqrt{XW.WZ}$
C. $\sqrt{2}\ \sqrt{XW.WZ}$ **D.** $\frac{\sqrt{2}}{2}\sqrt{XW.WZ}$

54. Each of two angles of an isosceles triangle is 30° and the longest side of the triangle is 9 cm. Each of the other sides is of length

A. $\frac{9\sqrt{2}}{2}$ cm **B.** $3\sqrt{3}$ cm
C. $3\sqrt{2}$ cm **D.** $4\frac{1}{2}$ cm

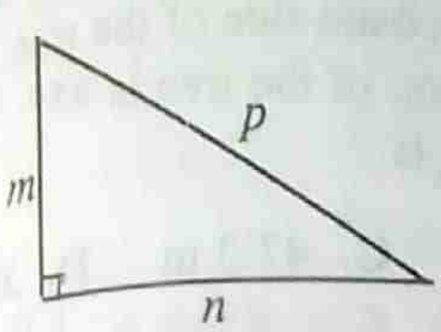
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55. The diagonals of a rhombus are of lengths 10 cm and 24 cm. The length of each side of the rhombus is

A. 5 cm **B.** 12 cm **C.** 13 cm **D.** 17 cm

56. If m and n are the lengths of the diagonals of a rhombus of side x, and $m^2 + n^2 = kx^2$, the value of k is

A. 4 **B.** 3 **C.** 2 **D.** 1

57. 

In the figure shown, $m:n = 3:4$ and $p = 25$ cm. The perimeter of the triangle is

A. 32 cm **B.** 45 cm
C. 54 cm **D.** 60 cm

58. A right-angled triangle has sides of lengths $(3m-1)$ cm, m cm and $(3m + 1)$ cm. The value of m is

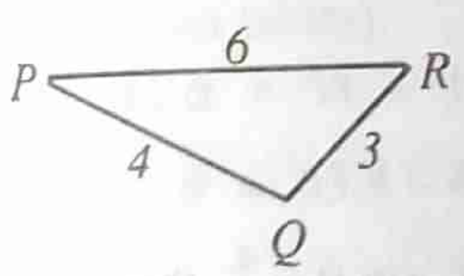
A. 2 **B.** 6 **C.** 12 **D.** 18

59. A rhombus has a side of length 5 cm. If one of the angles of the rhombus is $60°$, the longer diagonal is of length

A. $5\sqrt{2}$ cm **B.** $5\sqrt{3}$ cm **C.** $\frac{10\sqrt{3}}{3}$ cm **D.** 10 cm

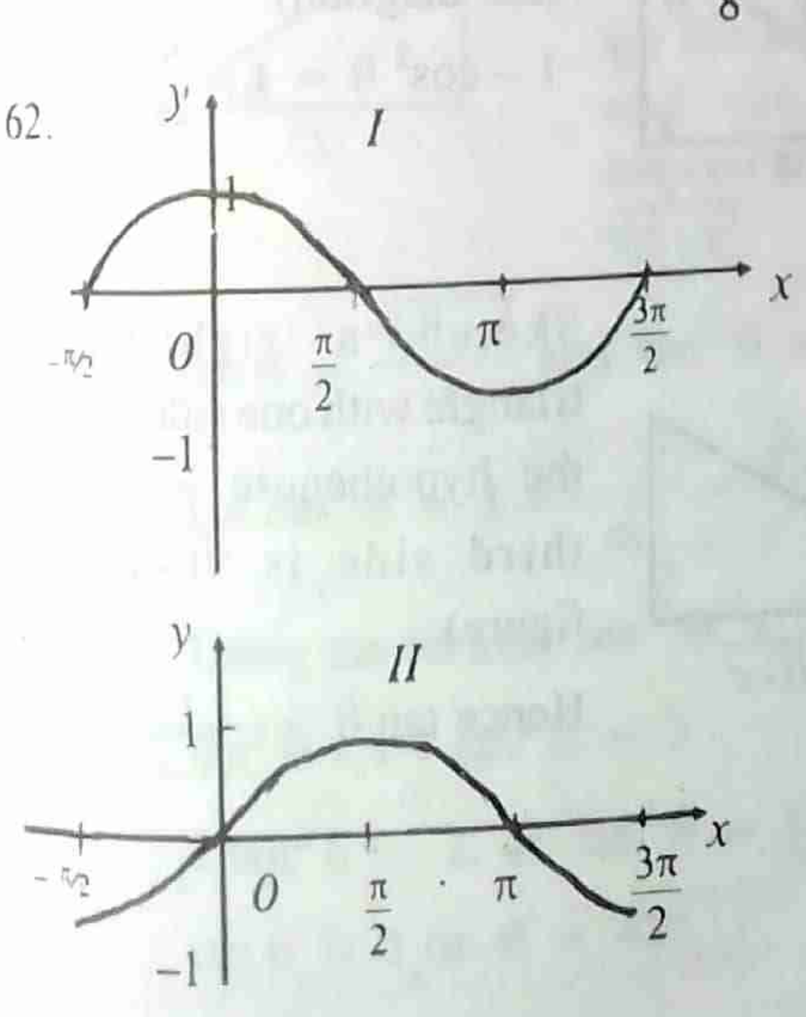
60. A triangle has sides of lengths y cm, $(y - 4)$ cm and $(y + 4)$ cm. If the largest angle of the triangle is $\cos^{-1}(1/5)$, find the value of y.

A. $12\frac{4}{7}$ cm **B.** 20 cm **C.** 24 cm **D.** 28 cm

61. 

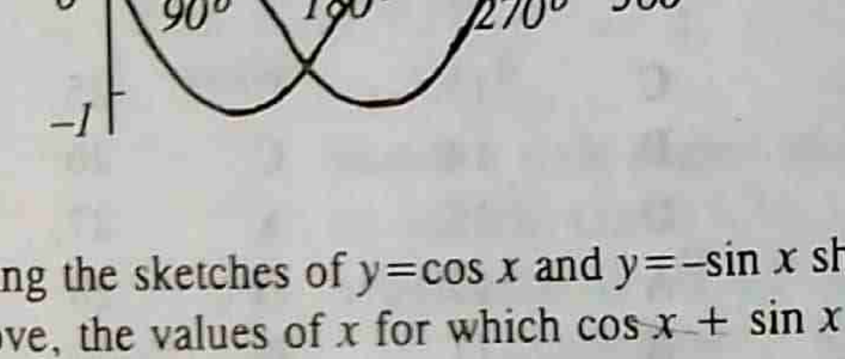
In the ΔPQR shown, $\cos Q$ is equal to

A. $-\frac{11}{24}$ **B.** $\frac{11}{24}$
C. $-\frac{3}{8}$ **D.** $-\frac{1}{2}$

62. 

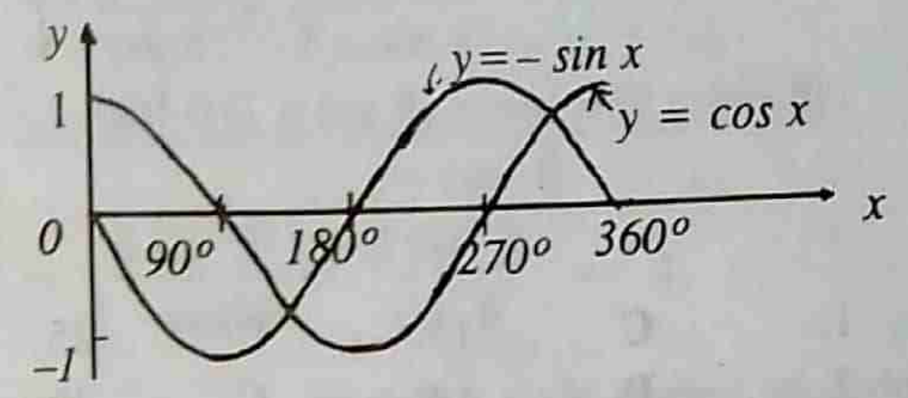
Figures I and II above show, respectively, the graphs of

A. $\sin x$ and $\cos x$ **B.** $\cos x$ and $\sin x$
C. $\tan x$ and $\cos x$ **D.** $\sin(-x)$ and $\cos(-x)$

63. 

The graph shown above is a sketch of

A. $y = \sin x$ **B.** $y = \sin 4x$
C. $y = \cos 4x$ **D.** $y = 4 \sin x$

64. 

Using the sketches of $y = \cos x$ and $y = -\sin x$ shown above, the values of x for which $\cos x + \sin x = 0$ are

A. $135°$ and $315°$ **B.** $45°$ and $225°$
C. $0°$, $180°$ and $360°$ **D.** $90°$ and $270°$

65. An observer on the ground views an airplane of altitude 1,000 m at an angle of elevation of $30°$. How far is the plane from the observer?

A. 500 m **B.** 1,000 m
C. 2,000 m **D.** $2{,}000/\sqrt{3}$ m

66. The angle of elevation of the top of a tower from an observer on the ground is $60°$. If the top of the tower is 100 m away from the observer, how high is the tower?

A. 50 m **B.** 86.6 m **C.** 173.2 m **D.** 200 m

67. An observer standing on the top of a building 40 m high views a stone on the ground level at an angle of depression of $38°$. The distance of the stone from the foot of the building in metres, is equal to

A. $40 \sin 38°$ **B.** $40 \cos 38°$
C. $40 \cot 38°$ **D.** $40 \tan 38°$

68. A tree on the bank of a river is 24 m tall. If the angle of depression of a point on the opposite bank of the river is $30°$, the width of the river is

A. $8\sqrt{3}$ m **B.** 12 m **C.** 24 m **D.** $24\sqrt{3}$ m

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69. The angle of elevation of the top of a tree from a point P an the ground is $30°$. From a point Q, 10 m closer to the foot of the tree, the angle of elevation is $45°$. How tall is the tree?

A. $5(\sqrt{3} + 1)$ m **B.** $10\sqrt{2}$ m

C. $10\sqrt{3}$ m **D.** $5(\sqrt{3} + 2)$ m

70. From the top of a 100 m –high building, a man observes the top of a tree at an angle of depression of $30°$. If the tree is 50 m tall, the angle of depression of the foot of the tree as viewed by the man is

A. $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ **B.** $\tan^{-1}(2\sqrt{3})$

C. $\tan^{-1}(\sqrt{3})$ **D.** $\tan^{-1}\left(\frac{2\sqrt{3}}{3}\right)$

71. The pilot of an airplane flying at an altitude of 2 km observes a landmark at an angle of depression of $25°$. A few seconds later, he observes the landmark at an angle of depression of $50°$. The distance between the two points of observation in km is

A. $2(\tan 50° - \tan 25°)$ **B.** $2(\cot 25° - \cot 50°)$

C. $2(\sin 50° - \sin 25°)$ **D.** $2(\cos 25° - \cos 50°)$

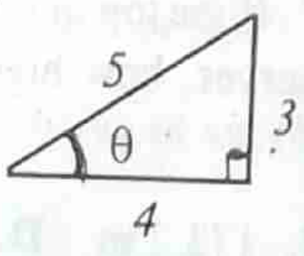
72. From a point P 30 m away from the foot of a tree, the angle of elevation of the top of the tree is $30°$. From a point Q on the opposite side of the tree, the angle of elevation of the top of the tree is $45°$. The distance between P and Q is

A. 17.3 m **B.** 42 m **C.** 47.3 m **D.** 50 m

ANSWER KEY

1.	C	13.	C	25.	C	37.	A	49.	B	61.	A
2.	D	14.	C	26.	D	38.	B	50.	A	62.	B
3.	D	15.	A	27.	D	39.	D	51.	C	63.	D
4.	A	16.	D	28.	A	40.	D	52.	D	64.	A
5.	B	17.	B	29.	B	41.	B	53.	B	65.	C
6.	A	18.	C	30.	B	42.	A	54.	B	66.	B
7.	B	19.	D	31.	A	43.	C	55.	C	67.	C
8.	C	20.	A	32.	C	44.	A	56.	A	68.	D
9.	C	21.	B	33.	D	45.	C	57.	D	69.	A
10.	A	22.	A	34.	A	46.	B	58.	C	70.	D
11.	B	23.	B	35.	C	47.	D	59.	B	71.	B
12.	D	24.	C	36.	D	48.	C	60.	C	72.	C

1. **(C)**



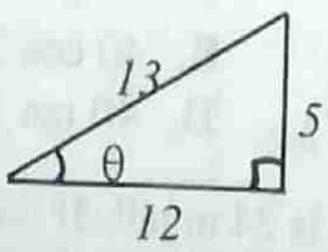
From the 3-4-5 triangle (*see figure*):

if $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$

2. **(D)** $\sin 50° = \cos(90° - 50°) = \cos 40°$.

Hence $\theta = 40°$.

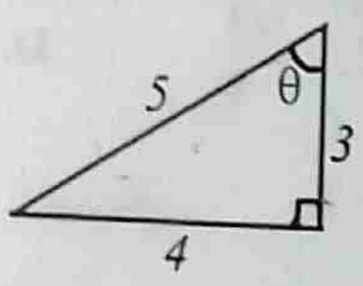
3. **(D)**



From the 5-12-13 triangle (see figure): If $\tan\theta = \frac{5}{12}$, $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$. Hence

$$\sin\theta - \cos\theta = \frac{5}{13} - \frac{12}{13} = -\frac{7}{13}.$$

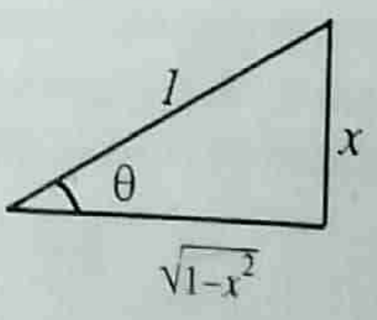
4. **(A)**



If $\sin\theta = \frac{4}{5}$, then $\cos\theta = \frac{3}{5}$ (see diagram):

$1 - \cos^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$.

5. **(B)**



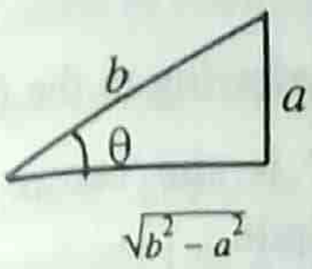
Sketch a right-angled triangle with one side = x and the hypothenuse = 1. The third side is $\sqrt{1-x^2}$ (see figure).

Hence $\tan\theta = \frac{x}{\sqrt{1-x^2}}$

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6. **(A)** $\sin 35^\circ = \cos(90^\circ - 35^\circ) = \cos 55^\circ$, and $\cos 15^\circ = \sin(90^\circ - 15^\circ) = \sin 75^\circ$

Hence $\dfrac{\sin 35^\circ}{\cos 55^\circ} + \dfrac{\cos 15^\circ}{\sin 75^\circ} = 1 + 1 = 2$

7. **(B)** 

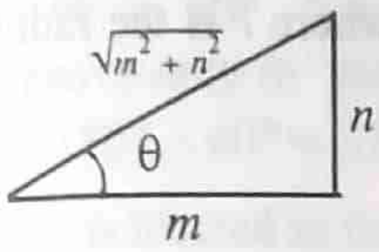
Sketch a right-angled triangle with one side $= a$ and the hypotenuse $= b$. The third side $= \sqrt{b^2 - a^2}$ (from Pythagoras)

$$\cot\theta = \frac{\text{adjacent side}}{\text{opp. side}} = \frac{\sqrt{b^2 - a^2}}{a}$$

$$= \frac{\sqrt{(b+a)(b-a)}}{a}$$

8. **(C)** $4\sin^2\theta = 3$, $\sin^2\theta = \dfrac{3}{4}$ and $\sin\theta = \dfrac{\sqrt{3}}{2}$

$\therefore\ \theta = 60^\circ$

9. **(C)** 

Sketch a right-angled triangle such that m and n are the adjacent and opposite sides to θ.

Hypotenuse $= \sqrt{m^2 + n^2}$

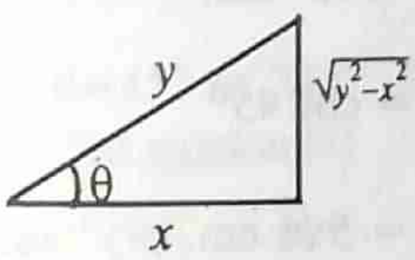
$$\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{\sqrt{m^2 + n^2}}{n}$$

10. **(A)** The ratio of the sides of a right-angled isosceles triangle is $1 : 1 : \sqrt{2}$. If the hypotenuse is 8 units, each of the other sides $= 8\left(\dfrac{1}{\sqrt{2}}\right) = 4\sqrt{2}$.

11. **(B)** $\tan(\theta + \alpha) = 4/2 = 2 \Rightarrow \theta + \alpha = 63.4^\circ$ (from tables).

$\tan\alpha = 4/4 = 1 \Rightarrow \alpha = 45^\circ$

$\therefore\ \theta = 63.4 - 45^\circ = 18.4^\circ$

12. **(D)** 

Sketch a right-angled triangle such that x and y are the adjacent side to θ and the hypotenuse, respectively. Third side $= \sqrt{y^2 - x^2}$

$$\tan\theta = \frac{\sqrt{y^2 - x^2}}{x} \text{ and } \tan^2\theta = \frac{y^2 - x^2}{x^2}$$

$$1 + \tan^2\theta = 1 + \frac{y^2 - x^2}{x^2} = \frac{y^2}{x^2}$$

13. **(C)** Using the identity $\sec^2\theta = 1 + \tan^2\theta$ gives

$\tan^2\theta + 1 + \tan^2\theta = 3$

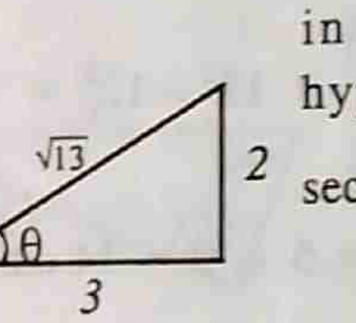
$2\tan^2\theta = 2$, or $\tan^2\theta = 1$

$\tan\theta = 1$, or $\theta = 45^\circ$

14. **(C)** $\dfrac{1 + \cos\theta}{1 - \cos\theta} \times \dfrac{1 + \cos\theta}{1 + \cos\theta} = \dfrac{(1 + \cos\theta)^2}{1 - \cos^2\theta}$

$= \dfrac{(1 + \cos\theta)^2}{\sin^2\theta}$ (making use of the identity $\cos^2\theta + \sin^2\theta = 1$).

Hence $\sqrt{\dfrac{1 + \cos\theta}{1 - \cos\theta}} = \dfrac{1 + \cos\theta}{\sin\theta}$

15. **(A)** 

Referring to the $1 - \sqrt{3} - 2$ triangle, $\sin\theta = 1/2$, $\cos\theta = \sqrt{3}/2$ and $\tan(90^\circ - \theta) = \sqrt{3}$.

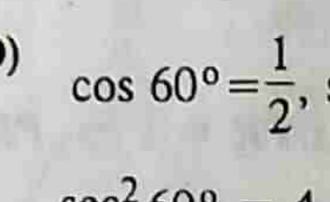
$$\frac{\cos^2\theta}{\tan(90^\circ - \theta)} = \frac{3/4}{\sqrt{3}} = \frac{\sqrt{3}}{4}.$$

16. **(D)** $\dfrac{\sin^2\theta}{1 - \cos\theta} + \dfrac{\sin^2\theta}{1 + \cos\theta}$

$$= \frac{\sin^2\theta(1 + \cos\theta) + \sin^2\theta(1 - \cos\theta)}{1 - \cos^2\theta}$$

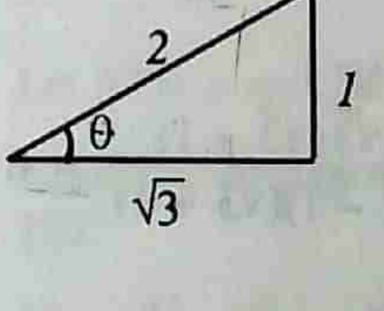
$$= \frac{2\sin^2\theta}{1 - \cos^2\theta} = \frac{2\sin^2\theta}{\sin^2\theta} = 2$$

17. **(B)** Sketch a right-angled triangle in which $\tan\theta = 2/3$. The hypotenuse is $\sqrt{3^2 + 2^2} = \sqrt{13}$



$\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{\sqrt{13}}{3}$.

$\sec^2\theta = \dfrac{13}{9}$.

18. **(C)** 

Using the identity $\cos^2\theta + \sin^2\theta = 1$ gives

$\cos^2\theta - (1 - \cos^2\theta) = \dfrac{1}{8}$

$2\cos^2\theta = 9/8$, or

$\cos^2\theta = 9/16$

$\cos\theta = 3/4$

and $\tan\theta = \sqrt{7}/3$ (see figure)

19. **(D)** $\cos 60^\circ = \dfrac{1}{2}$, $\sec 60^\circ = 1/\cos 60^\circ = 2$

$\sec^2 60^\circ = 4$ and $1 + \sec^2 60^\circ = 5$

20. **(A)**

Referring to the figure:

$\cot\theta = \sqrt{3}$, $\text{cosec}\,\theta = 2$

$\sec\theta = 2/\sqrt{3}$ and

$\tan\theta = 1/\sqrt{3}$

$$\frac{\cot\theta - \text{cosec}\,\theta}{\sec\theta + \tan\theta} = \frac{\sqrt{3} - 2}{\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{3 - 2\sqrt{3}}{3}$$

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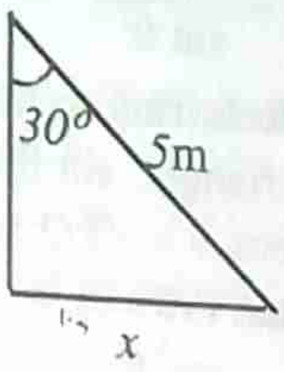
21. **(B)** $\sin Q\hat{S}R = 2/4 = 1/2 \Rightarrow Q\hat{S}R = 30°$

In the right-angled isosceles triangle PQS,

$Q\hat{P}S = P\hat{S}Q = 45°$

$P\hat{S}R = P\hat{S}Q + Q\hat{S}R = 45° + 30° = 75°$.

22. **(A)**



Distance required $= x$ (see diagram). $\frac{x}{5} = \sin 30°$

$x = 5 \sin 30° = 5(\frac{1}{2})$

$= 2\frac{1}{2}$ m

23. **(B)** In ΔRPS, $\cos P\hat{S}Q = 8/10 = 0.8$

In ΔQPS, $\cos P\hat{S}Q = 10/QS$

$\therefore 10/QS = 0.8$, or $QS = 10/0.8 = 12.5$ cm

$|QR| = |QS| - |RS| = 12.5 - 8 = 4.5$ cm

24. (C) $AC^2 = AD^2 - CD^2 = 225 - 81 = 144$

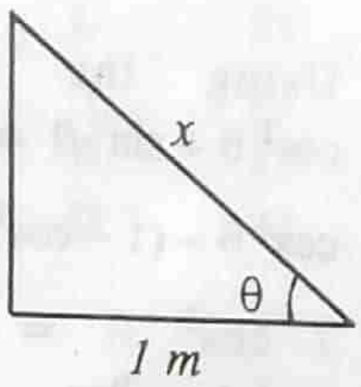
AC = 12 cm

$BC^2 = AB^2 - AC^2 = 13^2 - 12^2 = 25$

$BC = 5$ cm

$BD = BC + CD = 5 + 9 = 14$ cm

25. (C)



$\tan \theta = 2.4 = 12/5$

$\therefore \cos \theta = \frac{5}{13}$ (sketch the 5–12–13 Δ)

Referring to the figure, length of ladder $= x$

$\cos \theta = 1/x$, or $x = 1/\cos \theta = 13/5 = 2.6$ m

26. **(D)** In ΔSPQ, $\tan 60° = x/PQ = \sqrt{3}$, or

$PQ = \frac{x}{\sqrt{3}}$ ..... (i)

In ΔSPR, $\tan 45° = x/PR = 1 \Rightarrow PR = x$

$PQ = PR - QR = x - 4$ ... (ii)

From (i) and (ii): $\frac{x}{\sqrt{3}} = x - 4$, or

$x = x\sqrt{3} - 4\sqrt{3}$

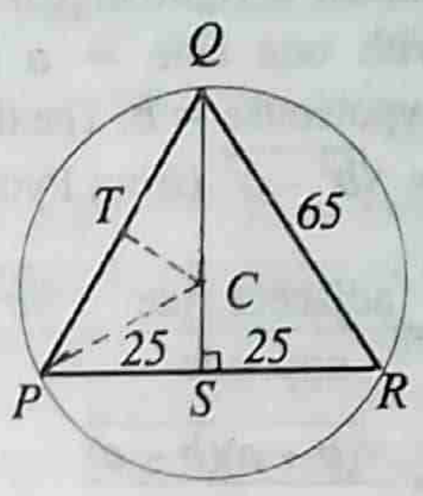
$x = \frac{4\sqrt{3}}{\sqrt{3} - 1} = \frac{4\sqrt{3}(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = 6 + 2\sqrt{3}$

27. **(D)** Recognize ΔPSQ as a 5–12–13 right-angled $\Delta \Rightarrow PQ = 12$ cm and $QR = PR - PQ$

$= 20 - 12 = 8$ cm

$\tan \theta = SQ/QR = 5/8$

28. **(A)**



Referring to the figure:

C is the centre of the circle,

$PQ = QR = 65$mm and CS is perpendicular to PR such that $PS = SR = 25$ mm.

$\sin P\hat{Q}S = 25/65 = 5/13$

$\therefore \cos P\hat{Q}S = 12/13$ (from 5–12–13 Δ)

The perpendicular CT from C to PQ bisects chord PQ. Hence in ΔCQT, $\cos P\hat{Q}S = QT/CQ = 65/2r$, where r is the radius of the circle.

$\frac{12}{13} = \frac{65}{2r}$,

$r = \frac{65 \times 13}{24} = 35\frac{5}{24}$ mm

29. **(B)** $\cot P\hat{S}R = RS/PR$

$\cot 60° = RS/12 \Rightarrow RS = 12 \cot 60°$

$= 12\sqrt{3} = 4\sqrt{3}$ cm

Since $Q\hat{S}R = 45°$, $QR = RS = 4\sqrt{3}$ cm

$PQ = PR - QR = 12 - 4\sqrt{3} = 4(3 - \sqrt{3})$ cm

30. **(B)** Recognize PQR as a 3–4–5 $\Delta \Rightarrow PR = 4$

$RS = QS - QR = 7 - 3 = 4$

$PS^2 = PR^2 + RS^2 = 4^2 + 4^2 = 16 \times 2$, or

$PS = 4\sqrt{2}$

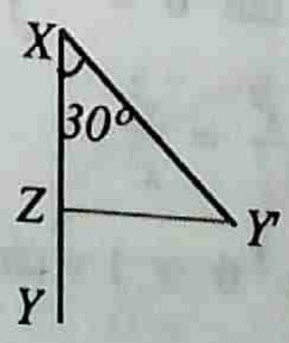
31. **(A)** $QR = PR \cos P\hat{R}Q = 10 \cos 30°$

$= 10(\sqrt{3}/2) = 5\sqrt{3}$ cm.

In ΔQRS, $\frac{QR}{QS} = \cos 45°$

$\Rightarrow QS = \frac{5\sqrt{3}}{1/\sqrt{2}} = 5\sqrt{6}$ cm.

32. **(C)**



XY and XY' are initial and final positions of the string. $Y'Z$ is perpendicular to XY. The required distance is YZ.

$XZ = XY' \cos 30°$

$= 1.0(\sqrt{3}/2) = \sqrt{3}/2$ m.

$YZ = XY - XZ = 1 - \frac{\sqrt{3}}{2}$

$= \frac{2 - \sqrt{3}}{2}$

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33. (D) In ΔPSR, $SR/PS = \cot 28°$, or $SR = 3 \cot 28°$

In ΔSQR, $QR/SR = \sin Q\hat{S}R = \sin 62°$

$QR = SR \sin 62° = 3 \cot 28° \sin 62°$.

34. (A) Cosine is *positive* in 1st and 4th quadrants

Sine is *negative* in 3rd and 4th quadrants

Hence $\cos \theta > 0$ and $\sin \theta < 0$ in the 4th quadrant i.e. $270° < \theta < 360°$.

35. (C) $\cos 270° = \cos (270° - 180°) = \cos 90° = 0$

$\sin 270° = \sin (270° - 180°) = \sin 90° = 1$

$2 \cos 270° - \sin 270° = 0 - 1 = -1$

36. (D) Sine is *negative* in the 3rd and 4th quadrants. The angles are $180° + 75° = 255°$, or $360° - 75° = 285°$.

37. (A) The angle whose sine is $\sqrt{3}/2$ and whose cosine is $1/2$ is $60°$. Sine is *positive* and cosine *negative* only in the second quadrant. Hence $\theta = 180° - 60° = 120°$.

38. (B) $225°$ is located in the 3rd quadrant

$\therefore \tan 225° = \tan (225° - 180°) = \tan 45° = 1$

$315°$ is located in the 4th quadrant

$\therefore \tan 315° = -\tan (360° - 315°)$

$= -\tan 45° = -1$

39. (D) Since $295°$ is located in the 4th quadrant,

$\cos 295° = \cos (360° - 295°) = \cos 65°$.

40. (D) $\cos 308° = \cos (360° - 308°) = \cos 52°$

$= \sin (90° - 52°) = \sin 38°$

41. (B) $\sin \theta = \cos \theta$

Divide both sides by $\cos \theta$ to give

$\tan \theta = 1$

$\therefore \theta = 45°$ or $225°$ (tangent is *positive* in lst and 3rd quadrants)

42. (A) $\cos^{-1}(1/2) = 60°$

Cosine is *negative* in the 2nd and 3rd quadrants. Hence $\theta = 180° - 60° = 120°$, or $180° + 60° = 240°$.

43. (C) $\sin (-1320°) = \sin [120° - (360° \times 4)]$

$= \sin 120° = \sin (180° - 120°)$

$= \sin 60° = \sqrt{3}/2$

44. (A) $4 \sin \theta = -2$, *or* $\sin \theta = -1/2$

But $\sin^{-1}(1/2) = 30°$ and sine is *negative* in the 3rd and 4th quadrants. Hence $\theta = 180° + 30° = 210°$, or $360° - 30° = 330°$.

45. (C) $\sin 870° = \sin [(360° \times 2) + 150°] = \sin 150°$

$= \sin (180° - 150°) = \sin 30° = 1/2$.

46. (B) $240°$ is located in the 3rd quadrant. Hence sine and cosine are *negative* while tangent is *positive*.

47. (D) I. $\sin 165° = \sin (180° - 165°) = \sin 15°$

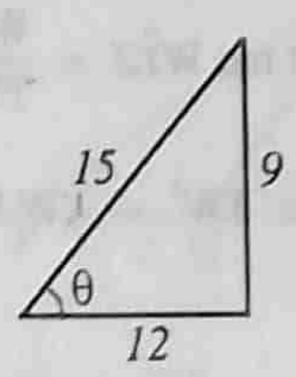
II. $\sin 285° = -\sin (360° - 285°) = -\sin 75°$

III. $\sin (-15°) = -\sin 15°$

IV. $\sin (-195°) = \sin (-195° + 360°)$

$= \sin 165° = \sin 15°$

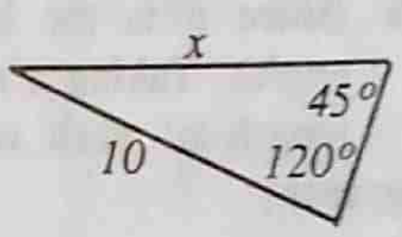
48. (C)



Recognize the triangle as a 3–4–5 right-angled Δ, since 9 and 12 are in the ratio 3:4. Hypotenuse = 15 cm.

The smallest angle (θ) is the angle facing the shortest side. Hence $\tan \theta = 9/12 = 3/4$, and $\theta = 36.9°$.

49. (B)



Referring to the figure:

$$\frac{x}{\sin 120°} = \frac{10}{\sin 45°}$$

$$= \frac{10}{1/\sqrt{2}} = 10\sqrt{2} \text{ cm}$$

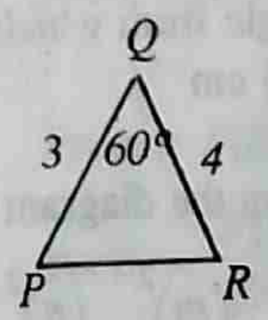
$$x = (\sin 120°)(10\sqrt{2}) = \left(\frac{\sqrt{3}}{2}\right)(10\sqrt{2}) = 5\sqrt{6} \text{ cm}$$

50. (A) $P\hat{R}Q = 180° - 150° = 30°$

In ΔPRQ, $\frac{\sin \theta}{2} = \frac{\sin 30°}{3}$

$$\sin \theta = \frac{2}{3} \sin 30° = \frac{2}{3}\left(\frac{1}{2}\right) = \frac{1}{3}.$$

51. (C)



Referring to the figure:

$PR^2 = 3^2 + 4^2 - 2(3)(4) \cos 60°$

$= 9 + 16 - 24(1/2) = 13$, or

$PR = \sqrt{13}$ cm.

52. (D) Let $XT = a$ and $TZ = 6 - a$.

Applying Pythagoras' theorem to Δs *XYT* and *YTZ*:

$h^2 = 5^2 - a^2 = 4^2 - (6 - a)^2$

$25 - a^2 = 16 - 36 + 12a - a^2$

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$$a = \frac{45}{12} = \frac{15}{4}$$

$$h^2 = 25 - a^2 = 25 - \frac{225}{16} = \frac{175}{16}$$

$$h = \frac{\sqrt{175}}{4} = \frac{5\sqrt{7}}{4} \text{ cm.}$$

53. **(B)** Since XZ is a diameter, $X\hat{Y}Z = 90°$

$X\hat{Y}W + Y\hat{X}W = 90°$, and

$X\hat{Y}W + W\hat{Y}Z = 90°$

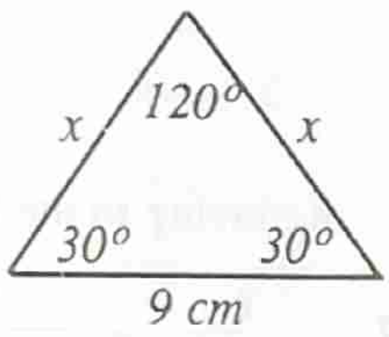
$\Rightarrow Y\hat{X}W = W\hat{Y}Z$

$\tan Y\hat{X}W = \frac{YW}{XW}$, and $\tan W\hat{Y}Z = \frac{WZ}{YW}$

Hence $\frac{YW}{XW} = \frac{WZ}{YW}$ or $YW^2 = XW.WZ$

and $YW = \sqrt{XW.WZ}$

54. **(B)**

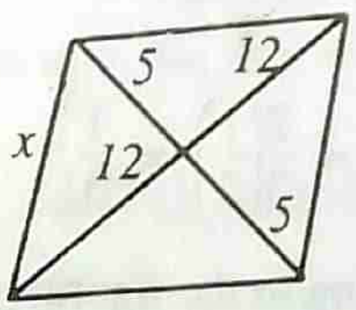


The third angle must be equal to $180° - 60° = 120°$, and the longest side must face this angle (since it is the largest angle). Taking x as the length of each of the other sides (see diagram):

$$\frac{x}{\sin 30°} = \frac{9}{\sin 120°} = \frac{9}{\sin 60°} = \frac{18}{\sqrt{3}}$$

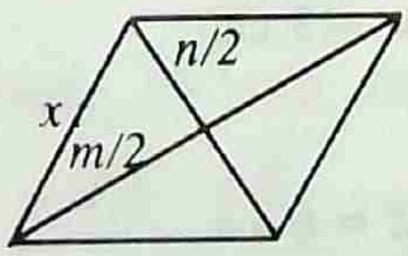
$$x = \frac{18}{\sqrt{3}}(\frac{1}{2}) = \frac{9}{\sqrt{3}} = 3\sqrt{3} \text{ cm}$$

55. (C)



Referring to the diagram, the diagonals of the rhombus bisect each other at right angles. Recognize the 5–12–13 right-angled triangle from which $x = 13$ cm

56. (A)



From the diagram:

$$x^2 = \left(\frac{m}{2}\right)^2 + \left(\frac{n}{2}\right)^2,$$

or $m^2 + n^2 = 4x^2$

Hence $k = 4$.

57. **(D)** Recognize the triangle as a 3–4–5 right-angled Δ

Hence $p : n : m = 5 : 4 : 3$

$n/p = 4/5$, or $n = \frac{4}{5}(25) = 20$ cm

$m/p = 3/5$, or $m = \frac{3}{5}(25) = 15$ cm

Perimeter of the $\Delta = m + n + p = 15 + 20 + 25 = 60$ cm.

58. **(C)** The longest side (i.e hypotenuse) must be $3m+1$. Applying Pythagoras' theorem:

$$(3m + 1)^2 = (3m - 1)^2 + m^2$$

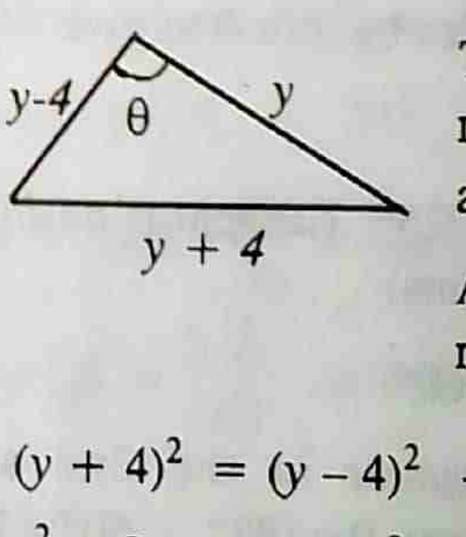
$$9m^2 + 6m + 1 = 9m^2 - 6m + 1 + m^2$$

$$m^2 - 12m = 0, \text{ or } m(m - 12) = 0$$

$m = 0$ or 12

[The 0 is ignored since the length of a triangle must be greater than 0].

59. **(B)**



$PQRS$ is the rhombus (use diagram). The required length is $|QS|$.

$Q\hat{P}S = 180° - 60° = 120°$

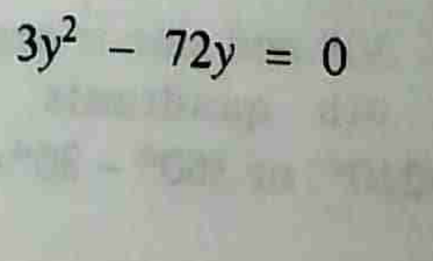
Applying cosine rule to $\Delta\ QPS$:

$$QS^2 = 5^2 + 5^2 - 2(5)(5)\cos 120°$$

$$= 25 + 25 - 50(-\frac{1}{2}) = 75$$

$$QS = \sqrt{75} = 5\sqrt{3} \text{ cm}$$

60. (C)



The longest side $(y+4)$ must face the largest angle $\theta = \cos^{-1}(1/5)$.

Applying the cosine rule:

$$(y + 4)^2 = (y - 4)^2 + y^2 - 2y(y - 4)\cos\theta$$

$$y^2 + 8y + 16 = y^2 - 8y + 16 + y^2 - (2y^2 - 8y)(\frac{1}{5})$$

$$y^2 - 16y - \frac{(2y^2 - 8y)}{5} = 0$$

$$5y^2 - 80y - 2y^2 + 8y = 0$$

$$3y^2 - 72y = 0$$

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$3y(y - 24) = 0$

$y = 0$ or 24 cm

61. **(A)** Applying *cosine rule* to the triangle PQR:

$6^2 = 4^2 + 3^2 - 2(4)(3)\cos Q$

$36 = 25 - 24\cos Q$, or

$\cos Q = -11/24$

62. **(B)** In fig. I, $y = \cos x$ since $y = 1$ when $x = 0$, $y = 0$ when $x = \pi/2 = 90°$, and $y = -1$ when $x = \pi = 180°$.

In fig. II, $y = \sin x$ since $y = 0$ when $x = 0°$ and $x = 180°$, $y = 1$ when $x = \pi/2 = 90°$ and $y = -1$ when $x = 3\pi/2 = 270°$.

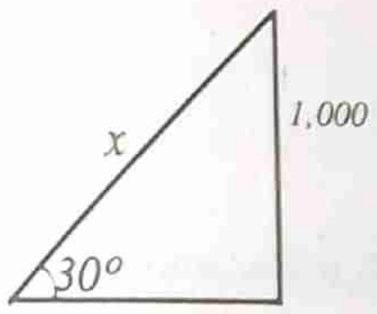
63. **(D)** The equation $y = 4\sin x$ is the only one consistent with

$y = 4$ when $x = \pi/2 = 90°$, and

$y = -4$ when $x = 3\pi/2 = 270°$

64. **(A)** The solution to $\cos x + \sin x = 0$ is given by the points of intersection of $y = \cos x$ and $y = -\sin x$. These are located between 90° and 180° (i.e. 135°) and between 270° and 360° (i.e. 315°).

65. **(C)**

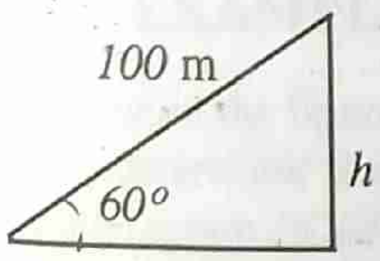


The distance required is x (see fig)

$\frac{1,000}{x} = \sin 30° = \frac{1}{2}$

$x = 2 \times 1,000 = 2,000$m

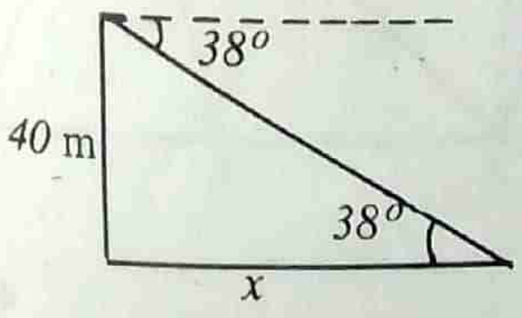
66. **(B)** The height required is h (see fig.)



$\frac{h}{100} = \sin 60° = \frac{\sqrt{3}}{2}$

$h = \frac{100\sqrt{3}}{2} = 86.6$ m

67. **(C)**

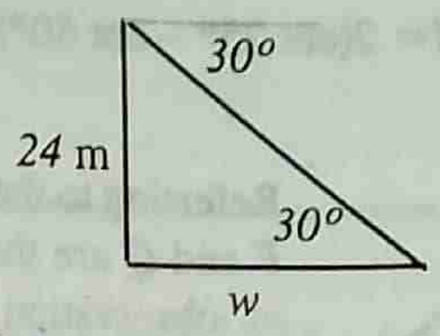


The distance required = x (see fig.)

$\frac{x}{40} = \cot 38°$, or

$x = 40\cot 38°$

68. **(D)**



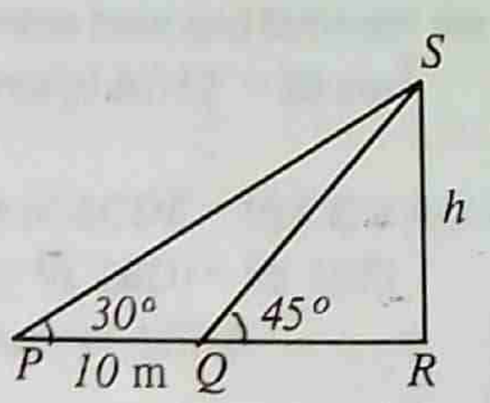
Referring to the figure,

w = width of river.

$\frac{w}{24} = \cot 30° = \sqrt{3}$, or

$w = 24\sqrt{3}$ m

69. **(A)**



Referring to the figure, the height of the tree = h.

Since ΔQSR is an isosceles right-angled triangle, $QR = SR = h$

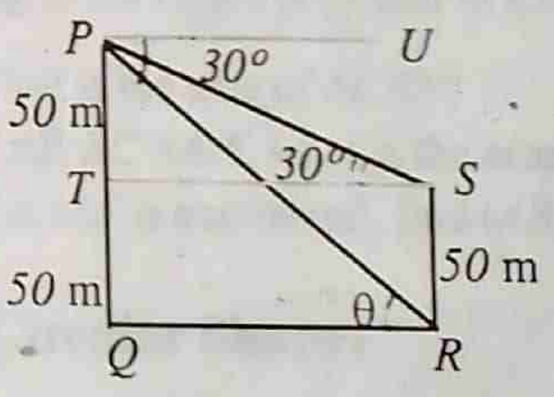
In ΔPSR, $\tan 30° = \frac{SR}{PR} = \frac{h}{h + 10}$

$\frac{1}{\sqrt{3}} = \frac{h}{h + 10}$

$h + 10 = h\sqrt{3}$, or $h = \frac{10}{\sqrt{3} - 1}$

$= \frac{10(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = 5(\sqrt{3} + 1)$

70. **(D)**



The man's eyes at P observe the top of the tree at S and the foot of the tree at R.

$U\hat{P}S = 30° = P\hat{S}T$, and the angle of depression of the foot of the tree = $U\hat{P}R = P\hat{R}Q = \theta$.

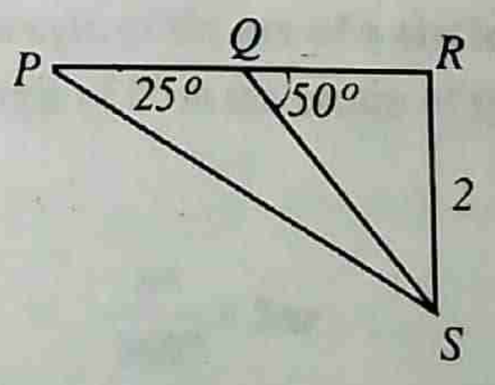
From ΔSPT, $ST = PT\cot 30° = 50\cot 30°$

$= 50\sqrt{3}$ m

From ΔPQR, $QR = ST = 50\sqrt{3}$ m

$\tan\theta = \frac{PQ}{QR} = \frac{100}{50\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$.

71. **(B)**



The landmark, S, is observed first at P and then at Q (see fig.). Distance required = PQ

From ΔPRS, $PR = RS\cot 25°$

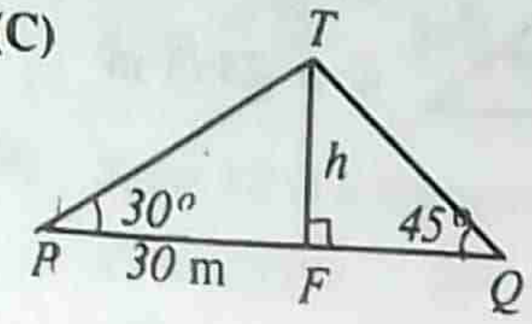
$= 2\cot 25°$

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From $\Delta\ QRS$, $QR = RS \cot 50° = 2 \cot 50°$

$PQ = PR - QR = 2(\cot 25° - \cot 50°)$.

72. (C)



Referring to the figure, P and Q are the points of observation of the tree TF.

From $\Delta\ PTF$, $h = PF \tan 30° = 30 \tan 30°$
$= 30/\sqrt{3} = 10\sqrt{3}$ m

From $\Delta\ QTF$, $FQ = TF = h = 10\sqrt{3}$ m

$PQ = PF + FQ$
$= 30 + 10\sqrt{3}$
$= 47.3$ m.

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16 Mensuration

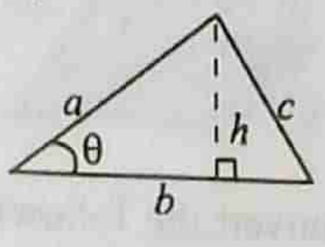
16.1 Triangles and Quadrilaterals

- **Perimeter** is the distance round the boundary of a closed, plane figure.
- **Area** is the size of the surface bounded by a figure.
- The following are the perimeters (P) and areas (A) of various plane shapes:

(i) **Triangle**, sides a,b,c, height h

$P = a + b + c$

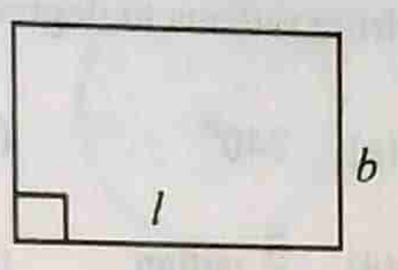
$A = \frac{1}{2}bh = \frac{1}{2}ab\sin\theta$



(ii) **Rectangle**, sides l,b

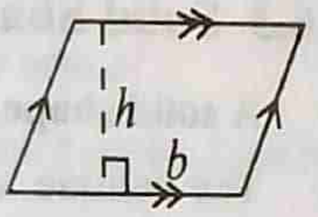
$P = 2(l + b)$

$A = lb$



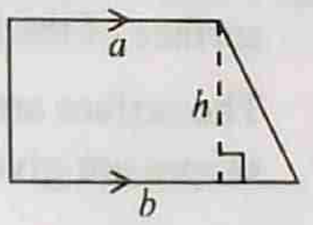
(iii) **Parallelogram**, base b, height h

$A = bh$



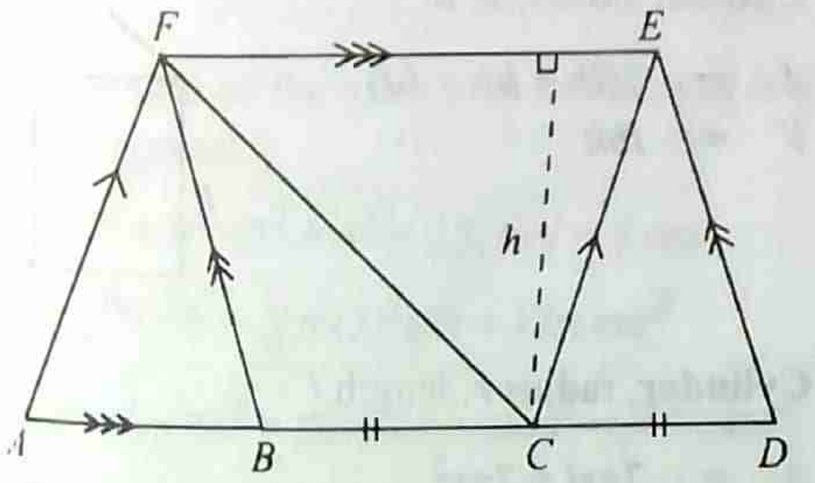
(iv) **Trapezium**, sides a, b, height h

$A = \frac{1}{2}(a + b)h$



EXAMPLE 16–1

If the area of ΔCEF in the figure provided below is 10 cm^2 and $BC = CD$, determine the area of (i) parallelogram $ACEF$ (ii) parallelogram $BDEF$ and (iii) trapezium $ADEF$.



Solutions

Let h be the perpendicular distance from C to EF.

(i) Area of $CEF = \frac{1}{2}\ EF \times h = 10$

$\Rightarrow$ $EF \times h = \mathbf{20\ cm^2}$ = area of parallelogram $ACEF$.

(ii) Area of $BDEF$ = area of $ACEF$ (parallelograms on the same base and between the same parallels).

$\Rightarrow$ area of $BDEF = \mathbf{20\ cm^2}$

(iii) Area of $\Delta CDE = \frac{1}{2}\ CD \times h$

$CD = \frac{1}{2}\ (BD) = \frac{1}{2}\ (EF)$

$\Rightarrow$ Area of $\Delta CDE = \frac{1}{2}\ (\frac{1}{2}EF \times h)$

$= \frac{1}{4} \times EF \times h$

$= \frac{1}{4} \times$ area of $BDEF$

$= \frac{1}{4} \times 20$

$= \mathbf{5\ cm^2}$

Area of trapezium $ADEF$ = area of $ACEF$ + area of CDE

$= 20 + 5 = \mathbf{25\ cm^2}$

DRILL 1

Referring to the figure provided in Example 16–1:

(a) What is the area of ΔCBF?

(b) If $AB:BC = 6:5$, what is the area of ΔABF?

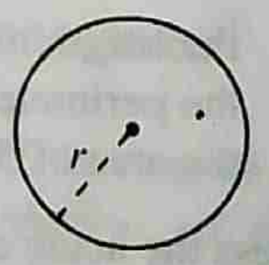
(c) If ΔABF is equilateral, find $|AB|$.

16.2 Circular Shapes

- The perimeter (or **circumference**) and area of a **circle** of radius r are given by

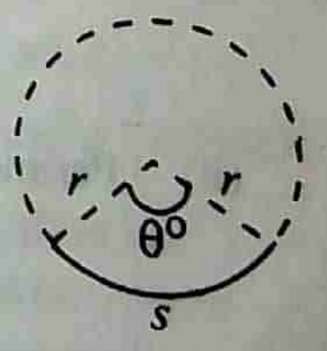
$P = 2\pi r$

$A = \pi r^2$



- The length of the **arc of a circle**, radius r, subtending an angle of θ^o at the centre of the circle is

$$s = \frac{\theta^o}{360^o} \times 2\pi r$$



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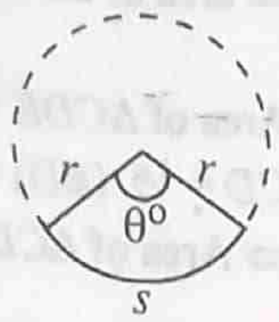
- The **radian**, a measure of an angle, is the angle which an arc of length 1 unit subtends at the centre of a circle of radius 1 unit, i.e. $\theta = 1$ radian when $s = 1$ and $r = 1$.
  Hence 2π radians $= 360^o$, or π **radians** $= 180^o$
  When θ is expressed in radians,

$$s = r\theta$$

- The perimeter and area of a **sector of a circle**, of radius r and arc of length s are given by

$$P = s + 2r$$

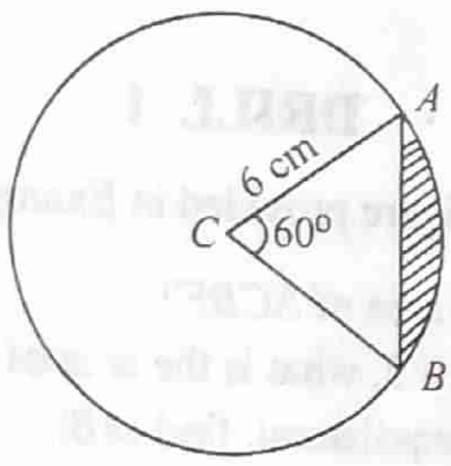
$$A = \frac{\theta^o}{360^o} \times \pi r^2$$



When θ is expressed in radians,

$$A = \frac{1}{2}r^2\theta$$

EXAMPLE 16–2



In the figure above, (a) find

(i) the length of arc AB.
(ii) the perimeter of the sector ABC.
(iii) the area of the sector ABC.

(b) express the acute and reflex angle ACB in radians.

Solutions

(a) (i) Length of arc $AB = \frac{60^o}{360^o} \times 2\pi\,(6)$

$= \mathbf{2\pi\ cm}$

(ii) Perimeter of sector ABC

$= 6\text{ cm} + 6\text{ cm} + 2\pi\text{ cm}$

$= \mathbf{(12 + 2\pi)\ cm}$

(iii) Area of sector $ABC = \frac{60^o}{360^o} \times \pi(6)^2$

$= \mathbf{6\pi\ cm^2}$

(b) Acute angle $ACB = 60^o = 60^o \times \frac{\pi}{180^o}$ rad.

$= \frac{\pi}{3}$ radians

Reflex angle $ACB = 360^o - 60^o = 300^o$

$= 300^o \times \frac{\pi}{180^o}$ rad.

$= \frac{5\pi}{3}$ radians

DRILL 2

Convert the following angles from degrees to radians or from radians to degrees:

(a) 240^o (b) $\frac{3\pi}{4}$ radians (c) 150^o

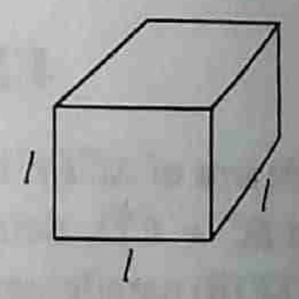
(d) $\frac{\pi}{6}$ radian (e) 15^o (f) 5 radians

16.3 Solid Shapes

- A **solid shape**, unlike a plane figure, occupies space.
- The **volume** of a solid body is the amount of space occupied by the body.
- The **surface area** of a solid body is the area of the surface of the body.
- The surface areas (A) of volumes (V) of common solid shapes are given below:

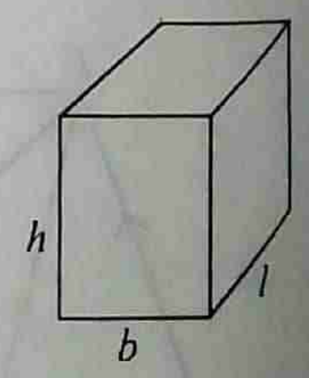
(i) **Cube**, side l

$$A = 6l^2$$
$$V = l^3$$



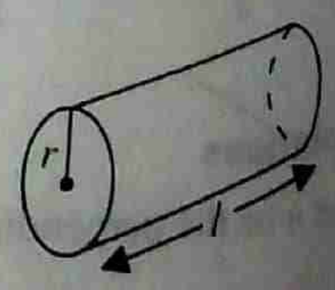
(ii) **Cuboid**, sides l, b, h

$$A = 2(lb + hb + hl)$$
$$V = lbh$$



(iii) **Cylinder**, radius r, length l

$$A = 2\pi rl + 2\pi r^2$$
$$V = \pi r^2 l$$

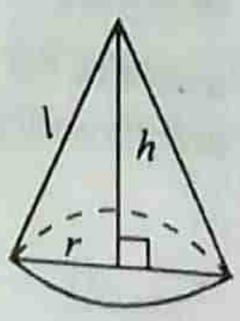


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(iv) Cone, base radius r, side l, height h

$$A = \pi rl + \pi r^2$$

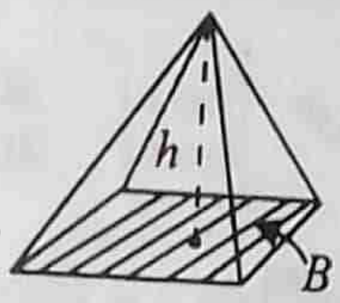
$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}Bh$$



where $B = \pi r^2$ = area of the base.

(v) Pyramid, height h, base area B

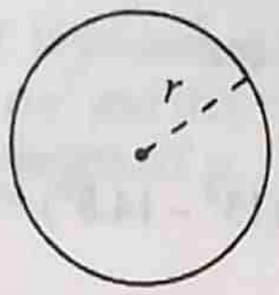
$$V = \frac{1}{3}Bh$$



(vi) Sphere, radius r

$$A = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$



EXAMPLE 16–3

Find the volume and the total surface area of

(a) a rectangular solid of dimensions 2 cm by 3 cm by 4 cm

(b) a solid cylinder of radius 2 cm and length 10 cm

(c) a solid cone of base radius 3 cm and height 4 cm.

Solutions

(a) $V = lbh = (2)(3)(4) = \mathbf{24\ cm^3}$

$A = 2(lb + bh + hl)$

$= 2[(2 \times 3) + (3 \times 4) + (2 \times 4)] = \mathbf{52\ cm^2}$

(b) $V = \pi r^2 l = \pi(2)^2(10) = \mathbf{40\pi\ cm^3}$

$A = 2\pi rl + 2\pi r^2$

$= 2\pi(2)(10) + 2\pi(2)^2 = \mathbf{48\pi\ cm^2}$

(c) The length of the slant side l of a cone is given by (Pythagoras)

$l^2 = r^2 + h^2 = 3^2 + 4^2 = 25$, or $l = 5$ cm

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2(4) = \mathbf{12\pi\ cm^3}$

$A = \pi rl + \pi r^2 = \pi(3)(5) + \pi(3)^2 = \mathbf{24\pi\ cm^2}$

DRILL 3

(a) If the rectangular solid in Example 16–3(a) is sitting on the side 2 cm by 3 cm, what is the total area of its vertical faces?

(b) What is the volume of metal in a 10 cm long cylindrical metal shell with internal and external diameters of 4 cm and 6 cm respectively?

(c) What is the area of the curved surface of the cone in Example 16–3(c) above?

EXAMPLE 16–4

Find the volume of:

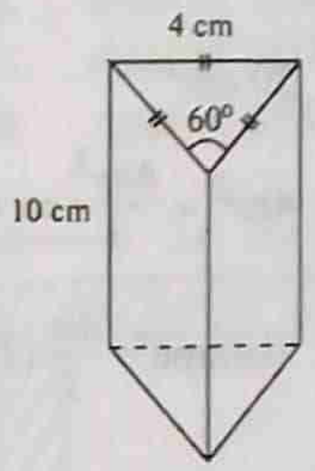
(a) a triangular prism of height 10 cm whose cross-section is an equilateral triangle of side 4 cm.

(b) a pyramid of height 10 cm whose base is a square of side 3 cm.

(c) the solid part of a hemispherical bowl of internal and external radii 5.4 cm and 6.0 cm respectively.

Solutions

(a)



Referring to the diagram, the base of the prism is an equilateral triangle with each side equal to 4 cm and each angle equal to 60°.

Base area of prism $= \frac{1}{2}ab\sin\theta$

$= \frac{1}{2}(4)(4)\sin 60^\circ$

$= 4\sqrt{3}\ \text{cm}^2$

Volume of prism = base area x height

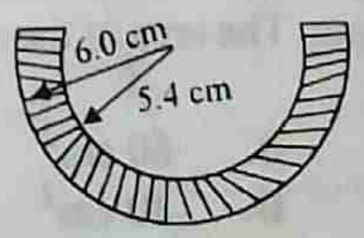
$= (4\sqrt{3})(10)$

$= 40\sqrt{3}\ \text{cm}^3$

(b) Base area of pyramid, $B = 3 \times 3 = 9\ \text{cm}^2$

Volume $= \frac{1}{3}Bh = \frac{1}{3} \times 9 \times 10 = \mathbf{30\ cm^3}$

(c)



Referring to the figure:

Volume of outer hemisphere of rad, r_o.

$= \frac{2}{3}\pi r_o^3 = \frac{2}{3}\pi(6.0)^3$

$= 144\pi\ \text{cm}^3$

Volume of inner hemisphere of rad, r_i

$= \frac{2}{3}\pi r_i^3 = \frac{2}{3}\pi(5.4)^3$

$\simeq 105\pi\ \text{cm}^3$

Volume of solid part $= 144\pi - 105\pi$

$= \mathbf{39\pi\ cm^3}$.

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DRILL 4

Find the volume of

(a) a pyramid of height 5 cm and rectangular base 3 cm by 6 cm.

(b) leather in a football of internal and external diameters of 14.8 cm and 15 cm respectively.

(c) a sphere for which the surface area and the volume are numerically equal.

ANSWERS TO DRILLS

1. (a) $5\ cm^2$ (b) $6\ cm^2$ (c) $3.72\ cm$

2. (a) $\frac{4\pi}{3}$ rad. (b) 135^o (c) $\frac{5\pi}{6}$ rad.

   (b) 30^o (e) $\frac{\pi}{12}$ rad (f) 286.5^o

3. (a) $40\ cm^2$ (b) $50\pi\ cm^2$ (c) $15\pi\ cm^2$

4. (a) $30\ cm^2$ (b) $\frac{4\pi}{3}(15^3 - 14.8^3) = 177.6\pi\ cm^3$

   (c) $4\pi r^2 = \frac{4\pi r^3}{3} \Rightarrow r = 3$

   $\text{Volume} = \frac{4\pi}{3}(3)^3 = 36\pi = \text{surface area.}$

SSCE/JME–TYPE OBJECTIVE QUESTIONS

1. The area of an equilateral triangle of side 8 cm is

   A. $16\sqrt{3}\ cm^2$ **B.** $32\sqrt{3}\ cm^2$
   B. $48\ cm^2$ **D.** $36\sqrt{3}\ cm^2$

2. A parallelogram of area $425\ cm^2$ has a height of 17 cm. The length of the base is

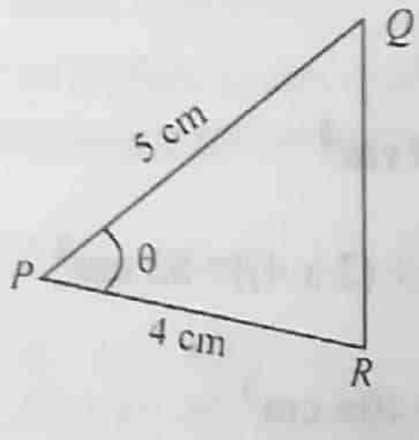
   A. 12.5 cm **B.** 20 cm
   C. 25 cm **D.** 50 cm

3. One of the sides of a rectangle measures 5 cm and the diagonal is 13 cm long. The area of the rectangle is

   A. $65\ cm^2$ **B.** $60\ cm^2$
   C. $32.5\ cm^2$ **D.** $30\ cm^2$

4. The two parallel sides of a trapezium are 4 cm and 8 cm and are separated by a distance of 5 cm. The area of the trapezium is

   A. $60\ cm^2$ **B.** $40\ cm^2$
   C. $30\ cm^2$ **D.** $20\ cm^2$

5. 

   The area of the triangle shown is $5\ cm^2$. The angle θ is

   A. 90^o **B.** 60^o
   C. 45^o **D.** 30^o

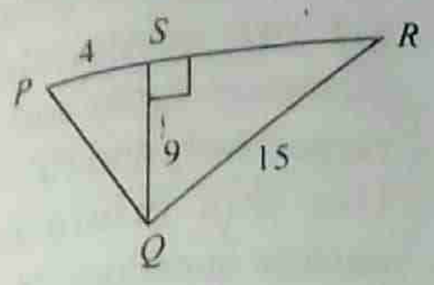
6. One of the two parallel sides of a trapezium is one-half the length of the other. If the height of the trapezium is 7 cm and its area is $52.5\ cm^2$, the lengths of the parallel sides are

   A. 10 cm, 20 cm **B.** 5 cm, 10 cm
   C. 7.5 cm, 15 cm **D.** 2.5 cm, 5 cm

7. A square tile measures 20 cm by 20 cm. How many of such tiles will cover a floor measuring 5 m by 4 m?

   A. 500 **B.** 400
   C. 320 **D.** 250

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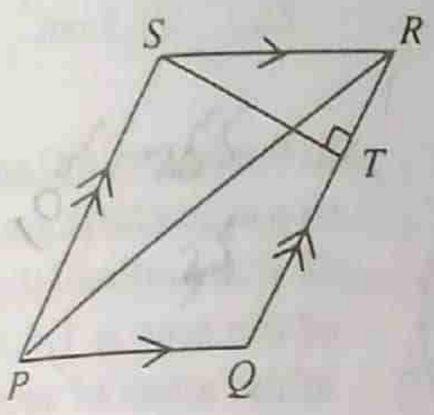
8. 

In the triangle PQR shown $\hat{QSR} = 90^\circ$, $|PS| = 4$ cm, $|SQ| = 9$ cm and $|QR| = 15$ cm. The area of the triangle is

A. 144 cm^2 **B.** 108 cm^2
C. 72 cm^2 **D.** 36 cm^2

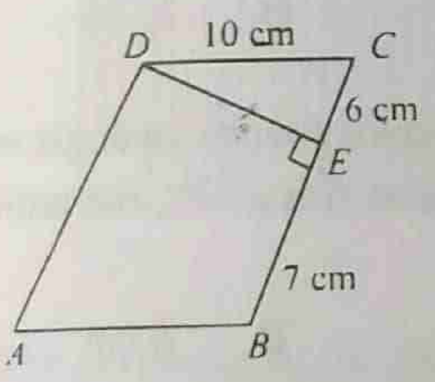
9. In a trapezium $PQRS$, $PQ//SR$, $|PQ| = 5$ cm, $|SR| = 7$ cm and the trapezium has an area of 24 cm^2. The area of the triangle PQS is

A. 16 cm^2 **B.** 14 cm^2
C. 12 cm^2 **D.** 10 cm^2

10. 

$PQRS$ is a parallelogram and ST is perpendicular to QR. If the area of ΔPSR is 25 cm^2 and $|PS| = 10$ cm, the length ST is

A. 10 cm **B.** 7.5 cm
C. 5 cm **D.** 2.5 cm

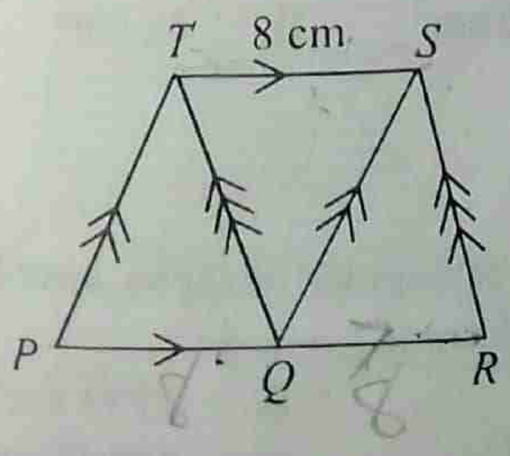
11. 

$ABCD$ is a parallelogram and DE is perpendicular to BC, $|CD| = 10$ cm, $|CE| = 6$ cm and $|BE| = 7$ cm. The area of the parallelogram is

A. 130 cm^2 **B.** 104 cm^2
C. 70 cm^2 **D.** 65 cm^2

12. One of the diagonals of a rhombus is 15 cm long. If the rhombus has an area of 60 cm^2, the length of the other diagonal is

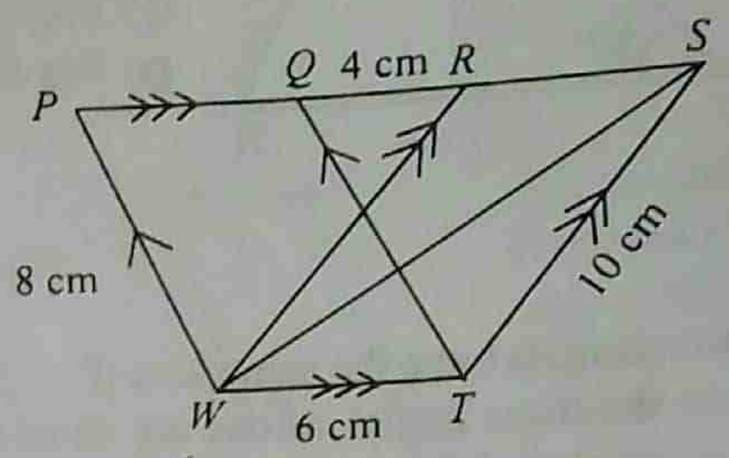
A. 12 cm **B.** 8 cm
C. 5 cm **D.** 4 cm

13. 

In the diagram $PRST$ is a trapezium, $PQ//TS$, $PT//QS$ and $QT//RS$. If $|TS| = 8$ cm and the height of the trapezium is 10 cm, its area is

A. 80 cm^2 **B.** 100 cm^2
C. 120 cm^2 **D.** 160 cm^2

In the diagram below, $|PW| = 8$ cm, $|WT| = 6$ cm, $|ST| = 10$ cm and $|QR| = 4$ cm. The area of ΔSWT is 21 cm^2. Use this information to answer questions **14** and **15.**

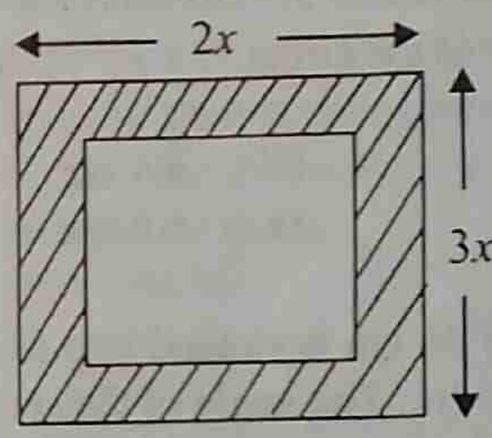


14. The area of the trapezium $PSTW$ is

A. 98 cm^2 **B.** 95 cm^2
C. 84 cm^2 **D.** 77 cm^2

15. Which of the following is **not** correct?

A. Area of $\Delta RSW = 21$ cm^2
B. Area of ΔPRW = Area of ΔQTS
C. Area of $PQTW = 42$ cm^2
D. Area of ΔPSW = Area of ΔQST

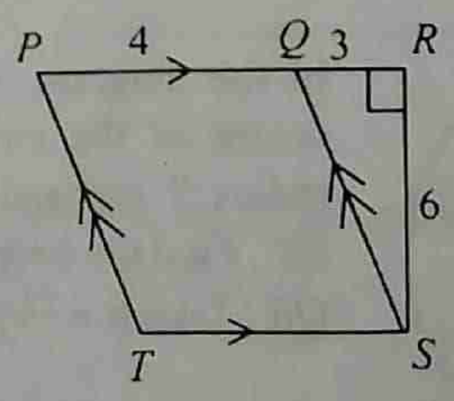
16. 

The figure shows a rectangle of dimensions $3x$ by $2x$. The shaded portion of uniform width 2 cm, has an area of 64 cm^2. Find the value of x.

A. 4 cm **B.** 6 cm
C. 7 cm **D.** 8 cm

17. A 32 cm long string just fits around the perimeter of a square board. The area of the board is

A. 32 cm^2 **B.** 64 cm^2
C. 100 cm^2 **D.** 144 cm^2

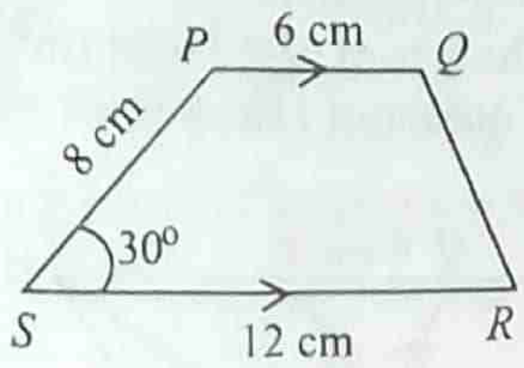
18. 

In the figure, $PQ//TS$, $TP//SQ$, $PQ = 4$ cm, $QR = 3$ cm and $RS = 6$ cm. The ratio of the area of $PQST$ to the area of $PQRST$ is

A. 8 : 11 **B.** 3 : 5
C. 2 : 3 **D.** 4 : 7

19. The area of the trapezium $PQRS$ shown in the figure below (next page) is

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A. $72\ cm^2$
B. $36\sqrt{3}\ cm^2$
C. $36\ cm^2$
D. $18\sqrt{3}\ cm^2$

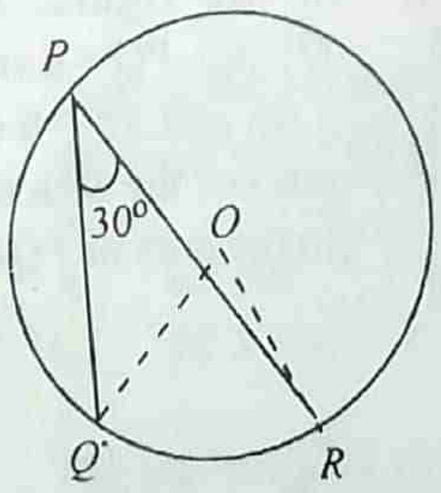
20. Two triangles have the same area if
A. the three angles of one are equal to the three angles of the other.
B. the three sides of one are equal to the three sides of the other.
C. they lie between the same parallels
D. they are similar.

21. The angle of a sector of a circle of radius 4.2 cm is 150^o. If $\pi = 22/7$, the perimeter of the sector is
A. 11 cm
B. 15.2 cm
C. 16.8 cm
D. 19.4 cm

22. An arc of a circle of radius 5.6 cm subtends an angle of 45^o at the centre of the circle. If $\pi = 22/7$, the length of the arc is
A. 2.2 cm
B. 4.4 cm
C. 6.0 cm
D. 8.8 cm

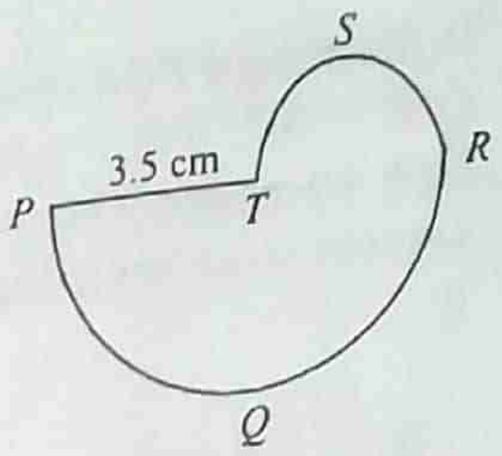
23. A string of length 66 cm is shaped into an arc of a circle of radius 28 cm. The angle of the sector is
A. 135^o
B. 120^o
C. 90^o
D. 45^o

24. The angle of a sector of diameter 12 cm is 75^o. Find the area of the sector.
A. $11\frac{5}{7}\ cm^2$
B. $18\frac{3}{7}\ cm^2$
C. $23\frac{4}{7}\ cm^2$
D. $25\frac{1}{7}\ cm^2$

25. 

In the diagram, O is the centre of the circle with radius 7 cm and $\hat{QPR} = 30^o$. Find the length of arc QR (Take $\pi = 22/7$).

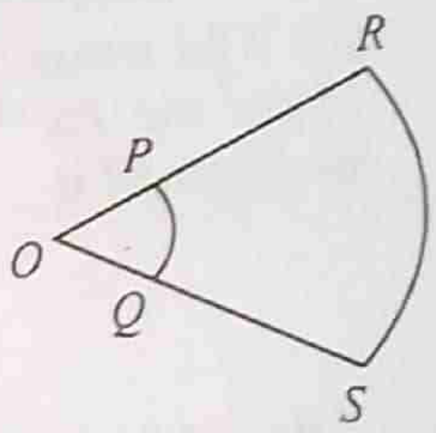
A. $3\frac{2}{3}$ cm
B. $7\frac{1}{3}$ cm
C. $10\frac{1}{2}$ cm
D. 12 cm



In the diagram, PQR and RST are two semicircles with diameters 7 cm and 3.5 cm respectively and $PT = 3.5$ cm. Use this information to answer questions **26** and **27**. (Take $\pi = 22/7$)

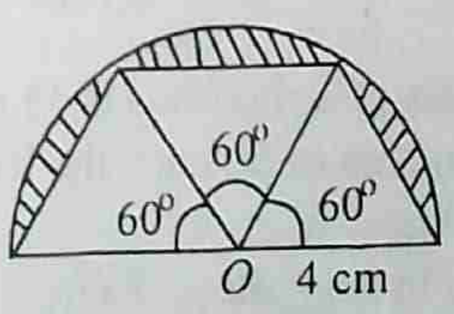
26. The area of the region $PQRST$ enclosed by the figure, to the nearest whole number, is
A. $24\ cm^2$
B. $48\ cm^2$
C. $96\ cm^2$
D. $193\ cm^2$

27. The perimeter of the figure is
A. 14 cm
B. 16.5 cm
C. 20 cm
D. 23.5 cm

28. 

In the figure, PQ and RS are two concentric arcs, centre O. If the ratio of the lengths of the arcs is 1:4, the ratio of the areas of sector OPQ to ORS is
A. 1 : 4
B. 1: 8
C. 3: 16
D. 1: 16

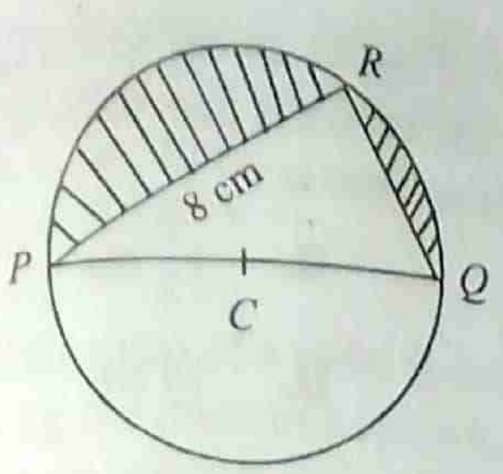
29. An arc of length 22 cm subtends an angle of 270^o at the centre of the circle. If $\pi = 22/7$, the radius of the circle is
A. $4\frac{2}{3}$ cm
B. 6 cm
C. 7 cm
D. $9\frac{1}{3}$ cm

30. 

The figure shows a semicircle of radius 4 cm. The area of the shaded portion is
A. $4(4\pi - 3\sqrt{3})\ cm^2$
B. $4(2\pi - 3\sqrt{3})\ cm^2$
C. $4(2\pi - \sqrt{3})\ cm^2$
D. $8\pi\ cm^2$

31. The area of a rectangular hexagon inscribed in a circle of radius 6 cm is
A. $9\sqrt{3}\ cm^2$
B. $27\sqrt{3}\ cm^2$
C. $54\sqrt{3}\ cm^2$
D. $108\sqrt{3}\ cm^2$

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32. 

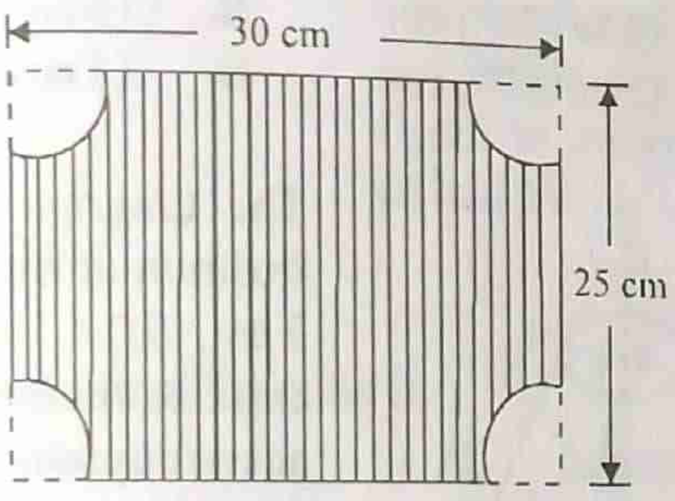
In the diagram, C is the centre of the circle of radius 5 cm, and $|PR| =$ 8 cm. The area of the shaded region is

A. $24\ cm^2$
B. $25\pi/2\ cm^2$
C. $(25\pi - 24)\ cm^2$
D. $\frac{1}{2}(25\pi - 48)\ cm^2$

33. A sector of a circle is bounded by two radii each of length 14 cm and an arc of length 12 cm. The area of the sector is

A. $168\ cm^2$
B. $84\ cm^2$
C. $63\ cm^2$
D. $42\ cm^2$

The diagram below shows a 30 cm by 25 cm rectangle. A quadrant of a circle of radius 7 cm has been cut away from each corner. Use this information to answer questions **34** and **35**.

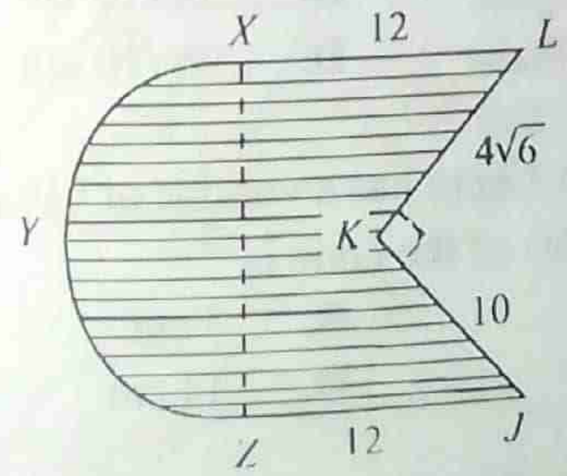


34. The perimeter of the shaded region is

A. 44 cm
B. 49 cm
C. 71 cm
D. 98 cm

35. The area of the shaded region is

A. $596\ cm^2$
B. $480\ cm^2$
C. $296\ cm^2$
D. $154\ cm^2$

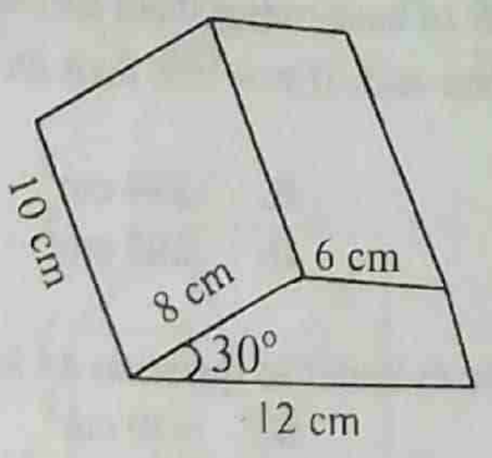
36. 

In the diagram below, XYZ is a semicircle, $XL = ZJ =$ 12 cm, $KJ =$ 10 cm and $KL = 4\sqrt{6}$ cm. If $L\hat{K}J = 90^o$, the area of the shaded region is approximately

A. $168\ cm^2$
B. $172\ cm^2$
C. $196\ cm^2$
D. $245\ cm^2$

37. A storage tank in the shape of a cuboid of base 2.5 m by 2 m can hold up to 7,500 litres of water. The height of the tank is

A. 0.15 m
B. 0.75 m
C. 1.0 m
D. 1.5 m

38. 

The diagram shows a prism which has as its cross-section a trapezium with the dimensions indicated. If the prism is 10 cm long, its volume is

A. $360\ cm^3$
B. $480\ cm^3$
C. $720\ cm^3$
D. $960\ cm^3$

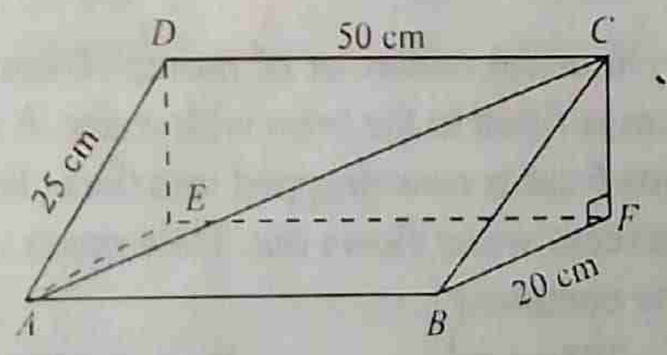
39. An open, rectangular wooden box of internal depth 40 cm has a base of internal dimensions 50 cm by 30 cm. If the box is made of wood of thickness 1 cm, the total volume of wood in the box is

A. $8{,}224\ cm^3$
B. $8{,}552\ cm^3$
C. $9{,}888\ cm^3$
D. $12{,}000\ cm^3$

40. Find the cost to the nearest naira of painting the box in question 39 *externally* if it is costs ₦100 to paint one square metre.

A. ₦855
B. ₦600
C. ₦102
D. ₦86

The figure below is a desk-top with the dimensions $AD = 25$ cm, $DC = 50$ cm and $BF = 20$ cm, $B\hat{F}C = 90^0$. Use the information to answer questions **41** and **42**.



41. The volume of the space occupied by the desktop is

A. $25{,}000\ cm^3$
B. $15{,}000\ cm^3$
C. $12{,}500\ cm^3$
D. $7{,}500\ cm^3$

42. The angle which the diagonal AC makes with the horizonal plane $ABFE$ has a tangent of

A. $\frac{4\sqrt{29}}{29}$
B. $\frac{3\sqrt{29}}{58}$
C. $\frac{3}{4}$
D. $\frac{3}{10}$

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43. The ratio of the sides of two similar rectangular blocks is 5:3. If the volume of the smaller block is 324 cm^3, that of the other block is

A. 1500 cm^3 **B.** 900 cm^3
C. 864 cm^3 **D.** 540 cm^3

44. A hollow cylinder of base radius 7 cm and height 10 cm is closed at one end. If $\pi = 22/7$, then its surface area is

A. 748 cm^2 **B.** 594 cm^2
C. 440 cm^2 **D.** 208 cm^2

45. The volume of the cylinder in question 44 is

A. 70 cm^3 **B.** 440 cm^3
C. 1540 cm^3 **D.** 3080 cm^3

46. An open cylindrical container has a base radius of 3.5 cm. If the ratio of the area of its base to that of its curved surface is 1:6, the height of the container is

A. 1.75 cm **B.** 3.5 cm
C. 7.0 cm **D.** 10.5 cm

47. A cylindrical metal pipe of wall thickness 1 cm has an internal radius of 15 cm. The volume of metal per metre of pipe is

A. 29π cm^3 **B.** 31π cm^3
C. $2{,}900\pi$ cm^3 **D.** $3{,}100\pi$ cm^3

48. The ratio of the heights of two circular cylinders is 1:3 and the ratio of their base radii is 5:4. The ratio of their volumes is

A. 25 : 48 **B.** 75 : 16
C. 45 : 4 **D.** 5: 36

49. A cylindrical container of radius 10 cm and height 20 cm is filled to the brim with water. A steel ball of radius 6 cm is now dropped into the cylinder so that the excess water flows out. How much water is left in the container?

A. 288π cm^3 **B.** $1{,}712\pi$ cm^3
C. $2{,}000\pi$ cm^3 **D.** $2{,}288\pi$ cm^3

50. The volume of a cone of radius 6 cm and vertical height 7 cm is ($\pi = 22/7$)

A. 21 cm^3 **B.** 26.4 cm^3
C. 42 cm^3 **D.** 264 cm^3

51. The total surface area of a cone of base radius 3 cm and height 4 cm is ($\pi = 22/7$)

A. $37\frac{5}{7}$ cm^2 **B.** 66 cm^2
C. $75\frac{3}{7}$ cm^2 **D.** $103\frac{5}{7}$ cm^2

52. A sector of a circle of radius 14 cm which subtends an angle of 120^o at the centre is bent to form a cone. The base radius of the cone is

A. $3\frac{1}{2}$ cm **B.** $4\frac{2}{3}$ cm
C. $9\frac{1}{3}$ cm **D.** 14 cm

53. The area of the curved surface of the cone in question 52 is

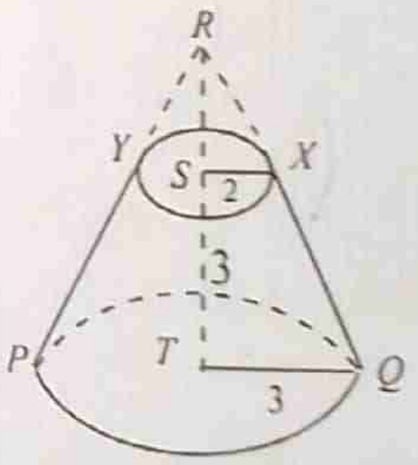
A. $\frac{196\pi}{3}$ cm^2 **B.** $\frac{92\pi}{3}$ cm^2
C. 196π cm^2 **D.** $\frac{196\pi}{9}$ cm^2

54. A cone of height 12 cm has a volume of 330 cm^3. The volume of a cone with the same radius but of height 8 cm is

A. 300 cm^3 **B.** 270 cm^3
C. 220 cm^3 **D.** $146\frac{2}{3}$ cm^3

55. A right circular cone has a radius of 5 cm and a vertical angle of 60^o. The height of the cone is

A. (5 tan 30^o) cm **B.** (5 sin 30^o) cm
C. (5 cot 30^o) cm **D.** (5 cos 30^o) cm



The diagram shows the frustrum of a cone. $|TQ| = 3$ cm, $|ST| = 3$ cm, $|SX| = 2$ cm. Use this information to answer questions **56** and **57**.

56. The height of the cone is

A. 9 cm **B.** 6 cm
C. 5 cm **D.** 3 cm

57. The curved surface area of the frustrum is

A. $4\pi\sqrt{10}$ cm^2 **B.** $5\pi\sqrt{10}$ cm^2
C. $6\pi\sqrt{10}$ cm^2 **D.** $9\pi\sqrt{10}$ cm^2

58. A cone of height 12 cm has a volume of 616 cm^3. If $\pi = 22/7$, the radius of the cone is

A. 3.5 cm **B.** 7 cm
C. 10.5 cm **D.** 14 cm

59. The surface area of a sphere of radius 3.5 cm is ($\pi = 22/7$)

A. 616 cm^2 **B.** 308 cm^2
C. $179\frac{2}{3}$ cm^2 **D.** 154 cm^2

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60. The volumes of two solid spheres are 32 cm^3 and 500 cm^3. The ratio of their radii is

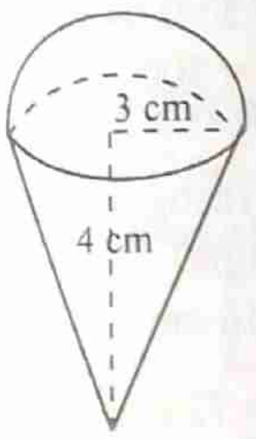
A. 8 : 125 B. 4 : 25
C. 3 : 5 D. 2 : 5

61. If the mass of a solid metal sphere of radius 3 cm is 9 kg, the mass of a spherical shell of the same metal whose internal and external radii are 2 cm and 3 cm respectively is

A. $4\frac{1}{2}$ kg B. 6 kg
C. $6\frac{1}{3}$ kg D. $7\frac{1}{2}$ kg

62. The volume of a certain sphere is numerically equal to twice its surface area. The diameter of the sphere is

A. 6 B. 9
C. 12 D. $\sqrt{6}$



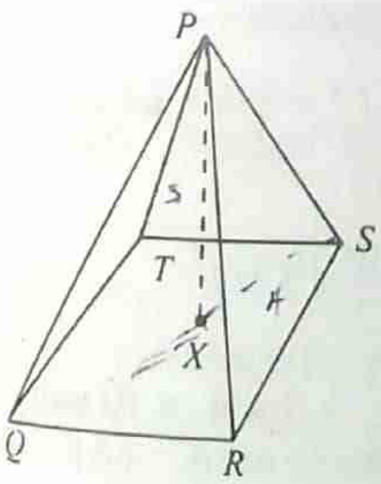
The figure shows a solid body consisting of a hemisphere mounted on a circular cone of radius 3 cm and height 4 cm. Use the figure to answer questions **63** and **64**.

63. The volume of the solid is

A. 30π cm^3 B. 24π cm^3
C. 18π cm^3 D. 12π cm^3

64. The surface area of the solid is

A. 51π cm^2 B. 33π cm^2
C. 30π cm^2 D. 18π cm^2



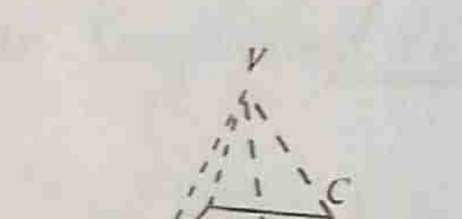
The figure shows a square pyramid in which the square base has a side of length 4 cm. Each of the slant sides is 5 cm long. Use this information to answer questions **65** and **66**.

65. The height PX of the pyramid is

A. $\sqrt{17}$ cm B. $\sqrt{34}$ cm
C. 4 cm D. $3\sqrt{2}$ cm

66. The volume of the pyramid is, to the nearest whole number,

A. 21 cm^3 B. 22 cm^3
C. 23 cm^3 D. 31 cm^3

67. 

A room has a cubic shape as shown in the figure. The cosine of the angle which the diagonal QM makes with the floor is

A. $\frac{1}{\sqrt{3}}$ B. $\frac{1}{\sqrt{2}}$
C. $\frac{\sqrt{3}}{2}$ D. $\sqrt{\frac{2}{3}}$

68. 

The figure shows a pyramid with a square base $EFGH$ of side 4 cm and height 12 cm. The pyramid is sliced into two by a plane $ABCD$ such that the height of pyramid $VABCD$ is 6 cm. The volume of the truncated section $ABCDHEFG$ is

A. 32 cm^3 B. 48 cm^3
C. 56 cm^3 D. 64 cm^3

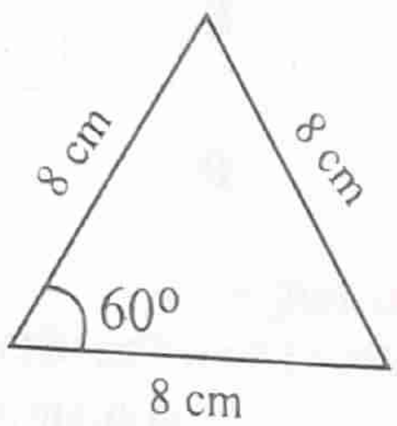
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ANSWER KEY

1.	A	13.	C	25.	B	37.	D	49.	B	61.	C
2.	C	14.	D	26.	A	38.	A	50.	D	62.	C
3.	B	15.	D	27.	B	39.	A	51.	C	63.	A
4.	C	16.	A	28.	D	40.	D	52.	B	64.	B
5.	D	17.	B	29.	A	41.	D	53.	A	65.	A
6.	B	18.	A	30.	B	42.	B	54.	C	66.	B
7.	A	19.	C	31.	C	43.	A	55.	C	67.	D
8.	C	20.	B	32.	D	44.	B	56.	A	68.	C
9.	D	21.	D	33.	B	45.	C	57.	B		
10.	C	22.	B	34.	D	46.	D	58.	B		
11.	B	23.	A	35.	A	47.	D	59.	D		
12.	B	24.	C	36.	C	48.	A	60.	D		

SOLUTIONS TO OBJECTIVE QUESTIONS

1. **(A)**



Each side of the equilateral Δ is 8 cm and each angle is 60^o

$$\text{Area} = \frac{1}{2}ab\sin\theta = \frac{1}{2}(8)(8)\sin 60^o$$

$$= 32\left(\frac{\sqrt{3}}{2}\right) = 16\sqrt{3}\ \text{cm}^2$$

2. **(C)** Area of parallelogram = base x height

$\therefore$ 425 = base x 17, or

base = 425/17 = 25 cm

3. **(B)** The other side of the rectangle is $\sqrt{13^2 - 5^2}$ = 12 cm.

Area of the rectangle = 5 x 12 = 60 cm^2.

4. **(C)** Area of trapezium $= \frac{1}{2}(4 + 8) \times 5$

$= 30\ \text{cm}^2$

5. **(D)** Area $= \frac{1}{2}(5)(4)\sin\theta$

$\therefore\ 5 = 10\sin\theta$

$\sin\theta = \frac{1}{2}$, or $\theta = 30^o$

6. **(B)** Let the sides be x and $2x$

$\frac{1}{2}(x + 2x) \times 7 = 52.5$

$3x = 15$,

$x = 5$ cm and $2x = 10$ cm.

7. **(A)** Area of floor = 5 x 4 = 20 m^2

Area of each tile = 0.2 x 0.2 = 0.04 m^2

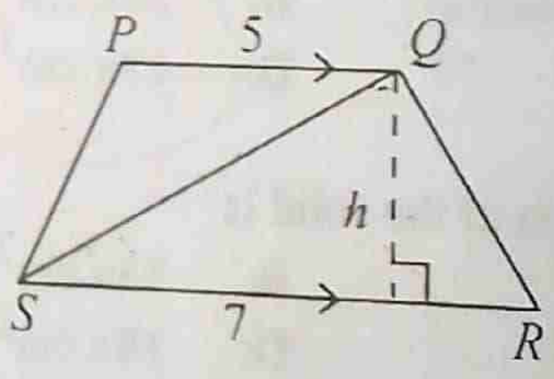
Number of tiles = 20/0.04 = 500

8. **(C)** From ΔSQR, SR = 12 cm (recognize as a 3 – 4 – 5 right-angled triangle).

$PR = PS + SR = 4 + 12 = 16$ cm

Area of $PQR = \frac{1}{2}(16)(9) = 72\ \text{cm}^2$

9. **(D)**



If h = height of the trapezium

Area of trapezium $= \frac{1}{2}(7 + 5)h = 6h\ \text{cm}^2$

$6h = 24$, or $h = 4$ cm

Area of $\Delta PSR = \frac{1}{2} \times 5 \times h$

$= \frac{1}{2} \times 5 \times 4 = 10\ \text{cm}^2$.

10. **(C)** Area of $\Delta PSR = \frac{1}{2}$ x PS x ST

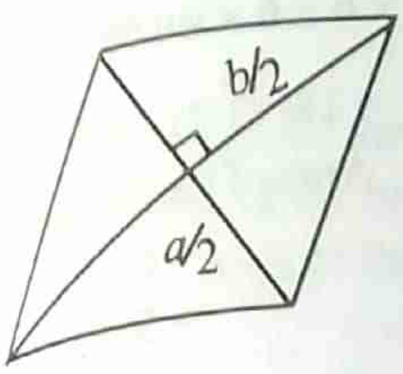
$25 = \frac{1}{2}$ x 10 x ST, or

ST = 5 cm

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11. **(B)** ΔDCE is a $3-4-5\ \Delta \Rightarrow DE = 8$ cm
Area of $ABCD = BC \times DE = 13 \times 8$
$= 104\ \text{cm}^2$

12. **(B)** Let the lengths of the diagonals be a and b. The diagonals of a rhombus bisect each other perpendicularly, hence the rhombus consists of four right angled triangles each of height $\frac{a}{2}$ and base $\frac{b}{2}$.



Area of each triangle
$= \frac{1}{2}\left(\frac{a}{2}\right)\left(\frac{b}{2}\right) = \frac{ab}{8}$

Area of rhombus $= \frac{ab}{8} \times 4 = \frac{ab}{2}$

i.e. *the area of a rhombus is equal to half the product of the diagonals*. In this case

$60 = \frac{1}{2}(15)(a)$, or $a = 8$ cm.

13. **(C)** $PQST$ and $QRST$ are parallelograms
$\therefore PQ = QR = 8$ cm, and $PR = 16$ cm
Area of trapezium $PRST$
$= \frac{1}{2}(8 + 16) \times 10 = 120\ \text{cm}^2$

14. **(D)** $PQ = WT = 6\ \text{cm} = RS$
$PS = PQ + QR + RS = 6 + 4 + 6 = 16$ cm
Area of $SWT = 21\ \text{cm}^2 = \frac{1}{2} \times 6 \times h$,
where h = height of the trapezium,
Hence $h = 7$ cm
Area of trapezium $PSTW$
$= \frac{1}{2}(WT + PS) \times h$
$= \frac{1}{2}(6 + 16) \times 7 = 77\ \text{cm}^2$

15. **(D)** Area of $\Delta PSW = \frac{1}{2} \times PS \times h$

Area of $\Delta QST = \frac{1}{2} \times QS \times h$

Since $PS \neq QS$, $\Delta PSW \neq \Delta QST$

16. **(A)** Area of outer rectangle $= 2x \times 3x = 6x^2$
Area of inner rectangle $= (2x - 4)(3x - 4)$
$= 6x^2 - 20x + 16$

Shaded area $= 6x^2 - (6x^2 - 20x + 16)$
$= 20x - 16$
$20x - 16 = 64$, or $x = 4$ cm.

17. **(B)** Let x = size of the board
Perimeter $= 4x = 32$ cm, or $x = 8$ cm
Area of board $= x^2 = 8^2 = 64\ \text{cm}^2$.

18. **(A)** Area of $PQST = PQ \times RS = 4 \times 6$
$= 24\ \text{cm}^2$

Area of $\Delta QRS = \frac{1}{2} \times QR \times RS$
$= \frac{1}{2} \times 3 \times 6 = 9\ \text{cm}^2$

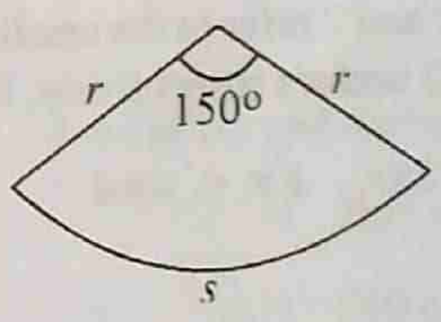
Area of $PQRST = 24 + 9 = 33\ \text{cm}^2$
$\therefore PQST : PQRST = 24 : 33 = 8 : 11$

19. **(C)** Height of trapezium = perpendicular distance from P to SR
$= PS \sin 30^\circ = 8 \sin 30^\circ$
$= 4$ cm

Area of trapezium $= \frac{1}{2}(6 + 12) \times 4$
$= 36\ \text{cm}^2$

20. **(B)** Two triangles have the same area if they are congruent, which is true if all the sides are equal, as stated in (B). Note that (A) implies that the triangles are similar, but not necessarily equal in area. In (C) it is not stated that the triangles have the *same base*.

21. **(D)** Length of arc, $s = \frac{150^\circ}{360^\circ} \times 2\pi r$
$= \frac{5}{12} \times 2 \times \frac{22}{7} \times 4.2$
$= 11$ cm



Perimeter of sector
$= s + 2r$
$= 11 + 4.2 + 4.2$
$= 19.4$ cm

22. **(B)** Length of arc $= \frac{\theta^\circ}{360^\circ} \times 2\pi r$
$= \frac{45^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 5.6$
$= 4.4$ cm.

23. **(A)** Length of arc= length of string
$s = 66 = \frac{\theta^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 28$ cm, or
$\theta = \frac{360^\circ \times 66 \times 7}{2 \times 22 \times 28} = 135^\circ$

24. **(C)** Radius of sector, $r = 6$ cm
Area of sector $= \frac{75^\circ}{360^\circ} \times \pi r^2$
$= \frac{75}{360} \times \frac{22}{7} \times 36$
$= 23\frac{4}{7}\ \text{cm}^2$

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25. **(B)** $Q\hat{O}R = 2(Q\hat{P}R) = 60^\circ$

Length of arc $QR = \frac{60^\circ}{360^\circ} \times 2\pi r$

$= \frac{1}{6} \times 2 \times \frac{22}{7} \times 7$

$= 7\frac{1}{3}$ cm

26. **(A)** Area of semicircle PQR

$= \frac{\pi r^2}{2} = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

$= 19.25 \text{ cm}^2$

Area of semicircle $TSR = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{4}\right)^2$

$= 4.81 \text{ cm}^2$

Area of $PQRST = 19.25 + 4.81$

$= 24.06 \text{ cm}^2$, or 24 cm^2

to the nearest whole number.

27. **(B)** Perimeter of $PQR = \pi r = \frac{22}{7} \times \frac{7}{2} = 11$ cm

Perimeter of $TSR = \frac{22}{7} \times \frac{7}{4} = 5.5$ cm

Perimeter of the figure $= 11 + 5.5$

$= 16.5$ cm

28. **(D)** Let subscripts 1 and 2 refer to the small (OPQ) and large (ORS) sectors respectively, Since

$s = \frac{\theta^\circ}{360^\circ} \times 2\pi r$, $s \propto r$ and

$\frac{r_1}{r_2} = \frac{s_1}{s_2} = \frac{1}{4}$

Area of a sector is directly proportional to r^2.

$\therefore \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{16}$

29. **(A)** $s = \frac{\theta^\circ}{360^\circ} \times 2\pi r$

$22 = \frac{270^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times r$

$r = 14/3 = 4\frac{2}{3}$ cm.

30. **(B)** Area of the semi-circle is $\frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 4^2 = 8\pi \text{ cm}^2$

The unshaded area consists of three equilateral triangles each of area

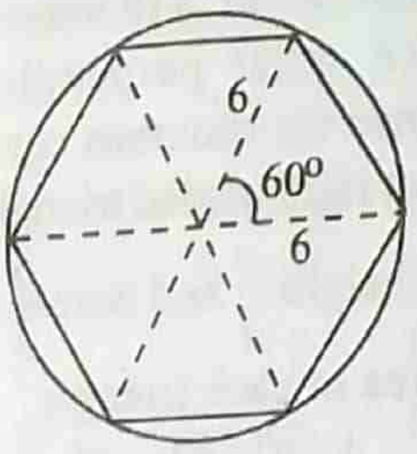
$\frac{1}{2} \times 4 \times 4 \times \sin 60^\circ = 4\sqrt{3} \text{ cm}^2$

Area of the three Δ s $= 3 \times 4\sqrt{3} = 12\sqrt{3} \text{ cm}^2$

Area of shaded region

$= 8\pi - 12\sqrt{3} = 4(2\pi - 3\sqrt{3}) \text{ cm}^2$

31. **(C)** Referring to the diagram, the hexagon consists of six equilateral triangles each of area



$\frac{1}{2} \times 6 \times 6 \times \sin 60^\circ$

$= 18(\sqrt{3}/2) = 9\sqrt{3} \text{ cm}^2$

$\therefore$ Area of the hexagon

$= 6 \times 9\sqrt{3}$

$= 54\sqrt{3} \text{ cm}^2$

32. **(D)** Diameter $= PQ = 10$ cm and $P\hat{R}Q = 90^\circ$

Hence $QR = 6$ cm (PRQ is a $3 - 4 - 5$ Δ)

Area of $\Delta PQR = \frac{1}{2} \times QR \times PR$

$= \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$

Area of semicircle $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 5^2 = 25\pi/2 \text{ cm}^2$

Area of shaded region $= \frac{25\pi}{2} - 24$

$= \frac{1}{2}(25\pi - 48) \text{ cm}^2$

33. **(B)** Length of arc, $s = \frac{\theta^\circ}{360^\circ} \times 2\pi r$ ...(i)

Area of sector, $A = \frac{\theta^\circ}{360^\circ} \times \pi r^2$ ...(ii)

Dividing (ii) by (i) gives

$\frac{A}{s} = \frac{r}{2}$, or $A = \frac{s \times r}{2}$

i.e. *the area of a sector of a circle is one-half the product of the radius and the length of the arc.* In this problem, $s = 12$ cm and $r = 14$ cm

$\therefore A = \frac{12 \times 14}{2} = 84 \text{ cm}^2$

34. **(D)** Each of the curved segments is an arc of a circle of radius 7 cm which subtends an angle of 90° at the centre. Hence the length of each segment

$= \frac{90^\circ}{360^\circ} \times 2\pi r$

$= \frac{1}{4} \times 2 \times \frac{22}{7} \times 7 = 11$ cm

Total length of the four curved segments

$= 44$ cm.

Length of the straight sides

$= 2(30 - 14) + 2(25 - 14)$

$= 54$ cm

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Perimeter of the shaded region
$= 44 + 54 = 98$ cm.

35. (A) Area of each of the four quadrants
$$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 7^2$$
$$= \frac{154}{4} \text{ cm}^2$$
$\therefore$ Area of the four quadrants $= 154$ cm^2
Area of the rectangle $= 30 \times 25 = 750$ cm^2
$\therefore$ Area of the shaded region $= 750 - 154$
$= 596$ cm^2.

36. (C) Join L to J : $JL^2 = 10^2 + (4\sqrt{6})^2 = 196$,
or $JL = 14$ cm.
Shaded region = semi-circle XYZ +
rectangle $XZJL$ – triangle LKJ.
Radius of semi-circle $XYZ = XZ/2 = JL/2$
$= 14/2 = 7$ cm
Area of semicircle $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7^2$
$= 77$ cm^2
Area of rectangle $XZJL = 12 \times 14$
$= 168$ cm^2
Area of $\Delta LKJ = \frac{1}{2} \times 10 \times 4\sqrt{6} = 20\sqrt{6}$ cm^2
Area of the shaded region $= 77 + 168 - 20\sqrt{6}$
$= 196$ cm^2

37. (D) Let h = height of the tank
Volume of the tank $= 2.5 \times 2 \times h = 5h$ m^3
This is equal to 7,500 litres
$= 7.5 \times 10^6$ cm^3 $= 7.5$ m^3
[1 litre $= 10^3$ cm^3 $= 10^{-3}$ m^3]
Hence $5h = 7.5$, or $h = 1.5$ m

38. (A) Height of trapezium $= 8 \sin 30^\circ = 4$ cm
Area of trapezium $= \frac{1}{2}(6 + 12) \times 4$
$= 36$ cm^2
Volume of the prism $= 36 \times 10 = 360$ cm^3

39. (A) External dimensions of the box are 52 cm by 32 cm by 41 cm. Total volume of box and wooden walls $= 52 \times 32 \times 41$
$= 68{,}224$ cm^3
Volume of space inside box
$= 50 \times 30 \times 40 = 60{,}000$ cm^3
Volume of wood $= (68{,}224 - 60{,}000)$ cm^3
$= 8{,}224$ cm^3

40. (D) Area of the base $= 52 \times 32 = 1{,}664$ cm^2
Area of two 52 by 41 surfaces
$= 2 \times 52 \times 41 = 4{,}264$ cm^2
Area of two 32 by 41 surfaces
$= 2 \times 32 \times 41 = 2{,}624$ cm^2
Total area of external surface
$= 1{,}664 + 4{,}264 + 2{,}624$
$= 8{,}552$ cm^2 $= 0.8552$ m^2
Cost of painting $= 0.8552 \times 100$
$= ₦85.52 = ₦86$, to the nearest naira.

41. (D) $BC = AD = 25$ cm and $CF = 15$ cm
(BCF is a $3 - 4 - 5$ Δ)
Area of $\Delta BCF = \frac{1}{2} \times 20 \times 15 = 150$ cm^2
Volume of desk top
= area of $\Delta BCF \times$ length
$= 150 \times 50 = 7{,}500$ cm^3

42. (B) Join A to F.
$AF^2 = AB^2 + BF^2 = 50^2 + 20^2 = 2900$
or $AF = 10\sqrt{29}$ cm
Angle required $= \hat{CAF}$
$\tan \hat{CAF} = \frac{CF}{AF} = \frac{15}{10\sqrt{29}} = \frac{3}{2\sqrt{29}} = \frac{3\sqrt{29}}{58}$

43. (A) Volume varies directly as the cube of the side.
hence volume of larger block
$= 324\left(\frac{5}{3}\right)^3 = 1{,}500$ cm^3

44. (B) Area of curved surface
$= 2\pi rl = 2 \times \frac{22}{7} \times 7 \times 10$
$= 440$ cm^2
Area of one end surface $= \pi r^2$
$= \frac{22}{7} \times 7^2 = 154$ cm^2
Total area of surface $= 440 + 154$
$= 594$ cm^2

45. (C) Volume $= \pi r^2 l = \frac{22}{7} \times 7^2 \times 10 = 1{,}540$ cm^3

46. (D) Let h = height of cylinder.
Area of base $= \pi r^2 = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{77}{2}$ cm^2
Area of surved surface
$= 2\pi rh = 2 \times \frac{22}{7} \times \frac{7}{2} \times h$
$= 22h$ cm^2
$\therefore \frac{77/2}{22h} = \frac{1}{6}$, or $h = \frac{77 \times 3}{22} = 10.5$ cm

47. (D) External radius of pipe $= 15 + 1 = 16$ cm
External voluime of pipe for 1 m length

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$= \pi r^2 l = \pi(16)^2\,(100)$

$= 25{,}600\pi \text{ cm}^3$

Internal volume of pipe $= \pi(15)^2(100)$

$= 22{,}500\pi \text{ cm}^3$

Volume of metal $= 25{,}600\,\pi - 22{,}500\pi$

$= 3{,}100\pi \text{ cm}^3$

48. **(A)** Let h_1, h_2 be the heights and r_1, r_2 the radii of the two cylinders

V_1 = Volume of $1 = \pi r_1^2 h_1$

V_2 = Volume of $2 = \pi r_2^2 h_2$

$$\frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{h_1}{h_2}\right)$$

$$= \left(\frac{5}{4}\right)^2 \left(\frac{1}{3}\right) = \frac{25}{48}$$

49. **(B)** Original volume of water = volume of cylindrical container

$= \pi r_1^2 h = \pi(10)^2\,(20)$

$= 2{,}000\,\pi \text{ cm}^3$

Volume of steel ball $= \frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi(6)^3$

$= 288\pi \text{ cm}^3$

= volume of water which overflows.

Volume of water left $= 2000\pi - 288\pi$

$= 1{,}712\pi \text{ cm}^3$

50. **(D)** Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 6^2 \times 7 = 264 \text{ cm}^3$

51. **(C)** Area of base $= \pi r^2 = \frac{22}{7}(3)^2 = 28\frac{2}{7} \text{ cm}^2$

Area of curved surface $= \pi r l = \frac{22}{7} \times 3 \times l$,

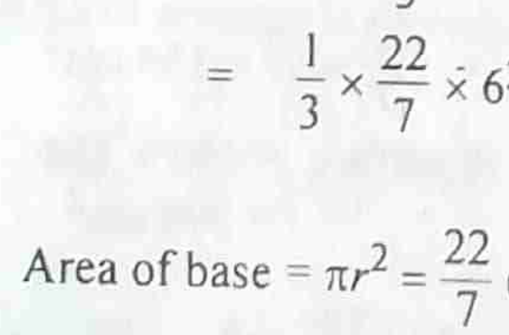
where $l = \sqrt{r^2 + h^2} = \sqrt{3^2 + 4^2} = 5$ cm

$\therefore$ Area of curved surface $= \frac{22}{7} \times 3 \times 5$

$= 47\frac{1}{7} \text{ cm}^2$

Total surface area $= 28\frac{2}{7} + 47\frac{1}{7} = 75\frac{3}{7} \text{ cm}^2$

52. **(B)**



When OPQ is bent to form a cone, the circumference of the circular base of the cone is PQ.

$|PQ| = \frac{120^\circ}{360^\circ} \times 2\pi\,(14) = \frac{28\pi}{3}$ cm

$\frac{28\pi}{3} = 2\pi R$, where R radius of cone,

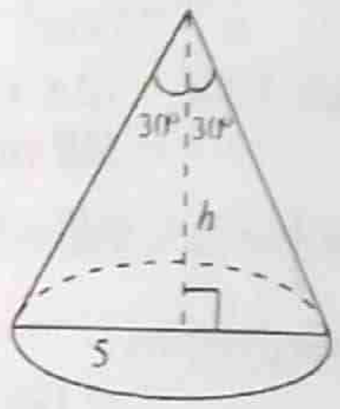
or $R = \frac{14}{3} = 4\frac{2}{3}$ cm.

53. **(A)** Area of curved surface of cone $= \pi R l$ where l = slant side = radius of the original sector = 14 cm.

$\therefore$ Area $= \pi\left(\frac{14}{3}\right)(14) = \frac{196\pi}{3} \text{ cm}^2$

54. **(C)** Since $V = \frac{1}{3}\pi r^2 h$, the volume of a cone is directly proportional to the height. Hence volume required is $(330)\left(\frac{8}{12}\right) = 220 \text{ cm}^3$.

55. **(C)** From the diagram,



$\frac{h}{5} = \cot 30^\circ$, or

$h = 5 \cot 30^\circ$

56. **(A)** Since $SX // TQ$,

$$\frac{SX}{TQ} = \frac{RS}{RT}$$

$$\frac{2}{3} = \frac{RS}{RS + 3}$$

$2RS + 6 = 3RS$, or $RS = 6$ cm

Height of the cone $= RT = 6 + 3 = 9$ cm.

57. **(B)** Curved surface area of a cone $= \pi r l$, where l = slant side.

For the larger cone (RPQ), $l = \sqrt{9^2 + 3^2}$

$= 3\sqrt{10}$ cm

Curved surface area $= \pi(3)(3\sqrt{10})$

$= 9\pi\sqrt{10} \text{ cm}^2$

For the smaller cone (RYX),

$l = \sqrt{6^2 + 2^2} = 2\sqrt{10}$ cm

Curved surface area $= \pi(2)\,(2\sqrt{10})$

$= 4\pi\sqrt{10} \text{ cm}^2$

Curved surface area of frustrum

$= 9\pi\sqrt{10} - 4\pi\sqrt{10} \text{ cm}^2$

$= 5\pi\sqrt{10} \text{ cm}^2$

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58. (B) Volume $= 616 = \frac{1}{3}\pi r^2(12)$,

$$r^2 = \frac{3 \times 616 \times 7}{12 \times 22} = 49, \text{ or } r = 7 \text{ cm}$$

59. (D) Surface area of sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = 154 \text{ cm}^2$$

60. (D) Since $V = \frac{4}{3}\pi r^3$ for a sphere

$$\frac{V_1}{V_2} = \frac{r_1^3}{r_2^3} = \left(\frac{r_1}{r_2}\right)^3$$

$$\therefore \frac{r_1}{r_2} = \left(\frac{V_1}{V_2}\right)^{1/3} = \left(\frac{32}{500}\right)^{1/3}$$

$$= \left(\frac{8}{125}\right)^{1/3} = \frac{2}{5}$$

61. (C) Volume of solid sphere

$$= \frac{4}{3}\pi(3)^3 = 36\pi \text{ cm}^3$$

Volume of spherical shell

$$= \frac{4}{3}\pi(3^3 - 2^3) = \frac{76\pi}{3} \text{ cm}^3$$

The mass varies directly as the volume, hence mass of spherical shell

$$= \frac{76\pi}{3 \times 36\pi} \times 9 = 6\frac{1}{3} \text{ kg}$$

62. (C) Let R = radius of the sphere.

Volume $= \frac{4\pi R^3}{3}$, and

surface area $= 4\pi R^2$

Hence $\frac{4\pi R^3}{3} = 2(4\pi R^2)$, or $R = 6$

Diameter $= 2R = 12$.

63. (A) Volume of hemispherical top

$$= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(3)^3 = 18\pi \text{ cm}^3$$

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2(4)$

$$= 12\pi \text{ cm}^3$$

Total volume of solid $= 18\pi + 12\pi$

$$= 30\pi \text{ cm}^3$$

64. (B) Slant side of cone $= l = \sqrt{4^2 + 3^2} = 5\text{cm}$

Surface area of cone $= \pi r l = \pi(3)(5) = 15\pi \text{ cm}^2$

Surface area of hemis. top $= 2\pi r^2 = 18\pi \text{ cm}^2$

Total surface area of solid $= 15\pi + 18\pi$

$$= 33\pi \text{ cm}^2$$

65. (A) Each of the angles of square $QRST$ is 90^o,

Join T to R

$TR^2 = ST^2 + SR^2 = 4^2 + 4^2 = 32$

or $TR = 4\sqrt{2}$ cm

$XR = TR/2 = 2\sqrt{2}$ cm

In ΔPXR, $P\hat{X}R = 90^o$

$\therefore PX^2 = PR^2 - XR^2 = 5^2 - (2\sqrt{2})^2 = 17$,

or $PX = \sqrt{17}$ cm

66. (B) Vol. of pyramid $= \frac{1}{3}$ x base area x height

$$= \frac{1}{3}(4^2)(\sqrt{17}) = 22 \text{ cm}^3, \text{ approx.}$$

67. (D) Join S to Q. Let each side of cube $= a$

$PQRS$ is a square, hence $SQ = a\sqrt{2}$

(Recognize ΔSQR as a $1 - 1 - \sqrt{2}$ $\hat{\Delta}$).

Triangle SQM is a right Δ with $Q\hat{S}M = 90^o$

Hence $QM^2 = SQ^2 + SM^2$

$$= (a\sqrt{2})^2 + a^2 = 3a^2$$

or $QM = a\sqrt{3}$

$$\cos S\hat{Q}M = \frac{SQ}{QM} = \frac{a\sqrt{2}}{a\sqrt{3}} = \sqrt{\frac{2}{3}}$$

68. (C) $\frac{AB}{EF} = \frac{\text{height of small pyramid}}{\text{height of large pyramid}} = \frac{6}{12} = \frac{1}{2}$

Hence AB $= 4/2 = 2$ cm

Volume of small pyramid

$$= \frac{1}{3}(2^2)(6) = 8 \text{ cm}^3$$

Volume of large pyramid

$$= \frac{1}{3}(4^2)(12) = 64 \text{ cm}^3$$

Volume of truncated section

$$= 64 - 8 = 56 \text{ cm}^3$$

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17 Bearings, Latitude and Longitude

17.1 Bearings

- The **bearing** of a point B from a point A is the angle (θ_B), measured in a clockwise direction, which the direction AB makes with the northern direction at A.
- In Fig. 17.1(a)
  (i) the bearing of B from A is θ_B.
  (ii) the bearing of A from B is θ_A.
- Bearing is sometimes specified relative to the North, South, East and West directions, e.g. the bearing of Q from P is $S\ \alpha^\circ$ E while that of P from Q is N α° W (Fig. 17.1(b)).

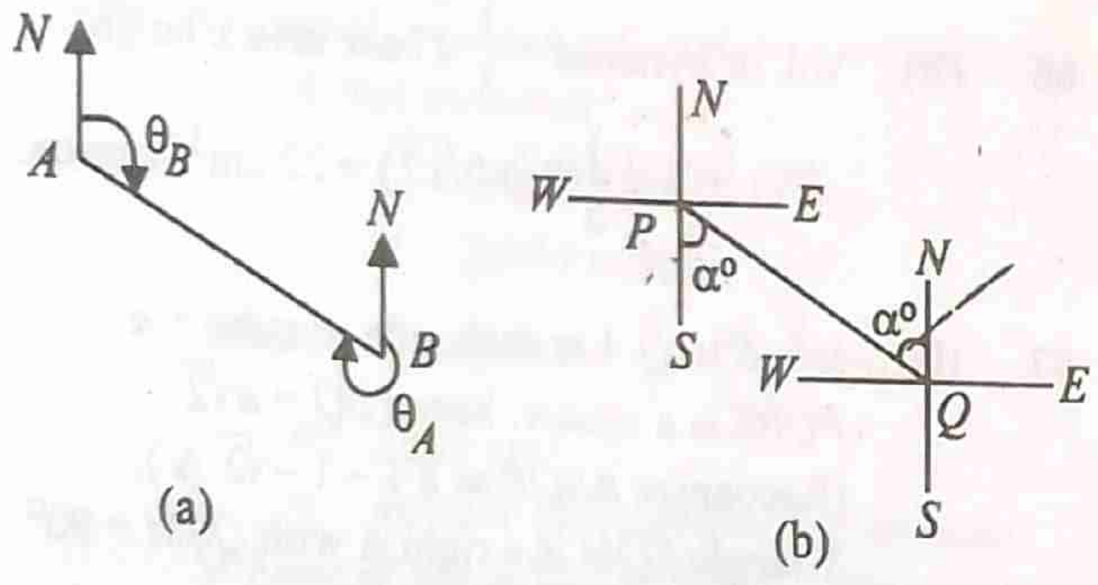


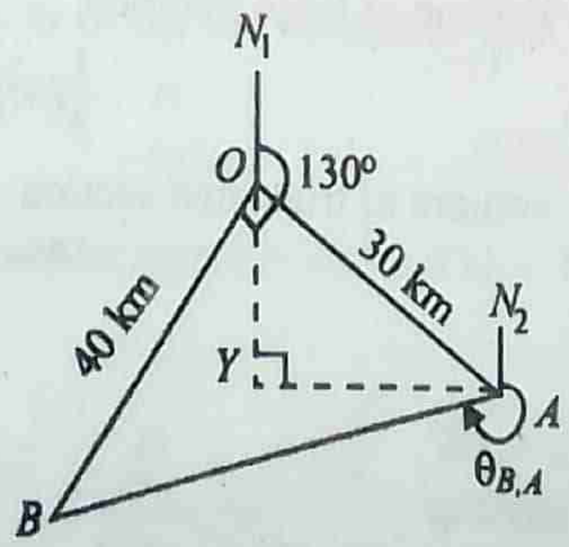
Fig. 17.1

EXAMPLE 17–1

The bearings of two points A and B from a reference point O are 130° and 220° respectively. Determine

(a) the distance AB if $|OA|$ =30 km and $|OB|$ = 40 km.
(b) The bearing of B from A.

Solutions



(a) Referring to the figure $N_1\hat{O}A = 130^\circ$ and reflex $N_1\hat{O}B = 220^\circ$.

$A\hat{O}B = 220^\circ - 130^\circ = 90^\circ$

Hence OAB is a right-angled triangle.

Pythagoras: $AB^2 = OA^2 + OB^2 = 30^2 + 40^2$
$= 2500$

$AB =$ **50 km**

(b) Construct a line YA which is perpendicular to N_1O and which meets N_1O produced at Y.

$Y\hat{O}A = 180^\circ - 130^\circ = 50^\circ$

$Y\hat{A}O = 90^\circ - Y\hat{O}A = 90^\circ - 50^\circ = 40^\circ$

$\tan O\hat{A}B = 40/30 = 1.333$, or $O\hat{A}B = 53.1^\circ$

$Y\hat{A}B = O\hat{A}B - Y\hat{A}O = 53.1^\circ - 40^\circ = 13.1^\circ$

Hence bearing of B from A is

$\theta_{B,A} = 270^\circ - Y\hat{A}B = 270^\circ - 13.1^\circ =$ **256.9°**

DRILL 1

(a) In Example 17–1, find the bearing of
(i) O from A (ii) O from B.

(b) The bearing of a point Y from X is 050°. Specify the bearing of X from Y in the two different conventions.

(c) A girl starting from a point O walks 60 m due South and then 45 m due West. What is her bearing from O?

17.2 Geometry of the Earth's Surface

- The earth is roughly spherical in shape with a radius of about 6,370 km (Fig. 17.2).

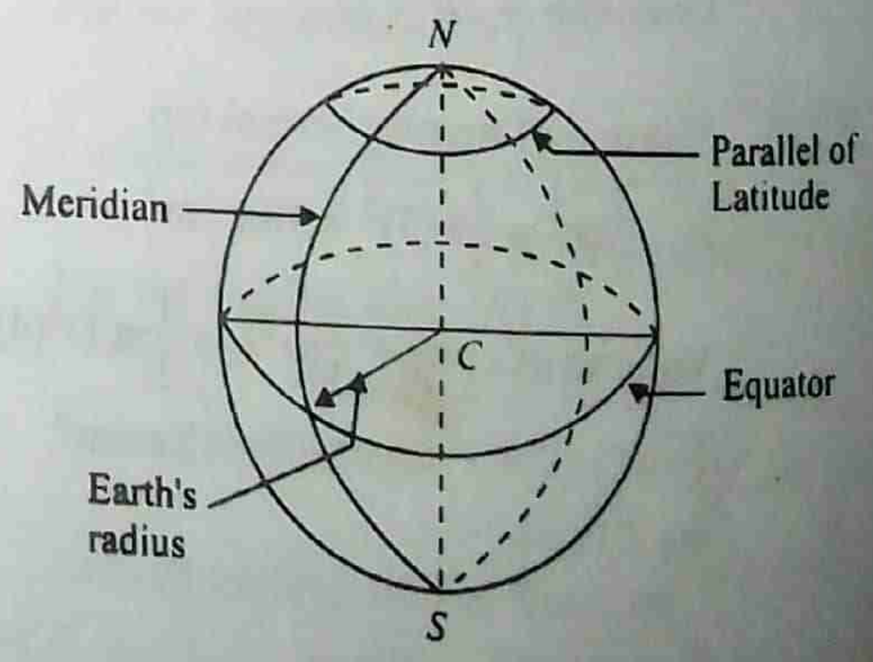


Fig. 17.2

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- It rotates about the north-south axis, the north end of the axis being the **north pole** (N) while the south end is the **south pole** (S).
- The **equator** is a circle formed by the intersection of the earth by a plane which is perpendicular to the N-S axis and which passes through the centre (C) of the earth.
- Other planes, parallel to the equator, intersecting the earth, form circles which are called **parallels of latitude.**
- A **great circle** is a circle formed by a plane which passes through the centre of the earth, e.g. the equator.
- A **small circle** is a circle formed by a plane which does not pass through the centre of the earth, e.g. a parallel of latitude.
- The closer a parallel of latitude is to a pole (or the farther from the centre of the earth), the smaller is its diameter.
- The **meridian** is the great circle which is formed by a plane passing through the axis of the earth.
- The **Greenwich meridian** is the reference meridian for all other meridians. It is the meridian which passes through *Greenwich,* a town in England.
- Latitude is a parallel of latitude half-circle.
- Longitude is a meridian half-circle.

EXAMPLE 17–2

Taking the earth's radius as 6,370 km, find

(a) the distance around the equator;
(b) the length of an arc of the equator which subtends an angle of 135^o at the centre of the earth.

Solutions

(a) The equator is a circle of radius 6,370 km.
Distance around the equator = circumference of the circle = $2\pi R = 2\pi\ (6{,}370)$

$$= 2 \times \frac{22}{7} \times 6{,}370 = \mathbf{40{,}040\ km}$$

(b) Length of the arc $= \dfrac{135^o}{360^o} \times 2\pi R = \dfrac{3}{8} \times 40{,}040$

$$= \mathbf{15{,}015\ km}$$

DRILL 2

(a) What angle is subtended at the centre of the earth by an arc of the equator of length
(i) 2,002 km (ii) 30,030 km?

(b) What is the length of the arc of the equator which subtends an angle at the centre of the earth of
(i) 15^o (ii) 240^o?

17.3 Latitude

- Latitude and longitude are used to describe the location of a point on the earth's surface.
- Latitude defines the parallel of latitude, relative to the equator, on which a point on the earth's surface is located.
- To determine the latitude of a point P, for instance (Fig. 17.3):

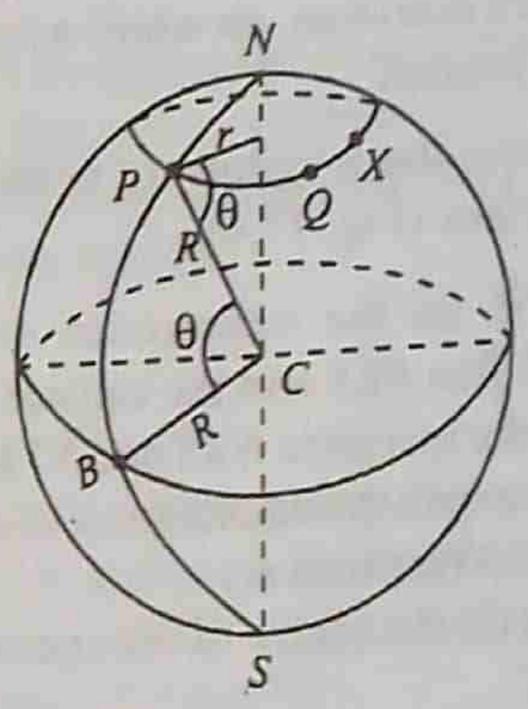


Fig. 17.3

(i) Let B be the point of intersection of the equator and the meridian which passes through P.
(ii) Join C, the centre of the sphere to B and to P.
(iii) Let angle $BCP = \theta$.
(iv) The latitude of P is written θ^oN, since P lies to the north of the equator.

- All the points such as Q and X which lie on the same parallel of latitude with P, have the same latitude.
- If r is the radius of the parallel of latitude while R is the radius of the earth, then

$$r = R \cos \theta \qquad (17.1)$$

i.e. *the radius of a parallel of latitude is simply equal to the radius of the earth multiplied by the cosine of the latitude.*

EXAMPLE 17–3

Find the radius and the circumference of the parallel of latitude which passes through

(a) Lat. 60^oN (b) Lat. 25^oS
[Radius of the earth = 6,370 km]

Solutions

(a) $r = R\cos\theta = 6{,}370 \cos 60^o = \mathbf{3{,}185\ km}$

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Circumference $= 2\pi r = \dfrac{2 \times 22 \times 3185}{7} =$ **20,020 km**

(b) $r = 6{,}370 \cos 25^\circ =$ **5,773.2 km**

Circumference $= 2 \times \dfrac{22}{7} \times 5{,}773.2 =$ **36,288.6 km**

DRILL 3

What are the two latitudes which have a circumference of

(a) 30,030 km? (b) 10,010 km?

17.4 Longitude

- Longitude defines the meridian, relative to the Greenwhich meridian, on which a point on the earth's surface is located.
- To determine the longitude of points X and Y on the earth's surface (Fig. 17.4):
  - (i) Let L be the intersection of the Greenwich meridian NLS and the equator.
  - (ii) Let the meridian NXS and $NYMS$ through X and Y intersect the equator at X (X is on the equator) and M respectively.
  - (iii) Join C, the centre of the sphere, to L, to M and to X.
  - (iv) Let $L\hat{C}X = \alpha$ and $L\hat{C}M = \beta$.
  - (v) The longitude of X is $\alpha^\circ W$ (X lies to the west of the Greenwich meridian) while that of Y (which is also that of M) is $\beta^\circ E$.
- All points on the same medirian have the same longitude.

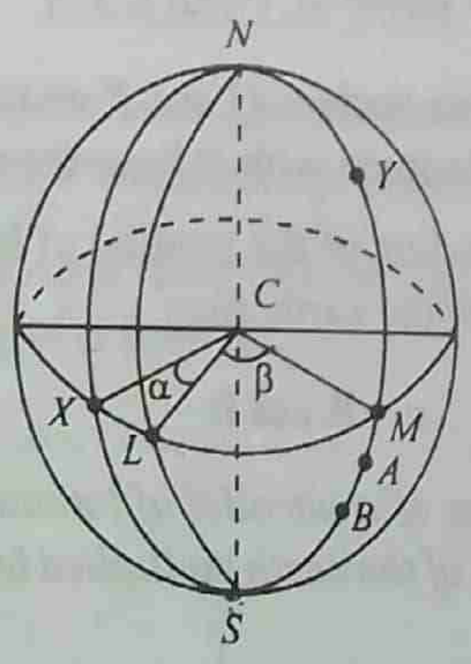
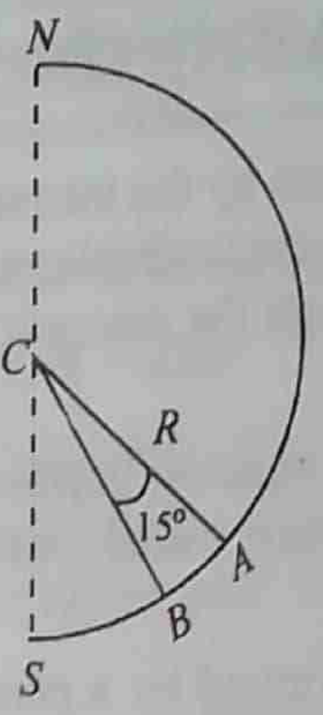


Fig.17.4

EXAMPLE 17–4

Two cities A and B lie on the same longitude (see Fig. 17.4) and B is 15° South of A. Taking the radius of the earth to be 6,370 km, what is the distance between A and B along the line of longitude? (Take $\pi = {}^{22}/_{7}$)

Solution



The locations A and B on the earth are shown on the meridian half-circle $NABS$, for clarity.

Distance between A and B = arc AB

$$= \frac{15^\circ}{360^\circ}(2\pi R)$$

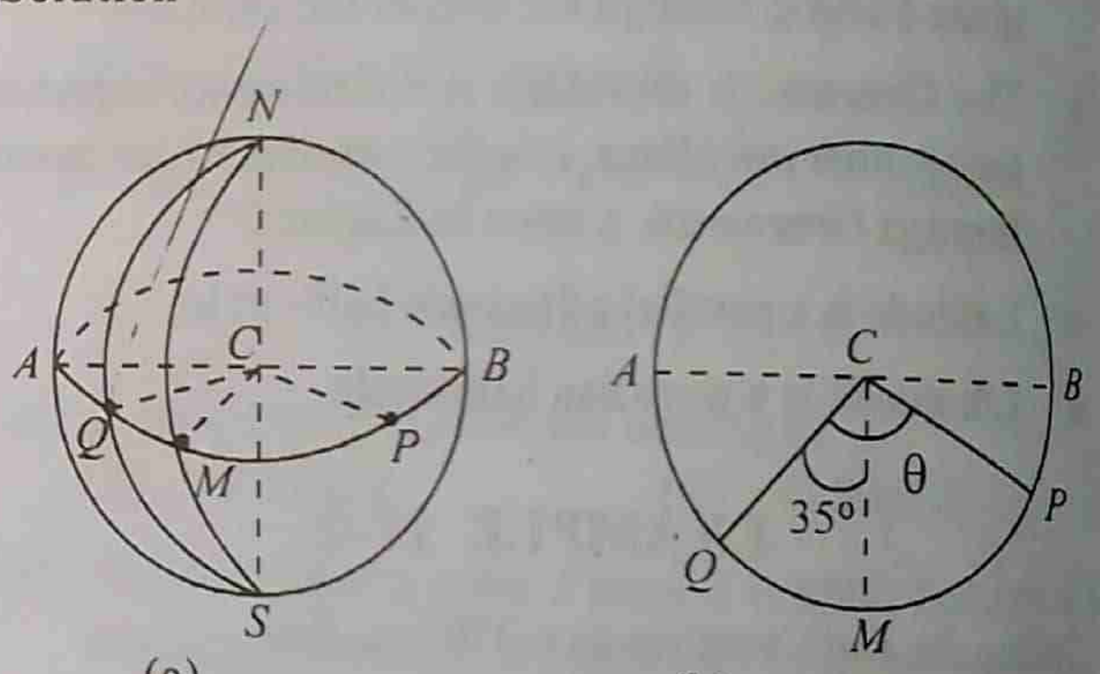
$$= \frac{15}{260} \times 2 \times \frac{22}{7} \times 6{,}370$$

$$= 1{,}668\tfrac{1}{3} \text{ km}$$

EXAMPLE 17–5

A point P lies 6,000 km east of Q (long. 35°W). If both are located on the equator, what is the longitude of P? (Take R = 6,400 km and $\pi = {}^{22}/_{7}$)

Solution



(a) (b)

In figure (a) NMS is the Greenwich meridian, NQS is the 35°W longitude on which Q lies, while the longitude of NPS is required. $AQMPB$ is the equator which is shown separately in (b) for clarity.

$$\text{arc PQ} = 6000 = \frac{\theta}{360} \times 2\pi R, \text{ or}$$

$$\theta = \frac{360 \times \text{arc}PQ}{2\pi R} = \frac{360 \times 6000}{2 \times \frac{22}{7} \times 6400} = 53.7^\circ$$

Hence $P\hat{C}M = 53.7^\circ - 35^\circ = 18.7^\circ$

i.e. P is the longitude **18.7°E.**

DRILL 4

Two cities P and Q, both on latitude 40°N, lie on longitudes 15°W and 45°E respectively. Find (a) the radius of the parallel of latitude which passes through P and Q; (b) the distance, along the parallel of latitude, between P and Q. [Take R = 6,370 km and $\pi = {}^{22}/_{7}$]

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ANSWERS TO DRILLS

(a) (i) 310^o (ii) 040^o
(b) 230^o, $S50^oW$
(c) 216.9^o or $S36.9^oW$

2. (a) (i) 18^o (ii) 270^o
(b) (ii) $1{,}668\frac{1}{3}$ km (ii) $2{,}669\frac{1}{3}$ km

3. (a) 41.4^oN or 41.4^oS
(b) 75.5^oN or 75.5^oS

4. (a) 4,880 km (b) 5,112 km

SSCE/JME–TYPE OBJECTIVE QUESTIONS

1. The bearing of a point P from a point Q is 065^o. What is the bearing of Q from P?
A. 115^o B. 155^o
C. 245^o D. 295^o

2. If the bearing of a town Y from X is 125^o, the bearing of X from Y is
A. 55^o B. 215^o
C. 235^o D. 305^o

3. A bearing of 310^o, expressed as a compass bearing is
A. $N50^oW$ B. $N40^oW$
B. $S40^oW$ D. $S50^oW$

4. The bearing of a point P from an observer O is $S37^oW$. The bearing of O from P is
A. $S53^oW$ B. $N53^oW$
C. $S37^oE$ D. $N37^oE$

5. The bearing of $N75^oW$ can also be expressed as
A. 075^o B. 165^o
C. 225^o D. 285^o

6. Points X and Y are respectively 48 m north and 20 m east of point Z. The distance XY is
A. 44 m B. 52 m
C. 60 m D. 68 m

7. The bearing of X from Y in question 6 is approximately
A. 067^o B. 157^o
C. 247^o D. 337^o

8. An airplane flies 100 km on a bearing of 330^o and then flies 300 km due west. How far west of the starting point is the plane?
A. 400 km B. 350 km
C. 300 km D. 200 km

9. Town X is on a bearing of 225^o from town Y. Town Z is directly north of X and directly west of Y. If Z is located 20 km away from Y, how far is X from Y?
A. 40 km B. 30 km
C. $20\sqrt{2}$ km D. 20 km

10. From an observation point close to the edge of the sea, a ship is 14 km away on a bearing of 130^o. A second ship is 48 km away on a bearing of 220^o. The distance separating the ships is
A. 34 km B. 50 km
C. 55 km D. 62 km

11. A motorist travels from a town P on a bearing of 150^o to a town Q 40 km away. From Q he travels to a town R on a bearing of 210^o. If R is located directly south of P, how far is R from P?
A. $40\sqrt{3}$ km B. $40\sqrt{2}$ km
C. 40 km D. 80 km

12. A hunter on the ground observes a bird perching on a tree on a bearing of $S15^oW$. The bearing of the hunter from the bird is
A. $N75^oE$ B. $N75^oW$
C. $N15^oW$ D. $N15^oE$

13. A girl walks 30 m from a point P on a bearing of 040^o to a point Q. She then walks 30 m on a bearing of 140^o to a point R. The bearing of R from P is
A. 90^o B. 50^o
C. 45^o D. 40^o

14. How far is R from P in question 13?
A. 25.2 km B. 30 km
C. 38.6 km D. 42.4 km

15. Three towns X, Y and Z are such that Y is located 50 km away from X on a bearing of 065^o. Z is 100 km away from X on a bearing of 305^o. The distance between Y and Z is
A. $50\sqrt{2}$ km B. $50\sqrt{3}$ km
C. $50\sqrt{5}$ km D. $50\sqrt{7}$ km

16. The bearing of Z from Y in question 15 is approximately

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A. 319^o B. 286^o
C. 221^o D. 041^o

17. The angle between latitudes 47^oS and 21^oN is
A. 26^o B. 34^o
C. 68^o D. 78^o

18. The angle between latitudes 20^oN and 74^oN is
A. 54^o B. 62^o
C. 74^o D. 94^o

19. If the radius of the earth is 6,370 km, the distance between two towns on latitudes 25^oN and 20^oS on the same meridian is ($\pi = 22/7$)
A. 3,185 km B. 4,004 km
C. 5,005 km D. 6,006 km

20. Two cities are both on longitude 55^oE. If their latitudes are 5^oS and 65^oS, and $R = 6{,}400$ km, their distance apart in km is
A. $\frac{6{,}400\pi}{3}$ B. $\frac{3{,}200\pi}{3}$
C. $\frac{22{,}400\pi}{9}$ D. $6{,}400\pi$

21. Three locations on the surface of the earth are $P(45^oN, 20^oE)$, $Q(45^oN, 40^oE)$ and $R(45^oS, 40^oE)$. Which of the following statements are **correct**?
I. Q and R are on the same longitude
II. Q and R are on the same latitude.
III. The difference between the longitudes of P and Q is 60^o.
IV. The difference between the latitudes of P and R is 90^o.
A. I and III only B. II and IV only
C. II and III only D. I and IV only

22. Two locations on the equator are on longitudes 20^oW and 70^oE respectively. If $R = 6{,}370$ km and $\pi = 22/7$, their distance apart along the equator, correct to 2 significant figures is
A. 5,600 km B. 10,000 km
C. 11,000 km D. 15,000 km

23. The difference in longitude between $P(55^oN, 12^oW)$ and $Q(15^oS, 65^oW)$ is
A. 40^o B. 53^o
C. 70^o D. 77^o

24. The latitudes on which the radius of the parallel of latitude is one-half of the earth's radius are
A. 30^oN and 30^oS B. 30^oN and 60^oS
C. 60^oN and 30^oS D. 60^oN and 60^oS.

25. Two towns are θ^o apart on the same meridian, If the earth's radius is 6,370 km and $\pi = 22/7$, their distance apart in km is approximately
A. 111θ B. 222θ
C. 56θ D. 167θ

26. Two locations on the parallel of latitude φ^oS have a difference in longitude of θ^o. If $R = 6{,}370$ km and $\pi = 22/7$, the circumference of the parallel of latitude in km, is
A. $6{,}370\cos\theta^o$ B. $20{,}020\cos\theta^o$
C. $40{,}040\cos\varphi^o$ D. $6{,}370\cos\varphi^o$

27. The distance between the two locations in question 26 in km, is approximately
A. $(111\varphi)\cos\theta^o$ B. $(111\theta)\cos\varphi^o$
C. $(222\theta)\cos\varphi^o$ D. $(56\varphi)\cos\theta^o$

28. Two points X and Y are on latitude 60^oN and their longitudes are 17^oW and 28^oE respectively. If $R = 6{,}370$ km and $\pi = 22/7$, the distance between X and Y, measured along the parallel of latitude is
A. 2,359.4 km B. 2,502.5 km
C. 4,718.8 km D. 5,005.0 km

29. Point $P(60^oS, 22^oE)$ lies on the same latitude as point Q. If the distance PQ measured along the parallel of latitude is 5,005 km, the radius of the earth is 6,370 km and $\pi = 22/7$, the longitude of point Q is
A. 68^oW B. 67^oE
C. 30^oE D. 30^oW

30. Two towns $P(\theta^oN, 7^oE)$ and $Q(\theta^oN, 29^oW)$ are separated by a distance of 2,002 km, measured along the parallel of latitude. If $R = 6{,}370$ km and $\pi = 22/7$, the value of θ is
A. 30^o B. 45^o
C. 60^o D. 90^o

31. An aircraft flies from point $X(30^oS, 44^oE)$ to a point Y $(30^oS, 28^oW)$ in $8\frac{2}{3}$ hours. If the radius of the earth is 6,370 km, $\pi = 22/7$, and the plane flies along the parallel of latitude, its average speed, to the nearest kilometre is
A. 462 km h^{-1} B. 620 km h^{-1}
C. 770 km h^{-1} D. 800 km h^{-1}

Starting from town $X(45^oN, 17^oE)$, an airplane flies to town $Y(45^oN, 47^oE)$ along the parallel of latitude and later to a third town $Z(60^oN, 47^oE)$ along the meridian. Use this information to answer questions **32** to **34**. [$R = 6{,}370$ km. $\pi = 22/7$]

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32. The distance between X and Y along the parallel of latitude in km, is

A. $\frac{5{,}005\sqrt{2}}{3}$ B. $\frac{5{,}005\sqrt{3}}{2}$

C. 5,005 D. $\frac{5{,}005\sqrt{2}}{2}$

33. The distance between Y and Z along the meridian is

A. 5,005 km B. $3{,}336\frac{2}{3}$ km

C. $2{,}502\frac{1}{2}$ km D. $1{,}668\frac{1}{3}$ km

34. If there is no stop-over at Y, and the average speed of the airplane is 500 km h^{-1}, the total time taken for the plane to fly from X to Z via Y is approximately

A. 10 h B. 8 h

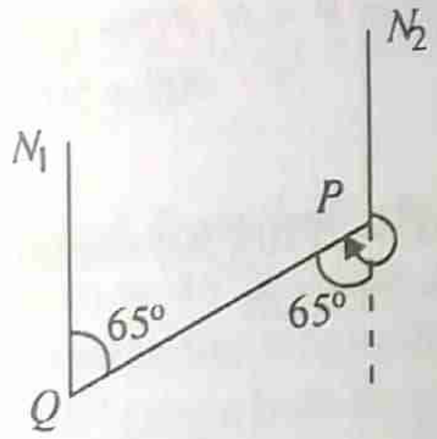
C. 7 h D. 5 h

ANSWER KEY

1.	C	7.	D	13.	A	19.	C	25.	A	31.	D
2.	D	8.	B	14.	C	20.	A	26.	C	32.	A
3.	A	9.	C	15.	D	21.	D	27.	B	33.	D
4.	D	10.	B	16.	B	22.	B	28.	B	34.	B
5.	D	11.	A	17.	C	23.	B	29.	A		
6.	B	12.	D	18.	A	24.	D	30.	C		

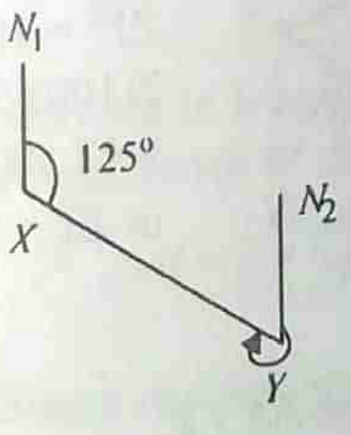
SOLUTIONS TO OBJECTIVE QUESTIONS

1. (C)



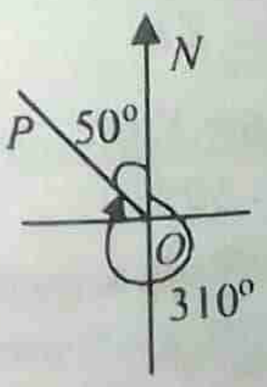
Referring to the figure, the bearing of Q from P is the reflex angle N_2PQ $= 180^o + 65^o = 245^o$.

2. (D)



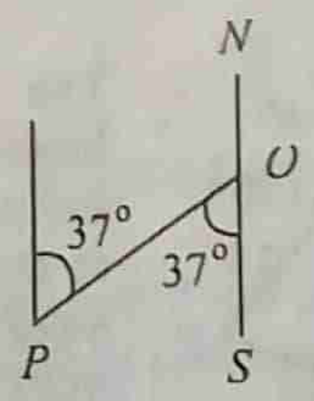
The bearing of X from Y is reflex angle N_2YX $= 360^o$ – acute $N_2YX =$ $360^o - 55^o = 305^o$.

3. (A)



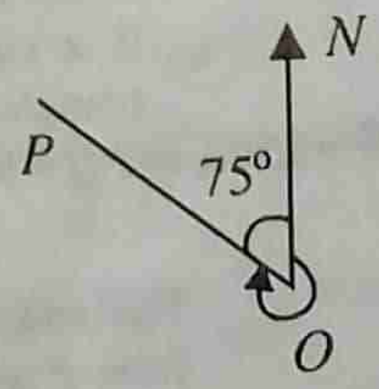
A bearing of 310^o corresponds to the direction OP shown in the figure. Since acute $N\hat{O}P = 360^o - 310^o = 50^o$, direction OP is also $N50^oW$.

4. (D)



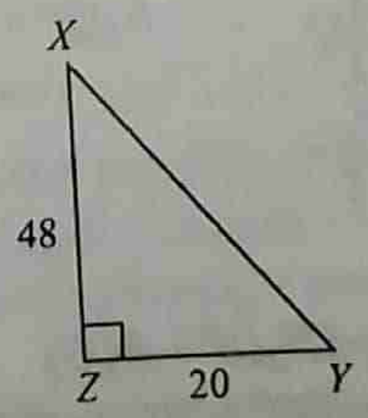
Referring to the figure, direction PO from a point P is $N37^oE$.

5. (D)



N75°W is direction OP, which is also given by reflex angle NOP = $360^o - 75^o = 285^o$.

6. (B)

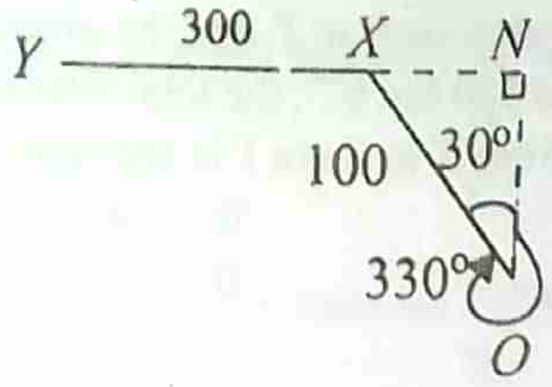


Recognize XYZ as a 5 – 12 – 13 Δ. Hence XY : ZY = 13 : 5, or XY = 13/5 x 20 = 52 m. [Alternatively, use *Pythagoras' theorem* to find XY.]

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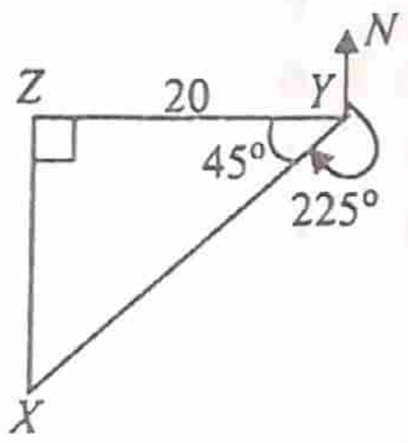
7. **(D)** $X\hat{Y}Z = \tan^{-1}(12/5) = \tan^{-1} 2.4 = 67^\circ$, approx.
Bearing of X from $Y = 270^\circ + 67^\circ = 337^\circ$.

8. **(B)**



The plane starts from O and ends at Y. Distance required
$= XN + XY = OX \sin 30^\circ + XY$
$= 100 \sin 30^\circ + 300$
$= 350$ km

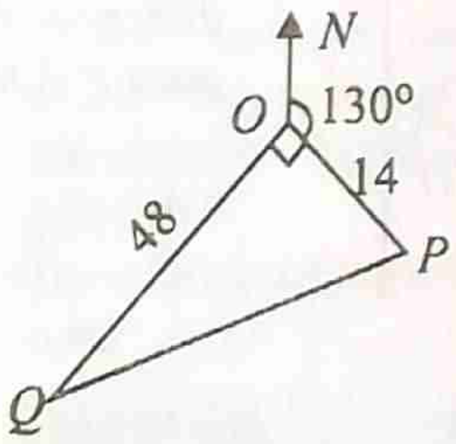
9. **(C)**



Referring to the figure, XYZ is $270^\circ - 225^\circ = 45^\circ$

$$XY = \frac{ZY}{\cos 45^\circ}$$
$$= ZY\sqrt{2}$$
$$= 20\sqrt{2} \text{ km}$$

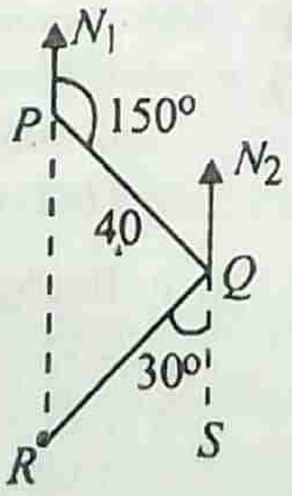
10. **(B)**



The ships are located at P and Q and the observer is at O (see figure). Since reflex angle $N\hat{O}Q$ is 220°, $POQ = 220^\circ - 130^\circ = 90^\circ$. Hence OPQ is a right-angled triangle.

$PQ^2 = 14^2 + 48^2 = 2500$, or $PQ = 50$ km.

11. **(A)**



The motorist travels from P to Q to R as shown in the figure.

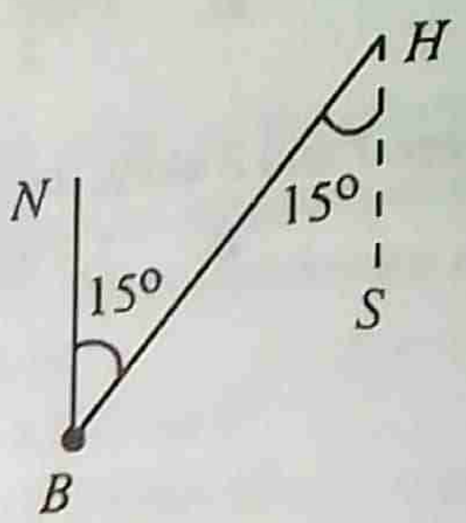
$N_2\hat{Q}P = 180^\circ - 150^\circ$
$= 30^\circ$

$$P\hat{Q}R = 180^\circ - (N_2\hat{Q}P + R\hat{Q}S)$$
$$= 180^\circ - (30^\circ + 30^\circ) = 120^\circ$$

$$R\hat{P}Q = Q\hat{R}P = 30^\circ$$

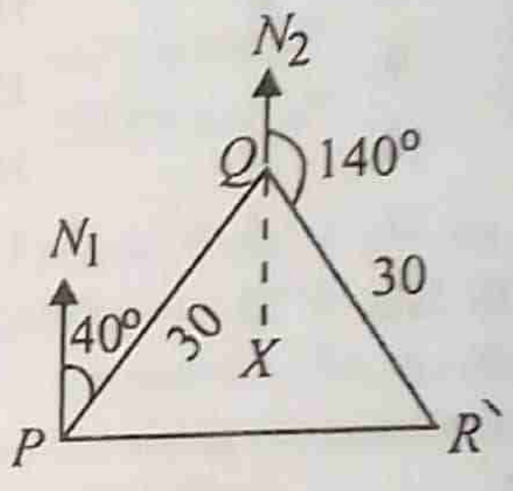
Hence PQR is an isosceles Δ, with $PQ = QR = 40$ km. Apply cosine rule:
$PR^2 = 40^2 + 40^2 - 2(40)(40) \cos 120^\circ$
$= 4800 = 1600 \times 3$, or $PR = 40\sqrt{3}$ km

12. **(D)**



The hunter is at H and the bird at B. From the diagram, bearing of H from B is $N15^\circ E$. [Note that the bearing is **not** affected by the fact that the hunter and the bird are not on the same level.]

13. **(A)**



$P\hat{Q}X = N_1\hat{P}Q = 40^\circ$ (see diagram)

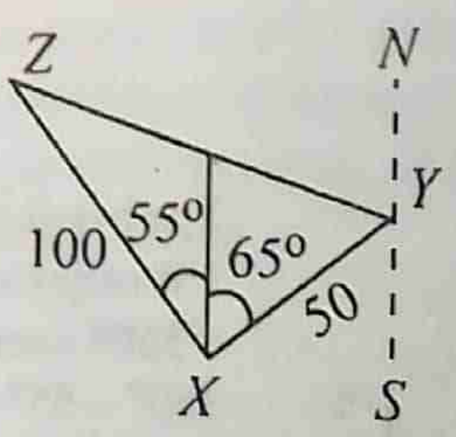
$R\hat{Q}X = 180^\circ - 140^\circ$
$= 40^\circ$

PQR is an isosceles Δ with $P\hat{Q}R$
$= 40^\circ + 40^\circ = 80^\circ$.

Hence $Q\hat{P}R = Q\hat{R}P = 50^\circ$
Bearing of R from P $= N_1PQ + Q\hat{P}R$
$= 40^\circ + 50^\circ = 90^\circ$

14. **(C)** *Cosine rule:*
$PR^2 = 30^2 + 30^2 - 2(30)(30) \cos 80^\circ$,
or $PR = 38.6$ km

15. **(D)**



Referring to the diagram, *cosine rule* gives
$ZY^2 = 100^2 + 50^2 - 2(100)(50) \cos 120^\circ$
$= 17{,}500 = 2{,}500 \times 7$
or $ZY = 50\sqrt{7}$ km

16. **(B)** Apply *sine rule* to ΔXYZ:

$$\frac{\sin X\hat{Y}Z}{XZ} = \frac{\sin Y\hat{X}Z}{ZY}$$
$$\frac{\sin X\hat{Y}Z}{100} = \frac{\sin 120^\circ}{50\sqrt{7}}$$
$$\sin X\hat{Y}Z = \sqrt{3}/\sqrt{7} = 0.65$$
$$X\hat{Y}Z = 40.9^\circ$$
But $X\hat{Y}S = 65^\circ$ *(alternate angles)*
Bearing of Z from Y

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$= 180^\circ + \hat{XYS} + \hat{XYZ}$

$= 180^\circ + 40.9^\circ + 65^\circ$

$= 285.9^\circ$

17. (C) $47^\circ S$ lies 47° *south* of the equator and $21^\circ N$ lies 21° *north* of the equator. Angle between the two latitudes $= 47^\circ + 21^\circ = 68^\circ$.

18. (A) Both latitudes lie to the *north* of the equator.
Angle between them $= 74^\circ - 20^\circ = 54^\circ$

19. (C) Circumference of the meridian $= 2\pi R$
Angle between the two latitudes
$= 25^\circ + 20^\circ = 45^\circ$
Distance between the two towns
$= \frac{45}{360} \times 2\pi R$
$= \frac{1}{8} \times 2 \times \frac{22}{7} \times 6{,}370$
$= 5{,}005$ km

20. (A) Angle between the latitudes
$= 65^\circ - 5^\circ = 60^\circ$
Circumference of the meridian $= 2\pi R$
Distance between the two cities
$= \frac{60^\circ}{360^\circ} \times 2\pi \times 6{,}400 = \frac{6{,}400\pi}{3}$ km

21. (D) Q and R are both on longitude $40^\circ E$. Q is on latitude $45^\circ N$ while R is on $45^\circ S$. Difference between the longitudes of P and Q is $40^\circ - 20^\circ = 20^\circ$ (since both lie to the east of the *Greenwich* meridian).
Difference between the latitudes of P and $R = 45^\circ + 45^\circ = 90^\circ$

22. (B) Difference in longitude $= 20^\circ + 70^\circ = 90^\circ$
Circumference of the equator
$= 2\pi R = 2 \times \frac{22}{7} \times 6{,}370$ km
Distance required $= \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 6{,}370$
$= 10{,}010$, or $10{,}000$ km to 2 s.f.

23. (B) Difference in longitude $= 65^\circ - 12^\circ = 53^\circ$

24. (D) Radius of parallel of latitude, $r = R \cos \theta$
Hence if $r = R/2$ then θ is given by
$\frac{R}{2} = R \cos \theta$, or $\cos \theta = \frac{1}{2}$ and $\theta = 60^\circ$
i.e. latitude $60^\circ N$ or $60^\circ S$.

25. (A) Circumference of the meridian $= 2\pi R$
Distance apart $= \frac{\theta^\circ}{360^\circ} \times 2\pi R$
$= \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 6{,}370$
$= 111\theta$, approx.

26. (C) Radius of the parallel of latitude,
$r = R \cos \varphi$
Circumference of the parallel of latitude
$= 2\pi r = 2 \times \frac{22}{7} \times 6{,}370 \cos \varphi^\circ$
$= 40{,}040 \cos \varphi^\circ$

27. (B) Distance required $= \frac{\theta^\circ}{360^\circ} \times 2\pi r$
$= \frac{\theta^\circ}{360^\circ} \times 40{,}040 \cos \varphi^\circ$
$= (111\theta) \cos \varphi^\circ$

28. (B) Difference in longitude $= 17^\circ + 28^\circ = 45^\circ$
Radius of parallel of latitude,
$r = R \cos \theta^\circ = 6{,}370 \cos 60^\circ$
Distance between X and Y $= \frac{45^\circ}{360^\circ} \times 2\pi r$
$= \frac{1}{8} \times 2 \times \frac{22}{7} \times 6.370 \times \frac{1}{2}$
$= 2{,}502.5$ km

29. (A) Latitude of P and $Q = 60^\circ S$
Radius of parallel of latitude, $r = R \cos \theta$
$= 6{,}370 \cos 60^\circ = 3{,}185$ km
Circumference of parallel of latitude
$= 2\pi r = 2 \times \frac{22}{7} \times 3{,}185 = 20{,}020$ km
Let θ = difference between longitudes of P and Q
Dist. $PQ = \frac{\theta^\circ}{360^\circ} \times 2\pi r = \frac{\theta^\circ}{360^\circ} \times 20{,}020$
$= 5{,}005$ km
or $\theta = 90^\circ$, i.e. longitude of Q is $90^\circ - 22^\circ$
$= 68^\circ W$

30. (C) Both P and Q lie on latitude $\theta^\circ N$ with radius of parallel of latitude $r = R \cos \theta$.
Circumference of parallel of latitude
$= 2\pi r = 2\pi R \cos \theta$
Difference in longitude $= 29^\circ + 7^\circ = 36^\circ$
Distance $PQ = \frac{36^\circ}{360^\circ} \times 2\pi r$

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$$= \frac{1}{10} \times 2 \times \frac{22}{7} \times 6{,}370 \cos\theta \text{ km}$$

Hence $2{,}002 = 4{,}004 \cos\theta$

$\cos\theta = \frac{1}{2}$, or $\theta = 60^\circ$.

31. **(D)** Radius of parallel of latitude,
$r = 6{,}370 \cos 30^\circ$ km.

Circumference $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 6{,}370 \cos 30^\circ \text{ km}$$

Difference in longitude $= 44^\circ + 28^\circ = 72^\circ$

$$\text{Distance } XY = \frac{72^\circ}{360^\circ} \times 2\pi r$$

$$= \frac{1}{5} \times 2 \times \frac{22}{7} \times 6{,}370 \cos 30^\circ$$

$$= 4{,}004\sqrt{3} \text{ km}$$

$$\text{Average speed} = \frac{\text{Dist. } XY}{\text{Time}} = \frac{4{,}004\sqrt{3}}{8\frac{2}{3}}$$

$= 800$ km h^{-1}, approx.

32. **(A)** Difference between longitudes of X and Y

$= 47^\circ - 17^\circ = 30^\circ$

Radius of parallel of latitude

$= 6{,}370 \cos 45^\circ$

Distance XY

$$= \frac{30^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 6{,}370 \cos 45^\circ$$

$$= \frac{5{,}005\sqrt{2}}{3} \text{ km}$$

33. **(D)** Y and Z both lie on longitude $47^\circ E$, with a difference in latitude of $60^\circ - 45^\circ$, or 15°.

$$\text{Dist. } YZ = \frac{15^\circ}{360^\circ} \times 2\pi R$$

$$= \frac{1}{24} \times 2 \times \frac{22}{7} \times 6{,}370 \text{ km}$$

$= 1{,}668\frac{1}{3}$ km

34. **(B)** Total distance covered

$$= \frac{5{,}005\sqrt{2}}{3} + 1{,}668\frac{1}{3}$$

$$= \frac{5{,}005}{3}(\sqrt{2} + 1)$$

$$\text{Time taken} = \frac{5{,}005}{500 \times 3}(2.414)$$

$= 8$ hours, approx.

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18 Construction and Loci

18.1 Construction

Construction of lines, angles and simple figures can be performed using a ruler and a pair of compasses only, as demonstrated in the following examples.

(a) Bisector of an angle

To construct the bisector of a given angle ABC (Fig. 18.1):

(i) With B as centre, describe an arc which cuts BA at X and BC at Y.

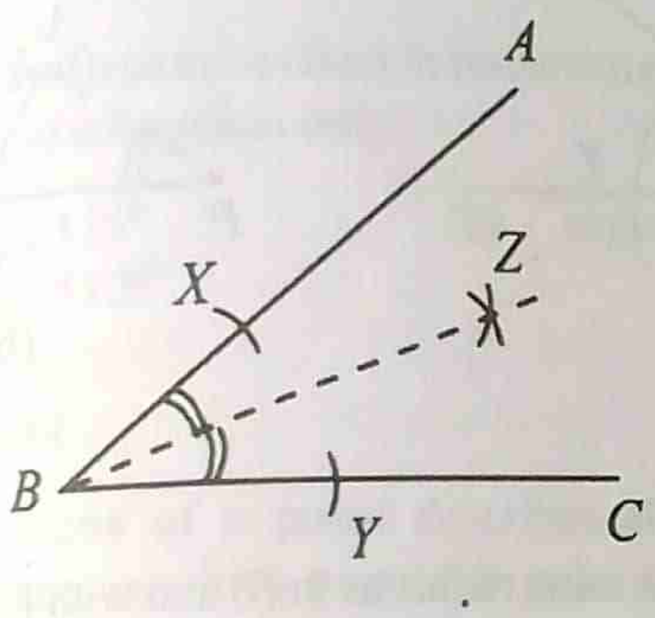


Fig.18.1

(ii) With X and Y as centres, describe two acrs of equal radii which intersect at Z.

(iii) BZ is the bisector of angle ABC.

(b) Perpendicular Bisector of a line

To construct the perpendicular bisector of a line AB (Fig. 18.2):

(i) With A as centre and with a radius greater than half of AB, describe two arcs, one above, and the other below the line AB.

(ii) With B as centre and with the same radius, describe two arcs which intersect the arcs in (i) at X and Y.

(iii) Line XY is the perpendicular bisector of line AB.

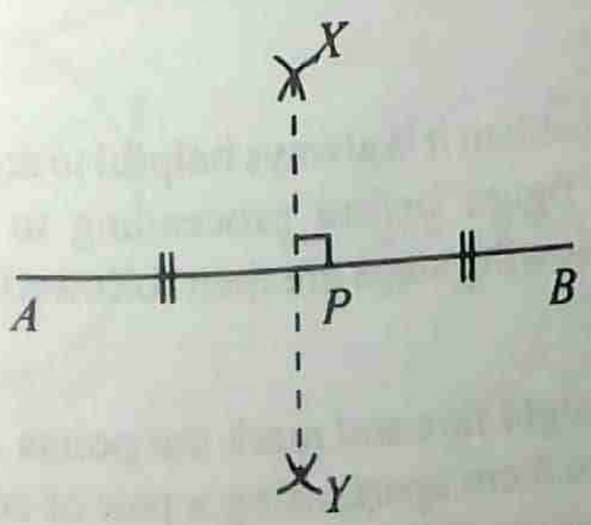


Fig. 18.2

(c) A line perpendicular to a given line from a point on the line

To construct a line through a point P perpendicular to a line AB (Fig. 18.3):

(i) With P as centre, describe two arcs which cut AB (produced if necessary) at X and Y.

(ii) With X and Y as centres, describe two arcs which intersect at K.

(iii) Line PK is perpendicular to AB.

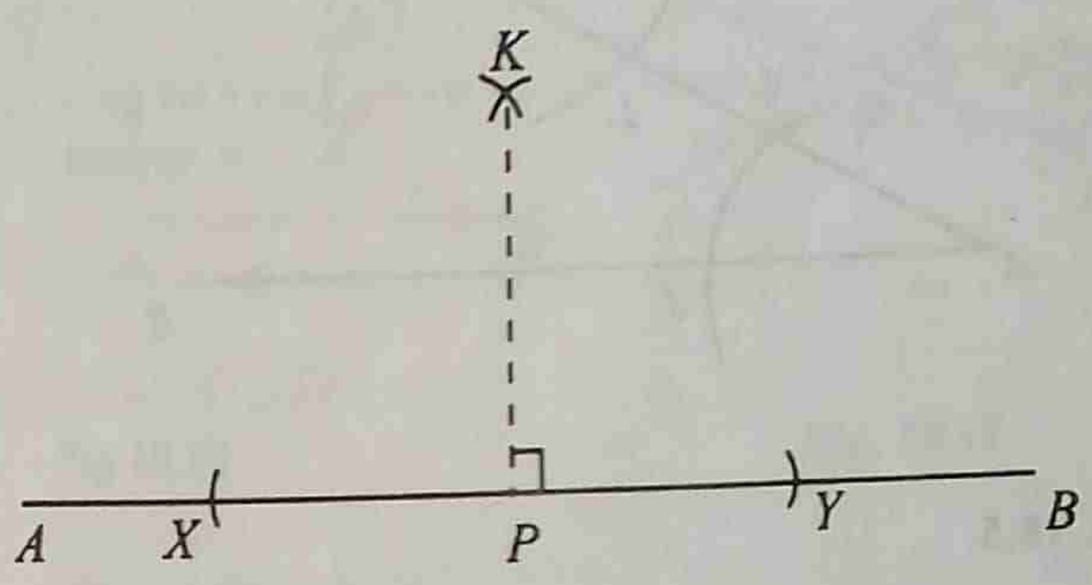


Fig. 18.3

(d) A line perpendicular to a given line from a point outside the line

To construct a line through R perpendicular to a line AB (Fig. 18.4):

(i) With R as centre, describe two acrs which cut AB (produced if necessary) at P and Q.

(ii) With P and Q as centres, describe two arcs which intersect at S.

(iii) Line RS is perpendicular to AB

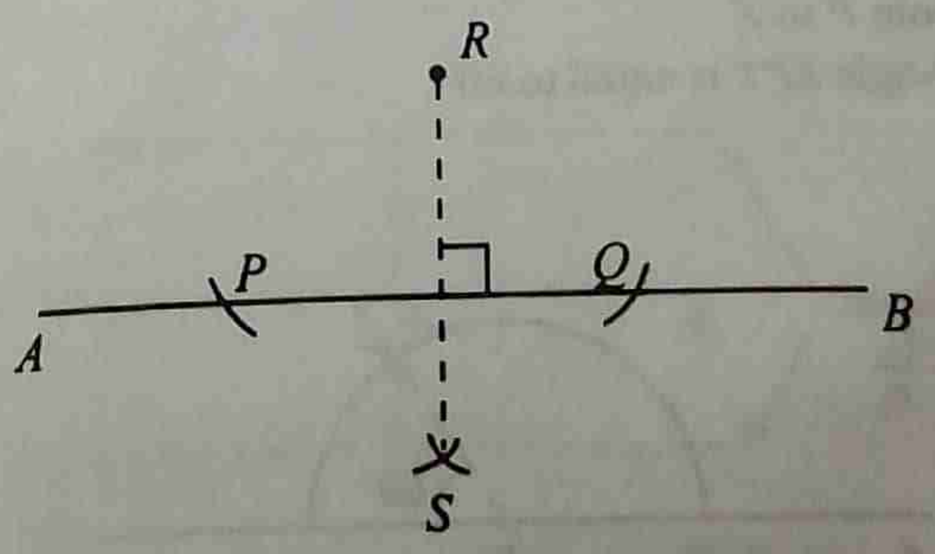


Fig. 18.4

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(e) A line parallel to a given line

To construct a line through a point P parallel to a line AB (Fig. 18.5):

(i) Join P to A.
(ii) With A as centre, describe an arc which cuts AB at J and AP at K.
(iii) With P as centre, and using the same radius as in (ii), describe an arc which cuts AP at L.
(iv) With L as centre, describe an arc, with radius equal to JK, which cuts the arc in (iii) at M.
(v) Line PM is parallel to AB.

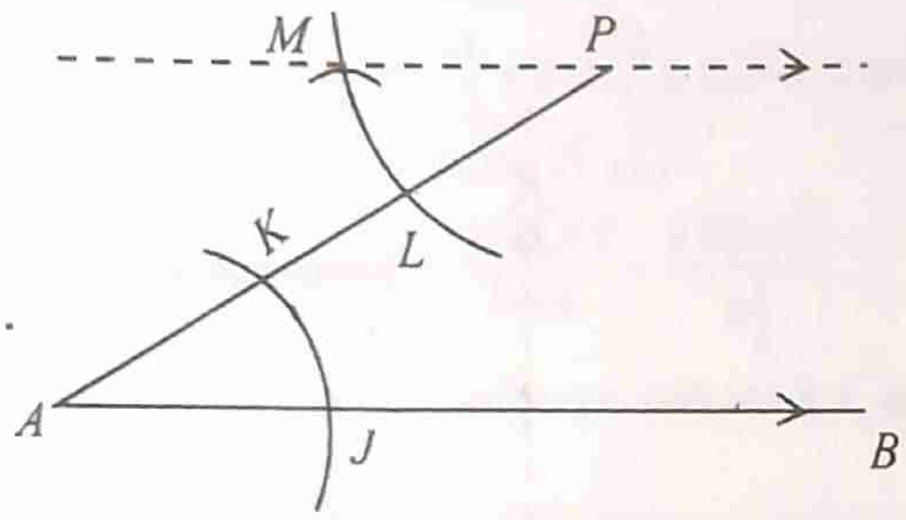


Fig. 18.5

DRILL 1

Show that

(a) BZ bisects $A\hat{B}C$ in Fig. 18.1
(b) KP is perpendicular to AB in Fig. 18.3.

(f) To construct angle 60°

To construct an angle of 60° on a given line AB (Fig. 18.6):

(i) With any point P along AB as centre, describe a semi-circle which cuts AB at X and Y.
(ii) With Y as centre and using the same radius as in (i), describe an arc which intersects the semi- circle at Z.
(iii) Join P to Z.
(iv) Angle ZPY is equal to 60°.

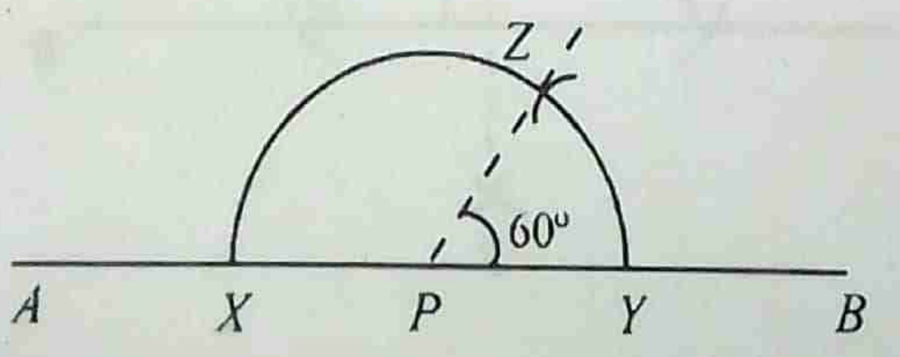


Fig. 18.6

(g) To construct angle 45°

First construct 90° (a perpendicular line) and then construct its bisector.

(h) To construct angle 30°

First construct 60° and then construct its bisector.

(i) To construct an angle equal to a given angle

To construct an angle on line PQ which is equal to $A\hat{B}C$ (Fig. 18.7):

(i) With B as centre, describe an arc which cuts AB at X and BC at Y (Fig. 18.7a).

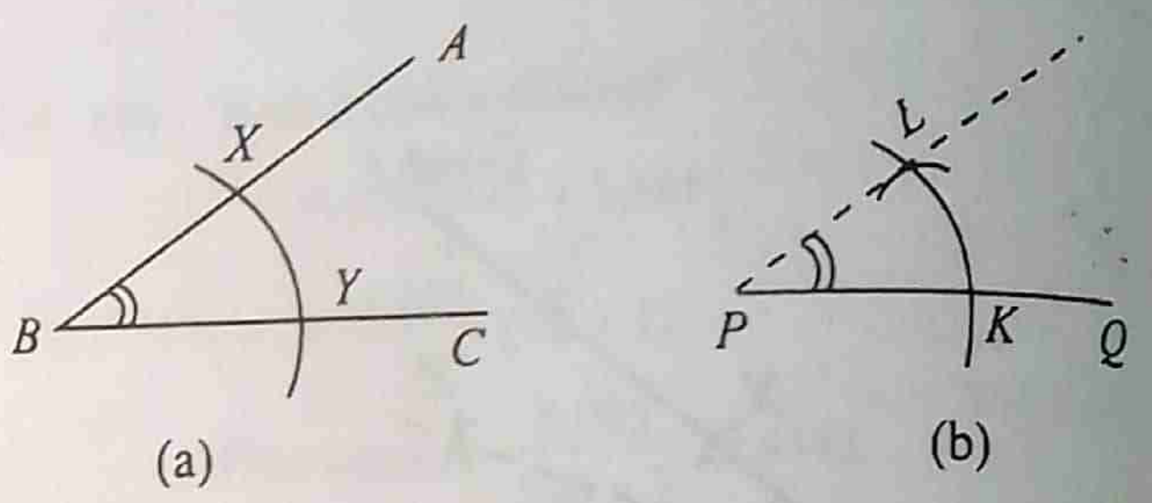


Fig. 18.7

(ii) Using the same radius as in (i) and with P as centre, describe an arc which cuts PQ at K (Fig. 18.7b).
(iii) With K as centre, and using a radius equal to YX, describe an arc which cuts the arc in (ii) at L.
(iv) Join L to P.
(v) Angle LPK is equal to angle ABC.

(j) **Triangles** and **Quadrilaterals** are constructed by combining two or more of the constructions described above along with other data provided.

EXAMPLE 18–1

Using a ruler and a pair of compasses only, construct a triangle ABC such that $|AB| = 4$ cm, $|BC| = 6$ cm and $C\hat{A}B = 150°$

Solution

For this type of problem it is always helpful to draw a sketch of the required figure before proceeding to the actual drawing. The following steps are then followed (see figure below):

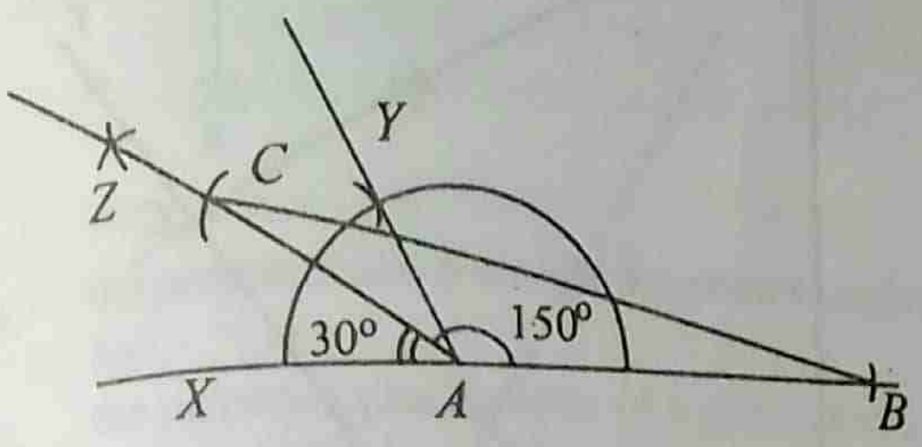
(a) Draw a straight line and mark out points A and B on it which are 4 cm apart, using a pair of compasses.
(b) Construct angle XAY which is equal to 60°.
(c) Construct the bisector AZ of angle XAY such that $XAZ = 30°$.

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(d) With B as centre, describe an arc of radius 6 cm which cuts AZ at C.

(e) Join B to C.

ABC is the required triangle.



DRILL 2

Describe the steps to be taken in constructing, using a ruler and a pair of compasses only:

(a) angle 135°
(b) angle 75°
(c) angle 67.5°

18.2 Loci

- The **locus of a point** describes all the possible locations in which a variable point could be found under a stated set of conditions.
- Loci can be determined by simple reasoning and/or making simple sketches as is demonstrated in the following examples.

(a) Locus of a point at a given distance from a given point

If a point P is at a fixed distance from O and is to remain in the plane of the paper, the locus of P is a **circle of radius** OP (Fig. 18.8). (Note that should P be placed at any point inside or outside the circle, the distance OP is no longer constant.)

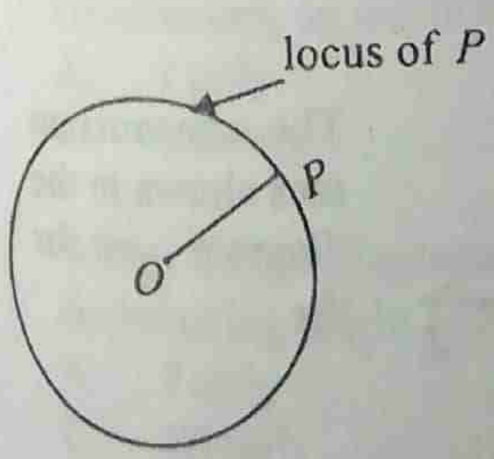


Fig. 18.8

(b) Locus of a point equidistant from two given points

For a point P to be equidistant from two points A and B, its locus is the **perpendicular bisector of line** AB (Fig. 18.9).

(c) Locus of a point equidistant from two parallel lines

If a point P is to be at equal distances from lines AB and CD which are parallel, its locus is a **third line which is parallel to** AB **and** CD **and which lies midway between them** (Fig. 18.10).

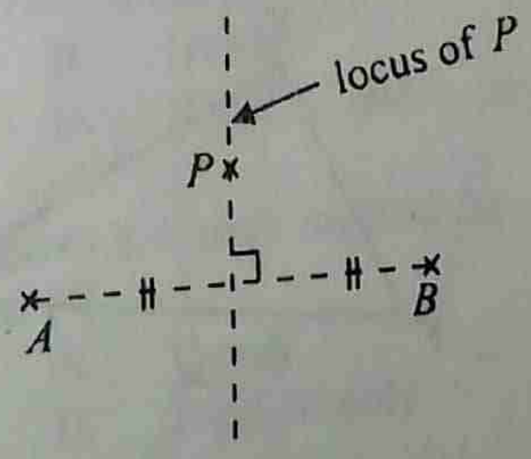


Fig.18.9

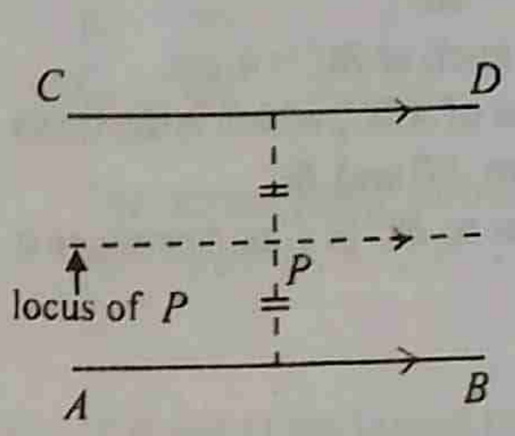


Fig.18.10

(d) Locus of a point equidistant from two intersecting lines

For a point P to maintain the same (perpendicular) distance from two intersecting lines AB and CD, it must lie on **the bisectors of the angles formed by the two lines**, i.e. the bisectors of angles BOD and BOC (Fig. 18.11).

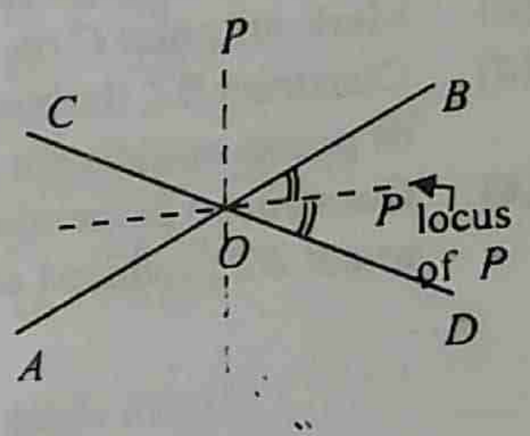


Fig. 18.11

(e) Locus of a point at a fixed distance from a given straight line

If a point P is to be at a constant perpendicular distance from line AB, it must lie on two **lines which are parallel to** AB **and placed at equal distances on opposite sides of it** (Fig. 18.12).

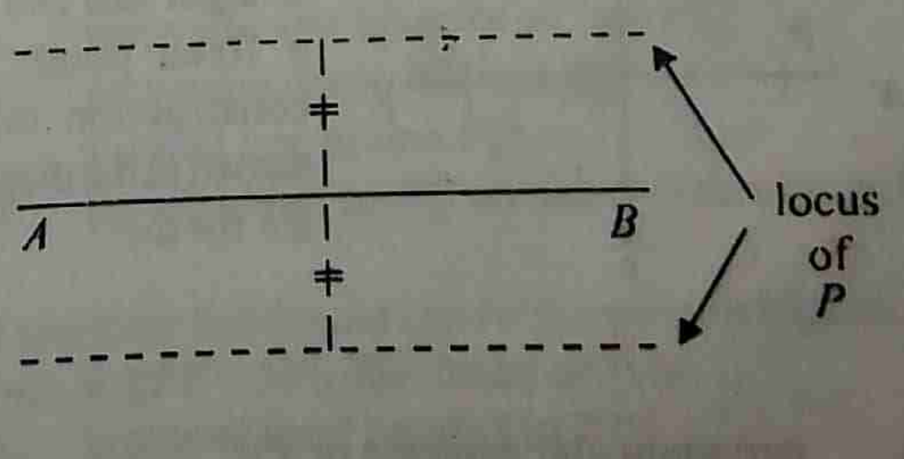


Fig. 18.12

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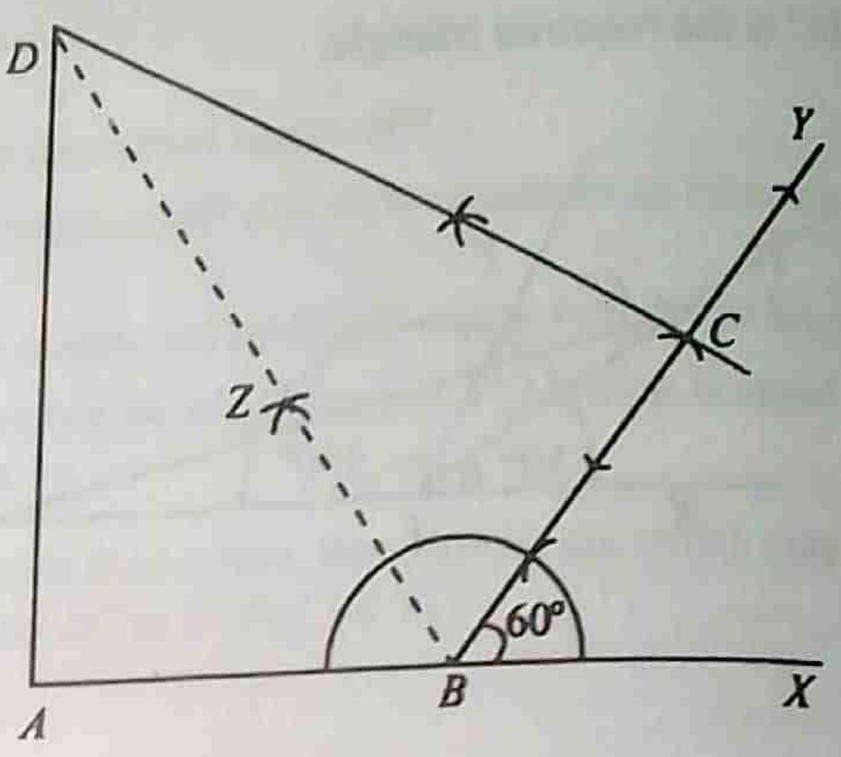
EXAMPLE 18–2

Using a ruler and a pair of compasses only, construct a quadrilateral $ABCD$ such that:

(i) $|AB| = |BC| = 4$ cm, $A\hat{B}C = 120^{\circ}$.
(ii) D is on the locus of a point equidistant from AB and BC.
(iii) CD is perpendicular to BC. Measure CD, AD, BD and $D\hat{A}B$.

Solution

(a) Mark off line AB of length 4 cm from a line AX (see diagram below).
(b) Construct an angle $XBY = 60^{\circ}$.
(c) Mark off point C on BY such at $BC = 4$ cm.
(d) Construct BZ, the bisector of ABC, which is the locus of points equidistant from AB and BC.
(e) Construct a perpendicular to BC at C and produce it to cut BZ produced at D.
(f) Join D to A. $ABCD$ is the required quadrilateral.



From the diagram: $|CD| = 6.9$ cm, $|AD| = 6.9$ cm $|BD| = 8.0$ cm and $D\hat{A}B = 90^{\circ}$.

ANSWERS TO DRILLS

1. (a) Join Z to X and to Y.
Δs BXZ and BYZ are congruent (SSS)
$X\hat{B}Z = Y\hat{B}Z$

(b) Join K to X and to Y.
Δs XKP and YKP are congruent (SSS)
$K\hat{P}X = K\hat{P}Y$ and $K\hat{P}X + K\hat{P}Y = 180^{\circ}$
$\therefore$ $K\hat{P}X = K\hat{P}Y = 90^{\circ}$

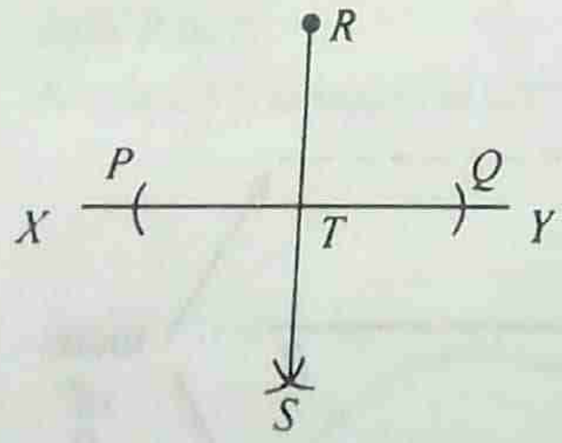
2. (a) *Construct angle 90° on a straight line.
*Bisect one of the two right angles (45°)
*$90^{\circ} + 45^{\circ} = 135^{\circ}$

(b) *Construct angle 90°
*Construct angle 60° at the same point.
*Construct the bisector of the 30° complementary angle (15°)
* $60^{\circ} + 15^{\circ} = 75^{\circ}$

(c) *Construct angle 90°.
*Construct the bisector of 90° (45°)
*Construct the bisector of one of the 45° angles (22.5°)
*$45^{\circ} + 22.5^{\circ} = 67.5^{\circ}$

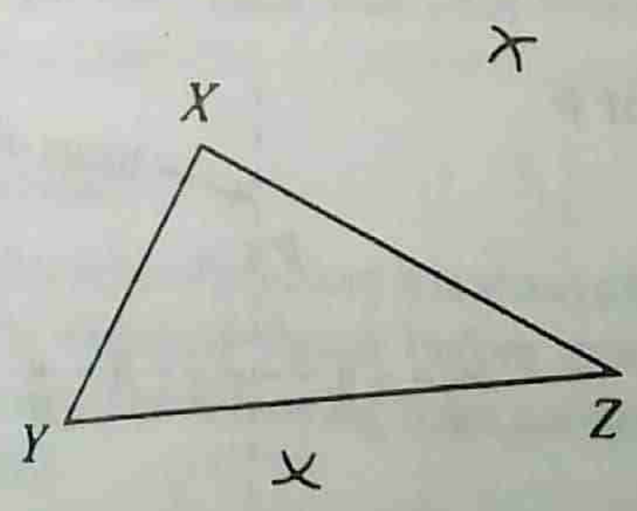
SSCE/JME–TYPE OBJECTIVE QUESTIONS

1. In the diagram, XY is straight line and R is a fixed point. The construction marks shown in the diagram are for the:



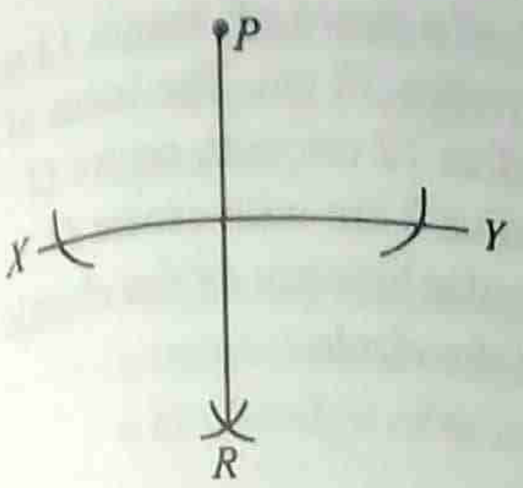
A. perpendicular bisector of XY.
B. perpendicular bisector of RS.
C. perpendicular from R to XY.
D. perpendicular to XY through the point T.

2. The construction arcs shown in the diagram are for the



A. bisector of $Y\hat{X}Z$
B. perpendicular bisector of YZ.
C. perpendicular bisector of XZ.
D. bisector of $X\hat{Y}Z$.

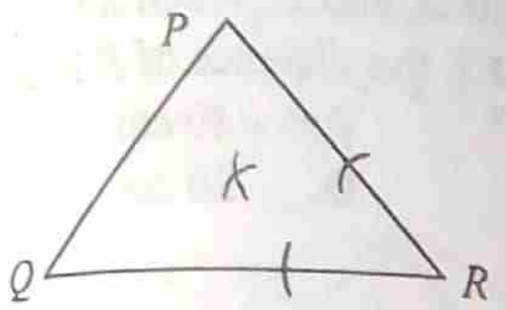
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3. The diagram shown is an example of the construction of

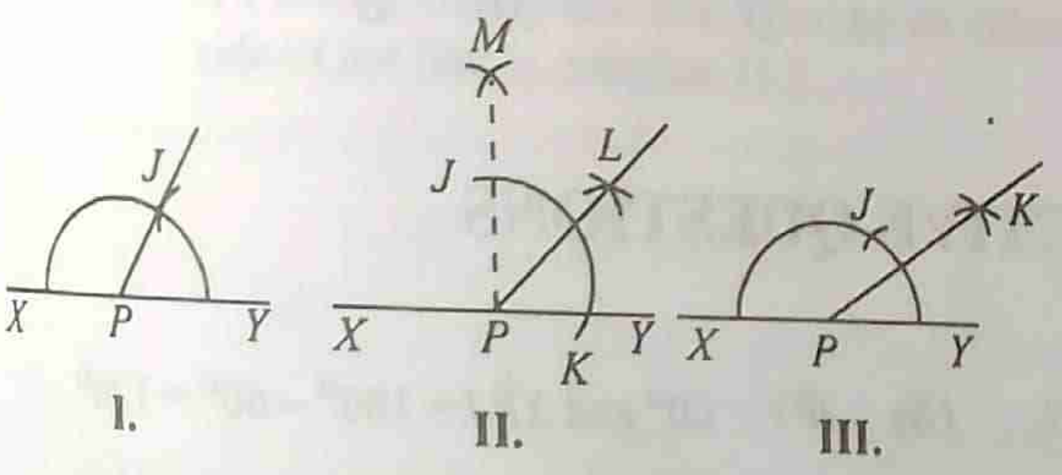
A. the perpendicular from a given point to a given line.
B. the perpendicular bisector of a given line.
C. the perpendicular to a line from a given point on the line.
D. a given angle.

4.



The diagram illustrates the construction of
A. the perpendicular bisector of QR
B. the perpendicular bisector of PR
C. the angular bisector of $P\hat{Q}R$
D. the angular bisector of $P\hat{R}Q$

Use the diagrams below to answer questions **5** to **7.**

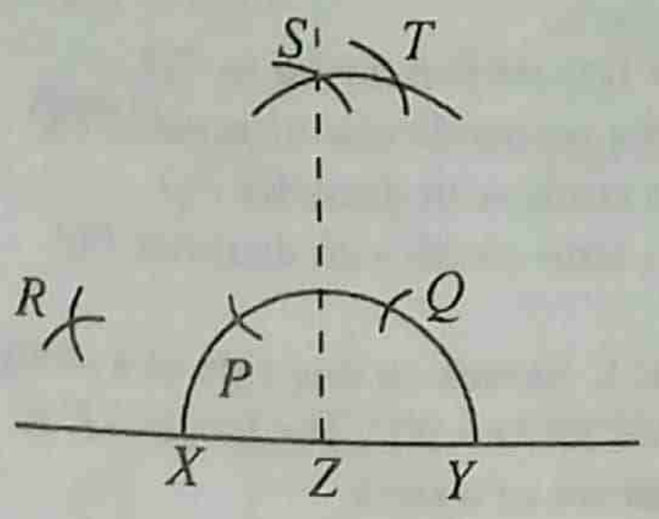


5. Which of the diagrams gives a correct method for constructing an angle of 150^o at the point P?
A. I only
B. II only
C. III only
D. II and III only

6. Which of the diagrams gives a correct method for constructing angle 135^o at P?
A. I only
B. II only
C. III only
D. I and III only

7. Which of the diagrams gives a correct method for constructing angle 120^o at P?
A I only
B. III only
C. I and II only
D. I and III only

Use the figure below to answer questions **8** to **10.**



8. The acute angle formed by joining T to Z is
A. 30^o
B. 45^o
C. 60^o
D. 75^o

9. The obtuse angle formed by joining R to Z is
A. 165^o
B. 150^o
C. 135^o
D. 120^o

10. If R and Q are joined to Z, angle RZQ is
A. 120^o
B. 105^o
C. 90^o
D. 75^o

11. Which of the following specified sets of data is **not necessarily** sufficient for the construction of a triangle?
A. Three angles
B. Two sides and a right angle
C. Two sides and an included angle
D. Three sides

12. Which of the following statements is **not true**? It is possible to construct a
A. triangle given one side and two angles
B. a quadrilateral given all four sides
C. a parallelogram given two adjacent sides and a diagonal
D. a rhombus given one side and an angle.

13. A point X is equidistant from two fixed points P and Q. The locus of X is the
A. perpendicular bisector of line PQ
B. perpendicular to the line PQ
C. parallel line to line PQ
D. parallel lines to PQ on either side of line PQ

14. X and Y are fixed points and P is a point which moves such at $XPY = 30^o$. The locus of P is
A. a pair of lines parallel to XY
B. the perpendicular bisector of XY
C. a circle with diameter XY
D. an arc of a circle passing through X and Y

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15. A variable point V moves relative to two fixed points P and Q such that PVQ is a right angle. The locus of V is
A. a line perpendicular to PQ
B. the perpendicular bisector of PQ
C. a circle with diameter PQ
D. a semi-circle with diameter PQ

16. A point K moves on one side of a straight line LM such that $L\hat{K}M = 90^o$. The locus of K is
A. an arc of a circle
B. a right-angled triangle
C. a circle
D. a semi-circle

17. The distance of a variable point X from a line PQ is always equal to the distance of X from a line PR which intersects PQ at P. The locus of X is
A. the perpendicular bisector of PQ and PR
B. the bisector of angle PQR
C. the bisector of angle QPR
D. a line parallel to PQ and PR

18. M is the mid-point of a chord of length 12 cm in a cirlce, centre O of radius 10 cm. The locus of M is
A. a circle of radius 12 cm with centre O
B. a circle of radius 8 cm with centre O
C. the perpendicular bisector of the chords
D. a diameter of the circle

19. X is a variable point which is equidistant from two parallel lines PQ and RS of equal lengths such that $PQRS$ forms a rectangle. If $X\hat{P}Q = 25^o$, then $R\hat{X}S$ is
A. 130^o B. 65^o
C. 50^o D. 25^o

20. A point P moves such that it is always equidistant from two fixed points X and Y. When XY is 12 cm and P is 8 cm from XY, the distance of P from Y is
A. 8 cm B. 10 cm
C. 12 cm D. 20 cm

ANSWER KEY

1.	D	5.	C	9.	B	13.	A	17.	C
2.	C	6.	B	10.	C	14.	D	18.	B
3.	A	7.	A	11.	A	15.	C	19.	A
4.	D	8.	D	12.	B	16.	D	20.	B

SOLUTIONS TO OBJECTIVE QUESTIONS

1. (D) See section 18.1(c) of revision notes.

2. (C) Note that the arcs are located on opposite sides of the line XZ and at about equal distances from it.

3. (A) See section 18.1(d) of revision notes.

4. (D) The intersecting arcs in the middle of the triangle must have been made with the marks on QR and PR as centres.

5. (C) In (III) $J\hat{P}Y = 60^o$ and PK is its bisector.
Hence $K\hat{P}Y = 30^o$ and $X\hat{P}K = 180^o - 30^o$
$= 150^o$

6. (B) $M\hat{P}Y = 90^o$ and PL is its bisector.
Hence $L\hat{P}Y = 45^o$ and $X\hat{P}L = 180^o - 45^o$
$= 135^o$

7. (A) $J\hat{P}Y = 60^o$ and $X\hat{P}J = 180^o - 60^o = 120^o$

8. (D) $Q\hat{Z}Y = 60^o$ and $S\hat{Z}Q = 30^o$
TZ bisects $S\hat{Z}Q$, hence $T\hat{Z}Q = 15^o$
$T\hat{Z}Y = 60^o + 15^o = 75^o$

9. (B) $X\hat{P}Z = 60^o$ and RZ bisects $X\hat{P}Z$
$\therefore\ R\hat{Z}X = 30^o$ and $R\hat{Z}Y = 180^o - 30^o$
$= 150^o$

10. (C) $Q\hat{Z}Y = 60^o$ and $R\hat{Z}X = 30^o$ (See answers to questions 8 and 9 above).
$R\hat{Z}Q = 180^o - Q\hat{Z}Y - R\hat{Z}X$
$= 180^o - 60^o - 30^o = 90^o$

11. (A) This problem is best solved by attempting a rough sketch of each of the figures. It will be

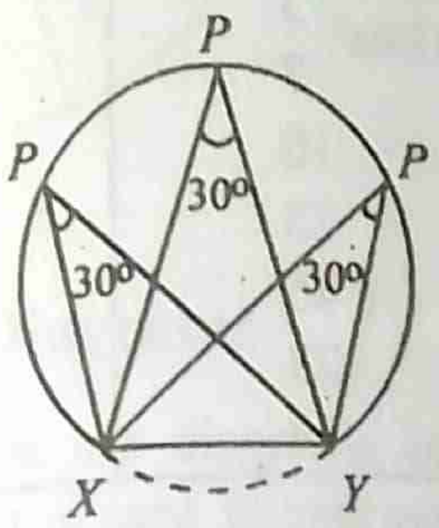
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clear from such exercise that it is *not* possible to draw a triangle if at least one side is not specified.

12. **(B)** Following the procedure in question 11 above, a quadrilateral cannot be constructed without a knowledge of at least one of the angles.

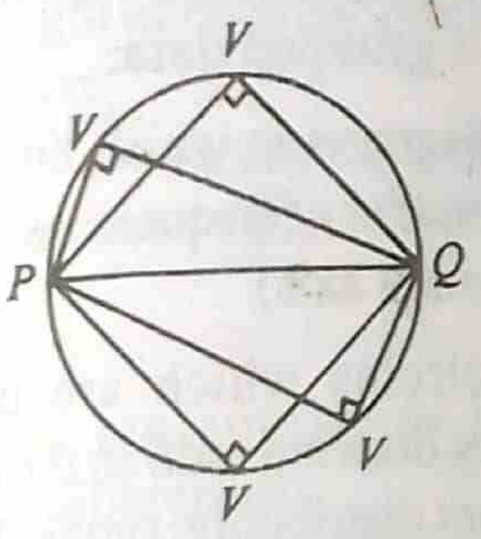
13. **(A)** See section 18–1(b) of revision notes.

14. **(D)**



XY must be the chord of a circle which subtends an angle of 30° at any point P on the *major arc* of the circle.

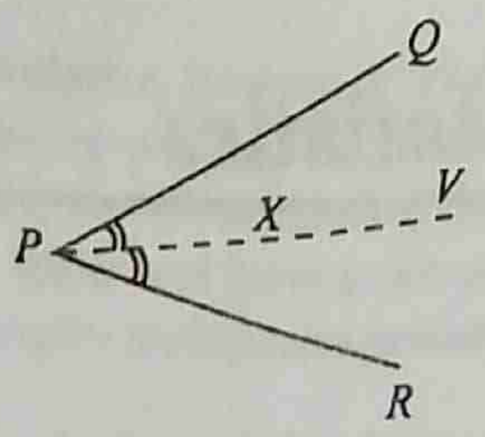
15. **(C)**



PQ must be the diameter of a circle which subtends an angle of 90° at any point *V* on the circle.

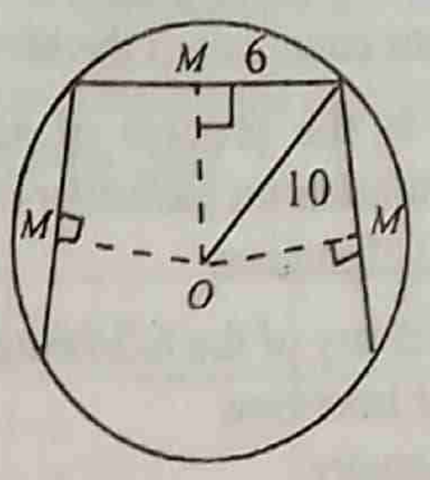
16. **(D)** This is similar to question 15 except that *K* is restricted to one side of the line. Hence its locus is a semi-circle. [Note that *V* can be on either side of the line in question 15.]

17. **(C)**



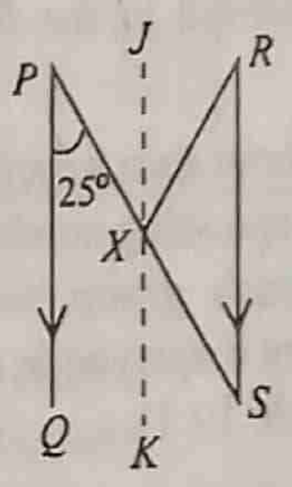
X can be any point on the bisector PV of angle *QPR* such that $X\hat{P}Q = X\hat{P}R$ (see diagram).

18. **(B)**



Referring to the figure, *M* must lie on a circle with *O* as centre and radius OM = $\sqrt{10^2 - 6^2} = 8$ cm, using Pythagoras' theorem.

19. **(A)**

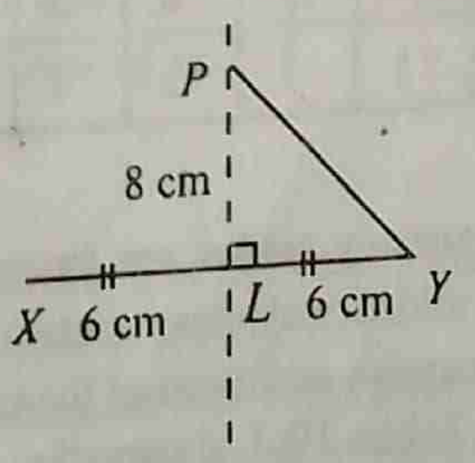


The point *X* must lie on line *JK* which is equidistant from, and parallel to *PQ* and *RS*. From the diagram:

$P\hat{X}J = X\hat{P}Q = 25° = R\hat{X}J$

$R\hat{X}S = 180° - P\hat{X}J - R\hat{X}J$
$= 180° - 25° - 25° = 130°$

20. **(B)**



P must lie on *PL*, the perpendicular bisector of *XY*
LY = 6 cm and
PL = 8 cm
$PY = \sqrt{8^2 + 6^2}$
= 10 cm.

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19 Statistics

19.1 Definitions

- **Statistics** is concerned with the methods of collection, organisation and analysis of numerical facts and the drawing of relevant conclusions therefrom.
- **Data collection** is the process of acquisition of (numerical) information on particular characteristics (called **variables**) of interest.
- Data is collected in any of the following ways:
  (i) By Personal Interview
  (ii) By Questionnaire
  (iii) By Telephone
- Data collected through any of the methods specified above can be represented in the form of tables or diagrams.
- **Frequency tables** show data arranged in groups or classes with the corresponding number of occurrences (or **frequency**) in each group indicated, e.g. the following numbers are displayed in a frequency table as shown below (Table 19.1).

  2, 3, 5, 10, 5, 2, 8, 10, 11, 15, 11, 2,
  5, 8, 8, 10, 2, 8, 8, 10

Table 19.1: Frequency Distribution

Number	2	3	5	8	10	11	15
Frequency	4	1	3	5	4	2	1

- **Cumulative frequency tables** are frequency tables which have an additional row containing entries formed from successive additions of the entries in the frequency row. Table 19.2 shows the cumulative frequency distribution of the numbers displayed in Table 19.1.

Table 19.2: Cumulative Frequency Distribution

Number	2	3	5	8	10	11	15
Frequency	4	1	3	5	4	2	1
Cumulative Frequency	4	5	8	13	17	19	20

- When the data collected involves a large number of observations, the frequency table is more conveniently based on **class intervals** than on pure numbers, as shown in Table 19.3.

Table 19.3: Cumulative frequency distribution of scores of students in a test

Class Interval (Scores)	Frequency (No. of students)	Cumulative Frequency
14–19	3	3
20–25	7	10
26–31	10	20
32–37	25	45
38–43	45	90
44–49	23	113
50–55	7	120

- The data represented in Table 19.2 is referred to as **discrete data** while that of Table 19.3 which is based on class intervals is **grouped data.**
- **Line charts** are diagrams in which lines are used to represent the magnitudes of frequencies (vertical axis) of numbers (horizontal axis).
- **Pie charts** are circles which are divided into component parts by drawing radii in the circles. They are used to represent distribution ratios. To draw a pie chart:
  (i) Calculate the sectorial angle θ for each entry as

$$\theta = \frac{\text{frequency of entry}}{\text{total frequency}} \times 360^{o}$$

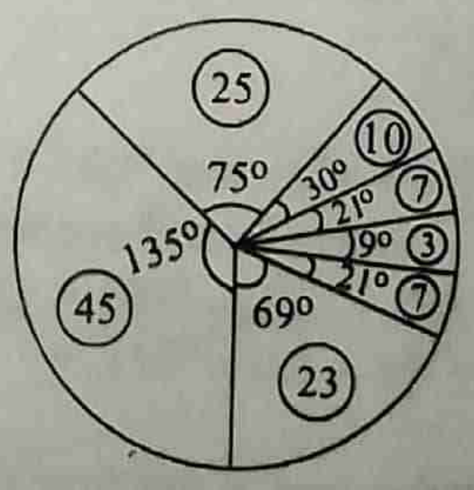
  (ii) Draw a circle of convenient radius.
  (iii) Carefully construct the angles calculated in (i).

EXAMPLE 19–1

Draw the pie chart for the data in Table 19.3.

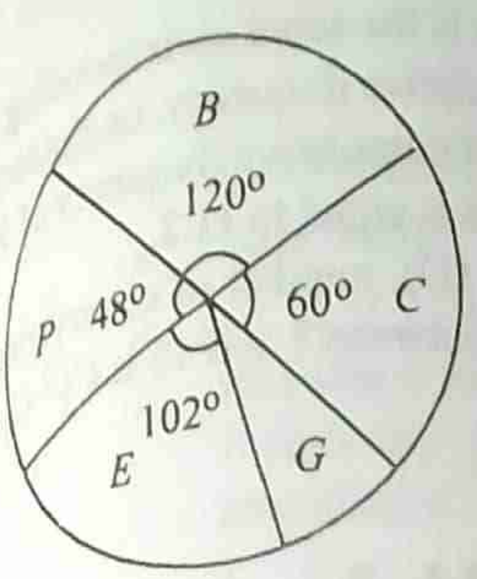
Solution

The total number of students (total frequency) is 120. The sectorial angle for the first entry with frequency 3 is (3/120) x 360^{o}, or 9^{o}. Similarly, the corresponding sectorial angles for the other rows are 21^{o}, 30^{o}, 75^{o}, 135^{o}, 69^{o}, 21^{o}. (Note that the angles must add up to 360^{o}.) The *pie chart* is shown below.



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DRILL 1



In a class of 60 students, the numbers offering Biology (B), Chemistry (C), Geography (G), Economics (E) and Physics (P) are displayed on the pie-chart shown. How many students offer (a) Biology (b) Physics and (c) Geography?

19.2 Line Diagrams

- **Bar charts** are diagrams in which a vertical bar (rectangle) is used to represent each entry with the following specifications:
  (i) the height of each bar represents the frequency of that class,
  (ii) the bars are separated, and
  (iii) the bars have equal width along the horizontal-axis.

- **Histograms** are similar to bar charts with two significant differences:
  (i) the bars are joined (not separated), and
  (ii) the end point of one class coincides with the beginning of the next class.

Fig. 19.1 shows the histogram for the data of Table 19.3.

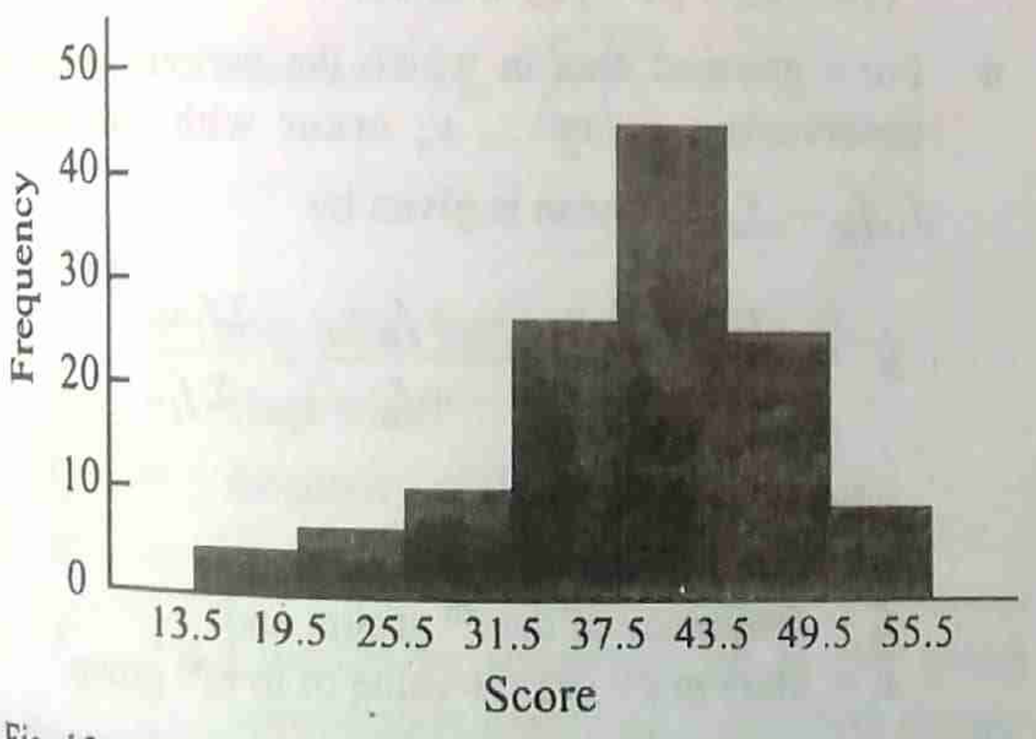


Fig. 19.1

- **Frequency polygons** show the shape of the distribution. They are constructed by either
  (i) joining the mid-points of the bar-widths in the histograms, or
  (ii) plotting the frequencies of the classes against their corresponding class marks (midpoints of the intervals) and joining the points.

- **A cumulative frequency curve** (or an **ogive**) is obtained by plotting the entries in the cumulative frequency row (or column) against the respective **true upper limits** and joining the points. It is useful for obtaining the **median** and **percentiles** of distributions.

EXAMPLE 19–2

The table below contains the distribution of the records of total scores of three dice:

Total Scores	Frequency
4–5	7
6–7	13
8–9	17
10–11	18
12–13	29
14–15	11
16–17	5

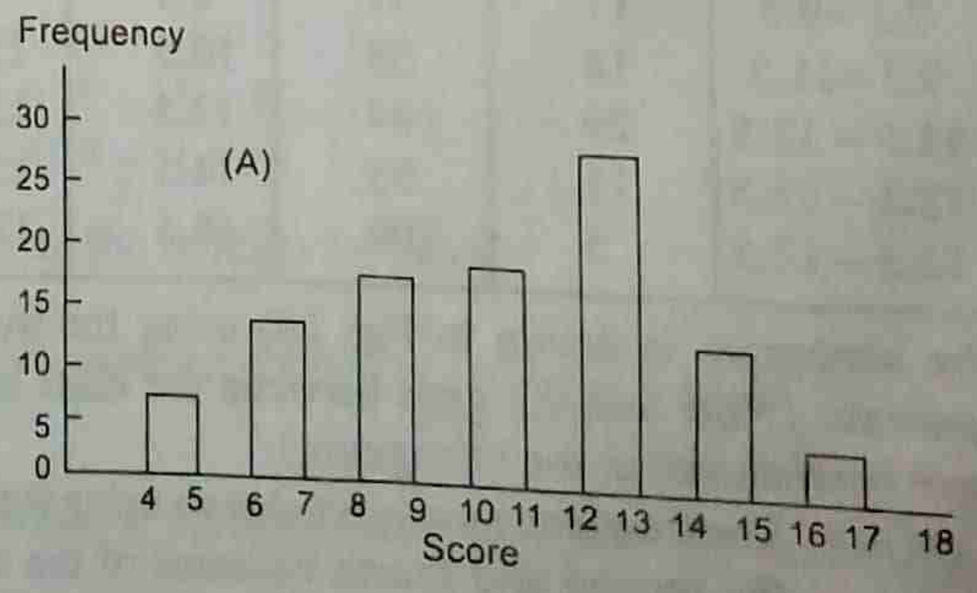
(a) Construct:
  (i) the bar chart
  (ii) the histogram
  (iii) the frequency polygon
  (iv) the ogive (cumulative frequency curve) for the distirbution.

(b) Use the ogive to estimate:
  (i) the median score
  (ii) the number of observations with sums of scores between 8 and 15.

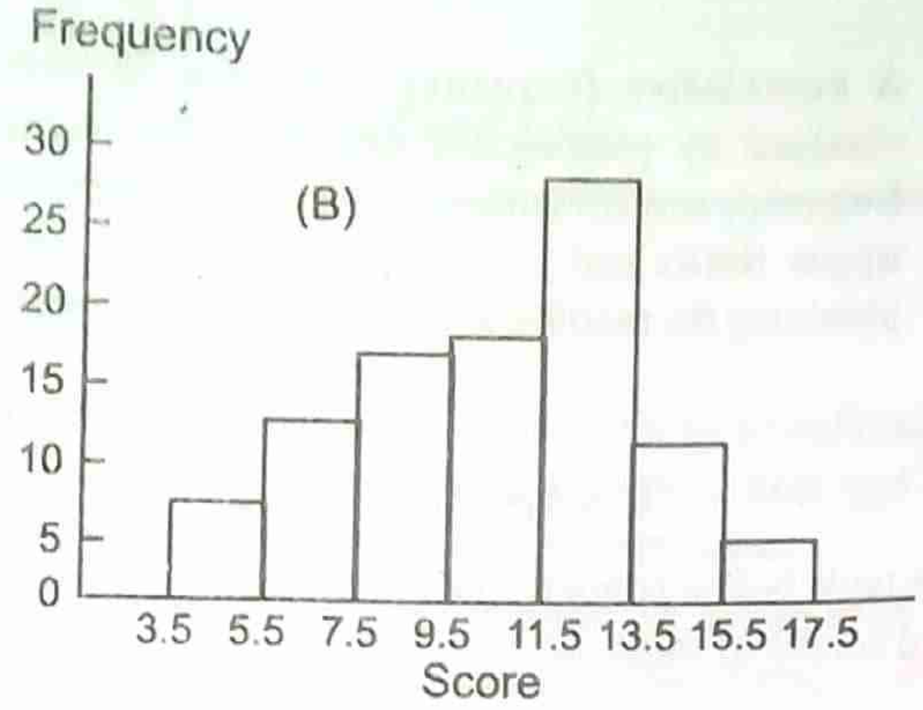
Solutions

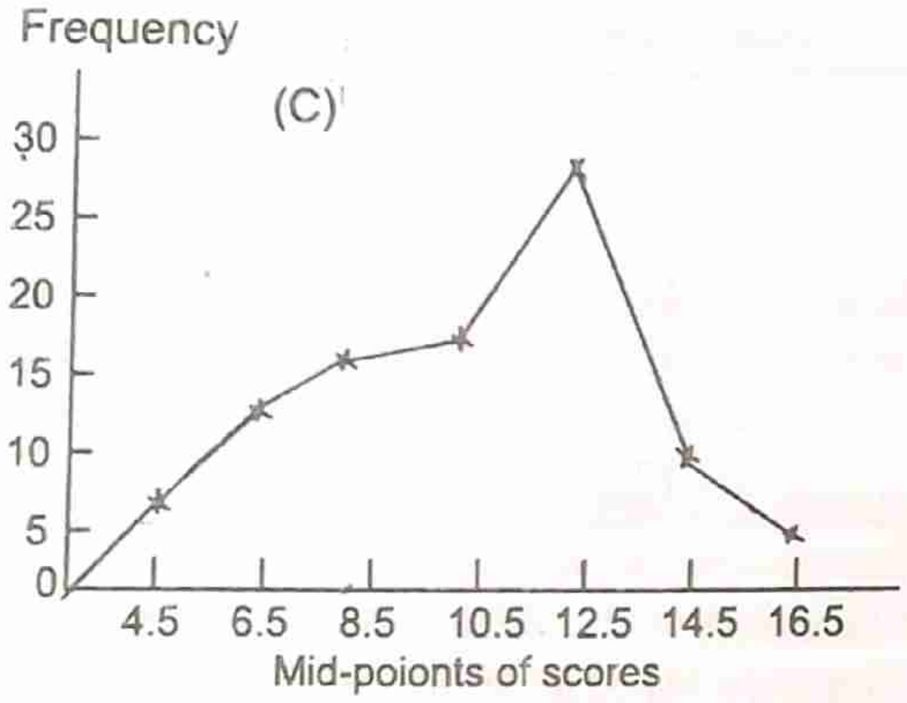
(a) (i) The bar chart is drawn directly from the data supplied, see Fig. (*A*).

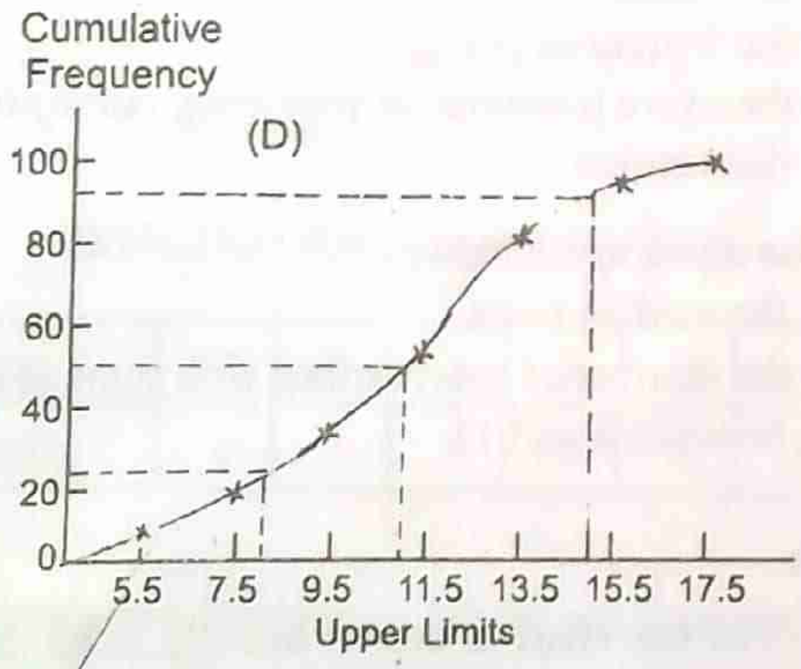
(ii) Obtain the *true class intervals* by adding half a unit on both sides of each class interval, e.g. 4 – 5 becomes 3.5 – 5.5, etc.



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True Class Intervals	Frequency f	Cumulative frequency, F	Mid-points x	Upper Limits
3.5–5.5	7	7	4.5	5.5
5.5–7.5	13	20	6.5	7.5
7.5–9.5	17	37	8.5	9.5
9.5–11.5	18	55	10.5	11.5
11.5–13.5	29	84	12.5	13.5
13.5–15.5	11	95	14.5	15.5
15.5–17.5	5	100	16.5	17.5

The histogram is drawn in Fig. (*B*) using the true class intervals. (Note that the gaps between the class intervals have been closed in the histogram.)

(iii) The frequency polygon, drawn using the data in the second and fourth columns of the table, is shown in Fig. (*C*).

(iv) The ogive is drawn using the data in the third and fifth columns of the table, see Fig. (*D*).

(b) (i) The median score is the score corresponding to half the total cumulative frequency, i.e. the score corresponding to a cumulative frequency of 50. From Fig. (*D*) this is equal to **11.2.**

(ii) Referring to Fig. (*D*), number of observations with total scores between 8 and 15 is $C_2 - C_1 = 94 - 24 = 70$.

DRILL 2

(a) A set of numbers is grouped as

1–5, 6–10, 11–15, 16–20.

Find (i) the size of the class interval.
(ii) the true class boundaries.

(b) From the ogive (Fig. *D*) of Example 19–2, find the number of observations with total scores
(i) less than 10
(ii) between 10 and 14.

19.3 Mean, Mode and Median

- The **mean**, $\bar{x}$, of a set of observations $x_1, x_2, \cdots, x_n$ is given by

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

where n is the total number of observations. For example, the mean of the numbers, 10, 12, 14, 16 is (10 + 12 + 14 + 16)/4, or 13.

- For a grouped data in which the various groups of observations $x_1, x_2 \ldots, x_n$ occur with frequencies $f_1, f_2, \cdots, f_n$ the mean is given by

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \cdots + f_n x_n}{f_1 + f_2 + \cdots + f_n} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

where
n = total number of groups
f_i = frequency of the i^{th} group; and
x_i = class mark or mid-value of the i^{th} group.

- The **mode** is the value that occurs most frequently in a given data.
- In a frequency table, it is the value which has the **highest** frequency.
- In a continuous (grouped) data, the mode is within the class interval which has the highest frequency. This class interval is called the **modal class**. The mode is given by the formula

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$$\text{Mode} = L + c\left(\frac{d_1}{d_1 + d_2}\right)$$

where

L = true lower limit of the modal class.

c = class width

d_1 = $f_m - f_{m-1}$ = difference between the frequency of the modal class and the frequency of the class immediately preceding it.

d_2 = $f_m - f_{m+1}$ = difference between the frequency of the modal class and the frequency of the class immediately succeeding it.

- The **median** is the value which lies in the middle of a set of data when the values are *arranged in order.*
- To find the median of a given set of data:
  (i) Arrange the data in ascending (or descending) order of magnitude.
  (ii) If the number (n) of observations is odd, the median is the value occupying the

$$\left(\frac{n+1}{2}\right)\text{th position.}$$

  (iii) If n is even, the median is the average of the two middle values, i.e.

$$\frac{1}{2}\left[\left(\frac{n}{2}\right)\text{th value} + \left(\frac{n+2}{2}\right)\text{th value}\right]$$

- In a grouped data, the median is given by

$$\text{Median} = L_m + \frac{c}{f_m}\left(\frac{N}{2} - F\right)$$

where

L_m = true lower limit of the median class

c = class width

f_m = frequency of the median class

N = sum of all the frequencies = Σf_i

F = Cumulative frequency, up to the group immediately preceding the median group.

EXAMPLE 19–3

Calculate the (a) mean, (b) mode and (c) median of the following set of numbers:

3, 5, 6, 7, 5, 4, 6, 4, 4, 5, 4, 7

Solutions

(a) Sum of the numbers = 3 + 5 + · · + 7 = 60
Number of observations, n = 12

$$\text{Mean, } \bar{x} = \frac{60}{12} = 5$$

(b) Mode = 4, which occurs four times.
(c) The data is re-arranged in ascending order:

3, 4, 4, 4, 4, 5, 5, 5, 6, 6, 7, 7

Median = average of the sixth and the seventh observations = (5 + 5)/2 = 5.

DRILL 3

Find the (a) mean, (b) mode and (c) median of the numbers

2, 0, 7, 3, 4, 8, 4, 6, 3, 3

EXAMPLE 19–4

Find the (a) mean, (b) mode and (c) median of the distribution given in Example 19–2.

Solutions

Total Scores	x_i	f_i	$f_i x_i$
4–5	4.5	7	31.5
6–7	6.5	13	84.5
8–9	8.5	17	144.5
10–11	10.5	18	189.0
12–13	12.5	29	362.5
14–15	14.5	11	159.5
16–17	16.5	5	82.5
		100	1054.0

(a) From the table, mean $\bar{x} = \Sigma f_i x_i / \Sigma f_i = 1054/100$
$= \mathbf{10.54}$

(b) The modal class is 12–13 (having the highest frequency of 29).
True lower limit, L = 11.5.
$c = 13.5 - 11.5 = 2$,
$d_1 = 29 - 18 = 11$,
$d_2 = 29 - 11 = 18$

Using the formula for *mode*,

$$\text{Mode} = 11.5 + 2\left[\frac{11}{11 + 18}\right] = \mathbf{12.26}$$

[Note that the mode must lie within the modal class.]

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(c) Median class is that which contains the 50th value (since $N/2 = 100/2 = 50$); this is the class 10– 11.
$L_m = 9.5$, $c = 2$, $f_m = 18$, $F = 7 + 13 + 17 = 37$

Using the formula for grouped data,

$$\text{Median} = 9.5 + \frac{2}{18}(50 - 37) = \mathbf{10.94}$$

DRILL 4

Calculate the (a) mean, (b) mode and (c) median for the data provided in the table below:

Number	0	2	6	8	9	10	12
Frequency	4	3	7	5	4	2	1

19.4 Range, Mean Deviation

- The **range** of a set of data is the difference between the largest value and the smallest value in the distribution.
- The **mean deviation** (*M.D.*) is a measure of the deviation of a set of numbers from the mean. For a discrete set of data it is given by

$$M.D. = \frac{1}{n}\sum_{i=1}^{n} |x_i - \bar{x}|$$

where x_i = i^{th} observation
$\bar{x}$ = mean, and
$|x_i - \bar{x}|$ = absolute value of the difference between the i^{th} observation and the mean.

- The mean deviation for a grouped set of data is

$$M.D. = \frac{\Sigma f_i |x_i - \bar{x}|}{\Sigma f_i}$$

where x_i = mid-point of the i^{th} class and
f_i = frequency of the i^{th} class.

EXAMPLE 19–5

Find the mean deviation for the distribution given in Example 19–2.

Solution

Since this is a grouped data, a table is prepared showing x_i, f_i, $f_i x_i$, $x_i - x$, and $f_i|x_i - x|$.
The mean $\bar{x}$ = 1054/100 = 10.54. Taking $\bar{x}$ =10.5 for convenience, the values of $|x_i - \bar{x}|$ are listed in the fourth column.

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
4.5	7	31.5	6.0	42
6.5	13	84.5	4.0	52
8.5	17	144.5	2.0	34
10.5	18	189.0	0.0	0
12.5	29	362.5	2.0	58
14.5	11	159.5	4.0	44
16.5	5	82.5	6.0	30
	100	1054.0		260

$$M.D. = \frac{\Sigma f_i |x_i - \bar{x}|}{\Sigma f_i} = \frac{260}{100} = \mathbf{2.6}$$

DRILL 5

Find the (a) range and (b) mean deviation for the set of numbers 2, 5, 6, 1, 5, 4, 3, 4, 3, 7.

19.5 Variance and Standard Deviation

- **Variance** is defined as the mean of the sum of the squares of the deviations from the mean.
- For a *discrete* set of data, the variance is given by

$$S^2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

- For a *grouped* set of data the variance is

$$S^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}$$

- **Standard deviation**, S, is the square root of the variance.
- To determine the variance and standard deviation for a *discrete* set of data:
  (i) Calculate the mean ..... $\bar{x}$
  (ii) Create a column for ..... $x_i - \bar{x}$
  (iii) Create a column for ..... $(x_i - \bar{x})^2$
  (iv) Find the sum of all entries in (iii) ..... $\Sigma (x_i - \bar{x})^2$
  (v) Divide (iv) by the number of entries to obtain the variance.
  (vi) Take the square root of (v) to obtain the standard deviation.

EXAMPLE 19–6

Find the variance and the standard deviation for the following set of numbers:

2, 4, 6, 8, 10, 12

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Solution

Since there are six entires, $n = 6$.

Mean $\bar{x} = (2 + 4 + 6 + 8 + 10 + 12)/6 = 42/6 = 7$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
2	–5	25
4	–3	9
6	–1	1
8	1	1
10	3	9
12	5	25
		70

Variance $= S^2 = 70/6 = \mathbf{11.7}$

Standard Deviation $= S = \sqrt{11.7} = \mathbf{3.4}$

- To determine the variance and standard deviation for a *grouped* set of data:
  - (i) Calculate the mean ..... $\bar{x}$
  - (ii) Create a column for ..... $x_i - \bar{x}$
  - (iii) Create a column for ..... $(x_i - \bar{x})^2$
  - (iv) Create a column listing $(x_i - x)^2$ multiplied by the corresponding frequency, f_i ..... $f_i (x_i - \bar{x})^2$
  - (v) Find the sum of all entries in (iv) ..... $\Sigma f_i (x_i - \bar{x})^2$
  - (vi) Find the sum of the entries in the frequency column ..... Σf_i
  - (vii) Divide (v) by (vi) to obtain the variance..... S^2
  - (viii) Take the square root of (vii) to obtain the standard deviation, S.

EXAMPLE 19–7

Find the variance and standard deviation for the distribution given in Example 19–2.

Solution

Taking the mean, $\bar{x} = 10.5$ (see Example 19–4) a table is drawn up as follows:

x_i	f_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
4.5	7	–6	36	252
6.5	13	–4	16	208
8.5	17	–2	4	68
10.5	18	0	0	0
12.5	29	2	4	116
14.5	11	4	16	176
16.5	5	6	36	180
	100			1000

Variance, $S^2 = 1000/100 = 10$

Standard deviation, $S = \sqrt{10} = \mathbf{3.16}$

DRILL 6

Calculate the (a) variance and (b) standard deviation for the numbers 1, 3, 5, 7, 9, 11.

ANSWERS TO DRILLS

1. (a) 20 (b) 8 (c) 5
2. (a) (i) 5 (ii) 0.5, 5.5, 10.5, 15.5, 20.5
   (b) (i) 40 (ii) 48
3. (a) 4 (b) 3 (c) 3.5
4. (a) 6 (b) 6 (c) 6
5. (a) 6 (b) 1.4
6. (a) 11.7 (b) 3.4

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SSCE/JME–TYPE OBJECTIVE QUESTIONS

The scores of students in a class test are given in the table below. Use the information to answer questions **1** to **5.**

Marks	1	2	3	4	5	6	7	8	9	10
Frequency	0	3	1	2	3	4	2	1	1	0

1. The number of students who took the test is
   A. 15 **B.** 17 **C.** 20 **D.** 55

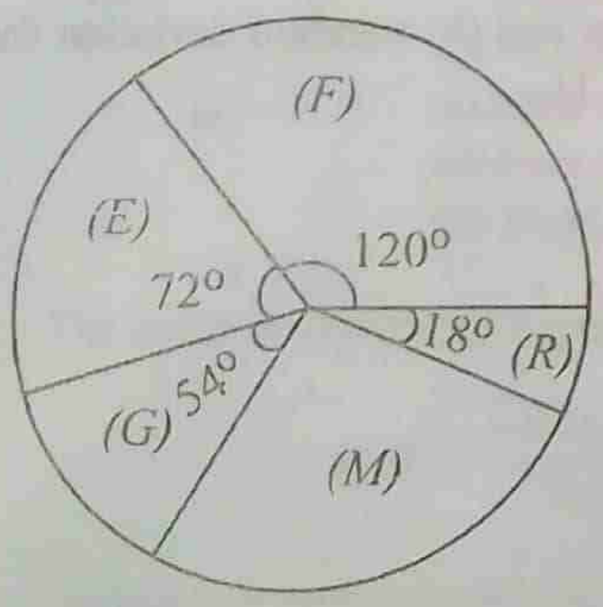
2. The number of students who scored at least 5 is
   A. 3 **B.** 9 **C.** 10 **D.** 11

3. The mean score is
   A. 5.1 **B.** 5.0 **C.** 4.9 **D.** 4.8

4. The modal score is
   A. 4 **B.** 5 **C.** 6 **D.** 7

5. The median score is
   A. 3 **B.** 4.5 **C.** 5 **D.** 5.5

6. The angles of the sectors representing the numbers 5, 8, 12, 15, 20 on a pie chart are, respectively,
   A. $5^o, 8^o, 12^o, 15^o, 20^o$
   B. $15^o, 24^o, 36^o, 45^o, 60^o$
   C. $30^o, 48^o, 72^o, 90^o, 120^o$
   D. $40^o, 64^o, 96^o, 120^o, 160^o$

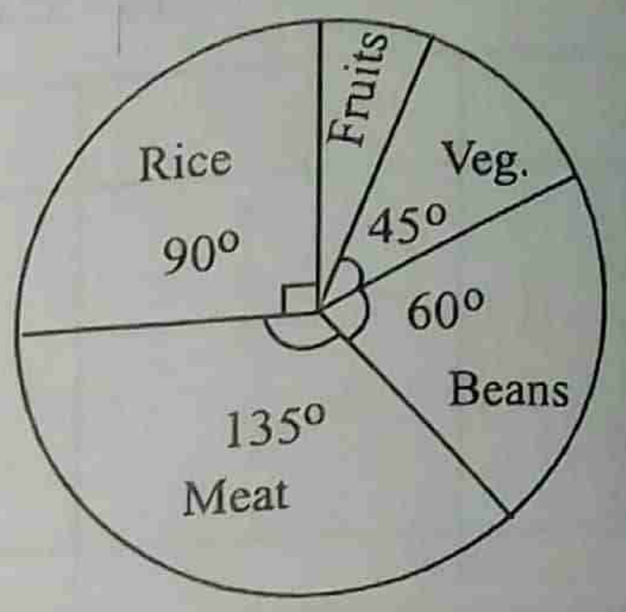


The total income of a household for one year is ₦60,000. This amount is spent on rent (*R*), food (*F*), education of the children (*E*), general maintenance (*G*) and miscellaneous items (*M*). Use this information to answer questions 7 and 8.

7. The amount spent on food items is
   A. ₦30,000 **B.** ₦20,000
   C. ₦15,000 **D.** ₦10,000

8. The amount spent on miscellaneous items is
   A. ₦25,000 **B.** ₦20,000
   C. ₦17,500 **D.** ₦16,000

9. The pie chart below shows the distribution of the expenditure of a household on various food items in a certain month. If ₦200 was spent on fruits, how much was spent on rice?



   A. ₦400 **B.** ₦600
   C. ₦800 **D.** ₦1,000

10. The distribution of grades in a Mathematics examination taken by a class of 72 students is represented on a pie chart. If the angle of the sector representing the number of students who scored the *A* grade is 30^o, the actual number of *A* grades scored is
    A. 6 **B.** 8 **C.** 9 **D.** 12

11. If 54 students passed the examination (question 10), the angle of the sector corresponding to the number of failures is
    A. 120^o **B.** 90^o **C.** 60^o **D.** 45^o

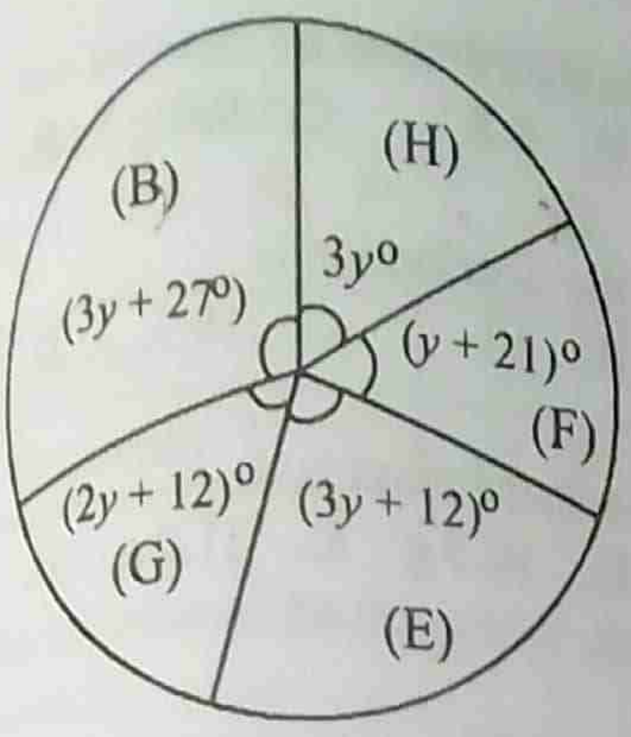
12. The numbers 2, 5, 7, 8, 10, 15 and 13 are represented on a pie chart. The sum of the angles representing all numbers equal to or greater than 8 is
    A. 276^o **B.** 228^o **C.** 84^o **D.** 48^o

13. If a 24-hour day is represented on a pie chart, what is the angle of the sector corresponding to the period between 3 p.m. on a Monday and 6 a.m. on the following Tuesday?
    A. 270^o **B.** 240^o **C.** 225^o **D.** 180^o

14. A sector on a pie chart representing the number of students offering Physics has an angle of 24^o. If 8 students offer Physics, the total number of students in the class is
    A. 80 **B.** 96 **C.** 100 **D.** 120

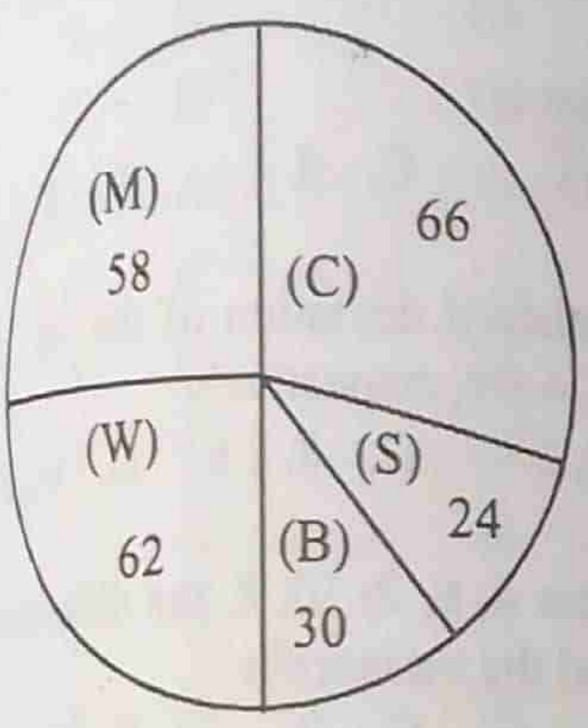
15. The distribution of a housewife's expendture on four food items is as follows: Item *A* (₦80), Item *B* (₦45), Item *C* (₦30), Item *D* (₦25). The sum of the angles of the sectors representing the expenditure on items *B* and *D* is
    A. 70^o **B.** 90^o **C.** 140^o **D.** 220^o

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16. 

The diagram shows the distribution of the number of candidates offering Biology (*B*), Geography (*G*), Economics (*E*), French (*F*) and History (*H*) in a school certificate class of 60 students. The number of candidates offering Economics is

A. 14 B. 16 C. 18 D. 20



The pie chart illustrates the distribution of the number of married men (*M*), married women (*W*), bachelors (*B*), spinsters (*S*) and children (*C*) at a social event. Use the information provided to answer questions **17** and **18.**

17. The angle of the sector representing spinsters is

A. 24° B. 36° C. 45° D. 48°

18. The angle representing the total number of married perons is

A. 87° B. 93° C. 120° D. 180°

The goals scored by 32 football teams in a world cup match are recorded below:

No. of goals	0	1	2	3	4	5
Frequency	5	6	12	10	2	1

Use this information to answer questions **19** and **20.**

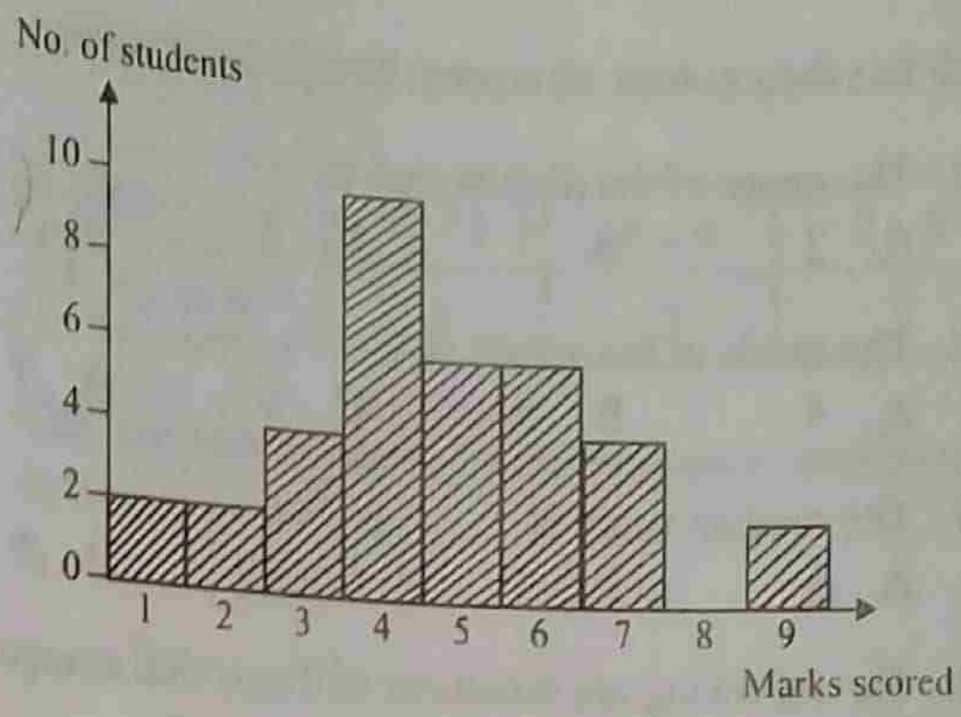
19. The total number of goals scored by all the teams is

A. 70 B. 72 C. 73 D. 78

20. If $cF(x)$ is the number of goals less than or equal to x, then $cF(1)$ and $cF(3)$ are, respectively

A. 5, 23 B. 11, 33 C. 6, 10 D. 11, 23

The diagram below illustrates the distribution of marks in a class test. Use the information in the diagram to answer questions **21** to **23.**



21. The total number of students who sat for the test is

A. 35 B. 36 C. 40 D. 48

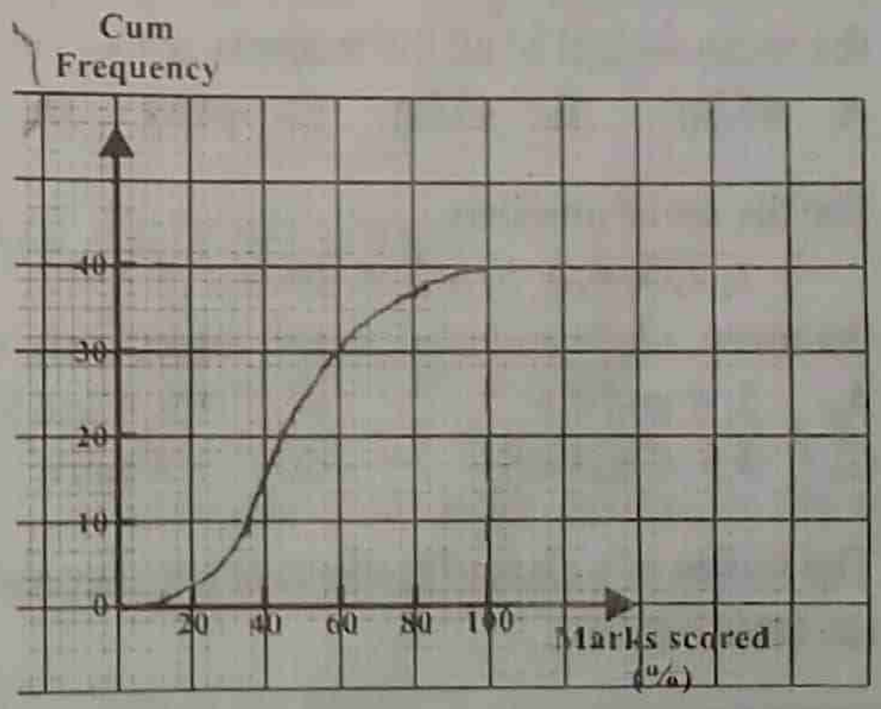
22. The percentage of students who scored less than 5 marks is

A. 75% B. 67% C. 50% D. 33%

23. The average and the modal marks scored are, respectively,

A. 4.7, 4 B. 4, 5 C. 5, 4 D. 4.7, 5

The diagram below shows the ogive for the scores obtained by 40 students in a Mathematics test. Use the figure to answer questions **24** to **27.**



24. The *median* score is

A. 20% B. 40% C. 44% D. 50%

25. The *upper quartile* is

A. 55% B. 60% C. 75% D. 80%

26. The *interquartile* range is

A. 50% B. 36% C. 30% D. 24%

27. The *percentile* corresponding to a score of 80% is

A. 66 B. 80 C. 85 D. 90

In a class of 20 students the scores in a Mathematics test out of 10 marks are as follows:

5, 3, 6, 7, 8, 4, 5, 4, 9, 3
4, 6, 4, 5, 7, 4, 5, 6, 7, 2

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Use this information to answer questions 28 to 30.

28. The range of the distribution is
A. 2 B. 7 C. 8 D. 9

29. The mode of the scores is
A. 4 B. 5 C. 6 D. 7

30. .The median score is
A. 3 B. 4 C. 4.5 D. 5

31. The following are measures of dispersion **except**
A. mean deviation B. mode
C. range D. standard deviation

32. The following are measures of central tendency **except**
A. mean B. median
C. mode D. mean deviation

33. In an experiment, the mean of 25 observations was 6. One of the observations, which has the value of 30, was cancelled. The mean of the remaining observations is
A. 3.5 B. 4.5 C. 4.8 D. 5

34. The mean weights of three groups of students are 40 kg, 45 kg and 50 kg respectively. If the number of students in the groups are 45, 30 and 25 respectively, the mean weight of all the students is
A. 46 kg B. 45 kg C. 44 kg D. 43 kg

35. For the set of numbers
1, 2, 3, 4, 4, 5, 6, 7, 8, 10
the mean, mode and median are, respectively
A. 5, 4 and 4.5 B. 4.5, 4 and 5
C. 4.5, 4 and 4.5 D. 5, 4 and 4

36. The scores of a class of students in a test are shown in the table below:

Scores	2	3	4	5	6	7
No. of students	1	3	5	x	4	2

If the average score is $4\frac{2}{3}$, the value of x is
A. 3 B. 4 C. 5 D. 6

37. The mean age of a class of twenty school pupils is 12. If their teacher's age is included, the mean becomes 13. The age of the teacher is
A. 25 B. 28 C. 33 D. 39

The table below shows the weight distribution of a group of potatoes. Use the table to answer questions **38** to **40.**

Weight (g)	1–5	6–10	11–15	16–20	21–25
No. of potatoes	3	7	5	3	2

38. The mode of the distribution is
A. 8.0 B. 8.8 C. 9.0 D. 9.8

39. The median of the distribution is
A. 9.5 B. 10 C. 10.5 D. 11

40. The mean of the distribution is
A. 10.0 g B. 10.5 g C. 11.0 g D. 11.5 g

The scores of a group of five students are 4, 5, 6, 7, 8. Use this information for questions **41** to **43.**

41. The mean deviation is
A. 1.0 B. 1.2 C. 1.5 D. 2.0

42. The standard deviation is
A. $\sqrt{2}$ B. $\sqrt{3}$ C. 2 D. 3

43. The variance and standard deviation of the set of numbers –4, –2, 0, 2, 4 are, respectively
A. 0, 0 B. 4, 2 C. 8, 2.8 D. 64, 8

44. For the set of numbers 4, 6, 2, 10, 8, the difference between the range and the variance is
A. –1 B. 1 C. 2 D. 0

45. The average age of the three children in a family is 9 yr. If the average age of their parents is 39 yr., the average age of the whole family is
A. 20 yr B. 21 yr C. 24 yr D. 27 yr

46. The heights of eight children are:
1.0 m, 1.2 m, 0.9 m, 0.8 m, 1.4 m, 0.8 m, 1.2 m and 1.1 m.
The sum of the mean and the range is
A. 2.45 m B. 1.85 m
C. 1.65 m D. 1.11 m

47. The following data were obtained from the analysis of six observations of a variable x.
$n = 6$, $\Sigma x = 42$, $\Sigma x^2 = 348$
$\Sigma |x - \bar{x}| = 18$, $\Sigma (x - \bar{x})^2 = 70$.
The mean deviation and the variance are, respectively
A. 3 and 11.7 B. 0.4 and 3
C. 2.3 and 5 D. 3 and 1.7

48. The table below shows the frequency distribution of marks scored by students in a class test :

Score	1	2	3	4	5
Frequency	$x + 2$	$x - 1$	$2x - 3$	$x + 4$	$3x - 4$

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If the average score is 43/14, how many students scored 3 points?
A. 1 B. 3 C. 5 D. 7

49.

Number	1	2	3	4	5
Frequency	1	3	2	3	1

The variance of the frequency distribution in the table above is
A. 1.0 B. 1.2 C. 1.4 D. 1.5

50.

Class	1 – 3	4 – 6	7 – 9
Frequency	5	8	5

The standard deviation of the frequency distribution in the table above is
A. 5 B. $\sqrt{5}$ C. $\sqrt{6}$ D. 1

ANSWER KEY

1.	B	10.	A	19.	C	28.	B	37.	C	46.	C
2.	D	11.	B	20.	B	29.	A	38.	B	47.	A
3.	A	12.	A	21.	B	30.	D	39.	C	48.	A
4.	C	13.	C	22.	C	31.	B	40.	D	49.	C
5.	C	14.	D	23.	A	32.	D	41.	B	50.	B
6.	C	15.	C	24.	C	33.	D	42.	A		
7.	B	16.	A	25.	B	34.	C	43.	C		
8.	D	17.	B	26.	D	35.	A	44.	D		
9.	B	18.	D	27.	D	36.	A	45.	B		

SOLUTIONS TO OBJECTIVE QUESTIONS

1. (B) This is the *cumulative frequency* for all scores,
or
$0 + 3 + 1 + 2 + 3 + 4 + 2 + 1 + 1 + 0 = 17$

2. (D) This is the sum of the frequencies of marks from 5 to 10
$= 3 + 4 + 2 + 1 + 1 + 0 = 11$

3. (A) Total marks scored
$= (1x0) + (2x3) + (3x1) + (4x2) + (5x3) + (6x4) + (7x2) + (8x1) + (9x1) + (10x0) = 87$
Total frequency = 17

$$\text{Mean score} = \frac{\text{Total marks scored}}{\text{Total frequency}} = \frac{87}{17} = 5.1$$

4. (C) Modal score is the score with the highest frequency. From the table, this is 6 (with a frequency of 4).

5. (C) Since the total number of observations (17) is not too large, the median is readily obtained by listing out the scores in order:
2, 2, 2, 3, 4, 4, 5, 5, 5, 6, 6, 6, 6, 7, 7, 8, 9
The median score is the middle, or 9th score, which is equal to 5.

6. (C) Sum of the numbers
$= 5 + 8 + 12 + 15 + 20 = 60$
The sectorial angles are:
For number 5, $(5/60 \times 360^o) = 30^o$
For number 8, $(8/60 \times 360^o) = 48^o$,
and so on.

7. (B) Angle of the sector for food = 120^o

$$\text{Amount spent on food} = \frac{120^o}{360^o} \times ₦60{,}000 = ₦20{,}000$$

8. (D) Angle of the sector for miscellaneous items
$= 360^o - (18^o + 120^o + 72^o + 54^o) = 96^o$
Amount spent on miscellaneous items

$$= \frac{96^o}{360^o} \times ₦60{,}000 = ₦16{,}000$$

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9. (B) Angle of the sector for fruits
$= 360^o - (90^o + 135^o + 60^o + 45^o) = 30^o$
If T = total expenditure, then
$\frac{30^o}{360^o} \times T = ₦200$, or $T = ₦2{,}400$
Amount spent on rice $= \frac{90^o}{360^o} \times ₦2{,}400$
$= ₦600.$

10. (A) Let x = number of A grades
Then $\frac{x}{72} \times 360^o = 30^o$, or $x = 6$

11. (B) Number of failures $= 72 - 54 = 18$
Sectorial angle for failures $= \frac{18}{72} \times 360^o$
$= 90^o$

12. (A) Sum of all the numbers
$= 2 + 5 + 7 + 8 + 10 + 15 + 13 = 60$
Sum of numbers ≥ 8 is
$8 + 10 + 15 + 13 = 46$
Sectorial angle $= \frac{46}{60} \times 360^o = 276^o$

13. (C) Number of hours from 3 pm to 6 am = 15
Sectorial angle $= \frac{15}{24} \times 360^o = 225^o$

14. (D) Let N = total number of students.
Then $\frac{8}{N} \times 360^o = 24^o$, or $N = 120$

15. (C) Total expenditure
$= ₦80 + ₦45 + ₦30 + ₦25 = ₦180$
Sum of expenditure on B and D
$= ₦45 + ₦25 = ₦70$
Sectorial angle $= 70/180 \times 360^o = 140^o$

16. (A) Sum of all the angles of the sectors
$= 3y^o + (3y + 27)^o + (2y + 12)^o$
$+ (3y + 12)^o + (y + 21)^o = 360^o$
$12y^o + 72^o = 360^o$, or $y = 24^o$
Sectorial angle for Economics
$= 3y + 12$
$= (3 \times 24) + 12 = 84^o$
Number offering Economics
$= \frac{84^o}{360^o} \times 60 = 14$

17. (B) Total number of people
$= 58 + 62 + 30 + 24 + 66 = 240$
Sectorial angle for spinsters
$= \frac{24}{240} \times 360^o = 36^o$

18. (D) Number of married men and women
$= 58 + 62 = 120$
Sectorial angle for married persons
$= \frac{120}{240} \times 360^o = 180^o$

19. (C) Total number of goals scored
$= (0x5) + (1x6) + (2x12) + (3x10)$
$+ (4x2) + (5x1) = 73$

20. (B) $cF(1) = 5 + 6 = 11$
$cF(3) = 5 + 6 + 12 + 10 = 33$

21. (B) Total number of students
$= 2 + 2 + 4 + 10 + 6 + 6 + 4 + 0 + 2$
$= 36$

22. (C) Number of students who scored less than 5
$= 2 + 2 + 4 + 10 = 18$
Percentage $= (18/36) \times 100 = 50\%$

23. (A) Total number of marks scored
$= (1x2) + (2x2) + (3x4) + (4x10)$
$+ (5x6) + (6x6) + (7x4) + (8x0)$
$+ (9x2) = 170$
Average mark $= \frac{\text{Total numberof marks}}{\text{Total number of students}}$
$= \frac{170}{36} = 4.7$
Modal score = score with highest frequency = 4

24. (C) Since the overall cumulative frequency = 40, the 50th percentile corresponds to a cumulative frequency of 20.
Median score = score corresponding to the 50th percentile
= 44%

25. (B) The upper quartile corresponds to the 75th percentile, i.e. a cumulative frequency
= 75% of 40 = 30.
Corresponding score = 60% (from the graph).

26. (D) The lower quartile corresponds to the 25th percentile, i.e. a cumulative frequency
= 25% of 40 = 10.
Corresponding score = 36% (from the graph).
Interquartile range = upper quartile
– lower quartile
= 60% – 36% = 24%

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27. (D) The score of 80% corresponds to a cumulative frequency of 36, i.e. a percentile of 36/40 x 100 = 90.

28. (B) Range = highest score – lowest score
= 9 – 2 = 7.

29. (A) Mode = score with highest frequency
= 4 (which occurs *five* times)

30. (D) Re-arrange the marks in order as:
2, 3, 3, 4, 4, 4, 4, 4, 5, 5,
5, 5, 6, 6, 6, 7, 7, 7, 8, 9
Since the total number of marks is 20, the median score is the average of the 10th and 11th scores = 5.

31. (B) *Mode* is a measure of **central tendency.**

32. (D) *Mean deviation* is a measure of **dispersion.**

33. (D) Sum of the original 125 observations
= 25 x 6 = 150
Sum of the remaining 24 observations
= 150 – 30 = 120
Mean = 120/24 = 5

34. (C) Total weight of first group of students
= 40 x 45 = 1,800 kg
Total weight of second group of students
= 45 x 30 = 1,350 kg
Total weight of third group of students
= 50 x 25 = 1,250 kg
Total weight of all three groups
= 1,800 + 1,350 + 1,250 = 4,400 kg
Total number of students
= 45 + 30 + 25 = 100
Mean weight of all students
$= \frac{4,400}{100} = 44$ kg

35. (A) Sum of the numbers = 50
Since there are ten numbers,
mean = 50/10 = 5
Mode = 4, which occurs twice.
Median = average of the 5th and 6th numbers = (4 + 5)/2 = 4.5

36. (A) Total score
= (2x1) + (3x3) + (4x5) + (5x) + (6x4) + (7x2) = 5x + 69.
Total number of students
= 1 + 3 + 5 + x + 4 + 2
= x + 15

Average score $= \frac{5x + 69}{x + 15} = \frac{14}{3}$
$15x + 207 = 14x + 210$, or $x = 3$.

37. (C) Total age of the 20 pupils = 12x20 = 240
Let x = teacher's age
Mean age of pupils and teacher is
$\frac{240 + x}{21} = 13$
$240 + x = 273$, or $x = 33$.

38. (B) Modal class is 6 – 10.
$L = 5.5$, c = 5, $d_1 = 7–3 = 4$, $d_2 = 7–5 = 2$

$$\text{Mode} = L + c\left[\frac{d_1}{d_1 + d_2}\right]$$

$$= 5.5 + 5\left[\frac{4}{4 + 2}\right] = 8.8$$

39. (C) Total number of potatoes = 20
Median class = class with the 10th potato
= 5.5 – 10.5
$L = 5.5, c = 5, f_m = 7, N = 20, F = 3$

$$\text{Median} = L_m + \frac{c}{f_m}\left[\frac{N}{2} - F\right]$$

$$= 5.5 + \frac{5}{7}\left[\frac{20}{2} - 3\right] = 10.5$$

40. (D) Re-draw the distribution table using the mid-values of the weight classes:

Weight, x_i	3	8	13	18	23
Frequency, f_i	3	7	5	3	2
$f_i x_i$	9	56	65	54	46

From the table,
$\Sigma f_i x_i = 9 + 56 + 65 + 54 + 46 = 230$
$\Sigma f_i = 3 + 7 + 5 + 3 + 2 = 20$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{230}{20} = 11.5 \text{ g}$$

41. (B) Mean, $\bar{x} = (4 + 5 + 6 + 7 + 8)/5 = 6$
$x_i - \bar{x} = -2, -1, 0, 1, 2$
$|x_i - \bar{x}| = 2, 1, 0, 1, 2$
$\Sigma |x_i - \bar{x}| = 2 + 1 + 0 + 1 + 2 = 6$, and $n = 5$

$$\text{Mean Deviation} = \frac{\Sigma |x_i - \bar{x}|}{n} = \frac{6}{5} = 1.2$$

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42. **(A)** $(x_i - \bar{x})^2 = 4, 1, 0, 1, 4$

$\Sigma (x_i - \bar{x})^2 = 4 + 1 + 0 + 1 + 4 = 10$

Variance, $S^2 = \dfrac{\Sigma (x_i - \bar{x})^2}{n} = \dfrac{10}{5} = 2$

Standard Deviation, $S = \sqrt{2}$

43. **(C)** Mean, $\bar{x} = (-4, -2 + 0 + 2 + 4)/5 = 0$

$x_i - \bar{x} = -4, -2, 0, 2, 4$

$(x_i - \bar{x})^2 = 16, 4, 0, 4, 16$

$\Sigma (x_i - \bar{x})^2 = 16 + 4 + 0 + 4 + 16 = 40$

Variance $= \dfrac{\Sigma (x_i - \bar{x})^2}{n} = \dfrac{40}{5} = 8$

Standard Deviation $= \sqrt{8} = 2\sqrt{2} = 2.8$

44. **(D)** Mean, $\bar{x} = (4 + 6 + 2 + 10 + 8)/5 = 6$

$x_i - \bar{x} = -2, 0, -4, 4, 2$

$(x_i - \bar{x})^2 = 4, 0, 16, 16, 4$

$\Sigma (x_i - \bar{x})^2 = 4 + 0 + 16 + 16 + 4 = 40$

Variance $= \dfrac{\Sigma (x_i - \bar{x})^2}{n} = \dfrac{40}{5} = 8$

Range $= 10 - 2 = 8$

Range – Variance $= 8 - 8 = 0$.

45. **(B)** Total age of the three children

$= 9 \times 3 = 27$ yr

Total age of the two parents

$= 39 \times 2 = 78$ yr

Total age of the family $= 27 + 78 = 105$ yr

Average age of the family $= 105/5 = 21$ yr

46. **(C)** Sum of the heights $= 8.4$ m

Mean height $= 8.4/8 = 1.05$ m

Range $= 1.4 - 0.8 = 0.6$ m

Mean + Range $= 1.05 + 0.6 = 1.65$ m

47. **(A)** Mean Deviation $= \dfrac{\Sigma (x_i - \bar{x})}{n} = \dfrac{18}{6} = 3$

Variance $= \dfrac{\Sigma (x_i - \bar{x})^2}{n} = \dfrac{70}{6} = 11.7$

48. **(A)** Total score

$= 1(x+2) + 2(x-1) + 3(2x-3)$

$+ 4(x+4) + 5(3x-4) = 28x - 13$

Total frequency

$= (x+2) + (x-1) + (2x-3) + (x+4)$

$+ (3x-4) = 8x - 2$

Average score $= \dfrac{28x - 13}{8x - 2} = \dfrac{43}{14}$

$392x - 182 = 344x - 86$

$48x = 96$, or $x = 2$

Number of candidates who scored 3 points

$= 2x - 3 = 2(2) - 3 = 1$

49. **(C)** Mean $\bar{x}$

$= \dfrac{(1\times1) + (2\times3) + (3\times2) + (4\times3) + (5\times1)}{1 + 3 + 2 + 3 + 1}$

$= 3$

Redraw the table as shown below:

Number, x_i	1	2	3	4	5
Frequency, f_i	1	3	2	3	1
$x_i - \bar{x}$	–2	–1	0	1	2
$(x_i - \bar{x})^2$	4	1	0	1	4
$f_i (x_i - \bar{x})^2$	4	3	0	3	4

Variance $= \dfrac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i} = \dfrac{4 + 3 + 0 + 3 + 4}{1 + 3 + 2 + 3 + 1}$

$= 14/10 = 1.4$

50. (B) Re-draw the distribution table as shown below using the class mid-values, x_i

x_i	2	5	8
f_i	5	8	5
$x_i - \bar{x}$	–3	0	3
$(x_i - \bar{x})^2$	9	0	9
$f_i (x_i - \bar{x})^2$	45	0	45

Mean is calculated as

$\bar{x} = \dfrac{(2\times5) + (5\times8) + (8\times5)}{5 + 8 + 5} = \dfrac{90}{18} = 5$

Variance $= \dfrac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i} = \dfrac{45 + 0 + 45}{5 + 8 + 5}$

$= 5$

Standard Deviation $= \sqrt{5}$

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20 Probability

20.1 Definitions

- The **probability** of an event is the chance that the event will occur.
- The **sample space** (S) is the list of all possible outcomes of an experiment, e.g. for a tossed coin, S = {H, T}, where H = Head and T = Tail. For a thrown die, S = {1, 2, 3, 4, 5, 6}.
- An **event** is a possible outcome or combination of outcomes of an experiment, e.g. {H} is the event that *head* appears in a tossed coin, and {3} is the event of obtaining a '3' on a thrown die.
- Two events are **equally likely** if the probability of one of them occurring is equal to the probability of the other occurring, e.g. for a tossed coin, the probability of obtaining a *head* is equal to the probability of obtaining a *tail*, hence {H} and {T} are equally likely.
- For an experiment with n *equally likely* outcomes, the probability of an event with m outcomes occurring is

$P(E) = m/n$

EXAMPLE 20–1

Find the probability of obtaining a '4' on a thrown die.

Solution

Sample space, S = {1, 2, 3, 4, 5, 6}, i.e. $n = 6$
Event = {4}, i.e. $m = 1$
$P(4)$ = $m/n = 1/6$.

EXAMPLE 20–2

If in a class of 100 students, 30 are under age sixteen, find the probability that a student selected at random from the class is under sixteen.

Solution

n = total number of students = 100
m = number of students under age sixteen = 30
$P(E)$ = $m/n = 30/100 = \mathbf{0.30}$.

DRILL 1

If the letters of the word **PROBABILITY** are cut and dropped into a bag and a letter is selected at random from the bag, find the probability that the selected letter is (a) a vowel, (b) an I, and (c) a P.

20.2 Properties of Probability

- The value of probability always lies between 0 and 1, i.e. $0 \le P(E) \le 1$
- If $P(E) = 1$, the event will SURELY occur.
  If $P(E) = 0$, it is impossible for the event to occur.
- Two events A and B are **mutually exclusive** if they cannot occur at the same time, e.g. {H} and {T} are mutually exclusive for a tossed coin.
- Two events are **independent** if the chance of one occurring is not affected by the occurrence of the other.
- **Notation**
  $P(A)$ = probability that event A will occur.
  $P(B)$ = probability that event B will occur.
  $P(A \cup B)$ = probability that A or B (or both) will occur.
  $P(A \cap B)$ = probability that A and B will occur together.
  $P(A|B)$ = probability of event A, given that B has occurred.

- If A and B are **mutually exclusive**, the probability of either A or B or both occurring is the sum of the individual probabilities, i.e.

$$P(A \cup B) = P(A) + P(B)$$

This is the **addition law** of probability.

- If A and B are **mutually exclusive and exhaustive**, i.e. no other outcome is possible besides A or B, then

$$P(A) + P(B) = 1$$

- If A and B are **not** mutually exclusive, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- If A and B are two **independent** events, the probability of both A and B occurring together is equal to the product of the individual probabiliites, i.e.

$$P(A \cap B) = P(A) \times P(B)$$

This is the **multiplicaton law** of probability.

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EXAMPLE 20–3

If in a football match the probability that a team will score 0 goal is 0.50, 1 goal is 0.30, 2 goals is 0.15 and 3 or more goals is 0.05, find

(a) the probability of the team scoring either 0 or 1 goal in the match, and
(b) the probability that the team will score at least one goal.

Solutions

(a) $P(0 \text{ or } 1) = P(0) + P(1) = 0.50 + 0.30 = \mathbf{0.80}$.

(b) Probability of scoring at least one goal
= probability of scoring 1, 2, 3 or more goals.
Hence P(at least one goal)
$= 0.30 + 0.15 + 0.05$
$= \mathbf{0.50}$

Alternatively: P(at least one goal)
$= 1 - P(0 \text{ goal})$
$= 1 - 0.50 = 0.50$

EXAMPLE 20–4

Suppose that of all new cars sold, 40% are blue and 30% have 2 doors. Find the probability of buying (a) a car that is blue and has 2 doors (b) either a blue car or a 2-door car.

Solution

$P(\text{blue}) = 0.40$, $P(2 \text{ doors}) = 0.30$

(a) $P(\text{blue and 2 doors}) = P(\text{blue}) \times P(2 \text{ doors})$
$= 0.40 \times 0.30 = \mathbf{0.12}$

(b) The events "blue car" and "2-door car" are **not** mutually exclusive, since a car can be blue and have 2 doors. Hence, P(blue or 2-door or both)
$= P(\text{blue}) + P(2 \text{ doors}) - P(\text{blue and 2 doors})$
$= 0.4 + 0.3 - 0.12 = \mathbf{0.58}$

EXAMPLE 20–5

In a single throw of two dice:
(a) what is the probability of obtaining two sixes?
(b) what is the probability of obtaining a total of 9 points?

Solutions

(a) For the first die, $P(6) = 1/6$ and for the second die, $P(6) = 1/6$. The two events are independent, hence probability of obtaining two sixes
$= 1/6 \times 1/6 = \mathbf{1/36}$.

(b) A total of 9 points can be obtained in the following ways:

Die A :	3	4	5	6
	or	or	or	
Die B :	6	5	4	3

But $P(3,6) = P(4,5) = P(5,4) = P(6,3) = 1/6 \times 1/6 = 1/36$
Hence $P(9 \text{ points}) = P(3,6) + P(4,5) + P(5,4) + P(6,3)$
$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{1}{9}$

Alternate Method

A sample space, S is drawn up as follows:

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

(a) From the sample space, two sixes appear **once** out of 36 possible outcomes.

(b) From the sample space there are 4 outcomes (encased) for which the sum is 9 out of 36 possibilities.
Hence $P(9 \text{ points}) = 4/36 = \mathbf{1/9}$.

DRILL 2

A coin is tossed and a die is thrown. Find the probability that the coin shows a head and the die shows a 1 or 6.

20.3 Sampling With and Without Replacement

- If 2 balls were to be selected at random from a basket containing 4 white balls, 3 blue balls and 5 red balls, the selection can be done in two ways:
  (i) Select one ball, replace it in the basket, reshuffle and draw a second ball — this is **sampling with replacement**; or
  (ii) Select the two balls consecutively without replacing the first in the basket — this is **sampling without replacement.**

- The probability of selecting two balls of types A and B when sampling with replacement is

$$P(A \cap B) = P(A) \times P(B)$$

- The probability of selecting two balls A and B when sampling **without replacement** is

$$P(A \cap B) = P(A) \times P(B|A)$$

where $P(B|A)$ is the probability of selecting B once A has been selected without replacement.

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EXAMPLE 20–6

Two balls are drawn from a bag containing 4 white and 6 black balls. Find the probability that:

(a) one is white and the other is black, if drawn without replacement.

(b) one is white and other is black, if drawn with replacement and

(c) both are white if drawn with replacement.

Solutions

(a) *Without replacement*

$P(W) = 4/10 = 2/5$

$P(B|W) = 6/9 = 2/3$

$P(W \text{ and } B) = P(W) \times P(B|W)$

$= 2/5 \times 2/3 = \mathbf{4/15.}$

(b) *With replacement*

$P(W) = 4/10 = 2/5$

$P(B) = 6/10 = 3/5$

$P(W \text{ and } B) = P(W) \times P(B)$

$= 2/5 \times 3/5 = \mathbf{6/25.}$

(c) *With replacement*

$P(W \text{ and } W) = P(W) \times P(W)$

$= 2/5 \times 2/5 = \mathbf{4/25}$

where $\bar{r}$ means "not a red ball".

P(exactly one red ball)

$= 66/455 + 66/455 + 66/455$

$= 198/455 = \mathbf{0.44}$

(c) For the three balls to be of the same colour, they must be either all white or all red or all black.

P(all white) $= 7/15 \times 6/14 \times 5/13 = 35/455$

P(all red) $= 3/15 \times 2/14 \times 1/13 = 1/455$

P(all black) $= 5/15 \times 4/14 \times 3/13 = 10/455$

$\Rightarrow P$(all of same colour) $= P$(all white) + P(all red) + P(all black) $= 35/455 + 14/455 + 10/455$

$= 46/455$

$= \mathbf{0.10}$

DRILL 3

Three balls are drawn without replacement from a bag containing 5 yellow and 7 red balls. Find the probability that:

(a) all three are yellow.
(b) all three are red.
(c) one is yellow.

20.4 Permutations and Combinations

- A **permutation** is an ordered arrangement of the elements of a finite set, e.g. the set $\{x, y, z\}$ can be arranged in six different ways as:

$\{x, y, z\}$; $\{x, z, y\}$; $\{y, x, z\}$
$\{y, z, x\}$; $\{z, x, y\}$; $\{z, y, x\}$

Each of the arrangements is a permutation of the set.

- A set containing n different elements has

$n(n-1)(n-2) \cdots (3)(2)(1) = n!$

permutations, ($n!$ is pronounced "n factorial")
e.g. $3! = 3 \times 2 \times 1 = 6$
and $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

- The number of permutations for a set of n elements, of which a elements are alike, while b elements of another kind are alike, and so on is

$$\frac{n!}{a!\,b!}$$

e.g. the number of permutations of all the letters in the word ELEMENT is

$$\frac{7!}{3!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 840$$

[Note that the word contains 7 elements, 3 of which are identical.]

EXAMPLE 20–7

A bag contains 7 white, 3 red and 5 black balls. Three balls are drawn, one after the other, without replacement from the bag. Find the probability that

(a) none of the three balls is red
(b) exactly one ball is red
(c) all are of the same colour.

Solutions

(a) Total no. of balls = 15
No. of balls **not** red = 12
Prob. that 1st ball is not red = 12/15
Prob. that 2nd ball is not red = 11/14
Prob. that 3rd ball is not red = 10/13
⇒ Prob. (no ball is red) $= 12/15 \times 11/14 \times 10/13$
$= \mathbf{44/91}$

(b) "Exactly one red ball" is obtained as
{red, not red, not red} or {not red, red, not red}
or {not red, not red, red} with probabilities

$P(r\ \bar{r}\ \bar{r}) = 3/15 \times 12/14 \times 11/13 = 66/455$

$P(\bar{r}\ r\ \bar{r}) = 12/15 \times 3/14 \times 11/13 = 66/455$

$P(\bar{r}\ \bar{r}\ r) = 12/15 \times 11/14 \times 3/13 = 66/455$

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EXAMPLE 20–8

In how many different ways can seven balls of identical size be arranged in a line if

(a) they are all of the same colour
(b) 4 are red and 3 are blue
(c) 3 are red, 2 are blue and 2 are white?

Solutions

(a) Number of different arrangements
$= 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5{,}040$

(b) Number of different arrangements

$$\frac{7!}{4!\,3!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(4 \times 3 \times 2 \times 1)(3 \times 2 \times 1)} = 35$$

(c) Number of different arrangements

$$\frac{7!}{3!\,2!\,2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(3 \times 2 \times 1)(2 \times 1)(2 \times 1)} = 210$$

DRILL 4

(a) What is the number of permutations of all the letters of the word
(i) INFINITE (ii) MISSISSIPPI?

(b) How many different numbers can be formed using all the digits in the number
(i) 3442 (ii) 22322 (iii) 55477?

- Suppose we have a set of five elements

$$S = \{a, b, c, d, e\}$$

The subsets of three elements of the set S are:

$\{a, b, c\}$; $\{a, b, d\}$; $\{a, b, e\}$; $\{a, c, d\}$;
$\{a, c, e\}$; $\{a, d, e\}$; $\{b, c, d\}$; $\{b, c, e\}$;
$\{b, d, e\}$; $\{c, d, e\}$

We thus have a total of 10 subsets.

- The number of combinations of n elements taken r at a time is given by

$$_nC_r = \frac{n!}{r!\,(n-r)!}$$

In the five-element set discussed above, the number of combinations of three-element subsets is

$$_5C_3 = \frac{5!}{3!\,2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{(3 \times 2 \times 1)(2 \times 1)}$$

$$= \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10$$

EXAMPLE 20–9

A committee of four is to be selected from a grouop of 6 men and 5 women such that the committee consists of two men and two women. In how many ways can this be done?

Solution

Number of ways of selecting 2 men from 6 men

$$= {}_6C_2 = \frac{6 \times 5}{1 \times 2} = 15$$

Number of ways of selecting 2 women from 5 women

$$= {}_5C_2 = \frac{5 \times 4}{1 \times 2} = 10$$

Number of ways of selecting 2 men **and** 2 women

$= 15 \times 10 = \mathbf{150}$.

EXAMPLE 20–10

A bag contains 6 black balls and 4 white balls. In how many ways can a selection be made consisting of

(a) 2 black or 2 white balls?
(b) 2 white and 2 black balls?

Solutions

No. of ways of selecting 2 black balls

$$= {}_6C_2 = \frac{6 \times 5}{1 \times 2} = 15$$

No. of ways of selecting 2 white balls

$$= {}_4C_2 = \frac{4 \times 3}{1 \times 2} = 6$$

(a) No. of ways of selecting 2 black **or** 2 white balls

$= 15 + 6 = \mathbf{21}$

(b) No. of ways of selecting 2 black **and** 2 white balls = $15 \times 6 = \mathbf{90}$

DRILL 5

A debating team of four is to be selected from a group of 8 boys and 6 girls. In how many ways can this be done if the team

(a) must be either all boys or all girls?
(b) must consist of 2 boys and 2 girls?

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ANSWERS TO DRILLS

1. (a) 4/11 (b) 2/11 (c) 1/11
2. 1/6
3. (a) 1/22 (b) 7/44 (c) 21/44
4. (a) (i) 3,360 (ii) 34,650
   (b) (i) 12 (ii) 5 (iii) 30
5. (a) 85 (b) 420

SSCE/JME–TYPE OBJECTIVE QUESTIONS

1. The probability of obtaining a number less tnan 5 in a single toss of a fair die is
   A. 1/6 B. 1/3 C. 1/2 D. 2/3

2. The probability of obtaining an odd number in a single toss of a fair die is
   A. 1/6 B. 1/3 C. 1/2 D. 2/3

3. The probability of obtaining a prime number in a single toss of a fair die is
   A. 1/2 B. 1/3 C. 1/4 D. 1/6

A number is selected randomly from the set of integers 1 to 20 inclusive. Use this information to answer questions 4 to 6.

4. The probability that the number is equal to or greater than 15 is
   A. 1/4 B. 3/10 C. 7/10 D. 3/4

5. The probability that the number is either a multiple of 3 or a multiple of 5 is
   A. 1/2 B. 11/20 C. 9/20 D. 3/10

6. The probability that the number is prime is
   A. 7/20 B. 2/5 C. 9/20 D. 1/2

7. The letters of the word MISSISSIPPI are cut and placed in a bag. If one letter is drawn randomly from the bag, the probability that it is **neither** an *I* **nor** an *S* is
   A. 8/11 B. 6/11 C. 1/2 D. 3/11

8. A figure is selected randomly from a group of figures consisting of a square, a triangle, a rectangle, a trapezium and a rhombus. The probability that the figure selected is a parallelogram is
   A. 4/5 B. 3/5 C. 2/5 D. 1/5

9. The probability that a number selected at random from 1 to 40 is a multiple of 7 is
   A. 1/10 B. 1/8 C. 3/20 D. 7/40

Use the information below to answer questions **10** and **11.**

The distribution of boys and girls in two classes in a certain school is as follows:

	Boys	Girls
Class *A*	25	15
Class *B*	20	10

10. If a boy is selected at random from a congregation of the two classes, the probability that he is from class *B* is
    A. 2/7 B. 3/7 C. 4/9 D. 2/3

11. If a student is selected at random, the probability of selecting a girl is
    A. 17/24 B. 3/8 C. 1/3 D. 5/14

The table below shows the scores of students in a class test on Mathematics:

Score	2	3	4	5	6	7	8
No. of students	1	4	6	3	3	2	1

Use the information in the table for questions **12** and **13.**

12. The probability that a student selected randomly from the class scored 5 marks is
    A. 0.15 B. 0.20 C. 0.30 D. 0.50

13. The probability that a student selected randomly scored at least 6 marks is
    A. 0.85 B. 0.70 C. 0.30 D. 0.15

The data below shows the distribution of workers in a certain secondary school. Use the information to answer questions **14** and **15.**

Teachers	30	Gardeners	15
Typists	9	Cleaners	6
Clerks	12	Messengers	3

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14. If one of the workers is absent from work on one particular day, the probability that he is a *clerk* is
A. 4/25 B. 3/25 C. 2/25 D. 1/25

15. If one of the workers is on leave, the probability that he is either a *teacher* or a *gardener* is
A. 3/25 B. 1/5 C. 2/5 D. 3/5

16. A bag contains 4 red, 8 green and 6 blue balls of identical size. The probability that a ball selected randomly is either red or green is
A. 2/3 B. 4/9 C. 1/3 D. 2/9

A die is rolled 100 times with the following outcomes:

Number	1	2	3	4	5	6
Frequency	15	20	18	14	17	16

Use the information to answer questions 17 and 18.

17. The experimental probability of obtaining a 2 is
A. 0.10 B. 0.20 C. 0.40 D. 0.80

18. The experimental probability of obtaining at most 4 is
A. 0.14 B. 0.47 C. 0.67 D. 0.84

Of the 30 students in a class, 8 surnames begin with *A* or *O* and 18 begin with a consonant. No surname begins with any of the letters *C, Q, U, V, X* and *Z*. Use this information for questions 19 and 20.

19. The probability that the surname of a student selected at random begins with a vowel is
A. 2/15 B. 4/15 C. 4/13 D. 2/5

20. The probability that a surname selected randomly begins with *E* or *I* is
A. 13/15 B. 3/5 C. 4/15 D. 2/15

21. A ball is selected and replaced in a bag containing blue, green and red identical balls. The results of 50 selections are shown below:

Colour of ball	Blue	Green	Red
No. of occurrences	8	15	27

The experimental probability of picking either a blue or a red ball is
A. 0.70 B. 0.54 C. 0.30 D. 0.16

Two events *M* and *N* are mutually exclusive and $P(M) = 1/4$ and $P(N) = 2/3$. Use this information for questions 22 and 23.

22. $P(M \cap N)$ is
A. 0 B. 1/6 C. 5/12 D. 1/3

23. $P(M \cup N)$ is
A. 1 B. 11/12 C. 1/2 D. 1/12

24. The probability that a baby born by a certain couple will grow up to be tall is 1/4. The probability that the couple will have a tall daughter is
A. 1/8 B. 1/4 C. 3/8 D. 1/2

25. The probability that a sum of 10 will appear in two tosses of a fair die is
A. 5/18 B. 1/6 C. 5/36 D. 1/12

26. Two fair dice are thrown together. The probability that the sum of the outcomes is at least 10 is
A. 5/36 B. 1/4 C. 1/6 D. 1/12

27. The probability that the two dice in question 26 will show the same number of points is
A. 1/36 B. 1/6 C. 1/3 D. 1/2

28. Two numbers are drawn randomly from the four numbers 1, 3, 5 and 8. The probability that the sum of the two is not odd is
A. 1/4 B. 1/3 C. 1/2 D. 2/3

29. The probability of an event *X* is 2/3 and that of another event *Y* is 1/4. If the probability of both *X* and *Y* is 1/6, the probability of either *X* or *Y* is
A. 1 B. 11/12 C. 3/4 D. 1/3

30. The chances of occurrence of three independent events *X, Y* and *Z* are 1/3, 1/4 and 2/5. What are the chances of occurrence of **only** *Y* and *Z*?
A. 1/30 B. 1/15 C. 1/10 D. 13/30

31. A group of 40 adults read either newspapers or magazines. If 20 read newspapers and 35 read magazines, the probability that one of them reads both newspapers and magazines is
A. 3/8 B. 2/3 C. 3/11 D. 1/2

32. *P* is a probability function of an exhaustive set $S = \{w, x, y, z\}$. If $P(x) = 1/4$, $P(y) = 1/3$, $P(z) = 1/6$, then $P(w)$ is
A. 1/72 B. 1/12 C. 1/4 D. 3/4

33. Four of the faces of a fair die are coloured black and the remaining two faces are coloured white. On throwing the die two consecutive times, the probability that both throws will show a black face is
A. 2/3 B. 4/9 C. 2/9 D. 1/9

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34. Two fair coins are tossed. The probability of getting at least one tail is
A. 3/4 B. 2/3 C. 1/2 D. 1/4

35. A box contains 2 green and 3 red balls of identical size. If two balls are picked randomly, one after the other, without replacement, the probability of picking two balls of different colours is
A. 3/5 B. 2/5 C. 7/20 D. 2/3

Three balls are drawn one after the other, without replacement from a bag containing 4 blue, 6 white and 5 red identical balls. Use this information for questons 36 to 38.

36. The probability that the three balls are: one blue, one white and one red is
A. $\frac{1}{3}$ B. $\frac{16}{225}$ C. $\frac{8}{225}$ D. $\frac{4}{91}$

37. The probability of picking 2 blue and 1 red ball is
A. $\frac{4}{225}$ B. $\frac{8}{273}$ C. $\frac{2}{91}$ D. $\frac{6}{91}$

38. The probability that all three balls are the same colour is
A. $\frac{34}{455}$ B. $\frac{4}{91}$ C. $\frac{2}{91}$ D. $\frac{4}{455}$

39. Two cards are drawn, one after the other, with replacement from a pack of 52 cards containing four aces and four queens. The probability that the cards are either both aces or both queens is
A. 4/13 B. 2/13 C. 2/221 D. 2/169

40. A bag contains 5 red balls and 3 green balls. Two balls are picked, one after the other, with replacement. The probability that both are the same colour is
A. $\frac{17}{32}$ B. $\frac{25}{64}$ C. $\frac{15}{64}$ D. $\frac{9}{64}$

41. The probability that the two balls selected in question 40 are of different colours is
A. $\frac{17}{32}$ B. $\frac{15}{32}$ C. $\frac{15}{64}$ D. $\frac{1}{8}$

42. Three guests X, Y and Z arrive in that order for a dinner party. If guests are served randomly, the probability that the three guests will be served in the sequence of their arrival is
A. 1/6 B. 1/3 C. 1/2 D. 2/3

43. In how many different ways can the letters of the word GEOLOGY be arranged in order?
A. 720 B. 1,260 C. 2,520 D. 5,040

44. A photograph is to be taken of five people including a married couple. If the married couple insist on sitting next to each other, the number of different arrangements is
A. 12 B. 24 C. 48 D. 60

45. How many different three-digit numbers can be formed using the integers 1 to 6 if no integer occurs twice in a number?
A. 20 B. 60 C. 120 D. 240

A university undergraduate has to take 5 courses from a pool of 4 Mathematics and 3 Physics courses. Use this information for questions **46** to **48.**

46. In how many ways can he select his courses so that he takes exactly two courses in Physics?
A. 24 B. 12 C. 7 D. 4

47. In how many ways can he select his courses so that he takes at least two courses in Physics?
A. 6 B. 12 C. 18 D. 72

48. In how many ways can he select his courses if he insists on taking all four Mathematics courses?
A. 3 B. 6 C. 9 D. 12

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ANSWER KEY

1.	D	9.	B	17.	B	25.	D	33.	B	41.	B
2.	C	10.	C	18.	C	26.	C	34.	A	42.	A
3.	A	11.	D	19.	D	27.	B	35.	A	43.	B
4.	B	12.	A	20.	D	28.	C	36.	D	44.	C
5.	C	13.	C	21.	A	29.	C	37.	D	45.	C
6.	B	14.	A	22.	B	30.	B	38.	A	46.	B
7.	D	15.	D	23.	B	31.	A	39.	D	47.	C
8.	B	16.	A	24.	A	32.	C	40.	A	48.	A

SOLUTIONS TO OBJECTIVE QUESTIONS

1. **(D)** Sample Space, $S = \{1, 2, 3, 4, 5, 6\}$ or $n = 6$
Event = $\{1, 2, 3, 4\}$, or $m = 4$
$P(E) = 4/6 = 2/3$

2. **(C)** Event = $\{1, 3, 5\}$, or $m = 3$
$P(\text{odd number}) = 3/6 = 1/2$

3. **(A)** Event = $\{2, 3, 5\}$, or $m = 3$
$P(\text{prime number}) = 3/6 = 1/2$
[Note that 1 is **not** a prime number.]

4. **(B)** Sample Space = $\{1, 2, 3, ..., 20\}$, or $n = 20$
Event = $\{15, 16, 17, 18, 19, 20\}$, or $m = 6$
$P(E) = 6/20 = 3/10$

5. **(C)** Event = $\{3, 5, 6, 9, 10, 12, 15, 18, 20\}$, or $m = 9$
$P(E) = 9/20$

6. **(B)** Event = $\{2, 3, 5, 7, 11, 13, 17, 19\}$, or $m = 8$
$P(E) = 8/20 = 2/5$

7. **(D)** Sample Space = $\{M, I, S, S, I, S, S, I, P, P, I\}$
$m = 11$, and the letter must be M or P
$\therefore$ Event = $\{M, P, P\}$, or $n = 3$
$P(E) = 3/11$

8. **(B)** Five figures are given, i.e. $m = 5$
Three of these (*square, rectangle* and *rhombus*) are parallelograms, i.e. $n = 3$
$P(E) = 3/5$

9. **(B)** $m = 40$
Event = $\{7, 14, 21, 28, 35\}$, or $n = 5$
$P(E) = 5/40 = 1/8$

10. **(C)** Sample Space = total number of boys = 45
Event = number of boys in Class $B = 20$
$P(E) = 20/45 = 4/9$

11. **(D)** Sample Space = total number of students = 70
Event = total number of girls = 25
$P(E) = 25/70 = 5/14$

12. **(A)** Total number of students = 20
Number who scored 5 marks = 3
$P(E) = 3/20 = 0.15$

13. **(C)** Number who scored 6 marks and above = 3 + 2 + 1 = 6
$P(E) = 6/20 = 0.30$

14. **(A)** Total number of workers = 75
Number of clerks = 12
$P(\text{clerk}) = 12/75 = 4/25$

15. **(D)** Number of teachers and gardeners = 45
$P(\text{teacher or gardener}) = 45/75 = 3/5$

16. **(A)** Total number of balls = 18
Number of red and green balls = 12
$P(\text{red or green ball}) = 12/18 = 2/3$

17. **(B)** Total frequency = 100
Frequency of 2 = 20
$P(2) = 20/100 = 0.20$

18. **(C)** Event = $\{1, 2, 3, 4\}$ with a total frequency of 15 + 20 + 18 + 14 = 67
$P(E) = 67/100 = 0.67$

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19. (D) Total number of students = 30
Number of surnames beginning with a vowel
= 30 − 18 = 12
$P(\text{vowel}) = {}^{12}\!/_{30} = {}^{2}\!/_{5}$

20. (D) Number of surnames beginning with a vowel = 12
Number beginning with E or I (U is not allowed)
= 12 − 8 = 4
$P(E \text{ or } I) = {}^{4}\!/_{30} = {}^{2}\!/_{15}$

21. (A) Total number of selections = 50
Total frequency of occurrence of blue and red balls = 8 + 27 = 35
$P(\text{blue or red ball}) = {}^{35}\!/_{50} = 0.70$

22. (B) $P(M \cap N) = P(M) \times P(N) = {}^{1}\!/_{4} \times {}^{2}\!/_{3} = {}^{1}\!/_{6}$

23. (B) $P(M \cup N) = P(M) + P(N) = {}^{1}\!/_{4} + {}^{2}\!/_{3} = {}^{11}\!/_{12}$

24. (A) $P(\text{female child}) = {}^{1}\!/_{2}$
$P(\text{tall child}) = {}^{1}\!/_{4}$
$P(\text{tall, female child}) = {}^{1}\!/_{2} \times {}^{1}\!/_{4} = {}^{1}\!/_{8}$

25. (D) Of the 36 possibilities (see Example 20–5), only three (4, 6; 5, 5; 6,4) will give a sum of 10.
$P(E) = {}^{3}\!/_{36} = {}^{1}\!/_{12}$

26. (C) Refer to the table under Example 20–5.
Number of possibilities which will give a sum of 10 and above = six. [These are 4,6; 6,4; 5,5; 5,6; 6,5 and 6,6.]
$P(E) = {}^{6}\!/_{36} = {}^{1}\!/_{6}$

27. (B) The possibilities are 1,1; 2,2; 3,3; 4,4; 5,5 and 6,6. There are six in number.
$P(E) = {}^{6}\!/_{36} = {}^{1}\!/_{6}$

28. (C) The possibilities are $1 + 3 = 4$; $1 + 5 = 6$; $1 + 8 = 9$; $3 + 5 = 8$; $3 + 8 = 11$; $5 + 8 = 13$. Of the six possibilities, three yield sums which are even.
$P(E) = {}^{3}\!/_{6} = {}^{1}\!/_{2}$

29. (C) $P(X) = {}^{2}\!/_{3}$, $P(Y) = {}^{1}\!/_{4}$, $P(X \cap Y) = {}^{1}\!/_{6}$
$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$
$= {}^{2}\!/_{3} + {}^{1}\!/_{4} - {}^{1}\!/_{6} = {}^{3}\!/_{4}$

30. (B) $P(X) = {}^{1}\!/_{3}$, $P(Y) = {}^{1}\!/_{4}$, $P(Z) = {}^{2}\!/_{5}$
Since **only** Y and Z occur, X does not occur.
Probability of X not occurring
$= P(\bar{X}) = 1 - {}^{1}\!/_{3} = {}^{2}\!/_{3}$
$P(Y \cap Z \cap \bar{X}) = P(Y) \cdot P(Z) \cdot P(\bar{X})$
$= {}^{1}\!/_{4} \times {}^{2}\!/_{5} \times {}^{2}\!/_{3} = {}^{1}\!/_{15}$

31. (A) Number of adults who read **both** newspapers and magazines
= (20 + 35) − 40 = 15
[Sketch a Venn diagram if not sure.]
$P(E) = {}^{15}\!/_{40} = {}^{3}\!/_{8}$

32. (C) Since the set is *exhaustive*,
$P(w) + P(x) + P(y) + P(z) = 1$
$P(w) = 1 - ({}^{1}\!/_{4} + {}^{1}\!/_{3} + {}^{1}\!/_{6}) = {}^{1}\!/_{4}$

33. (B) Prob. that each throw shows a black face
$= {}^{4}\!/_{6} = {}^{2}\!/_{3}$
Prob. that both throws show a black face
$= {}^{2}\!/_{3} \times {}^{2}\!/_{3} = {}^{4}\!/_{9}$

34. (A) The possible outcomes are H,H; T,T; H,T; and T,H. Of these four, three contain at least one tail.
$P(E) = {}^{3}\!/_{4}$

35. (A) Probability of picking two balls of different colours = Prob. (Green and Red, or Red and Green).
Since there is no replacement,
$P(G) = {}^{2}\!/_{5}$, $P(R|G) = {}^{3}\!/_{4}$ for the first selection, and
$P(R) = {}^{3}\!/_{5}$, $P(G|R) = {}^{2}\!/_{4} = {}^{1}\!/_{2}$ for the second selection.
$P(\text{Event}) = ({}^{2}\!/_{5} \times {}^{3}\!/_{4}) + ({}^{3}\!/_{5} \times {}^{1}\!/_{2}) = {}^{3}\!/_{5}$

36. (D) Prob. that first ball is blue = ${}^{4}\!/_{15}$
Prob. that second ball is white = ${}^{6}\!/_{14}$
Prob. that third ball is red = ${}^{5}\!/_{13}$
$$P(E) = \frac{4}{15} \times \frac{6}{14} \times \frac{5}{13} = \frac{4}{91}$$

37. (D) The balls are picked as BRB or RBB or BBR
$$P(BRB) = \frac{4}{15} \times \frac{5}{14} \times \frac{3}{13} = \frac{2}{91}$$
$$P(RBB) = \frac{5}{15} \times \frac{4}{14} \times \frac{3}{13} = \frac{2}{91}$$
$$P(BBR) = \frac{4}{15} \times \frac{3}{14} \times \frac{5}{13} = \frac{2}{91}$$
$$P(\text{Event}) = \frac{2}{91} + \frac{2}{91} + \frac{2}{91} = \frac{6}{91}$$

38. (A) The balls are drawn as BBB or WWW or RRR
$$P(BBB) = \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13} = \frac{4}{455}$$
$$P(WWW) = \frac{6}{15} \times \frac{5}{14} \times \frac{4}{13} = \frac{4}{91}$$
$$P(RRR) = \frac{5}{15} \times \frac{4}{14} \times \frac{3}{13} = \frac{2}{91}$$

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$$P(\text{Event}) = \frac{4}{455} + \frac{4}{91} + \frac{2}{91} = \frac{34}{455}$$

39. **(D)** Since the cards are drawn **with** replacement:

Prob. (both aces) $= \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

Prob. (both queens) $= \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

Prob. (both aces **or** both queens)

$= \frac{1}{169} + \frac{1}{169} = \frac{2}{169}$

40. **(A)** The balls are drawn **with** replacement.

Prob. (both red) $= \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$

Prob. (both green) $= \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$

Prob. (both red **or** both green)

$= \frac{25}{64} + \frac{9}{64} = \frac{17}{32}$

41. **(B)** The balls are either *RG* or *GR*

$P(RG) = \frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$

$P(GR) = \frac{3}{8} \times \frac{5}{8} = \frac{15}{64}$

Prob. (red, green or green, red)

$= \frac{15}{64} + \frac{15}{64} = \frac{15}{32}$

42. **(A)** The guests can be served in the order *XYZ, XZY, YXZ, YZX, ZXY* or *ZYX* for a total of 6 possibilities. Only the first possibility tallies with the order of arrival.

$P(E) = 1/6$

43. **(B)** The word GEOLOGY contains 7 letters, two of which occur twice.

Number of arrangements

$= \frac{7!}{2!\,2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2}{2 \times 2}$

$= 1{,}260$

44. **(C)** Taking the married couple as one, we have $4! = 4 \times 3 \times 2$, or 24 arrangements. Since the couple can also swap positions, this gives a total of 24×2, or 48 arrangements.

45. **(C)** Number of ways of choosing 3 digits from 6 digits

$= {}_6C_3 = \frac{6 \times 5 \times 4}{1 \times 2 \times 3} = 20$

Each 3-digit number can be arranged in 3! ways $= 3 \times 2 = 6$ ways.

Total number of arrangements

$= 20 \times 6 = 120.$

46. **(B)** If he takes 2 courses in Physics, he must take 3 in Mathematics.

Number of ways of choosing 2 out of 3 courses in Physics $= {}_3C_2 = 3.$

Number of ways of choosing 3 out of 4 courses in Mathematics $= {}_4C_3 = 4$

Total number of ways $= 3 \times 4 = 12.$

47. **(C)** He either takes

2 Physics courses and 3 Math courses

or

3 Physics courses and 2 Math courses.

The first option can be done in 12 ways (see problem 46).

The second option can be done in

${}_3C_3 \times {}_4C_2 = 1 \times 6 = 6$ ways

Total number of possibilities for first **or** second option $= 12 + 6 = 18.$

48. **(A)** If he takes 4 Math courses, he can only take 1 course in Physics.

Number of ways $= {}_4C_4 \times {}_3C_1 = 1 \times 3 = 3.$

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21 Miscellaneous Topics

21.1 Binary Operations

Binary Operations are arithmetic operations in which two elements are combined according to a well-defined rule. Examples of binary operations include *addition, subtraction, multiplication* and *division.*

Properties of Binary Operations

1. **Closure**
   For a set of elements $S = \{a, b, c, d, ...\}$, the set S is *closed* under the operation $*$ if $\{a * b\} \in S$.

2. **Commutativity**
   The operation $*$ is *commutative* if $a * b = b * a$.

3. **Associativity**
   The operation $*$ is *associative* if
   $(a * b) * c = a * (b * c)$.

4. **Distributivity**
   The operation $*$ is *distributive* if
   $a * (b + c) = (a * b) + (a * c)$.

5. There is an **identity** or **netural element** e such that
   $a * e = e * a = a$.
   e.g the identity element for multiplication is given by $a \times e = e \times a = 1$, or $e = 1$.

6. The **inverse** i of an element a is such that
   $a * e = e * a = a$.
   e.g. the inverse element for *multiplication* is given by $a \times e = e \times a = 1$, or $i = 1/a = a^{-1}$.

EXAMPLE 21–1

For which of the following binary operations is the set of positive integers *closed:* (i) multiplication (ii) division (iii) addition, and (iv) subtraction?

Solutions

$S = \{1, 2, 3, 4, 5 ...\}$

(i) *Multiplication:* Any two positive integers multiplied together will yield a number which is also a positive integer, e.g. $3 \times 4 = 12$, hence S is closed under multiplication.

(ii) *Division:* One integer divided by another will not necessarily yield an integer, e.g. $13 \div 5$ is **not** an integer, hence S is **not** closed under division.

(iii) *Addition:* The addition of two positive integers always yields another positive integer, hence S is closed under addition.

(iv) *Subtraction:* The difference between two positive integers will not necessarily yield another positive integer, e.g. $2 - 5 = -3$. Hence S is **not** closed under subtraction.

EXAMPLE 21–2

A binary operation $*$ is defined over a set of real numbers $(a, b, c ...)$ such that $a * b = a + (a + b)$.

(a) Find $3 * 2$.
(b) Determine whether or not $*$ is (i) commutative (ii) associative.

Solutions

(a) $3 * 2 = 3 + (3 + 2) = 3 + 5 = 8$

(b) (i) $a * b = a + (a + b) = 2a + b$
$b * a = b + (b + a) = 2b + a$
Since $a * b \neq b * a$, $*$ is **not** commutative.

(ii) $(a * b) * c = (2a + b) * c$
$= 2a + b + (2a + b + c)$
$= 4a + 2b + c$
$a * (b * c) = a * (2b + c)$
$= a + (a + 2b + c)$
$= 2a + 2b + c$
Since $(a * b) * c \neq a * (b * c)$, $*$ is **not** associative.

DRILL 1

(a) Test the binary operations: *addition, subtraction, multiplication* and *division* for commutativity and distributivity.
(b) Find the identity (e) and inverse (i) elements for *addition.*
(c) Test the binary operation $*$ which is defined by
$a * b = a \times (2b)$
for (i) commutativity; (ii) associativity

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21.2 Differentiation

- If the value of a fucntion

$$y = f(x) \qquad (1)$$

changes by a small amount Δy as x changes by Δx, then

$$y + \Delta y = f(x + \Delta x) \qquad (2)$$

From Eqs. (1) and (2):

$$\Delta y = (y + \Delta y) - y = f(x + \Delta x) - f(x)$$

- The **derivative** of y with respect to x is defined as $\Delta y/\Delta x$ in the limit $\Delta x \to 0$, i.e.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

As an example consider the function $y = x^3$

$$\begin{aligned} y + \Delta y &= (x + \Delta x)^3 \\ &= x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 \\ \Delta y &= (y + \Delta y) - y \\ &= x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3 \\ &= 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 \end{aligned}$$

$$\text{or } \frac{\Delta y}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2$$

In the limit $\Delta x \to 0$, the last two terms also approach zero, hence

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 3x^2$$

- The **gradient** of a curve at a point, which is also the gradient of the tangent to the curve at that point, is given by the derivative of the function at the point, e.g. the gradient of the curve $y = x^3$ at $x = 2$ is

$$\left.\frac{dy}{dx}\right|_{x=2} = 3(2)^2 = 12$$

- The derivatives of some simple functions are listed below:

Function	Derivative
constant , k	0
x^n, n = constant	nx^{n-1}
$(ax + b)^n$	$na(ax + b)^{n-1}$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$
$\tan ax$	$a \sec^2 ax$
e^{ax}	ae^{ax}
$\log_e x$ (or $\ln x$)	$1/x$
$1/x$	$-1/x^2$

- Simple rules of differentiation:

(i) If $y = kf(x)$, where k is a constant, then

$$\frac{dy}{dx} = k\frac{d}{dx}f(x)$$

(ii) If $y = u(x) + v(x)$,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

(iii) If $y = u(x) \cdot v(x)$,

$$\frac{dy}{dx} = u(x)\frac{dv}{dx} + v(x)\frac{du}{dx}$$

(iv) If $y = \dfrac{u(x)}{v(x)}$,

$$\frac{dy}{dx} = \frac{v(x)\dfrac{du}{dx} - u(x)\dfrac{dv}{dx}}{[v(x)]^2}$$

EXAMPLE 21–3

(a) Find the gradient of the curve $y = x^2 - 5x + 6$ at $x = 1$.

(b) Find the derivative of

(i) $3x^2 - \dfrac{5}{x}$ (ii) $3\sqrt{x} - \dfrac{2}{\sqrt{x}}$

(iii) $x \cos 3x$

Solutions

$$\frac{dy}{dx} = 2x - 5 = 2(1) - 5 = -3$$

(b) (i) $y = 3x^2 - 5x^{-1}$

$$\frac{dy}{dx} = 6x + 5x^{-2} = 6x + \frac{5}{x^2}$$

(ii) $y = 3x^{1/2} - 2x^{-1/2}$

$$\frac{dy}{dx} = \frac{3}{2}x^{-1/2} + x^{-3/2} = \frac{3}{2\sqrt{x}} + \frac{1}{\sqrt{x^3}}$$

(iii) $y = x \cos 3x$

$$\begin{aligned} \frac{dy}{dx} &= x(-3 \sin 3x) + 1(\cos 3x) \\ &= -3x \sin 3x + \cos 3x \end{aligned}$$

DRILL 2

Find the derivatives of the following functions with respect to x:

(a) $(2x - 3)^2$

(b) $\dfrac{4}{(3x + 1)^2}$

(c) $\sec ax$

(d) $\sin x \tan x$

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21.3 Integration

- **Integration** is the reverse of differentiation. If the derivative of a function $f(x)$ with respect to x is $f'(x)$, then the **integral** of $f'(x)$ with respect to x is $f(x)$, i.e. if

$\frac{d}{dx}[f(x)] = f'(x)$, then

$$\int f'(x)\,dx = f(x) + c$$

where c is the constant of integration.

As an example,

$\frac{d}{dx}(x^3) = 3x^2$, hence

$$\int 3x^2\,dx = x^3 + c$$

- The integrals of some simple functions are listed below:

Function	Integral
constant k	$kx + c$
x^n	$\frac{1}{n+1}x^{n+1} + c$
$(ax + b)^n$	$\frac{(ax+b)^{n+1}}{a(n+1)} + c$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$
e^{ax}	$\frac{1}{a}e^{ax} + c$
$\frac{1}{x}$	$\ln x + c$

EXAMPLE 21–4

Integrate the following functions with respect to x:

(a) $x^2 + 4x - 6$ (b) $(x - 3)(3x + 1)$

(c) $3/x + 5$ (d) $\sec^2 x - 2\cos 2x$

Solutions

(a) $\int (x^2 + 4x - 6)\,dx = \int x^2dx + \int 4xdx - \int 6dx$

$= \frac{1}{3}x^3 - 2x^2 - 6x + c$

(b) Express the function as a polynomial, i.e.

$(x - 3)(3x + 1) = 3x^2 - 8x - 3$

$\int (3x^2 - 8x - 3)dx = x^3 - 4x^2 - 3x + c$

(c) $\int (3/x + 5)\,dx = \int 3/x\,dx + \int 5dx$

$= 3\ln x + 5x + c$

(d) $\int (\sec^2 x - 2\cos 2x)\,dx$

$= \int \sec^2 x dx - \int 2\cos 2x\,dx$

$= \tan x - \sin 2x + c$

DRILL 3

Integrate the following functions with respect to x:

(a) $(3x + 2)^2$ (b) $\frac{4}{\sqrt{x}} - 3x^2$

(c) $\cos x - 2\sin x$ (d) $\frac{3}{x} - e^{2x}$

21.4 Matrices and Determinants

- An m x n **matrix** is a rectangular array of numbers in m rows and n columns. Thus:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

is a 2 x 3 matrix (having 2 rows and 3 columns) where a_{ij} is the element occupying the *ith* row and the *jth* column.

- A **square matrix** is one containing equal numbers of rows and columns, i.e. $m = n$.
- A **null** or **zero matrix** is a matrix in which every element is zero, e.g.

$$0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- A **unit** or **identity matrix** (I) is a square matrix in which every element in the leading diagonal (from upper left to lower right) is equal to 1, while all other elements are zero, i.e.

$$0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- To add two matrices A and B, simply add the elements a_{ij} and b_{ij} in corresponding positions in each matrix:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} k & l \\ m & n \end{bmatrix} = \begin{bmatrix} a+k & b+l \\ c+m & d+n \end{bmatrix}$$

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- The difference between A and B is obtained by subtracting the elements in corresponding positions, i.e.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} k & l \\ m & n \end{bmatrix} = \begin{bmatrix} a-k & b-l \\ c-m & d-n \end{bmatrix}$$

- To multiply a matrix A by a real number k, multiply each element of A by k, i.e.

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

- The product of two matrices A and B is found as follows:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- The prdouct of an m x n matrix and an n x p matrix is an m x p matrix.
- The **transpose** (A^T) of a matrix A is obtained by interchanging the rows and columns of A, i.e.

  if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

- The **inverse** (A^{-1}) of a matrix A is the matrix which, on multiplying by A, gives the identity matrix I, i.e. $A^{-1}A = I$

$$\text{If } A^{-1} = \begin{bmatrix} r & s \\ t & u \end{bmatrix} \text{ and } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ then}$$

$$\begin{bmatrix} r & s \\ t & u \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- For a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the **determinant** of A is given by $\det A = |A| = ad - bc$.

- For a 3 x 3 matrix

$$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix}$$

the determinant is given by

$$\det M = |M| = a(ej - hf) - b(dj - gf) + c(dh - ge)$$

$$= a\begin{vmatrix} e & f \\ h & j \end{vmatrix} - b\begin{vmatrix} d & f \\ g & j \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}, \text{ or}$$

$$\det M = |M| = a\begin{vmatrix} e & f \\ h & j \end{vmatrix} - d\begin{bmatrix} b & c \\ h & j \end{bmatrix} + g\begin{bmatrix} b & c \\ e & f \end{bmatrix}$$

EXAMPLE 21–5

Find (a) $A + 3B$ (b) AB (c) $\det A$ (d) A^{-1}

if $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$

Solutions

(a) $A + 3B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 4 & -1 \end{bmatrix}$

(b) $AB = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2+3 & 0-3 \\ 1+2 & 0-2 \end{bmatrix}$

$$= \begin{bmatrix} 5 & -3 \\ 3 & -2 \end{bmatrix}$$

(c) $\det A = (2 \times 2) - (3 \times 1) = 4 - 3 = 1$

(d) Let $A^{-1} = \begin{bmatrix} r & s \\ t & u \end{bmatrix}$

$$\text{Hence } \begin{bmatrix} r & s \\ t & u \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Upon multiplying out, we have:
$2r + s = 1$ for 1st row and 1st column, and
$3r + 2s = 0$ for 1st row and 2nd column.
Solving these two equations simultaneously gives
$r = 2$ and $s = -3$
Also $2t + u = 0$ for 2nd row and 1st column, and
$3t + 2u = 1$ for 2nd row and 2nd column.
Solving simultaneously gives $t = -1$ and $u = 2$.

$$\text{Hence } A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

EXAMPLE 21–6

Find the determinant of the matrix

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 2 \\ 0 & 3 & 2 \end{bmatrix}$$

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Solution

The determinant is

$$2[(-1 \times 2) - (2 \times 3)] - 0[(1 \times 2) - (0 \times 2)] + 1[(1 \times 3) - (0 \times -1)]$$
$$= 2(-2 - 6) - 0(2 - 0) + 1(3 - 0)$$
$$= 2(-8) - 0 + 1(3)$$
$$= -13$$

DRILL 4

Given that $X = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $Y = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, find

(a) XY (b) YX
(c) YX^T (d) X^{-1}

21.5 Coordinate Geometry

- The distance d between two points (x_1, y_1) and (x_2, y_2) in the x–y coordinate axes is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- It follows that the distance between the origin (0, 0) and a point (a, b) is $\sqrt{a^2 + b^2}$

- The coordinates of the mid-point M of the line joining two points (x_1, y_1) and (x_2, y_2) are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- The equation of a straight line of slope m passing through a given point (h, k) is

$$\frac{y - k}{x - h} = m$$

- The equation of a line which has intercepts a and b on the x and y axes respectively is

$$\frac{x}{a} + \frac{y}{b} = 1$$

where $a \neq 0$ and $b \neq 0$.

- The (perpendicular) distance D of a point (h, k) from the line $ax + by + c = 0$ is

$$D = \left| \frac{ah + bk + c}{\sqrt{a^2 + b^2}} \right|$$

- The angle θ between two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is given by

$$\theta = \tan^{-1}\left[\frac{m_1 - m_2}{1 + m_1 m_2}\right]$$

where m_1 and m_2 are the slopes of the lines.

EXAMPLE 21–7

Find:
(a) the distance between the points (1, 5) and (–2, 1):
(b) the equation of the line passing through the point (1, 2) and having a slope of –2.
(c) the distance between the point (3, 1) and the line $4x + 3y = 5$.

Solutions

(a) $d = \sqrt{(-2 - 1)^2 + (1 - 5)^2} = \sqrt{9 + 16} = 5$

(b) The equation required is

$$\frac{y - 2}{x - 1} = -2$$
$$y - 2 = -2x + 2, \text{ or}$$
$$y + 2x = 4$$

(c) The distance required is

$$D = \frac{(4 \times 3) + (3 \times 1) - 5}{\sqrt{4^2 + 3^2}} = \frac{10}{5} = 2$$

DRILL 5

Two points A and B have coordinates of (–1, 3) and (1, 7) respectively. Find:

(a) the distance between A and B.
(b) the equaton of the line AB.
(c) the perpendicular distance of the origin (0, 0) from the line AB.
(d) the coordinates of the midpoint of line AB.

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ANSWERS TO DRILLS

1. (a)

Operation	Comm.	Assoc.	Distr.
+	Yes	Yes	No
–	No	No	No
x	Yes	Yes	Yes
÷	No	No	No

(b) For a real number a, $e = 0$ and $i = -a$ for *addition*.

(c) (i) Yes (ii) Yes

2. (a) $4(2x - 3)$ (b) $-\frac{24}{(3x+1)^3}$

(c) $a \sec ax \tan ax$ (d) $\sin x (\sec^2 x + 1)$

3. (a) $3x^3 + 6x^2 + 4x + c$

(b) $8\sqrt{x} - x^3 + c$

(c) $\sin x + 2 \cos x + c$

(d) $3 \ln x - \frac{1}{2} e^{2x} + c$

4. (a) $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

5. (a) $2\sqrt{5}$ (b) $y = 2x + 5$

(c) $\sqrt{5}$ (d) $(0, 5)$

SSCE/JME–TYPE OBJECTIVE QUESTIONS

1. The binary operation $*$ is defined over the set of real numbers by

$m * n = 2(m + n) + mn$

If $5 * y = 2 * y$, the value of y is

A. 5 **B.** –2 **C.** 2 **D.** 5

2. A binary operation $*$ is defined over the set of real numbers a and b by

$a * b = ab + a + b$

The identity element under this operation is

A. 0 **B.** –1 **C.** –2 **D.** 1

3. If a binary operation $*$ is defined over the set of real numbers by

$a * b = a^b$

and $m * 2 = m$, the possible values of m are

A. 0, 2 **B.** 1, 2 **C.** 2, 2 **D.** 0, 1

4. A binary operation $*$ is defined over the set of positive integers m, n by

$m * n = mn$

The following properties hold **EXCEPT**

A. Closure **B.** Associativity

C. Identity **D.** Inverse

5. The multiplication table below has modulo 10 over the set $S = \{2, 4, 6, 8\}$

⊗ mod 10	2	4	6	8
2	4	8	2	6
4	8	6	4	2
6	2	4	6	8
8	6	2	8	4

The inverse of 8 is

A. 8 **B.** 6 **C.** 4 **D.** 2

6. Which of the following binary operations performed over the set of integers is *commutative*?

A. $m * n = m^2 + n$

B. $m * n = m + 2n$

C. $m * n = m + n - mn$

D. $m * n = m(n + 1)$

7. If $m * n = \sqrt{mn}$, the value of $3 * (6 * 24)$ is

A. 3 **B.** 6 **C.** 12 **D.** 24

8. The binary operations ⊙ and ⊗ are defined over the set of real numbers by.

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$a \circledcirc b = ab - b - 1$, and

$a \otimes b = ab + b - 2$

The value of $3 \otimes (4 \circledcirc 5)$ is

A. 42 B. 54 C. 57 D. 60

9. If $m * n = m + n - mn$ and $(y * 2) + (y * 3) = 68$, the value of y is

A. −21 B. −12 C. 22 D. 24

10. The derivative of $\frac{2}{\sqrt{x}}$ with respect to x is

A. $4\sqrt{x}$ B. $\frac{1}{\sqrt{x}}$

C. $-\frac{1}{x\sqrt{x}}$ D. $-\frac{3}{x\sqrt{x}}$

11. Find the value of x for which the function $y = 2x^3 - 3x^2 - 36x + 6$ is a *minimum*

A. 3 B. −2 C. 6 D. −1

12. The *maximum* value of the function given in question 11 is

A. −2 B. −75 C. 36 D. 50

13. The gradient of the curve at $x = -1$ for the function given in question 11 is

A. −36 B. −24 C. 1 D. 0

14. For what value of x is the tangent to the curve $y = x^2 - 6x + 5$ parallel to the x-axis?

A. 0 B. 1 C. 3 D. 5

15. The gradient of the curve $y = 4\sqrt{x} - \frac{3}{x}$ at the point $x = 1$ is

A. 1 B. 2 C. 3 D. 5

16. The derivative of $\frac{4x^3 - 6x^2 + 1}{2x^2}$ with respect to x is

A. $2 - \frac{1}{x^3}$ B. $2 + \frac{1}{x^3}$

C. $2 - \frac{1}{4x}$ D. $2 + \frac{1}{4x}$

17. The rate of consumption of fuel by a car is given by the expression $3v + 4800/v$, where v is the speed of the car in kmh^{-1}. The speed at which rhe rate of fuel consumption is least is

A. 30 B. 40 C. 120 D. 1,600

18. For the function of $y = x\cos x$, $\frac{d^2y}{dx^2}$ is equal to

A. $x\cos x + 2\sin x$ B. $x\cos x - 2\sin x$

C. $\cos x - x\sin x$ D. $-x\cos x - 2\sin x$

19. If $y = \text{cosec } x$, $\frac{dy}{dx}$ is

A. $-\cot x \sec x$ B. $\tan x \text{ cosec } x$

C. $-\cot x \text{ cosec } x$ D. $-\cot x \sec x$

20. The derivative with respect to x of $\cos(2x - 3x^2)$ is

A. $(6x - 2)\sin(2x - 3x^2)$

B. $-\sin(2x - 3x^2)$

C. $(-6x + 2)\sin(2x - 3x^2)$

D. $-\sin(2 - 6x)$

21. The value of $\int_1^3 (3x - 2)^2\, dx$ is

A. 36 B. 38 C. 40 D. 48

22. The integral of $\frac{x-2}{x^3}$ with respect to x is

A. $\frac{1}{x^2} + \frac{2}{x^3} + k$ B. $\frac{x^2 - 2x}{x^4} + k$

C. $\frac{1}{x^2} - \frac{1}{x} + k$ D. $-\frac{1}{x} - \frac{1}{x^2} + k$

23. The value of $\int_{-1}^{1} (2x - 1)^2\, dx$ is

A. 0 B. $\frac{2}{3}$ C. 4 D. $4\frac{2}{3}$

24. The function y is expressed in terms of a variable x such that $\frac{dy}{dx} = 3x^2 - 2$ and $y = 12$ when $x = 3$. The function y is

A. $x^3 - 2x$ B. $x^3 - 2x + 9$

C. $x^3 - 2x - 9$ D. $6x - 6$

25. If $y = x(x^3 - x + 1)$, then the value of $\int_{-1}^{1} y\, dx$ is

A. $-\frac{4}{15}$ B. $\frac{11}{30}$ C. 1 D. 0

26. The value of $\int_{\pi/12}^{\pi/4} 4\cos 2x\, dx$ is

A. −1 B. −2 C. 1 D. 2

27. The integral with respect to x of $\frac{2}{x} - \sin x$ is

A. $-\frac{2}{x^2} - \cos x + k$ B. $-\frac{2}{x^2} + \cos x + k$

C. $2\ln x - \cos x + k$ D. [illegible]

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28. The area bounded by the curve $y = (3x + 1)(x - 1)$, the x-axis, $x = 1$ and $x = 3$ is
A. 14 B. 16 C. 18 D. 20

29. The area bounded by the curve $y = x^2 - 5x + 4$, the x-axis, the y-axis and $x = 4$ is
A. $6\frac{1}{3}$ B. $4\frac{1}{2}$ C. $-2\frac{2}{3}$ D. $-1\frac{5}{6}$

30. The rate of ice formation in the freezer compartment of a refrigerator is $(2 - 0.3t)$ g per min; where t is the time in minutes. If there was initially 10 g of ice in the ice-maker, the mass of ice present after 10 minutes is
A. 5 g B. 10 g C. 15 g D. 25 g

31. The volume of a balloon which is being blown increases at the rate of $(60 - 3t^2)$ cm^3 s^{-1}, where t is the time in seconds. If the initial volume of the balloon is zero, its volume after 3 seconds is
A. 33 cm^3 B. 153 cm^3
C. 180 cm^3 D. 207 cm^3

32. The determinant of the matrix
$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 3 \\ 2 & -5 & 4 \end{bmatrix}$$ is
A. 65 B. 33 C. 29 D. 3

33. If X^T is the transpose of the matrix
$$X = \begin{bmatrix} 0 & 3 & 0 \\ 2 & 1 & 3 \\ 4 & 2 & 2 \end{bmatrix}$$
the value of $|X^T|$ is
A. –48 B. –24 C. 24 D. 48

34. If
$$Y = \begin{bmatrix} 0 & 2 & 4 \\ 3 & 1 & 2 \\ 0 & 3 & 2 \end{bmatrix}$$
and X is defined as in question 33, then XY is equivalent to
A. XX^T B. XX^{-1} C. YX D. XX

35. The values of x and y which satisfy the equation
$$\begin{bmatrix} 5 & x \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$ are
A. $x = 2, y = 3$ B. $x = 3, y = -2$
C. $x = -3, y = 2$ D. $x = -2, y = 3$

36. Find the value of x if
$$\begin{vmatrix} x & 2 & -2 \\ x & 1 & 1 \\ 0 & 2 & 3 \end{vmatrix} = 6$$
A. –2 B. 1 C. $-\frac{2}{3}$ D. $\frac{1}{3}$

37. If $X = \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$ and $Y = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, find XY.
A. $\begin{bmatrix} -1 & 3 \\ 3 & 6 \end{bmatrix}$ B. $\begin{bmatrix} -1 & 6 \\ 3 & 3 \end{bmatrix}$
C. $\begin{bmatrix} 8 & -1 \\ 9 & -3 \end{bmatrix}$ D. $\begin{bmatrix} 3 & -1 \\ 6 & 3 \end{bmatrix}$

38. Find the value of $x + y$ if
$$\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$
A. –1 B. 1 C. –3 D. 3

39. Find the non-zero positive value of y in the equation
$$\begin{vmatrix} y & 1 & 0 \\ 1 & y & 1 \\ 0 & 1 & y \end{vmatrix} = 0$$
A. 1 B. $\sqrt{2}$ C. $\sqrt{3}$ D. 2

40. Find the value of x in the equation
$$\begin{vmatrix} -2 & 2 & 1 \\ 1 & 1 & 3 \\ 1 & x & -1 \end{vmatrix} = 16$$
A. 4 B. 3 C. 2 D. 1

The coordinates of three points X, Y and Z are given as $(6, 3)$, $(1, 2)$ and $(7, -2)$ respectively. Use this information for questions **41** to **43**.

41. The coordinates of the midpoint of the line XY are
A. $(7, 5)$ B. $(5, 1)$
B. $\left(\frac{7}{2}, \frac{5}{2}\right)$ D. $\left(\frac{5}{2}, \frac{1}{2}\right)$

42. The gradient of the line XZ is
A. –5 B. –6 C. 5 D. 6

43. The equation of the line YZ is
A. $3y + 2x = 8$ B. $3y - 2x = 4$
C. $2x + 3y = 8$ D. $3y + 2x = 7$

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44. The perpendicular distance of the point (2, –1) from the line $3x - 4y + 5 = 0$ is
A. 2 B. 5 C. 4 D. 3

45. The equation of the line which passes through the point (–1, 3) and is parallel to the line $2x + 3y = 5$ is
A. $3x + 2y = 3$ B. $2x + 3y = 7$
C. $2x + 3y = 11$ D. $3x - 2y = -9$

46. The point (x, y) at which the curve $y = 3x^2 - 4x + 1$ has a gradient of –10 is
A. (–1, 8) B. {2, 341)
C. (0, 1) D. (1, 0)

47. The distance between the point (6, 5) and the point of intersection of the lines $x + y = 4$ and $x - y = 2$ is
A. $\sqrt{3}$ B. 2 C. $\sqrt{5}$ D. 5

48. A line which makes an angle of 135° with the positive x-axis passes through the point (5, –3). The equation of the line is
A. $x - y = 8$ B. $x + y = 2$
C. $x - y = 2$ D. $x + y = 8$

ANSWER KEY

1.	B	9.	A	17.	B	25.	A	33.	C	41.	C
2.	A	10.	C	18.	D	26.	C	34.	A	42.	A
3.	D	11.	A	19.	C	27.	D	35.	D	43.	A
4.	D	12.	D	20.	A	28.	B	36.	C	44.	D
5.	D	13.	B	21.	B	29.	C	37.	A	45.	B
6.	C	14.	C	22.	C	30.	C	38.	B	46.	A
7.	B	15.	D	23.	D	31.	B	39.	B	47.	D
8.	B	16.	A	24.	C	32.	B	40.	D	48.	B

SOLUTIONS TO OBJECTIVE QUESTIONS

1. **(B)** $5 * y = 2(5 + y) + 5y = 10 + 7y$
$2 * y = 2(2 + y) + 2y = 4 + 4y$
$10 + 7y = 4 + 4y$, or $y = -2$

2. **(A)** $a * e = ae + a + e = a$
$e(1 + a) = 0$, or $e = 0$

3. **(D)** $m * 2 = m^2 = m$
$m^2 - m = 0$, or $m(m - 1) = 0$
$m = 0$ or 1.

4. **(D)** The binary operation, as defined, is *multiplication.* It is *closed* since the product of any two positive integers is also a positive integer.
It is *associative* since $(a * b) * c = a * (b * c) = abc$.
It has an *identity* element e such that
$a * e = a$, i.e. $ae = a$, or $e = 1$.
The *inverse* element i is such that
$a * i = ai = 1$, or $i = 1/a$, which is **not** an integer.

5. **(D)** The identity element i is given by
$8 \otimes i = 8$, or $i = 6$ (from the table)
The inverse e is given by
$8 \otimes e = 6$, or $e = 2$.

6. **(C)** For option (*C*), $m * n = m + n - mn$, and $n * m = n + m - nm$. Hence $m * n = n * m$
[As an exercise, the candidate should check the operations given in the other options for commutativity.]

7. **(B)** $3 * (6 * 24) = 3 * (\sqrt{6 \times 24}) = 3 * 12$
$= \sqrt{3 \times 12} = 6.$

8. **(B)** $4 \odot 5 = (4)(5) - 5 - 1 = 14$
$3 \otimes 14 = (3)(14) + 14 - 2 = 54$

9. **(A)** $y * 2 = y + 2 - 2y = 2 - y$
$y * 3 = y + 3 - 3y = 3 - 2y$
$(y * 2) + (y * 3) = (2 - y) + (3 - 2y)$
$= 5 - 3y$
$5 - 3y = 68$, or $y = -21$

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10. **(C)** Let $y = \frac{2}{\sqrt{x}} = 2x^{-1/2}$

$$\frac{dy}{dx} = 2(-1/2)\,x^{-3/2} = -x^{-3/2} = -\frac{1}{x\sqrt{x}}$$

11. **(A)** The function y is a *minimum* when $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2}$ is positive.

$$\frac{dy}{dx} = 6x^2 - 6x - 36 = 0$$

$x^2 - x - 6 = 0$, or $(x-3)(x+2) = 0$

$x = 3$ or -2.

$\frac{d^2y}{dx^2} = 12x - 6$, which is *positive* for $x = 3$.

12. **(D)** The function is *maximum* when $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2}$ is *negative*. From problem 11, this occurs for $x = -2$, and the value of the function is then $2(-2)^3 - 3(-2)^2 - 36(-2) + 6 = 50$.

13. **(B)** Gradient $= \frac{dy}{dx} = 6x^2 - 6x - 36$

At $x = -1$, $\frac{dy}{dx} = 6(-1)^2 - 6(-1) - 36 = -24$

14. **(C)** The tangent is parallel to the x-axis if it has a gradient of zero, i.e. if

$\frac{dy}{dx} = 2x - 6 = 0$, or $x = 3$.

15. **(D)** Let $y = 4\sqrt{x} - \frac{3}{x} = 4x^{1/2} - 3x^{-1}$

$$\frac{dy}{dx} = 2x^{-1/2} + 3x^{-2} = \frac{2}{\sqrt{x}} + \frac{3}{x^2}$$

At $x = 1$, $\frac{dy}{dx} = 2 + 3 = 5$

16. **(A)** Let $y = \frac{4x^3 - 6x^2 + 1}{2x^2} = 2x - 3 + \frac{1}{2}x^{-2}$

$$\frac{dy}{dx} = 2 - x^{-3} = 2 - \frac{1}{x^3}.$$

17. **(B)** Rate of fuel consumption, $F = 3v + 4800/v$

or $F = 3v + 4800v^{-1}$

$$\frac{dF}{dv} = 3 - 4800v^{-2} = 3 - \frac{4800}{v^2}$$

The least (or minimum) fuel consumption rate occurs when $\frac{dF}{dv} = 0$, i.e.

$3 - \frac{4800}{v^2} = 0$, or $v = 40$.

18. **(D)** $y = x\cos x$

$$\frac{dy}{dx} = \cos x - x\sin x$$

$$\frac{d^2y}{dx^2} = -\sin x - x\cos x - \sin x$$

$$= -x\cos x - 2\sin x$$

19. **(C)** If $y = \frac{1}{f(x)}$, then $\frac{dy}{dx} = \frac{-f'(x)}{[f(x)]^2}$

For $y = \operatorname{cosec} x = \frac{1}{\sin x}$

$$\frac{dy}{dx} = \frac{-\cos x}{\sin^2 x} = -\cot x \operatorname{cosec} x$$

20. **(A)** If $y = \cos[f(x)]$, $\frac{dy}{dx} = -f'(x)\sin[f(x)]$

For $y = \cos(2x - 3x^2)$

$$\frac{dy}{dx} = -(2-6x)\sin(2x-3x^2)$$

$$= (6x-2)\sin(2x-3x^2)$$

21. **(B)** $\int_1^3 (3x-2)^2 = \int_1^3 (9x^2 - 12x + 4)\,dx$

$$= \left[3x^3 - 6x^2 + 4x\right]_1^3$$

$$= [3(3)^3 - 6(3)^2 + 4(3)] - [3(1)^3 - 6(1)^2 + 4(1)]$$

$$= 39 - 1 = 38.$$

22. **(C)** $\frac{x-2}{x^3} = \frac{1}{x^2} - \frac{2}{x^3} = x^{-2} - 2x^{-3}$

$$\int (x^{-2} - 2x^{-3})\,dx = -x^{-1} + x^{-2} + k$$

$$= -\frac{1}{x} + \frac{1}{x^2} + k$$

23. **(D)** $\int_{-1}^{1} (2x-1)^2\,dx = \int_{-1}^{1} (4x^2 - 4x + 1)\,dx$

$$= \left[\frac{4}{3}x^3 - 2x^2 + x\right]_{-1}^{1}$$

$$= \left[\frac{4}{3} - 2 + 1\right] - \left[-\frac{4}{3} - 2 - 1\right]$$

$$= 4\tfrac{2}{3}$$

24. **(C)** $\frac{dy}{dx} = 3x^2 - 2$

$y = \int (3x^2 - 2)\,dx = x^3 - 2x + k$

But $y = 12$ when $x = 3$

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Hence $12 = 3^3 - 2(3) + k$, or $k = -9$
$\therefore\ y = x^3 - 2x - 9$

25. (A) $y = x(x^3 - x + 1) = x^4 - x^2 + x$

$$\int_{-1}^{1} y\,dx = \int_{-1}^{1} (x^4 - x^2 + x)\,dx$$

$$= \left[\frac{1}{5}x^5 - \frac{1}{3}x^3 + \frac{1}{2}x^2\right]_{-1}^{1}$$

$$= \left[\frac{1}{5} - \frac{1}{3} + \frac{1}{2}\right] - \left[-\frac{1}{5} + \frac{1}{3} + \frac{1}{2}\right]$$

$$= -\frac{4}{15}$$

26. (C) $\int_{\pi/12}^{\pi/4} 4\cos 2x\,dx = \left[2\sin 2x\right]_{\pi/12}^{\pi/4}$

$= 2\sin \pi/2 - 2\sin \pi/6$
$= 2(1) - 2(1/2)$
$= 2 - 1 = 1$

27. (D) $\int \left(\frac{2}{x} - \sin x\right) dx = 2\ln x + \cos x + k$

[Note that the integral of $1/x$ is $\ln x$.]

28. (B) This area is equal to $\int_1^3 (3x^2 - 2x - 1)\,dx$

$$= \left[x^3 - x^2 - x\right]_1^3$$

$$= [3^3 - 3^2 - 3] - [1 - 1 - 1] = 16$$

29. (C) The y-axis is the line $x = 0$. Hence the area in question is

$$\int_0^4 (x^2 - 5x + 4)\,dx$$

$$= \left[\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x\right]_0^4$$

$$= \left[\frac{1}{3}(4)^3 - \frac{5}{2}(4)^2 + 4(4)\right]$$

$$= \frac{64}{3} - 40 + 16 = -2\frac{2}{3}$$

30. (C) Let m = mass of ice present at any time t

$$\frac{dm}{dt} = 2 - 0.3t$$

$$m = \int (2 - 0.3t)\,dt$$

$$= 2t - \frac{0.3}{2}t^2 + k$$

At $t = 0$, $m = 10$ g $\Rightarrow$ $k = 10$
$\therefore\ m = 2t - 0.15t^2 + 10$
At $t = 10$, $m = 2(10) - 0.15(10)^2 + 10$
$= 15$ g

31. (B) Let v = volume of balloon at any time t

$$\frac{dv}{dt} = 60 - 3t^2$$

$$v = \int (60 - 3t^2)\,dt = 60t - t^3 + k$$

Since $v = 0$ at $t = 0$, $\Rightarrow$ $k = 0$
Hence $v = 60t - t^3$
At $t = 3$, $v = 60(3) - 3^3 = 153\ \text{cm}^3$

32. (B) Determinant
$= 1(16 + 15) - 3(0 - 6) + 2(0 - 8)$
$= 31 + 18 - 16 = 33$

33. (C) $X^T = \begin{bmatrix} 0 & 2 & 4 \\ 3 & 1 & 2 \\ 0 & 3 & 2 \end{bmatrix}$

$|X^T| = 0(2 - 6) - 2(6 - 0) + 4(9 - 0) = 24$
[Note that the determinant of a matrix is also equal to the determinant of its transpose.]

34. (A) It is not necessary to perform the actual expansion, once it is recognized that Y is the transpose of X. Hence $XY = XX^T$.

35. (D) $(5 \times 2) + xy = 4$, or $xy = -6$
$(4 \times 2) + (-1 \times y) = 5$, or $y = 3$
Hence $x = -6/y = -2$.

36. (C) $x(3 - 2) - 2(3x - 0) - 2(2x - 0) = 6$
$x - 6x - 4x = 6$, or $x = -2/3$

37. (A) $XY = \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

where $a = (2 \times 1) + (-1 \times 3) = -1$
$b = (2 \times 2) + (-1 \times 1) = 3$
$c = (3 \times 1) + (0 \times 3) = 3$
$d = (3 \times 2) + (0 \times 1) = 6$

Hence $XY = \begin{bmatrix} -1 & 3 \\ 3 & 6 \end{bmatrix}$

38. (B) $3x + 2y = 1$ ... (i)
$2x + 3y = 4$ ...(ii)
Multiply (i) by 2 and (ii) by 3 to give

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$6x + 4y = 2$, and
$6x + 9y = 12$

$5y = 10$, or $y = 2$
From (i), $3x = 1 - 2y = 1 - 4 = -3$, or $x = -1$
$x + y = -1 + 2 = 1$

39. **(B)** $y(y^2 - 1) - 1(y - 0) = 0$
$y^3 - 2y = 0$
$y(y^2 - 2) = 0$
$y = 0$ or $\pm\sqrt{2}$
The non-zero, positive value of y is $\sqrt{2}$.

40. **(D)** $-2(-1 - 3x) - 2(-1 - 3) + 1(x - 1) = 16$
$2 + 6x + 8 + x - 1 = 16$
$7x = 16 - 9 = 7$, or $x = 1$

41. **(C)** Coordinates of the midpoint of XY are

$$\left(\frac{6+1}{2}, \frac{3+2}{2}\right), \text{ or } \left(\frac{7}{2}, \frac{5}{2}\right)$$

42. **(A)** Gradient of $XZ = \frac{-2-3}{7-6} = -5$

43. **(A)** Slope of $YZ = \frac{-2-2}{7-1} = -\frac{2}{3}$

Equation of YZ is $y = -\frac{2}{3}x + k$, where k is a constant.
Hence $3y + 2x =$ constant.
Substituting the point $(1, 2)$
$\Rightarrow$ constant $= 8$
Equation of YZ is $3y + 2x = 8$.

44. **(D)** $D = \frac{ah + bk + c}{\sqrt{a^2 + b^2}} = \frac{(3 \times 2) + (-4 \times -1) + 5}{\sqrt{3^2 + (-4)^2}}$
$= \frac{15}{5} = 3.$

45. **(B)** The equation of the line must be of the form $2x + 3y =$ constant.
Substituting the point $(-1, 3)$ gives the constant as $2(-1) + 3(3) = 7$.
Hence the equation is $2x + 3y = 7$.

46. **(A)** Gradient $= \frac{dy}{dx} = 6x - 4$
Hence $6x - 4 = -10$, or $x = -1$
$y = 3(-1)^2 - 4(-1) + 1 = 8$
$\therefore$ The point is $(-1, 8)$.

47. **(D)** The point of intersection of the lines $x + y = 4$ and $x - y = 2$ is obtained by solving the two equations simultaneously to give $x = 3$, $y = 1$.
Distance between $(6, 5)$ and $(3, 1)$ is
$\sqrt{(6-3)^2 + (5-1)^2} = 5.$

48. **(B)** Slope of the line $= \tan 135^\circ = -\tan 45^\circ = -1$
Equation of the line is of the form $y = -x + k$, where k is a constant.
Substituting the point $(5, -3)$ gives
$k = y + x = -3 + 5 = 2$
Hence the desired equation is $x + y = 2$.

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MODEL MATRICULATION EXAMINATION 1

1. Determine the value of base x if
$343_x - 244_x = 111111_2$
A. 5 B. 6 C. 7 D. 8

2. A number P is divisible exactly by 3 and 5 and a number Q by 3, 5 and 7. The highest common factor of P and Q is
A. 15 B. 105
C. a factor of 15 D. a multiple of 15

3. Evaluate $\left[\frac{1}{0.3} \div \frac{1}{3.03}\right] - (0.5 \times 0.25)$
correct to two decimal places.
A. 9.90 B. 9.97 C. 9.98 D. 10.00

4. The length of a rod was given as 35 cm correct to the nearest centimetre. The actual length lies within the range
A. 34.5 – 35.5 cm B. 34.1 – 35.9 cm
C. 34.9 – 35.1 cm D. 34.95 – 35.05 cm

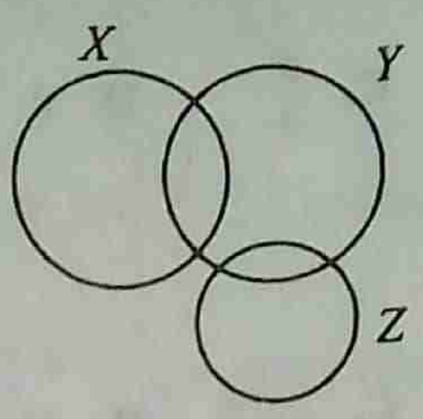
5. The number whose logarithm to base 10 is $\bar{1}.5$ is
A. $10^{-1.5}$ B. $10^{0.5}$
C. $10^{-0.5}$ D. $10^{-1} + 10^{0.5}$

6. Simplify $\left[\frac{1}{x^{-2}} \times \frac{1}{y^2}\right]^{-\frac{1}{2}}$
A. $\frac{x}{y}$ B. $\frac{y}{x}$ C. xy D. 1

7. Evaluate $3 \log \sqrt{2} + \log \sqrt{3} - \log \sqrt{6}$
A. $\log 2$ B. $\log 3$ C. $\log \sqrt{2}$ D. $\log 6$

8. Simplify $\left(3^{1+\sqrt{2}}\right)^{1-\sqrt{2}}$
A. 3 B. $\frac{1}{3}$ C. $\frac{1}{9}$ D. 27

9. If P = {positive integers less than 20, and divisible by 3} and Q = {odd numbers from 1 to 20 inclusive}, then $P \cap Q$ equals
A. $\varnothing$ B. P
C. {1, 3, 9, 15} D. {3, 9, 15}

10. 

In the figure X, Y and Z are the subsets of a universal set U. Which of the following statements are correct?
I. $X \cap Y \cap Z = \varnothing$
II. $(X \cup Y) \cap Z = X \cup (Y \cap Z)$
III. $X \cup Z' = X \cup Y \cap Z'$
A. I and II only B. II and III only
C. I and III only D. I, II and III

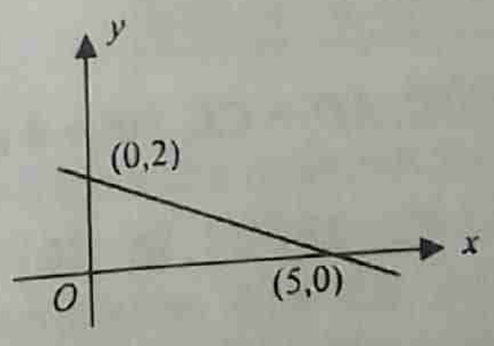
11. If $(x-y)^2 = 1$ and $xy = 2$, the positive value of $\frac{1}{x} + \frac{1}{y}$ is
A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{3}{2}$ D. 1

12. If $x^2 - 5x + 6 = (x-a)^2 + b$, then the value of b is
A. $-\frac{1}{4}$ B. $\frac{5}{2}$ C. 2 D. 3

13. If $f\left(\frac{1}{x}\right) = x^2 - 2x + 3$, then $f(2)$ is
A. 3 B. $2\frac{1}{4}$ C. 2 D. $1\frac{1}{2}$

14. Solve for y:
$y^{2/3} - 2y^{1/3} - 3 = 0$
A. –1, 3 B. 1, 3 C. –1, 9 D. –1, 27

15. When the length of a rectangle is doubled and its width is increased by 4 m, its perimeter increases from 22 m to 48 m. The original length and width of the rectangle are, respectively
A. 7 m, 4 m B. 5 m, 3 m
C. 6 m, 4 m D. 9 m, 2 m

16. 

The equation of the line shown in the graph is
A. $5y + 2x = 10$ B. $y + 5x = 2$
C. $2y + 5x = 4$ D. $y - 5x = 2$

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17. If $\frac{x}{y}=\frac{3}{5}$, then $\frac{x-3}{y-5}$ is

A. $\frac{2}{5}$ B. $\frac{3}{5}$ C. $\frac{5}{3}$ D. 0

18. Solve the equation

$$\frac{4}{10+x}+\frac{4}{10-x}=\frac{5}{6}$$

A. $-2, 1,$ B. $1, 2$ C. $-2, 2$ D. $-1, -2$

19. A pump P can fill a water tank in 5 h. With a second pump Q also operating, the tank can be filled in 2 h. Pump Q operating alone can fill the tank in

A. 3 h B. $3\frac{1}{3}$ h C. $3\frac{1}{2}$ h D. 4 h

20. If $m > n$, then which of the following is not correct for all values of m and n?

A. $-m < -n$ B. $-n^2 < m^2$
C. $m^2 > n^2$ D. $0 < m - n$

21. The greatest value of an integer x which satisfies the inequality $2 - 4x > 15$ is

A. -3 B. 3 C. 4 D. -4

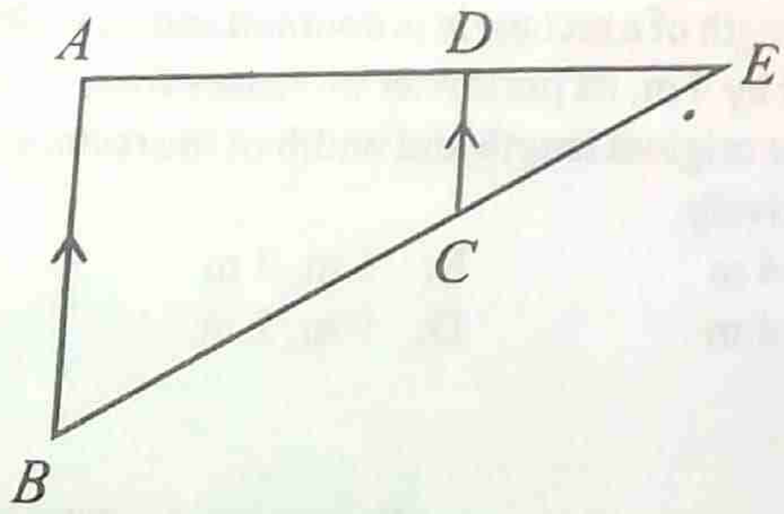
22. A reception hall is arranged such that the first row contains 10 seats and each successive row has two more seats than the preceding row. The total number of seats in the first twelve rows is

A. 252 B. 240 C. 222 D. 120

23. If $x - 1$, x, $x + 2$ form a geometric sequence, then the value of x is

A. 1 B. -1 C. 2 D. -2

24.



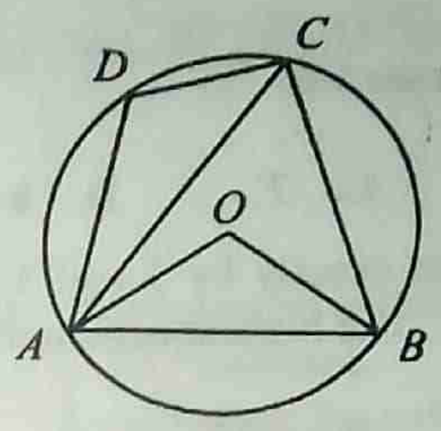
In the figure above, $AB//DC$, $AD = CE$, $DE = 4$ and $BC = 9$. What is BE?

A. 18 B. 15 C. 13 D. 10

25. Four of the interior angles of a pentagon are equal and each is twice the fifth angle. Each of the four equal angles is

A. 60° B. 90° C. 120° D. 135°

26.



The figure shows a circle, centre O. If $A\hat{O}B = 120°$ and $CAB = 50°$, angles CAO and ADC are, respectively.

A. 30° and 120° B. 30° and 110°
C. 20° and 110° D. 20° and 120°

27. $\sin\theta \cot\theta$ is equal to

A. $\cos\theta$ B. $\sec\theta$
C. $\operatorname{cosec}\theta$ D. $\tan\theta$

28. If x, y and z are angles which lie between 90° and 180° and $x < y < z$, then

A. $\sin x < \sin y < \sin z$
B. $\sin x > \sin y > \sin z$
C. $\sin y < \sin x < \sin z$
D. $\sin y > \sin x > \sin z$

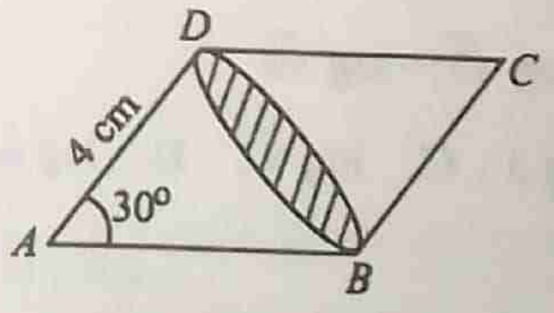
29. If the lengths of the sides of a right-angled triangle are given as $x - 1$, x and $x + 1$, then the hypotenuse is

A. 13 B. 4 C. 7 D. 5

30. Two sectors of the same circle subtend angles of 75° and 30° at the centre of the circle. Their areas are in the ratio

A. 25 : 4 B. 5 : 2
C. $\sqrt{5} : \sqrt{2}$ D. 3 : 1

31.



The figure shows a rhombus $ABCD$ of side 4 cm and $B\hat{A}D = 30°$. B is joined to D by two circular arcs centred at A and C. The shaded area is

A. $4\left(\frac{\pi}{3}-1\right)$ cm^2 B. $\frac{4\pi}{3}-2$ cm^2

C. $8\left(\frac{\pi}{3}-1\right)$ cm^2 D. $\frac{8\pi}{3}-4$ cm^2

32. A cylindrical metal rod of radius 10 cm and length 35 cm is to be melted down and recast into circular cones of base radius 7 cm and height 15 cm. If $\pi = \frac{22}{7}$, the maximum number of cones which can be cast from the original material is

A. 18 B. 14 C. 12 D. 7

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33. The locus of a point which is equidistant from the lines $y - x = 0$ and $y + x = 0$, for $x > 0$, is the
A. x – axis
B. y – axis
C. line $x = 1$
D. line $y = 1$

34. A binary operation $*$ is defined over the set of real numbers such that $a * b = a \times (b + 2)$. Find $3 * (4 * 2)$.
A. 54
B. 60
C. 72
D. 80

35. Which of the following binary operations is (are) ***NOT*** closed over the set of *odd* integers?
I. Addition
II. Subtraction
III. Multiplication
IV. Division

A. I and III only
B. II and IV only
C. IV only
D. I, II and IV only

36. $\lim_{x \to 1} \dfrac{x^3 - 1}{x^2 - 1}$ is equal to

A. 0
B. 1
C. $1\frac{1}{2}$
D. infinity

37. Differentiate $y = \sqrt{x} - \dfrac{1}{\sqrt{x}}$ with respect to x

A. $\dfrac{1}{2\sqrt{x}} + \dfrac{1}{2x\sqrt{x}}$
B. $\dfrac{1}{\sqrt{x}} - \dfrac{1}{2x\sqrt{x}}$
C. $x + \dfrac{1}{x\sqrt{x}}$
D. $x^{-1/2} + x^{-3/2}$

38. If $y = x^2 \sin x$, then $\dfrac{d^2y}{dx^2}$ equals
A. $2x\sin x + x^2\cos x$
B. $4x\cos x + (2 - x^2)\sin x$
C. $(2 - x^2)\sin x - 4x\cos x$
D. $2\sin x - x^2\sin x$

39. The vlaue of $\int_0^1 (1 - 2x)^3\, dx$ is

A. 0
B. $-\frac{1}{8}$
C. $-\frac{1}{4}$
D. $\frac{1}{4}$

40. The coordinates of points X and Y on the line XY are $X = (-2, -1)$ and $Y = (2, 1)$. The equation of the perpendicular bisector of XY is
A. $y - 2x = 0$
B. $y + 2x = 0$
C. $y + 2x = 5$
D. $y - 2x = 3$

41. The point at which the tangent to the curve $y = x^2 - 5x + 4$ makes an angle of 135° with the positive x-axis is
A. $(1, 0)$
B. $(3, -2)$
C. $\left(\frac{5}{2}, -\frac{9}{4}\right)$
D. $(2, -2)$

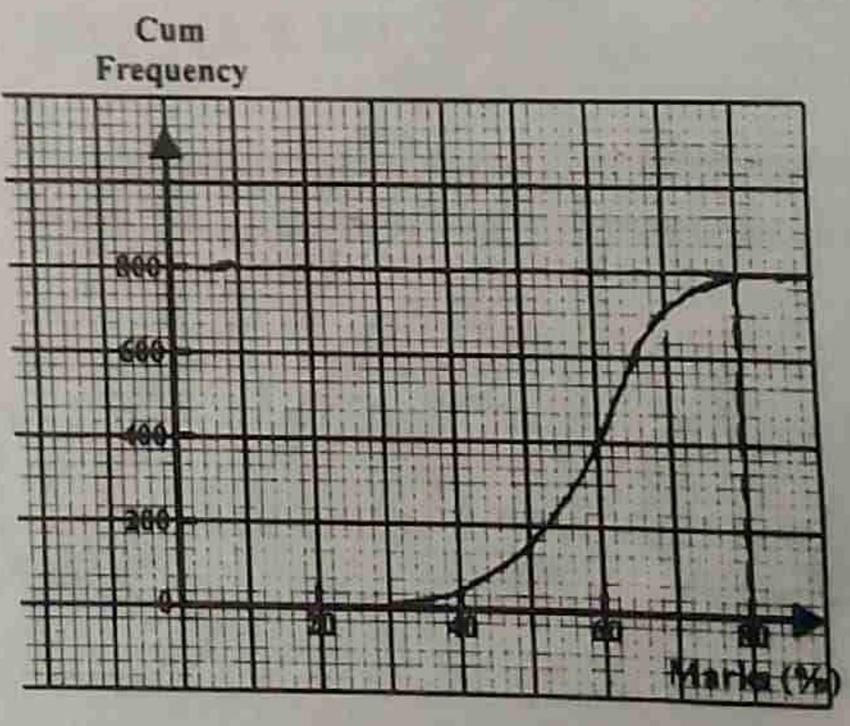
42. If $P = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and the inverse of P is $\begin{bmatrix} 2 & x \\ y & 2 \end{bmatrix}$, then the values of x and y are
A. $x = 3, y = 1$
B. $x = -3, y = 1$
C. $x = -3, y = -1$
D. $x = -1, y = -3$

43. In a test conducted on the three arms of a class, class A with 30 students scored an average of 5.5 marks, class B with 25 students scored an average of 6.4 marks, and class C got an average of 6.25 marks. If the average mark for the whole class is 6.0, the number of students in class C is
A. 35
B. 30
C. 25
D. 20

44. The distribution of marks scored by students in a class test is given below:

Mark	2	3	4	5	6	7	8	9
No. of students	1	2	3	4	5	3	1	1

From the table, the **mean** and the **modal** marks are respectively
A. 5, 6
B. 5.4, 6
C. 5.5, 5
D. 5.4, 5.5



The figure above shows the ogive drawn for the marks scored by 500 candidates in a matriculation examination. Use the information in the figure to answer questions **45** and **46**.

45. The median and the upper quartile are, respectively,
A. 60, 65
B. 50, 75
C. 60, 70
D. 55, 75

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46. The score of 50% corresponds to a percentile of
A. 60.0 B. 50.0 C. 18.8 D. 15.5

47. When the monthly budget for a household is displayed on a pie chart, the sectorial angles for the various items of expenditure are as follows: Food (162^o), Education (90^o), Transportation (36^o), Clothing (x^o), and Miscellaneous (18^o). If ₦900 is the amount budgeted for clothing, then the education buget is
A. ₦1,250 B. ₦1,500
C. ₦1,800 D. ₦2,000

48. The standard deviation of the set of numbers $-3, -2, -1, 0, 1, 2, 3$, is
A. 4 B. $2\sqrt{2}$ C. 2 D. 4

49. Four books whose authors' surnames start with the letters *A*, *J*, *P* and *W* are placed randomly on a shelf. The probability that the books will be arranged in the alphabetical order of the authors' surnames is
A. $\frac{1}{4}$ B. $\frac{1}{6}$ C. $\frac{1}{12}$ D. $\frac{1}{24}$

50. A die is thrown and a coin is tossed. Find the probability that the die shows an even number and the coin shows a head.
A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{1}{6}$ D. 1

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ANSWER KEY

1.	D	11.	C	21.	D	31.	C	41.	D
2.	D	12.	A	22.	A	32.	B	42.	C
3.	C	13.	B	23.	C	33.	A	43.	D
4.	A	14.	D	24.	B	34.	A	44.	B
5.	C	15.	D	25.	C	35.	D	45.	A
6.	B	16.	A	26.	C	36.	C	46.	C
7.	A	17.	B	27.	A	37.	A	47.	B
8.	B	18.	C	28.	B	38.	B	48.	C
9.	D	19.	B	29.	D	39.	A	49.	D
10.	C	20.	C	30.	B	40.	B	50.	A

SOLUTIONS TO MODEL MATRICULATION EXAMINATION 1

1. (D) $343_x = 3x^2 + 4x + 3$
$244_x = 2x^2 + 4x + 4$
$111111_2 = 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 1 = 63$
$343_x - 244_x$
$= (3x^2 + 4x + 3) - (2x^2 + 4x + 4)$
$= x^2 - 1$
Hence $x^2 - 1 = 63$, $x^2 = 64$ and $x = 8$.

2. (D) P is a multiple of 15 and Q is a multiple of 105. Note that 15 is a common factor but not necessarily the *highest* common factor. The numbers $P = 60$ and $Q = 210$, for example, satisfy the given conditions and their HCF is 30, which is multiple of 15.

3. (C) $\left(\frac{1}{0.3} \times 3.03\right) - 0.125 = \frac{30.3}{3} - 0.125$
$= 10.1 - 0.125 = 9.975 = 9.98$

4. (A) Although (C) and (D) will also approximate to 35 cm, only (A) contains the *whole range* of numbers which can be so approximated.

5. (C) The antilog of $\bar{1}.5$ is $10^{-1} \times 10^{0.5} = 10^{-0.5}$

6. (B) $\left[\frac{1}{x^{-2}} \times \frac{1}{y^2}\right]^{-1/2} = \left[\frac{x^2}{y^2}\right]^{-1/2} = \left[\frac{y^2}{x^2}\right]^{1/2} = \frac{y}{x}$

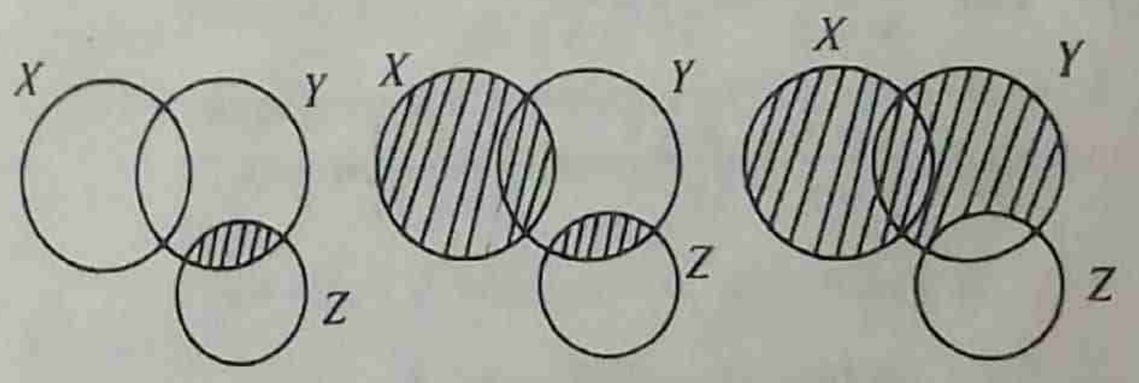
7. (A) Re-write the expression as

$$\log \frac{(\sqrt{2})^3 \times \sqrt{3}}{\sqrt{6}} = \log \frac{2\sqrt{2} \times \sqrt{3}}{\sqrt{6}}$$
$$= \log \frac{2\sqrt{6}}{\sqrt{6}} = \log 2$$

8. (B) $\left(3^{1+\sqrt{2}}\right)^{1-\sqrt{2}} = 3^{(1+\sqrt{2})(1-\sqrt{2})} = 3^{1-2}$
$= 3^{-1} = \frac{1}{3}$

9. (D) $P = \{3, 6, 9, 12, 15, 18\}$
$Q = \{1, 3, 5, 7, 9, 11, 13, 15, 17. 19\}$
$P \cap Q = \{3, 9, 15\}$

10. (C) I is correct because there is no member common to all three subsets. The others are shown below:



$(X \cup Y) \cap Z$ $X \cup (Y \cap Z)$ $X \cup Z' = X \cup Y \cap Z'$

11. (C) $x^2 + y^2 - 2xy = 1$, or $x^2 + y^2 = 1 + 2xy$
$= 1 + 4 = 5$
$(x + y)^2 = (x^2 + y^2) + 2xy = 5 + 4 = 9$
$\therefore \quad x + y = \pm 3$
$\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{3}{2}\left(\text{or } -\frac{3}{2}\right)$

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12. **(A)** $\left(x-\frac{5}{2}\right)^2 = x^2 - 5x + \frac{25}{4}$

Hence $x^2 - 5x + 6 = \left(x-\frac{5}{2}\right)^2 - \frac{1}{4}$

$\therefore\ b = -\frac{1}{4}$

13. **(B)** $f\left(\frac{1}{x}\right) = x^2 - 2x + 3$

$f\left(\frac{1}{x}\right) = f(2)$ when $\frac{1}{x} = 2$ or $x = \frac{1}{2}$

$\therefore\ f(2) = \frac{1}{4} - 2\left(\frac{1}{2}\right) + 3 = 2\frac{1}{4}$

14. **(D)** Setting $y^{1/3} = x$ gives

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0,\ x = 3$ or -1

$\therefore\ y^{1/3} = 3$ or -1, and $y = 27$ or -1

15. **(D)** Let l, w = original length and width, respectively.

$2(l + w) = 22$, or $l + w = 11$ ...(i)

$2(2l + w + 4) = 48$, or $2l + w = 20$ ...(ii)

Solving (i) and (ii) simultaneously gives

$l = 9$ m and $w = 2$ m.

16. **(A)** Slope, $m = \frac{0-2}{5-0} = -\frac{2}{5}$

Equation of the line is

$y = -\frac{2}{5}x + c$, where c = intercept on y-axis = 2

Hence, $y = -\frac{2}{5}x + 2$, or $5y + 2x = 10$

17. **(B)** $\frac{x}{y} = \frac{3}{5} \Rightarrow y = \frac{5x}{3}$

$$\frac{x-3}{y-5} = \frac{x-3}{\frac{5x}{3}-5} = \frac{3(x-3)}{5(x-3)} = \frac{3}{5}$$

18. **(C)** $\frac{4(10-x)+4(10+x)}{100-x^2} = \frac{5}{6}$

$\frac{80}{100-x^2} = \frac{5}{6} \Rightarrow 480 = 500 - 5x^2$

$5x^2 = 20,\ x^2 = 4$ and $x = \pm 2$.

19. **(B)** In 1 h, P fills $\frac{1}{5}$ of the tank.

In 1 h, P and Q fill $\frac{1}{2}$ of the tank.

Hence Q alone fills $\frac{1}{2} - \frac{1}{5}$, or $\frac{3}{10}$ of the tank in 1 h, and fills the whole tank in $\frac{10}{3}$ h $= 3\frac{1}{3}$ h.

20. **(C)** is not correct for negative values of m and n, e.g. $-3 > -5$ but $(-3)^2 < (-5)^2$.

21. **(D)** $4x - 2 < -15$

$4x < -13$, or $x < -3\frac{1}{4}$

and since x is an integer, its greatest value must be -4.

22. **(A)** This is the arithmetic series

$10 + 12 + 14 + \ldots$

Hence $a = 10$, $n = 12$ and $d = 2$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= 6[20 + (11 \times 2)] = 252$

23. **(C)** Common ratio, $r = \frac{x}{x-1} = \frac{x+2}{x}$

$x^2 = (x-1)(x+2) = x^2 + x - 2$

$\therefore\ x = 2$

24. **(B)** Let $AD = CE = x$

$\frac{DE}{AD} = \frac{CE}{BC} \Rightarrow \frac{4}{x} = \frac{x}{9}$

$x^2 = 36$, or $x = 6$

$BE = 9 + x = 9 + 6 = 15$

25. **(C)** Let each of the four equal angles $= x$.

The fifth angle $= x/2$

The sum of the interior angles of a pentagon is

$2(5) - 4$ rt. angles $= 6$ rt. angles $= 540^\circ$

Hence $4x + \frac{x}{2} = 540$, $9x = 1080$

or $x = 120^\circ$

26. **(C)** $A\hat{C}B = \frac{1}{2}(A\hat{O}B) = 60^\circ$

ΔAOB is isosceles with

$O\hat{A}B = O\hat{B}A = \frac{1}{2}(180^\circ - 120^\circ) = 30^\circ$

$C\hat{A}O = C\hat{A}B - O\hat{A}B = 50^\circ - 30^\circ = 20^\circ$

$C\hat{B}A = 180^\circ - (C\hat{A}B + A\hat{C}B) = 180^\circ - 110^\circ = 70^\circ$

$A\hat{D}C = 180^\circ - C\hat{B}A = 180^\circ - 70^\circ = 110^\circ$

27. **(A)** $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

$\sin\theta\ \cot\theta = \sin\theta\left(\frac{\cos\theta}{\sin\theta}\right) = \cos\theta$

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28. (B) $\sin\theta$ decreases as θ increases from 90^o to 180^o (e.g. $\sin 90^o = 1$ and $\sin 180^o = 0$). Hence the smallest angle x, has the largest sine.

29. (D) Hypotenuse = longest side $= x + 1$

Pythagoras: $(x-1)^2 + x^2 = (x+1)^2$

$$x^2 - 2x + 1 + x^2 = x^2 + 2x + 1$$

$$x^2 - 4x = 0 \Rightarrow x(x-4) = 0$$

$x = 4$ and $x + 1 = 5$

30. (B) Area of a sector of angle $\theta = \frac{1}{2} r^2\theta$, i.e. the area varies directly as the angle θ. Ratio of the areas of the sectors = 75 : 30 = 5 : 2.

31. (C) Join B to D with a straight line.

Area of sector $ADB = \frac{1}{2} r^2\theta$

$$= \frac{1}{2} \times 4^2 \times \frac{\pi}{6} = \frac{4\pi}{3} \text{ cm}^2$$

Area of $\Delta ADB = \frac{1}{2} \cdot AD \cdot AB \sin 30^o$

$$= \frac{1}{2} \times 4^2 \times \frac{1}{2} = 4 \text{ cm}^2$$

Shaded area = 2(area of sector ADB − area of ΔADB)

$$= 2\left(\frac{4\pi}{3} - 4\right) = 8\left(\frac{\pi}{3} - 1\right) \text{cm}^2$$

32. (B) Volume of cylindrical rod $= \pi r^2 h$

$$= \frac{22}{7} \times 10^2 \times 35 = 11{,}000 \text{ cm}^3$$

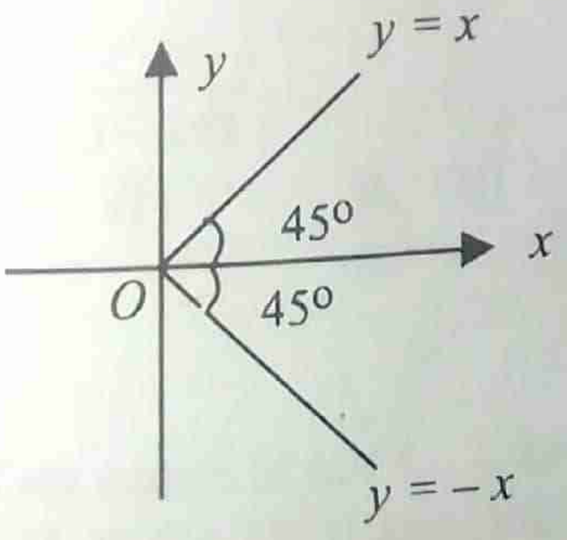
Volume of each cone

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 15$$

$$= 770 \text{ cm}^3$$

Number of cones $= \frac{11{,}000}{770} = 14.3 \approx 14.$

33. (A)



Each of the lines $y = x$ and $y = -x$ makes angle 45^o with the x-axis (see figure). Hence all points on the x-axis are equidistant from the two lines.

34. (A) $a * b = a \times (b + 2)$

$4 * 2 = 4 \times (2 + 2) = 16$

$3 * (4 * 2) = 3 * 16 = 3 \times (16 + 2) = 54$

35. (D) Try each operation with some odd numbers:

Addition: $3 + 5 = 8$ (not closed)

Subtraction: $5 - 3 = 2$ (not closed)

Multiplication: $5 \times 3 = 15$, $3 \times 7 = 21 \ldots$ (closed)

Division: $5 \div 3 = 1\frac{2}{3}$ (not closed)

36. (C) $$\frac{x^3 - 1}{x^2 - 1} = \frac{(x-1)(x^2 + x + 1)}{(x-1)(x+1)} = \frac{x^2 + x + 1}{x + 1}$$

Hence $\lim\limits_{x \to 1} = \frac{1 + 1 + 1}{1 + 1} = \frac{3}{2}$

37. (A) $y = x^{1/2} - x^{-1/2}$

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2} = \frac{1}{2\sqrt{x}} + \frac{1}{2x\sqrt{x}}$$

38. (B) $y = x^2 \sin x$

$$\frac{dy}{dx} = x^2 \cos x + 2x \sin x$$

$$\frac{d^2y}{dx^2} = -x^2 \sin x + 2x \cos x + 2 \sin x + 2x \cos x$$

$$= 4x \cos x + (2 - x^2) \sin x$$

39. (A) $$\int_0^1 (1-2x)^3 \, dx = \left[-\frac{1}{8}(1-2x)^4\right]_0^1$$

$$= \left[-\frac{1}{8}(1)\right] - \left[-\frac{1}{8}(1)\right]$$

$$= -\frac{1}{8} + \frac{1}{8} = 0.$$

40. (B) Mid-point of $XY = \left(\frac{-2+2}{2}, \frac{-1+1}{2}\right)$, or (0, 0).

Slope of $XY = \frac{1 - (-1)}{2 - (-2)} = \frac{1}{2}$

The perpendicular bisector of XY has a slope of −2 and must pass through (0, 0). The equation is thus

$y = -2x + c$ where $c = 0$, or $y + 2x = 0$.

41. (D) Slope of tangent $= \frac{dy}{dx} = 2x - 5$

Singe the tangent makes an angle of 135^o with the x-axis, its slope = $\tan 135^o = -1$

Hence $2x - 5 = -1$, or $x = 2$ and

$$y = x^2 - 5x + 4 = 2^2 - 5(2) + 4 = -2$$

42. (C) $PP^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

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$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & x \\ y & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

1st row x 1st column:
$(2)(2) + 3y = 1$, or $y = -1$

1st row x 2nd column:
$2x + 3(2) = 0$, or $x = -3$

43. **(D)** Total marks scored by class A
$= 30 \times 5.5 = 165$

Total marks scored by class B
$= 25 \times 6.4 = 160$

Total marks scored by class C
$= N \times 6.25 = 6.25N$

where N = number of students in class C.
Total score in all three classes
$= 6.25N + 160 + 165$

Total number of students $= N + 30 + 25 = N + 55$
Overall average score = 6.0
Hence $6(N + 55) = 6.25N + 160 + 165$,
or $N = 20$.

44. **(B)** Total marks scored by all students
$= (2 \times 1) + (3 \times 2) + (4 \times 3) + \ldots + (9 \times 1) = 108$

Total number of students
$= 1 + 2 + 3 + 4 + 5 + 3 + 1 + 1 = 20$

Mean mark $= \frac{108}{20} = 5.4$

Modal mark = mark scored by highest number of students = 6

45. **(A)** Median = mark corresponding to one-half of the total cumulative frequency, i.e. the mark corresponding to a cumulative frequency of 400. This is 60%.

Upper quartile = mark corresponding to 75% of total cumulative frequency, i.e. the mark corresponding to a cumulative frequency of 600. This is 65%.

46. **(C)** A score of 50% corresponds to a cumulative frequency of 150, i.e. a percentile of
$\frac{150}{800} \times 100 = 18.8$.

47. **(B)** Clothing sectorial angle.
$x = 360^o - (162^o + 90^o + 36^o + 18^o)$
$= 54^o$
The budget for each item is directly proportional to the sectorial angle. Hence education budget is
$\frac{90^o}{54^o} \times ₦900 = ₦1{,}500$.

48. **(C)** Mean $= (-3 - 2 - 1 + 0 + 1 + 2 + 3)/7 = 0$
$x_i - \bar{x} = -3, -2, -1, 0, 1, 2, 3$
$(x_i - \bar{x})^2 = 9, 4, 1, 0, 1, 4, 9$
$\Sigma(x_i - \bar{x})^2 = 9 + 4 + 1 + 0 + 1 + 4 + 9 = 28$
Variance $= \frac{\Sigma (x_i - \bar{x})^2}{n} = \frac{28}{7} = 4$
Standard deviation $= \sqrt{4} = 2$.

49. **(D)** Total number of possible arrangements
$= 4! = 4 \times 3 \times 2 \times 1 = 24$
Only 1 of these will be in alphabetical order.
Hence $P(E) = \frac{1}{24}$

50. **(A)** Probability that the die shows an even number (2, 4 or 6) is $P(E) = \frac{3}{6} = \frac{1}{2}$.
Probability that the coin shows a head is
$P(H) = \frac{1}{2}$.
Probability of an even number *and* a head is
$P(E) \times P(H) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

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MODEL MATRICULATION EXAMINATION 2

If $1P43_7 = 549_{10}$, the value of P is

A. 3 B. 4 C. 5 D. 6

Simplify $\dfrac{3\frac{1}{3} - 5\frac{2}{3}}{1\frac{1}{4} \text{ of } 1\frac{2}{5}}$

A. $-\frac{4}{3}$ B. $-\frac{3}{4}$ C. $-\frac{49}{12}$ D. $\frac{1}{2}$

If n is an odd integer, which of the following is also odd?

A. $n(n+1)$ B. $n(n-1)$
C. $(n-1)(n+1)$ D. $(n-1)^2 + 1$

Find the percentage error, correct to 1 decimal place, in the area of a square which has its side measured as 2.1 cm instead of the actual 2.0 cm.

A. 10.0% B. 10.3%
C. 12.5% D. 25.0%

5. Solve for x: $16^x = \left(\frac{1}{8}\right)^{x-2}$

A. 6 B. 3 C. $\frac{2}{3}$ D. $\frac{6}{7}$

6. If $\log_3 (x+2)^2 = 4$, then the positive value of x is

A. 6 B. 7 C. 8 D. 9

7. If $\log X = \bar{3}.2504$ and $\log Y = \bar{1}.7782$, evaluate $\frac{1}{4}\log X - \frac{1}{3}\log Y$.

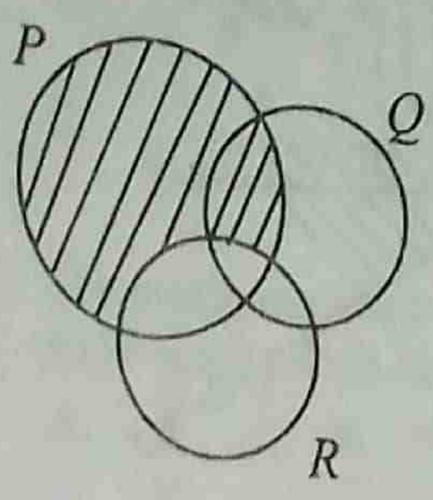
A. 0.6135 B. $\bar{1}.6135$
C. $\bar{1}.3865$ D. $\bar{1}.7801$

8. Simplify $1 - \dfrac{1}{3 - 2\sqrt{2}}$

A. $-2(1+\sqrt{2})$ B. $2(1+\sqrt{2})$
C. $\sqrt{2} - 1$ D. $1 - \sqrt{2}$

9. X, Y and Z are subsets of $U = \{1, 2, 3, 4, 5\}$. If $X \cup Z = U$, $X \cap Z = \varnothing$, $X \cup Y = \{2, 3\}$ and $Y \cap Z = \varnothing$, then Z is

A. $\{1\}$ B. $\{1, 5\}$
C. $\{1, 4, 5\}$ D. $\varnothing$

10. 

The figure shows the subsets P, Q and R of a universal set U. The shaded portion is

A. $P \cup Q' \cap R'$ B. $P \cap Q' \cap R'$
C. $P \cup Q' \cup R'$ D. $P \cup Q \cap R$

11. Divide $2x^3 - xy^2 + y^3$ by $x + y$

A. $2x^2 + xy$ B. $2x^2 - y^2$
C. $2x^2 + xy + y^2$ D. $2x^2 - 2xy + y^2$

12. If $ax^2 + 6x + k$ is a perfect square, then k equals

A. 3 B. 9 C. $9/a$ D. $3/a$

13. Solve the equation $\sqrt{x+6} = \dfrac{2}{\sqrt{x+1}} + \sqrt{x+1}$

A. 2 B. $\frac{5}{2}$ C. 3 D. 4

14. The maximum value of the function $f(x) = -x^2 - 6x + 10$ is

A. –3 B. 1 C. 10 D. 19

15. The line $3y - x = 7$ makes an angle of θ with the positive x-axis. The value of $\sin\theta$ is

A. $\frac{3\sqrt{10}}{10}$ B. $\frac{\sqrt{10}}{10}$ C. $\frac{1}{3}$ D. $\frac{3}{7}$

16. The expression $\dfrac{x^2 - 6x + 9}{x(x^2 - 9)}$ is undefined for

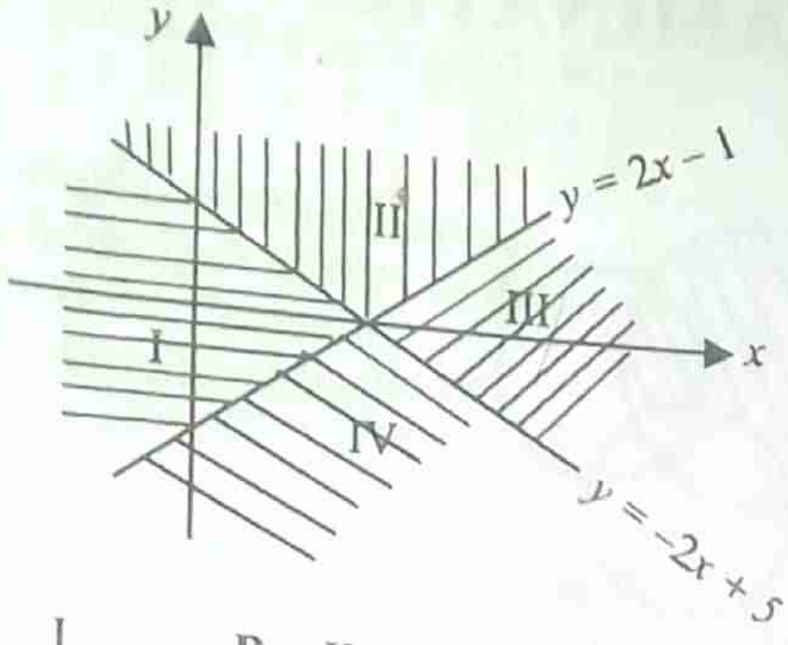
A. $x = 0$ or 9 B. $x = 0$ or –3
C. $x = 0$ or 3 D. $x = 0$, 3 or –3

17. If $\dfrac{2x-3}{2x(2x+1)} = \dfrac{P}{2x+1} + \dfrac{Q}{2x}$ the values of P and Q are, respectively

A. 4, –3 B. 2, 3 C. 4, 3 D. 2, –3

18. Which of the regions in the graph (next page) represents the solution set for the inequalities $2x + y \leq 5$ and $2x - y \leq 1$?

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A. I **B.** II **C.** III **D.** IV

19. The twelfth term of the sequence $x - a, x, x + a, \ldots$ is

A. $x + 12a$ **B.** $x + 11a$
C. $x + 10a$ **D.** $x + 9a$

20. A worker employed in 1992 on a salary of ₦50,000 a year had his salary increased annually in arithmetic progression. If his salary in 1997 was ₦80,000, what was his salary in 1995?

A. ₦65,000 **B.** ₦62,000
C. ₦70,000 **D.** ₦68,000

21. The second term of an infinite geometric series is $-\frac{1}{2}$ and the third term is $\frac{1}{4}$. The sum of the series is

A. 2 **B.** 1 **C.** $\frac{2}{3}$ **D.** $\frac{3}{2}$

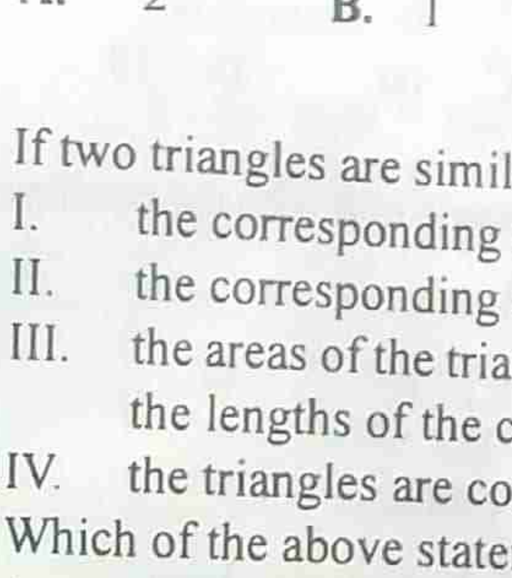
22. If two triangles are similar, then

I. the corresponding angles are equal
II. the corresponding sides are proportional
III. the areas of the triangles are in the same ratio as the lengths of the corresponding sides.
IV. the triangles are congruent.

Which of the above statements are CORRECT?

A. I and II only **B.** III and IV only
C. I, II and III only **D.** I, III and IV only

23.



In the figure, BE is perpendicular to AC and BF is perpendicular to BD. If $C\hat{B}D = 3D\hat{B}E$, then EBF is equal to

A. 70^o **B.** 60^o
C. 45^o **D.** $67\frac{1}{2}^o$

24. An arc of a circle subtends an angle of 45^o at the circumference. The arc must be

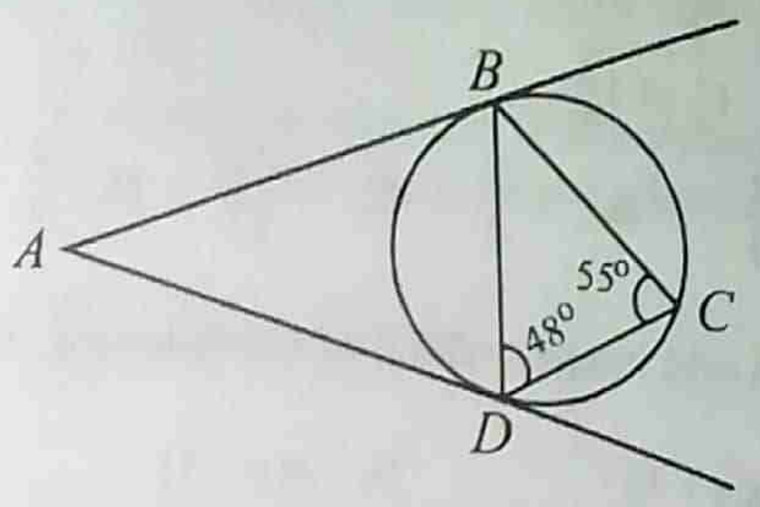
I. a major arc
II. a minor arc

III. a semicircle
IV. one-quarter of the circumference of the circle.

Which of the above is (are) correct?

A. I only **B.** II only
C. III only **D.** II and IV only

25.



In the figure above, AB and AD are tangents to the circle. If $BCD = 55^o$ and $BDC = 48^o$, then BAD =

A. 55^o **B.** 70^o **C.** 77^o **D.** 84^o

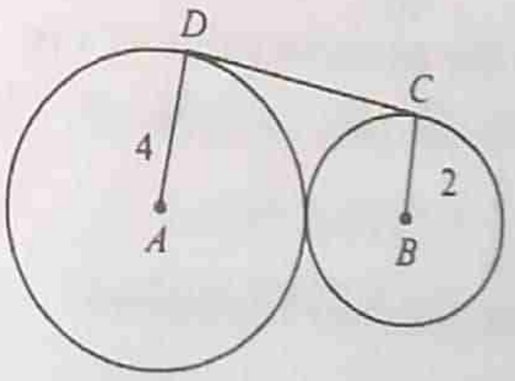
26. If $\sin\theta = -0.6$ and $\tan\theta > 0$, then $\sec\theta$ is equal to

A. 0.8 **B.** −0.8 **C.** −1.25 **D.** 1.25

27. Both $\sin x$ and $\cos x$ decrease as x increases in the range

A. $0 < x < \pi/2$ **B.** $\pi/2 < x < \pi$
C. $\pi < x < 3\pi/2$ **D.** $3\pi/2 < x < 2\pi$

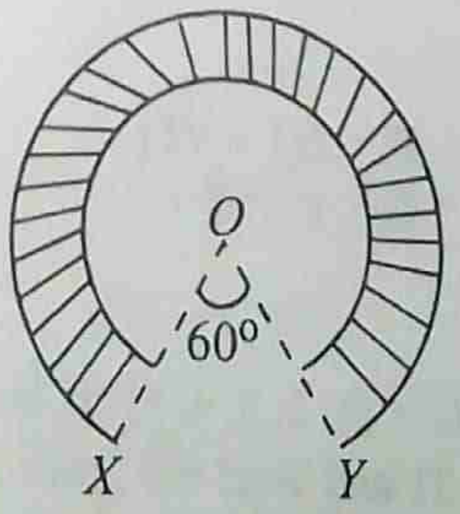
28.



The figure shows two circles of radii 4 cm and 2 cm with centres A and B respectively. If CD is a common tangent to the circles, calculate $|CD|$.

A. $2\sqrt{5}$ cm **B.** $4\sqrt{2}$ cm
C. 6 cm **D.** $4\sqrt{3}$ cm

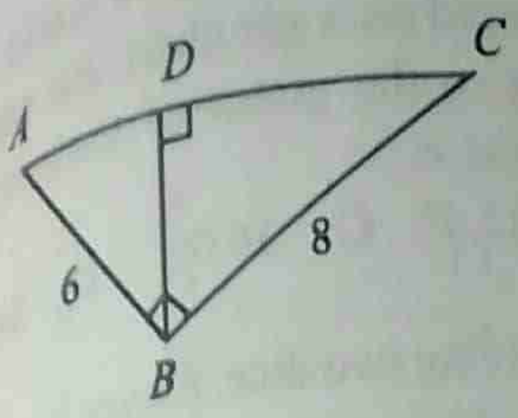
29.



The figure shows a section of a ring made up of two concentric circles of radii 4 cm and 5 cm. A portion of the ring has been cut off such that angle XOY is 60^o. The shaded area is

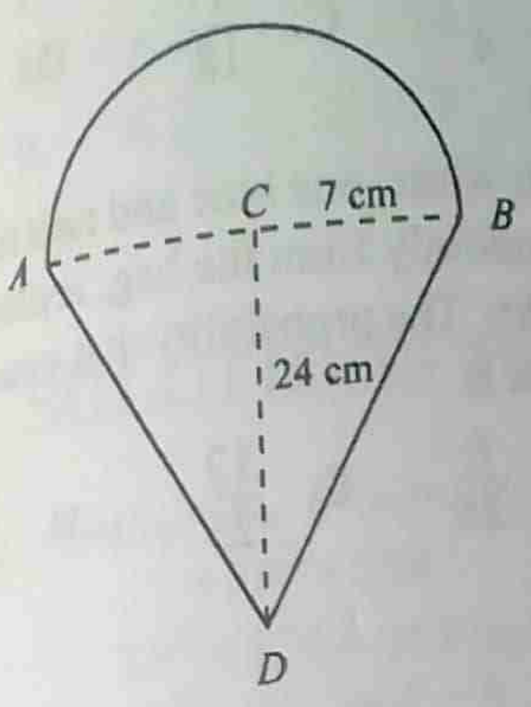
A. $\frac{15\pi}{2}$ cm^2 **B.** 9π cm^2
C. $\frac{3\pi}{2}$ cm^2 **D.** 2π cm^2

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30. 

The figure shows a right-angled triangle ABC in which $AB = 6$, $BC = 8$ and BD is perpendicular to AC. The length of BD is

A. 4.8 B. 5 C. 7.2 D. 10

31. 

The figure shows a wooden toy with a hemispherical top having a radius of 7 cm The height CD is 24 cm. The toy is to be painted with a paint which comes in small cans such that each can will paint an area of 200 cm^2. How many cans of paint are required? (Take $\pi = \frac{22}{7}$)

A. 6 B. 5 C. 4 D. 3

32. The distance between points P (Lat. 60°S, Long. 25°E) and Q (Lat. 60°S, Long. X) on the earth's surface along the parallel of latitude is 2502.5 km. If $\pi = 22/7$ and the earth's radius is 6,370 km, then X could be

A. 22.5°W or 22.5° E
B. 45°W or 45°E
C. 70° W or 70°E
D. 20°W or 70°E

33. Which of the following angles cannot be constructed with a ruler and a pair of compasses only?

A. 105° B. $112\frac{1}{2}^{\circ}$
C. 115° D. 120°

34. A binary operation $*$ is defined over the set of real numbers as $m * n = m \times 2n$. The inverse of 1 is

A. 0 B. 1 C. $\frac{1}{4}$ D. $\frac{1}{2}$

35. Which of the following binary operations is NOT commutative?

A. $a * b = a + (b + 1)$
B. $a * b = ab + a + b$
C. $* b = a \times 3b$
D. $b = (a + b) + b + 2$

36. The derivative of $\sqrt{5 - 4x}$ with respect to x is

A. $-\frac{4}{5}(5 - 4x)^{3/2}$ B. $\frac{1}{\sqrt{5 - 4x}}$
C. $\frac{4}{\sqrt{5 - 4x}}$ D. $-\frac{2}{\sqrt{5 - 4x}}$

37. The maximum value of the function $f(x) = \cos 2x + \sin 2x$ is

A. 2 B. $\sqrt{2}$ C. 1 D. $-\sqrt{2}$

38. Evaluate $\int_{\pi/8}^{\pi/2} 2\sin 2x dx$

A. $\frac{\sqrt{2}}{2}$ B. $\frac{1 - \sqrt{2}}{2}$
C. $\frac{2 + \sqrt{2}}{2}$ D. $\frac{2 - \sqrt{2}}{2}$

39. The population P of a certain town increases at a rate which is directly proportional to the existing population. If P_0 is the population of the town at time $t = 0$, the population after a time t is

A. $P = P_0 e^{kt}$ B. $P = P_0 ln(kt)$
C. $P = P_0 e^{-kt}$ D. $P = P_0 (1 + kt)$

40. If the point $(m, -n)$ lies in the second quadrant, which of the following is true?

A. $m > 0, n > 0$ B. $m > 0, n < 0$
C. $m < 0, n > 0$ D. $m < 0, n < 0$

41. The line $y = 2x - 6$ is a tangent to a curve at the point $(1, -1)$. The equation of the normal to the curve at $(1, -1)$ is

A. $2y - x + 3 = 0$ B. $2y + x + 1 = 0$
C. $y - 2x + 3 = 0$ D. $2y + x - 1 = 0$

42. If $\begin{bmatrix} 2 & m \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & n \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$

then the values of m and n are, respectively

A. 3 and –2 B. –1 and 0
C. 1 and –2 D. 3 and 2

43. Find the possible values of y if

$$\begin{vmatrix} y & 1 \\ 2 & y \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ -1 & 2 \end{vmatrix}$$

A. 3, 0 B. 2, –2 C. 2, 3 D. 3, –3

Use the distribution below to answer questions **44–46.**

Class	1–5	6–10	11–15	16–20
Frequency	1	2	3	2

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44. The mean of the distribution is

A. 10.5 B. 10.55 C. 11.75 D. 12

45. The mode of the distribution is

A. 13.0 B. 12.5 C. 12.0 D. 11.5

46. The mean deviation is

A. 4.0 B. 4.1 C. 4.3 D. 4.5

47. For the following set of numbers:

8, 3, 7, 5, 4, 2, 6, 5, 1, 3

the range and the median are, respectively,

A. 7, 4.5 B. 7, 5 C. 8, 4.5 D. 10, 5

48. The distribution of marks in a class test is shown below for a class of 100 students:

Class	30–39	40–49	50–59	60–69	70–79
No. of students	x	24	28	20	10

No student scored below 30 or above 79 marks. If the distribution is displayed on a pie chart, the sectorial angle corresponding to the group of students who scored below 40 marks is

A. 18.0^o B. 32.4^o C. 36.0^o D. 64.8^o

49. In a *Ludo* game involving two dice, player A will win if he throws at least one six, or the sum of the numbers is 10. The probability of A winning at the next throw is

A. $\frac{1}{3}$ B. $\frac{1}{4}$ C. $\frac{7}{18}$ D. $\frac{4}{9}$

50. A bag contains two white, one blue and two red balls. A ball is drawn randomly from the bag, replaced and another ball is drawn. The probability that exactly one of the balls is white is

A. $\frac{36}{225}$ B. $\frac{6}{25}$ C. $\frac{12}{25}$ D. $\frac{2}{5}$

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ANSWER KEY

1.	C	11.	D	21.	C	31.	B
2.	A	12.	C	22.	A	32.	D
3.	D	13.	C	23.	D	33.	C
4.	B	14.	D	24.	D	34.	C
5.	D	15.	B	25.	B	35.	D
6.	B	16.	B	26.	C	36.	D
7.	C	17.	A	27.	B	37.	B
8.	A	18.	A	28.	B	38.	C
9.	C	19.	C	29.	A	39.	A
10.	A	20.	D	30.	A	40.	D

41.	B
42.	A
43.	D
44.	C
45.	A
46.	B
47.	A
48.	D
49.	C
50.	C

SOLUTIONS TO MODEL MATRICULATION EXAMINATION 2

1. (C) $1P43_7 = 1 \times 7^3 + P \times 7^2 + 4 \times 7 + 3$
$= 7P^2 + 374$
$\therefore\ 7P^2 + 374 = 549 \Rightarrow 7P^2 = 175$
$P^2 = 25,$ or $P = 5$

2. (A) $\dfrac{3\frac{1}{3} - 5\frac{2}{3}}{1\frac{1}{4} \text{ of } 1\frac{2}{5}} = \dfrac{-2\frac{1}{3}}{\frac{5}{4} \times \frac{7}{5}} = \dfrac{-\frac{7}{3}}{\frac{7}{4}}$
$= -\frac{7}{3} \times \frac{4}{7} = -\frac{4}{3}$

3. (D) If n is odd, $n-1$ and $n+1$ are even.
Hence $n(n+1)$, $n(n-1)$ and $(n-1)(n+1)$ are all even. Also, $(n-1)^2$ is even and $(n-1)^2 + 1$ is odd.

4. (B) Measured area $= 2.1 \times 2.1 = 4.41 \text{ cm}^2$
Actual area $= 2.0 \times 2.0 = 4.00 \text{ cm}^2$
Error $\doteq \dfrac{4.41 - 4.00}{4.00} \times 100\% = 10.3\%$

5. (D) $16^x = \left(\frac{1}{8}\right)^{x-2}$
$2^{4x} = (2^{-3})^{x-2} = 2^{-3(x-2)}$
$4x = -3(x-2)$
$7x = 6,$ or $x = \frac{6}{7}$

6. (B) $(x+2)^2 = 3^4 = 81$
$x + 2 = 9,$ or $x = 7$

7. (C) $\frac{1}{4}\log X = \frac{1}{4}(\bar{3}.2504) = \frac{1}{4}(\bar{4} + 1.2504) = \bar{1}.3126$
$\frac{1}{3}\log Y = \frac{1}{3}(\bar{1}.7782) = \frac{1}{3}(\bar{3} + 2.7782)$
$= \bar{1}.9261$
$\frac{1}{4}\log X - \frac{1}{3}\log Y = (\bar{1}.3126) - (\bar{1}.9261)$
$= \bar{1}.3865$

8. (A) $1 - \dfrac{1}{3 - 2\sqrt{2}} = \dfrac{3 - 2\sqrt{2} - 1}{3 - 2\sqrt{2}} = \dfrac{2 - 2\sqrt{2}}{3 - 2\sqrt{2}}$
$= \left(\dfrac{2 - 2\sqrt{2}}{3 - 2\sqrt{2}}\right)\left(\dfrac{3 + 2\sqrt{2}}{3 + 2\sqrt{2}}\right)$
$= \dfrac{6 + 4\sqrt{2} - 6\sqrt{2} - 8}{9 - 8}$
$= -2 - 2\sqrt{2} = -2(1 + \sqrt{2})$

9. (C) Since $X \cup Z = U$, all the elements of U are contained in either X or Z. Also, since no element is common to X and Z, or to Y and Z, Y must be an empty set, i.e. $Y = \varnothing$. Hence $X \cup Y = \{2, 3\} = X$, and $Z = \{1, 4, 5\}$.

10. (A) The shaded portion is the union of P and Q', excluding elements contained in R, i.e. $P \cup Q' \cap R'$.

11. (D)
$$\begin{array}{r|l} & 2x^2 - 2xy + y^2 \\ \hline x+y & 2x^3 + 0x^2y - xy^2 + y^3 \\ & \underline{2x^3 + 2x^2y} \\ & -2x^2y - xy^2 \\ & \underline{-2x^2y - 2xy^2} \\ & xy^2 + y^3 \\ & xy^2 + y^3 \end{array}$$

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12. **(C)** $(\sqrt{a}x + \sqrt{k})^2 = ax^2 + 2\sqrt{a} \cdot \sqrt{k}\, x + k$
Comparing this with $ax^2 + 6x + k$, we have $2\sqrt{a} \cdot \sqrt{k} = 6$, $ak = 9$ and $k = 9/a$.

13. **(C)** Multiply through by $\sqrt{x+1}$

$$\sqrt{(x+6)(x+1)} = 2 + x + 1 = x + 3$$
$$(x+6)(x+1) = (x+3)^2$$
$$x^2 + 7x + 6 = x^2 + 6x + 9$$
$$\text{or } x = 3$$

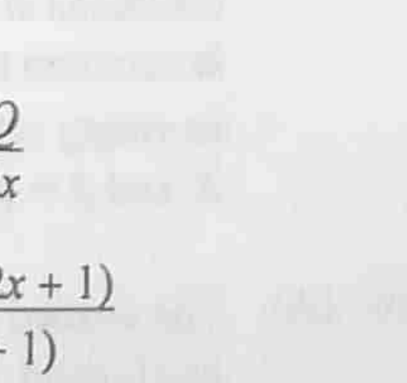
14. **(D)** Re-write the function as
$f(x) = -(x^2 + 6x + 9) + 19 = 19 - (x+3)^2$
$f(x)$ is maximum when the quantity inside the parenthesis is zero.
Hence maximum value of $f(x) = 19$

15. **(B)** Re-write the equation as $y = \frac{1}{3}x + \frac{7}{3}$

Gradient $= \frac{1}{3} = \tan\theta$

From the right-angled Δ (see figure), the hypotenuse $= \sqrt{1^2 + 3^2} = \sqrt{10}$ and

$$\sin\theta = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$



16. **(B)** $\dfrac{x^2 - 6x + 9}{x(x^2 - 9)} = \dfrac{(x-3)^2}{x(x-3)(x+3)}$

$$= \frac{x-3}{x(x+3)}$$

This is undefined when the denominator is zero, i.e. when $x(x+3) = 0$
$\therefore\ x = 0$ or $x = -3$

17. **(A)** $\dfrac{2x-3}{2x(2x+1)} = \dfrac{P}{2x+1} + \dfrac{Q}{2x}$

$$= \frac{2Px + Q(2x+1)}{2x(2x+1)}$$

Equate numerators:
$2x - 3 = 2Px + Q(2x+1)$

When $x = -\frac{1}{2}$: $-4 = -P$, and $P = 4$

When $x = 0$: $-3 = Q$.

18. **(A)** $2x + y \le 5 \Rightarrow y \le -2x + 5$
$2x - y \le 1 \Rightarrow y \ge 2x - 1$
The required region lies below the line $y = -2x + 5$ and above $y = 2x - 1$. This is region I on the graph.

19. **(C)** 3rd term $= x + a$, 4th term $= x + 2a$,
5th term $= x + 3a$, ... *nth* term $= x + (n-2)a$.
Hence 12th term $= x + 10a$

20. **(D)** His salary is represented by the arithmetic sequence, 50,000 ,..., 80,000
where $a = 50{,}000$, $t_n = 80{,}000$ and $n = 6$
$t_n = a + (n-1)d$
$80{,}000 = 50{,}000 + (6-1)d$, or $d = 6{,}000$
Hence salary in 1995 ($n = 4$) is
$50{,}000 + 3(6{,}000) = ₦68{,}000$.

21. **(C)** The infinite geometric series is
$a, -\frac{1}{2}, \frac{1}{4}, \ldots$

with $r = \dfrac{\frac{1}{4}}{-\frac{1}{2}} = -\frac{1}{2}$

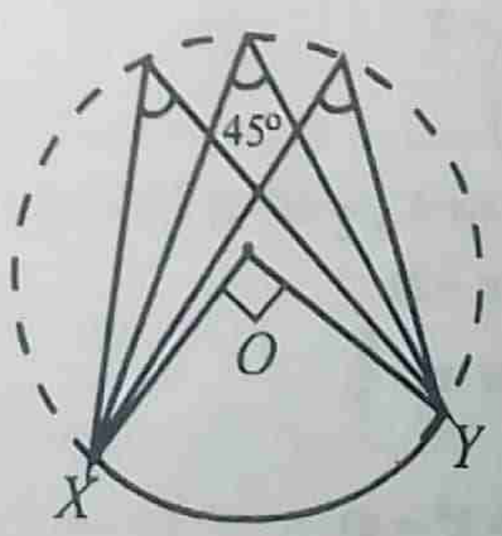
Hence $\dfrac{-\frac{1}{2}}{a} = -\frac{1}{2}$, or $a = 1$

Sum of the series, $S = \dfrac{a}{1-r} = \dfrac{1}{1+\frac{1}{2}} = \frac{2}{3}$

22. **(A)** The ratio of the areas is equal to the ratio of the squares of the lengths of the corresponding sides. Also, the triangles are congruent only if corresponding sides are *equal*.

23. **(D)** Let $D\hat{B}E = x$ and $C\hat{B}D = 3x$
$3x + x = 90^\circ$, or $x = 22\frac{1}{2}^\circ$
Since $B\hat{D}F = 90^\circ$, $E\hat{B}F = 90^\circ - D\hat{B}E$
$= 90^\circ - 22\frac{1}{2}^\circ = 67\frac{1}{2}^\circ$

24. **(D)**



This is an arc (XY) which subtends an angle of 90° at the centre (O) of the circle. The length of the arc is one-quarter of the circle's circumference (see figure). Since the arc is less than one-half of the circumference, it is a *minor* arc.

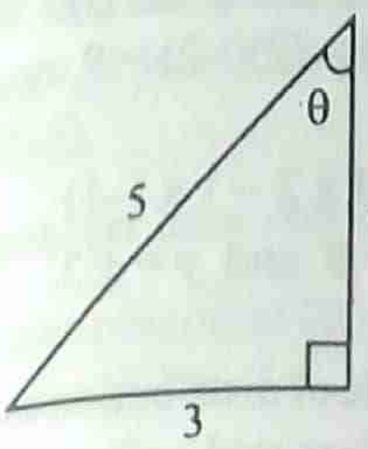
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25. (B) $A\hat{B}D = A\hat{D}B = B\hat{C}D = 55^\circ$

$B\hat{A}D = 180^\circ - A\hat{B}D - A\hat{D}B$

$= 180^\circ - 55^\circ - 55^\circ = 70^\circ$

26. (C)



From the 3 – 4 – 5 right-angled triangle,

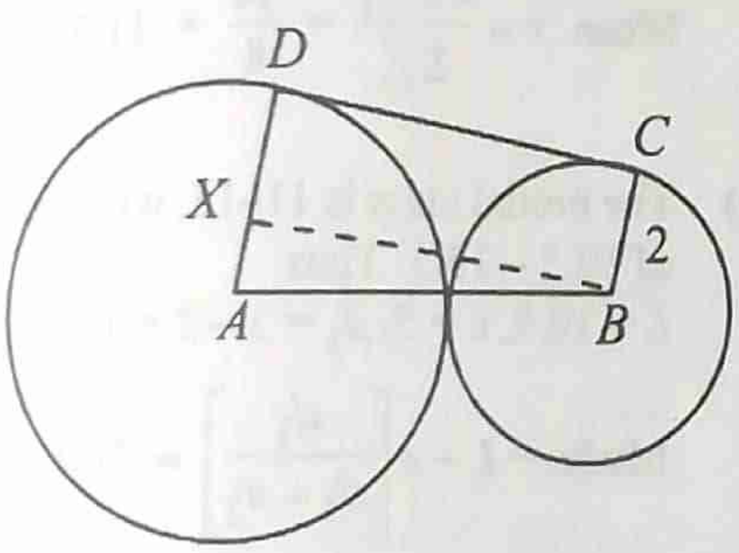
$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{4} =$ 1.25 when $\sin\theta = \frac{3}{5}$

Since $\sin\theta < 0$ and $\tan\theta > 0$, θ lies in the third quadrant, where $\cos\theta < 0$. Hence $\sec\theta < 0$, i.e. $\sec\theta = -1.25$.

27. (B) $\sin x$ *decreases* from 1 to 0 as x increases from $\pi/2$ to π.

$\cos x$ *decreases* from 0 to –1 as x increases from $\pi/2$ to π.

28. (B)



Join B to AD such that BX is perpendicular to AD. $BCDX$ is a rectangle with

$DX = CB = 2$ cm

$AX = AD - DX = 4 - 2 = 2$ cm

and $AB = 4 + 2 = 6$ cm

Pythagoras: $BX^2 = AB^2 - AX^2$

$= 6^2 - 2^2 = 32$

$BX = \sqrt{32} = 4\sqrt{2}$ cm $= CD$.

29. (A) Area of sector formed by outer cirlce

$= \frac{1}{2} r^2\theta = \frac{1}{2}(5)^2 \times \frac{5\pi}{3} = \frac{125\pi}{6}$ cm^2

$\left[\text{Note that } \theta = 300^\circ \times \frac{2\pi}{360^\circ} = \frac{5\pi}{3}\right]$

Area of sector formed by inner circle

$\frac{1}{2}(4)^2 \times \frac{5\pi}{3} = \frac{80\pi}{6}$ cm^2

Shaded area $= \frac{125\pi}{6} - \frac{80\pi}{6} = \frac{15\pi}{2}$ cm^2

30. (A) $AC^2 = AB^2 + BC^2 = 6^2 + 8^2 = 100$, or

$AC = 10$.

Area of $\Delta ABC = BD \times AC/2 = AB \times BC/2$

$BD = \frac{AB \times BC}{AC} = \frac{6 \times 8}{10} = 4.8$

31. (B) Surface area of hemispherical top

$= 2\pi r^2 = 2 \times \frac{22}{7} \times 7^2 = 308$ cm^2

Length of slanted side of cone,

$BD = \sqrt{24^2 + 7^2} = 25$ cm

Area of slanted surface

$= \pi rl = \frac{22}{7} \times 7 \times 25 = 550$ cm^2

Total area of toy's surface

$= 308 + 550 = 858$ cm^2

Number of cans of paint required

$= \frac{858}{200} = 4.29$, i.e. 5 cans are needed.

32. (D) Radius of parallel of latitude

$= R\cos\theta = 6{,}370 \cos 60^\circ = 3{,}185$ km

Circumference of parallel of latitude

$= 2\pi(3{,}185) = 2 \times \frac{22}{7} \times 3{,}185$

$= 20{,}020$ km

Distance $PQ = 2{,}502.5 = 20{,}020 \times \frac{\theta}{360}$

where θ = angle subtended by the arc PQ at the centre of the earth.

$\theta = \frac{2{,}502.5 \times 360}{20{,}020} = 45^\circ$

Hence Q is 45°E of P (i.e. Long. 70°E) or 45°W of P (Long. 20°W).

33. (C) Angles 90° and 60°, as well as their bisectors, can be constructed using a ruler and a pair of compasses. Angles 45°, 30°, $22\frac{1}{2}^\circ$, 15°, ... and any combination of these can be constructed. Hence 105° can be constructed as 90° + 15°, $112\frac{1}{2}^\circ$ as $90^\circ + 22\frac{1}{2}^\circ$ and 120° as 60° + 60°

34. (C) $m * n = m \times 2n$

Identity element is given by

$m * e = e * m = m$

Hence $m \times 2e = m$, or $e = 1/2$

Inverse i of 1 is given by

$1 * i = e \Rightarrow 1 \times 2i = 1/2$, or $i = 1/4$

35. (D) $a * b = a + 2b + 2$

$b * a = b + 2a + 2$

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Hence $a * b \neq b * a$
[The candidate should check that (A), (B) and (C) are commutative.]

36. **(D)** Let $y = (5 - 4x)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}(5 - 4x)^{-1/2} \times -4$$

$$= -2(5 - 4x)^{-1/2}$$

$$= \frac{-2}{\sqrt{5 - 4x}}$$

37. **(B)** $y = \cos 2x + \sin 2x$

$\frac{dy}{dx} = -2 \sin 2x + 2 \cos 2x = 0$ for maximum or minimum point.
Hence $\tan 2x = 1$, or $2x = 45^\circ$ or 225°
For $2x = 45^\circ : y = \cos 45^\circ + \sin 45^\circ = \sqrt{2}$ (max).
For $2x = 225^\circ : y = \cos 225^\circ + \sin 225^\circ$
$= -\sqrt{2}$ (min).

38. **(C)** $\int_{\pi/8}^{\pi/2} 2\sin 2x \, dx = [-\cos 2x]_{\pi/8}^{\pi/2}$

$$= -\cos \pi - (-\cos \pi/4) = 1 + \frac{\sqrt{2}}{2}$$

$$= \frac{2 + \sqrt{2}}{2}$$

39. **(A)** $\frac{dP}{dt} = kP$

$$\int \frac{dP}{P} = \int k\,dt$$

$$\ln P = kt + c$$

At $t = 0$, $P = P_0 \Rightarrow c = \ln P_0$
$\ln P = \ln P_0 + kt$

$$\ln \frac{P}{P_0} = kt \text{ or } P = P_0 e^{kt}$$

40. **(D)** The x-coordinate, m, is negative in the second quadrant, i.e. $m < 0$.
The y-coordinate, $-n$, is positive in the second quadrant, i.e. $-n > 0$, and $n < 0$.

41. **(B)** The slope of the tangent is 2, hence the normal to the curve must have a slope m such that $m \times 2 = -1$, or $m = -\frac{1}{2}$. Equation of the normal is thus

$$y = -\frac{1}{2}x + c \text{ or } 2y + x = k$$

Substituting the point $(1, -1)$ gives $k = -1$
Hence $2y + x + 1 = 0$.

42. **(A)** 1st row x 2nd column:

$$2n + 3m = 5 \quad \text{...(i)}$$

2nd row x 2nd column:

$$n + 6 = 4 \quad \text{...(ii)}$$

From (ii), $n = -2$ and from (i)
$3m = 5 - 2n = 5 - (2)(-2) = 9$, or $m = 3$.

43. **(D)** $y \times y - 2 \times 1 = 3 \times 2 - 1 \times (-1)$
$y^2 - 2 = 7$, $y^2 = 9$ and $y = \pm 3$.

44. **(C)** Redraw the table of distribution as shown below including the class mid-values, x_i.

Class	x_i	f_i	$f_i x_i$
1–5	3	1	3
6–10	8	2	16
11–15	13	3	39
16–20	18	2	36
		8	94

Mean $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{94}{8} = 11.75$

45. **(A)** The modal class is 11–15, with true class limits of 10.5 – 15.5. Thus
$L = 10.5$, $c = 5$, $d_1 = 3 - 2 = 1$, $d_2 = 3 - 2 = 1$

$$\text{Mode} = L + c\left[\frac{d_1}{d_1 + d_2}\right] = 10.5 + 5\left(\frac{1}{2}\right)$$

$$= 13.0$$

46. **(B)**

x_i	f_i	$\lvert x_i - \bar{x} \rvert$	$f_i(x_i - \bar{x})$
3	1	8.75	8.75
8	2	3.75	7.50
13	3	1.25	3.75
18	2	6.25	12.50
	8		32.50

$$\text{Mean Deviation} = \frac{\Sigma f_i (x_i - \bar{x})}{\Sigma f_i} = \frac{32.50}{8}$$

$$= 4.06 \approx 4.1$$

47. **(A)** Re-arrange the numbers in ascending order:
1, 2, 3, 3, 4, 5, 5, 6, 7, 8
Range = 8 – 1 = 7

$$\text{Median} = \frac{1}{2}(5\text{th value} + 6\text{th value})$$

$$= \frac{1}{2}(4 + 5) = 4.5$$

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48. (D) $x = 100 - (24 + 28 + 20 + 10) = 18$

Sectorial angle $= \frac{18}{100} \times 360^\circ = 64.8^\circ$

49. (C) Of the thirty-six possible outcomes, 11 will show at least one six (see table under Example 20–5).

Prob. (at least one six) $= \frac{11}{36}$

Three outcomes (6, 4), (4, 6) and (5, 5) will give a sum of 10.

Prob. (sum of 10) $= \frac{3}{36}$

Probability of at least one six *or* sum of 10

$= \frac{11}{36} + \frac{3}{36} = \frac{7}{18}$

50. (C) Either 1st ball is white and second *not* white $= P(w, \overline{w})$

or 1st ball is *not* white and second is white $= P(\overline{w}, w)$

$P(w, \overline{w}) = \frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

$P(\overline{w}\ w) = \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

Probability required $= \frac{6}{25} + \frac{6}{25} = \frac{12}{25}$

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MODEL MATRICULATION EXAMINATION 3

1. Calculate $2034_5 + 4213_5 - 3112_5$
   A. 3240_5 **B.** 3130_5
   C. 3140_5 **D.** 14414_5

2. Evaluate $\frac{56{,}000 \times 0.07}{0.014}$, leaving your answer in standard form
   A. 2.8×10^7 **B.** 2.8×10^5
   C. 2.8×10^4 **D.** 2.8×10^{-3}

3. A housewife spent a quarter of her food budget on provisions, half the balance on staples and one-third of the remainder on meat. If she was left with ₦350, how much was her food budget?
   A. ₦700 **B.** ₦1,050
   C. ₦1,400 **D.** ₦2,100

4. A man opened a savings account with ₦1,000.00. One year later he deposited ₦500.00 in the account and deposited a further ₦500.00 two years after opening the account. If the interest rate is 8% p.a. computed annually, calculate the total amount, to the nearest naira, in his account three years after opening the account.
   A. ₦2,480 **B.** ₦2,383
   C. ₦2,360 **D.** ₦2,348

5. Solve for x : $9^x = 27 \times 3^{1-x}$
   A $\frac{4}{3}$ **B.** $\frac{3}{2}$ **C.** -1 **D.** $\frac{2}{3}$

6. Which of the following is correct?
   A. $(x^3)^2 = x^5$ **B.** $x^3 + x^4 = x^7$
   C. $x^2 \times x^{-3} = \frac{1}{x}$ **D.** $x\sqrt{y} = y\sqrt{x}$

7. If $\log 4.5 = 0.6532$ and $\log 8.1 = 0.9085$, find $\log\left(\frac{0.045}{81}\right)$ correct to 3 decimal places.
   A. $\bar{2}.745$ **B.** $\bar{3}.255$ **C.** $\bar{4}.225$ **D.** $\bar{4}.745$

8. Solve for x : $2 \log x - 3 \log x + 4 \log x = \log 27$
   A. 2 **B.** 3 **C.** 6 **D.** 9

9. The antilog of a number is written as -2.3010. This is the same as
   A. $\bar{3}.6990$ **B.** $\bar{2}.3010$
   C. $\bar{2}.6990$ **D.** $\bar{3}.3010$

10. If $U = \{\text{integers } x : 0 < x < 10\}$
    $P = \{\text{prime numbers } y : 0 < y < 10\}$, and
    $Q = \{\text{odd numbers } z : 0 < z < 10\}$
    then $P' \cap Q'$ is
    A. $\varnothing$ **B.** $\{1, 4, 6, 8\}$
    C. $\{2, 4, 6, 8\}$ **D.** $\{4, 6, 8\}$

11. In a senior secondary school certificate class, 48 students offer at least one of Economics and Physics. If 36 students offer Economics while 25 offer Physics, find the number of students who offer Economics but not Physics.
    A. 25 **B.** 23 **C.** 13 **D.** 11

12. Simplify $\frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}}$
    A. $\frac{y}{x}$ **B.** $\frac{xy}{y - x}$
    C. $\frac{x - y}{xy}$ **D.** $-\frac{x - y}{xy}$

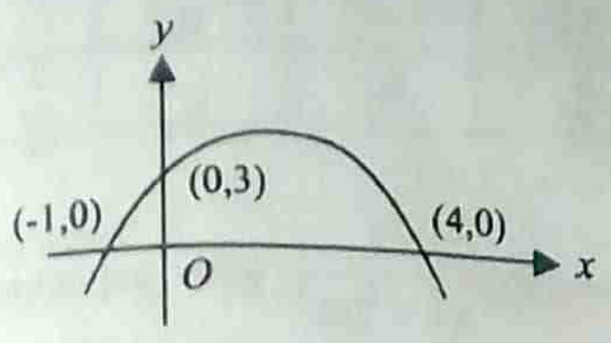
13. The quantity x varies inversely as y, and y varies directly as the square of z. If z is halved, then the value of x is
    A. halved
    B. doubled
    C. reduced by a factor of 4
    D. quadrupled

14. If $f(x + 2) = x^2 + 3x + 1$, then $f(-2)$ is
    A. -1 **B.** 1 **C.** 5 **D.** 11

15. When a polynomial $f(x)$ is divided by $2x - 3$, the quotient is $x^2 - x + 2$ and the remainder is -1. Find $f(x)$.
    A. $2x^3 - 5x^2 + 7x - 5$
    B. $2x^3 - 5x^2 + 7x - 7$
    C. $2x^3 + 5x^2 - 3x + 7$
    D. $2x^3 + 5x^2 - 7x + 5$

16. If -1 is a root of the equation $-cx^2 + 6x + 8 = 0$, the value of c is
    A. 4 **B.** 3 **C.** 2 **D.** 1

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The equation of the parabola shown in the figure above is

A. $y = 3 + 4x - x^2$ B. $y = x^2 - 4x + 3$

C. $y = \frac{3}{4}x^2 - \frac{9}{4}x + 3$ D. $y = 3 + \frac{9}{4}x - \frac{3}{4}x^2$

18. Two years ago, a woman was six times as old as her son. Four years from now, she will be three times as old as the son. Their present ages are

A. 32, 8 B. 26, 6 C. 28, 7 D. 30, 7

19. Solve the simultaneous equations

$\frac{2}{x} + \frac{1}{y} = 3$ and

$\frac{3}{x} - \frac{2}{y} = 8$

A. $x = \frac{1}{2}, y = -1$ B. $x = 1, y = 2$

C. $x = \frac{1}{2}, y = 1$ D. $x = -1, y = 2$

20. Simplify $[x + (1 + x^{-1})^{-1}]^{-1}$

A. $\frac{1+x}{x+2}$ B. $\frac{x(x+2)}{1+x}$

C. $\frac{1+x}{x(x+2)}$ D. 1

21. If $x^2 > 4x - 3$, then

A. $x < 1$ or $x > 3$ B. $x > 3$

C. $1 < x < 3$ D. $x < 1$

22. If $\log_a (x^2 + x) - \log_a 2 - 2\log_a x > 0$, then

A. $x > 0$ or $x < 1$ B. $x < 0$ or $x > 1$

C. $0 < x < 1$ D. $0 < x < 2$

23. An arithmetic sequence has five terms between 2 and 20. The fourth term of the sequence is

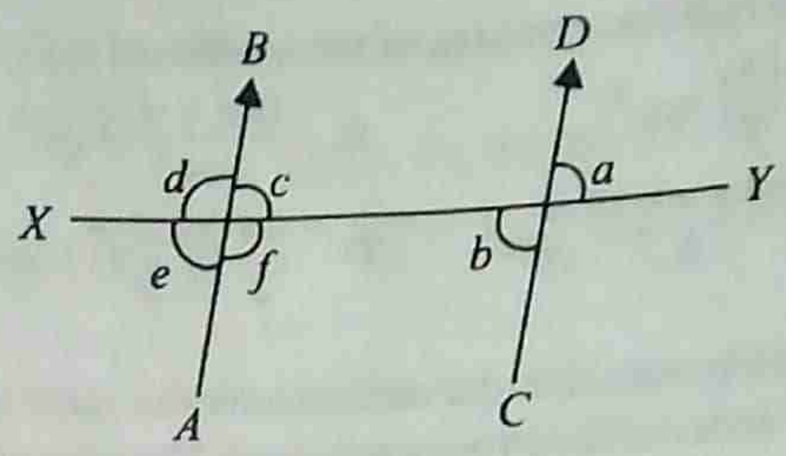
A. 8 B. 11 C. 12 D. 14

24. A boy received a cash gift from his parents on his 10th birthday. On each succeeding birthday, he received twice the amount received in the previous year. If he had received a total of ₦31,500 by the time he was 15, the cash gift for his 12th birthday was

A. ₦4,000 B. ₦3,000

C. ₦2,000 D. ₦1,000

25.

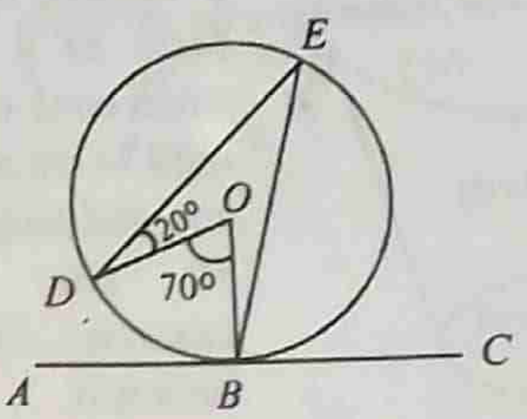


In the figure above $AB // CD$ and XY intersects both AB and CD. If $a = 85^\circ$, then

A. $d = 85^\circ$ B. $e = 95^\circ$

C. $b + e = 180^\circ$ D. $a + d = 180^\circ$

26.



In the figure, ABC is a tangent to the circle centre O. If $DOB = 70^\circ$ and $EDO = 20^\circ$, then CBE is

A. 60° B. 65° C. 70° D. 75°

27. $\sec\theta \sin(90^\circ - \theta)$ equals

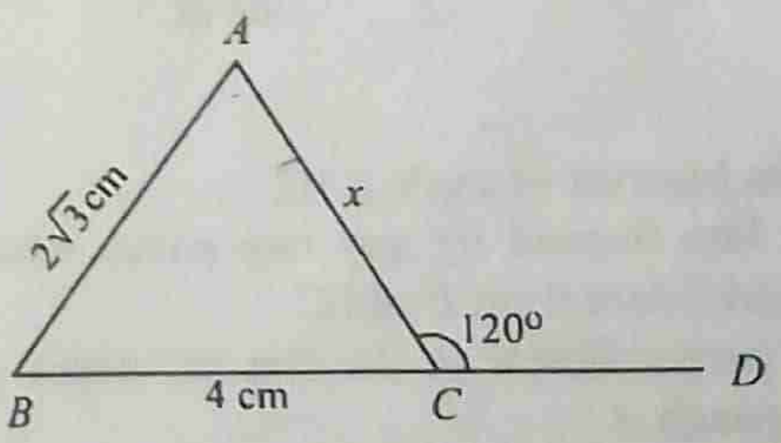
A. 1 B. 0 C. $\tan\theta$ D. $\cot\theta$

28. If $2\cos^2\theta = -\cos\theta$ and $0 < \theta < 180^\circ$, then θ is

A. 90° or 120° B. 60° or 90°

C. 60° or 120° D. 30° or 150°

29.

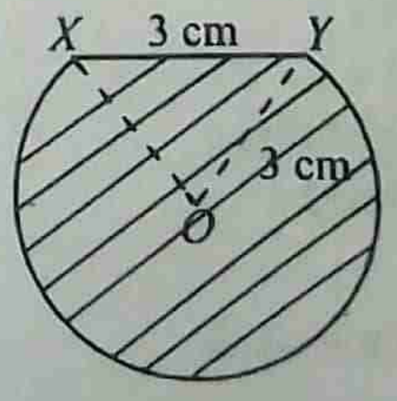


The figure above shows a triangle ABC with $AB = 2\sqrt{3}$ cm, $BC = 4$ cm and $AC = x$. BC is produced to D such that angle $ACD = 120^\circ$. Find the value of x.

A. $3\sqrt{2}$ cm B. 3 cm

C. $2\sqrt{2}$ cm D. 2 cm

30.



The figure shows a cardboard in the form of a circle, centre O, which has been truncated along XY. If $XY = 3$ cm and the radius of the circular

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section is 3 cm, the area of the cardboard is

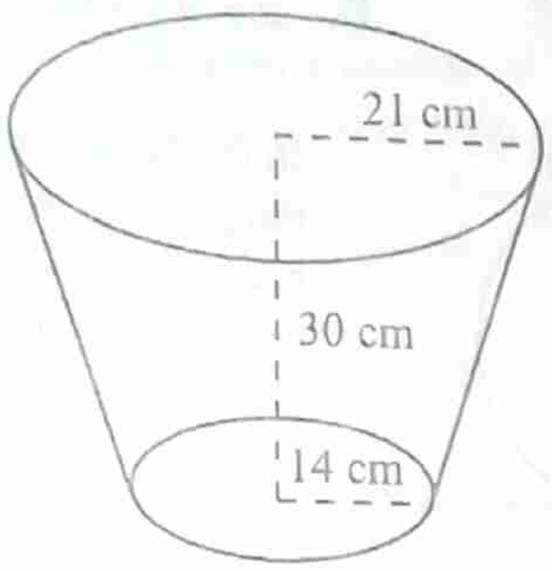
A. $\frac{15\pi}{2}$ cm^2 B. $\frac{9(\pi+\sqrt{3})}{4}$ cm^2

C. $\frac{30\pi+9\sqrt{3}}{4}$ cm^2 D. $\frac{15}{2}(\pi+\sqrt{3})$ cm^2

31. A square pyramid and a cuboid have the same height and the areas of their bases are equal. The ratio of their volumes is

A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$

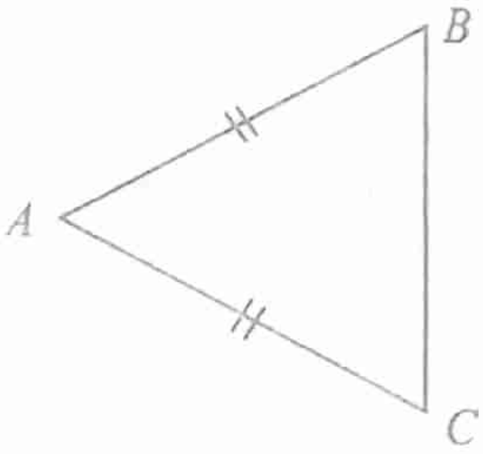
32.



The figure shows a bucket in the form of a truncated cone of end radii 14 cm and 21 cm, and height of 30 cm. The capacity of the bucket is

A. 12,320 cm^3 B. 29,260 cm^3
C. 41,580 cm^3 D. 53,900 cm^3

33.



In the figure, ABC is an isosceles triangle with $AB = AC$. Which of the following constructions will yield the perpendicular bisector of side BC?

I. the bisector of angle BAC.
II. a line formed by any two points which are equidistant from B and C.
III. a perpendicular to the line BC which passes through A.

A. I and II only B. I and III only
C. II and III only D. I, II and III

34. A binary operation $*$ is defined over the set of real numbers such that $m * n = m + n + 2$ and $m \odot n = m - n + 1$. Which of the following equations is ***not*** true?

A. $a * (b * c) = (a * b) * c$
B. $a * (b * c) = (a * c) * b$
C. $a \odot (b \odot c) = (a \odot b) \odot c$
D. $a * (b \odot c) = (a * b) \odot c$

35. If $y = \sqrt{x^2+1}$, then $\frac{dy}{dx}$ at x = –1 is

A. $\frac{\sqrt{2}}{2}$ B. $-\frac{\sqrt{2}}{2}$
C. $\sqrt{2}$ D. $-\sqrt{2}$

36. Differentiate $\frac{3}{(1-2x)^2}$ with respect to x.

A. $\frac{12}{(1-2x)^3}$ B. $-\frac{12}{(1-2x)^3}$
C. $12(1-2x)^{-4}$ D. $\frac{6}{(1-2x)^3}$

37. Integrate $\left(x-\frac{1}{x}\right)^2$ with respect to x.

A. $\frac{1}{3}\left(x-\frac{1}{x}\right)^3+c$ B. $\frac{x^3}{3}-x-\frac{1}{x^3}+c$

C. $\frac{x^3}{3}-2x+\frac{1}{x^3}+c$ D. $\frac{x^3}{3}-2x-\frac{1}{x}+c$

38. The area bounded by the curve $y = \sin 2x$, the x-axis, the y-axis and the line $x = \pi/2$ is

A. 0. B. $\frac{1}{2}$ C. 1 D. 2

39. Find the value of the acute angle x for which the tangent to the curve $y = \sin 2x$ is parallel to the line $x - y + 2 = 0$.

A. 15° B. 30° C. 45° D. 60°

40. If $X = (-1, 2)$ and $Y = (0, 1)$, which of the following points lies on the line XY produced?

A. (1, 3) B. (3, 4) C. (3, –2) D. (2, –2)

41. The coordinates of the vertices P and Q of a square $PQRS$ are $P = (1, 3)$ and $Q = (5, 1)$. The coordinates of R could be

A. (3, 7) B. (3, 0) C. (6, 3) D. (7, 5)

42. If $P = \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix}$ and P^T is the transpose of P, then PP^T is equal to

A. 0 B. 1 C. 7 D. 49

43. If the mean of the numbers 6, 8, 10, x, $2x - 2$ is 14, the largest number in the group is

A. 30 B. 24 C. 20 D. 16

The following distribution shows the marks scored by students in a class test. Use the distribution to answer questions **44–46.**

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Class	10–19	20–29	30–39	40–49
Frequency	2	4	3	1

44. The mean score is
A. 27.5 B. 28.0 C. 28.6 D. 28.9

45. The modal score is
A. 27.0 B. 26.2 C. 25.5 D. 24.5

46. The median score is
A. 24.5 B. 25.5 C. 26.5 D. 27.0

47. The variance for the set of numbers
3, 6, 9, 12, 15 is
A. 36 B. 24 C. 18 D. 12

48. Find the lower and upper quartiles for the set of numbers
4, 7, 8, 10, 12, 15, 17, 20, 26, 32, 40
A. 8, 20 B. 10, 20
C. 8, 26 D. 7, 32

49. What is the probability that the sum of any two numbers drawn from the set {1, 2, 3, 4} is odd?
A. $\frac{5}{6}$ B. $\frac{2}{3}$ C. $\frac{1}{2}$ D. $\frac{1}{3}$

50. A group photograph of a family of six (four children and their parents) is to be taken. If the parents must occupy the two middle seats, then the number of different arrangements is
A. 48 B. 120 C. 24 D. 720

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ANSWER KEY

1.	C	11.	B	21.	A	31.	C	41.	D
2.	B	12.	D	22.	C	32.	B	42.	D
3.	C	13.	D	23.	B	33.	D	43.	A
4.	B	14.	C	24.	C	34.	C	44.	A
5.	A	15.	B	25.	D	35.	B	45.	B
6.	C	16.	C	26.	D	36.	A	46.	D
7.	D	17.	D	27.	A	37.	D	47.	C
8.	B	18.	B	28.	A	38.	C	48.	C
9.	A	19.	A	29.	D	39.	B	49.	B
10.	D	20.	C	30.	C	40.	C	50.	A

SOLUTIONS TO MODEL MATRICULATION EXAMINATION 3

1. (C)
$$\begin{array}{r} 2034 \\ +\ 4213 \\ \hline 11302 \\ -\ 3112 \\ \hline 3140_5 \end{array}$$

2. (B) $$\frac{56 \times 10^3 \times 7 \times 10^{-2}}{14 \times 10^{-3}} = 28 \times 10^4$$
$$= 2.8 \times 10^5$$

3. (C) On provisions she spent ... $\frac{1}{4}$ of budget.

On staples she spent ... $\frac{1}{2}\left(1 - \frac{1}{4}\right) = \frac{3}{8}$ of budget.

Balance $= 1 - \left(\frac{1}{4} + \frac{3}{8}\right) = \frac{3}{8}$ of budget.

On meat she spent $\frac{1}{3} \times \frac{3}{8} = \frac{1}{8}$ of budget.

Total spent $= \frac{1}{4} + \frac{3}{8} + \frac{1}{8} = \frac{3}{4}$

Balance $= \frac{1}{4}$ of budget $=$ ₦350

$\Rightarrow$ Budget $= 4 \times 350 =$ ₦1,400

4. (B) Opening ... ₦1,000.00

One year later, $I = \frac{PRT}{100} = \frac{1{,}000 \times 8 \times 1}{100}$
$=$ ₦80.00

Total amount after 1 year
$= 1{,}000 + 80 + 500 =$ ₦1,580

Two years later, $I = \frac{PRT}{100} = \frac{1580 \times 8 \times 1}{100}$
$=$ ₦126.40

Total amount after 2 years
$= 1{,}580 + 126.40 + 500 =$ ₦2,206.40

One more year later,
$$I = \frac{2206.40 \times 8 \times 1}{100}$$
$=$ ₦176.51

Total amount $=$ ₦2206.40 + 176.51
$=$ ₦2,383, approximately.

5. (A) $9^x = 27 \times 3^{1-x}$
$3^{2x} = 3^3 \times 3^{1-x} = 3^{4-x}$
$2x = 4 - x$, or $x = \frac{4}{3}$

6. (C) $x^2 \times x^{-3} = x^{-1} = \frac{1}{x}$

[Note that $(x^3)^2 = x^6$ and $x^3 \times x^2 = x^5$].

7. (D) $\log\left(\frac{0.045}{81}\right) = \log 0.045 - \log 81$
$= \bar{2}.6532 - 1.9085$
$= \bar{4} + 2.6532 - 1.9085$
$= \bar{4}.745$

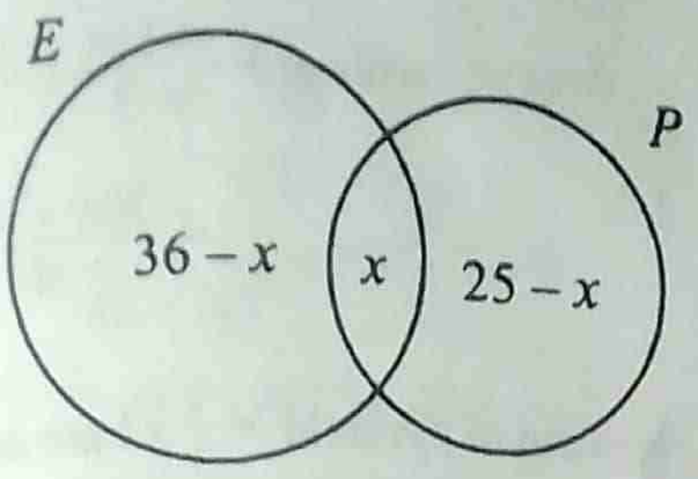
8. (B) $\log x^2 - \log x^3 + \log x^4 = \log 27$
$\log \frac{x^2 \times x^4}{x^3} = \log 27$
$\log x^3 = \log 27 \Rightarrow x^3 = 27$ and $x = 3$

9. (A) $-2.3010 = -3 + 0.6990 = \bar{3}.6990$

10. (D) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$P = \{2, 3, 5, 7\}$, $P' = \{1, 4, 6, 8, 9\}$
$Q = \{1, 3, 5, 7, 9\}$, $Q' = \{2, 4, 6, 8\}$
$P' \cap Q' = \{4, 6, 8\}$

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11. (B)



In the Venn diagram above, E and P are the subsets of students who offer Economics and Physics respectively.

$36 - x + x + 25 - x = 48$

$61 - x = 48$, or $x = 13$

Number offering Economics only

$= 36 - x = 36 - 13 = 23.$

12. (D) $$\frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}} = \frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x} + \frac{1}{y}} = \frac{y^2 - x^2}{x^2 y^2} \times \frac{xy}{x + y}$$

$$= \frac{(y + x)(y - x)}{xy(x + y)}$$

$$= \frac{y - x}{xy} = -\frac{x - y}{xy}$$

13. (D) $x \propto \frac{1}{y}$ and $y \propto z^2 \Rightarrow x \propto \frac{1}{z^2}$, or $x = \frac{k}{z^2}$

If z is halved,

new $x = \frac{k}{(z/2)^2} = 4\frac{k}{z^2}$

i.e. x is quadrupled.

14. (C) $f(x + 2) = x^2 + 3x + 1$

$f(x + 2) = f(-2)$ when $x + 2 = -2$,

or $x = -4$

$\therefore\ f(-2) = f(-4 + 2)$

$= (-4)^2 + 3(-4) + 1 = 5$

15. (B) $f(x) = (2x - 3)(x^2 - x + 2) - 1$

$= 2x^3 - 2x^2 + 4x - 3x^2 + 3x - 6 - 1$

$= 2x^3 - 5x^2 + 7x - 7$

16. (C) Substitute $x = -1$ into the equation:

$-c(-1)^2 + 6(-1) + 8 = 0$, or $c = 2$

17. (D) The intercepts on the x-axis are -1 and 4.

Hence the equation is of the form

$y = k(x + 1)(x - 4) = kx^2 - 3kx - 4k$

The intercept on the y-axis is 3, hence

$-4k = 3$ or $k = -\frac{3}{4}$

The equation is $y = -\frac{3}{4}x^2 + \frac{9}{4}x + 3.$

18. (B) Let x, y be the present ages of the mother and son, respectively.

$x - 2 = 6(y - 2)$, or $x - 6y = -10$ ...(i)

$x + 4 = 3(y + 4)$, or $x - 3y = 8$ ...(ii)

Subtract (i) from (ii): $3y = 18$, or $y = 6$

From (ii), $x = 8 + 3y = 8 + 3(6) = 26.$

19. (A) Multiply the first equation by 2:

$\frac{4}{x} + \frac{2}{y} = 6$ ...(i)

$\frac{3}{x} - \frac{2}{y} = 8$ ... (ii)

(i) + (ii): $\frac{4}{x} + \frac{3}{x} = 14$

$\frac{7}{x} = 14$, or $x = \frac{1}{2}$

From (i): $\frac{2}{y} = 6 - \frac{4}{1/2} = 6 - 8 = -2$, or

$y = -1$

20. (C) $$[x + (1 + x^{-1})^{-1}]^{-1} = \left[x + \left(1 + \frac{1}{x}\right)^{-1}\right]^{-1}$$

$$= \left[x + \frac{x}{1 + x}\right]^{-1} = \left[\frac{x + x^2 + x}{1 + x}\right]^{-1}$$

$$= \frac{1 + x}{x(x + 2)}$$

21. (A) $x^2 - 4x + 3 > 0$

$(x - 3)(x - 1) > 0$

Either $x > 3$ and $x > 1 \Rightarrow x > 3$

or $x < 3$ and $x < 1 \Rightarrow x < 1$

Hence $x < 1$ or $x > 3$

22. (C) $\log_a(x^2 + x) - \log_a 2 - 2\log_a x > 0$

$\log_a(x^2 + x) - \log_a 2 - \log_a x^2 > 0$

$\log_a \frac{x^2 + x}{2x^2} > 0$

$\frac{x^2 + x}{2x^2} > 1 \Rightarrow x^2 + x > 2x^2$

$x^2 - x < 0$

$x(x - 1) < 0$

Either $x > 0$ and $x < 1 \Rightarrow 0 < x < 1$

or $x < 0$ and $x > 1$ ... (not feasible)

23. (B) The arithmetic sequence is

2, ..., 20

The nth term, $t_n = a + (n - 1)d$

First term is 2 and 7th term is 20

$\therefore\ 20 = 2 + 6d$, or $d = 3$

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The sequence is thus 2, 5, 8, 11, 14, 17, 20 and the fourth term is 11.

24. (C) Birthday: 10th 11th 12th 13th 14th 15th
Gift: a $2a$ $4a$ $8a$ $16a$ $32a$
The gift forms a geometric sequence with $r = 2$.

$$\text{Sum, } S_n = \frac{a(1-r^n)}{1-r} = \frac{a(1-2^6)}{1-2} = 63a$$

[Alternatively, add all the gifts together to get $63a$].
Hence $63a = 31{,}500$, or $a = ₦500$.
$\therefore$ Gift on 12th birthday $= 4a = ₦2{,}000$.

25. (D) $a = c$ and $c + d = 180^\circ$
Hence $a + d = 180^\circ$

26. (D) Join B to D
ODB is an isosceles Δ

$$\hat{ODB} = \hat{OBD} = \frac{1}{2}(180^\circ - 70^\circ) = 55^\circ$$

$$\hat{EDB} = \hat{EDO} + \hat{ODB} = 20^\circ + 55^\circ = 75^\circ$$

$$\hat{CBE} = \hat{EDB} = 75^\circ \text{ (angle in alternate segment)}$$

27. (A) $\sec\theta = \frac{1}{\cos\theta}$ and $\sin(90^\circ - \theta) = \cos\theta$
Hence $\sec\theta \sin(90^\circ - \theta)$

$$= \left(\frac{1}{\cos\theta}\right)(\cos\theta) = 1$$

28. (A) $2\cos^2\theta + \cos\theta = 0$
$\cos\theta(2\cos\theta + 1) = 0$
$\cos\theta = 0$ or $-\frac{1}{2}$
$\theta = 90^\circ$ or 120°

29. (D) $\hat{ACB} = 180^\circ - 120^\circ = 60^\circ$
Apply cosine rule to ΔABC

$$AB^2 = BC^2 + AC^2 - 2(BC)(AC)\cos\hat{ACB}$$

$12 = 16 + x^2 - 8x\cos 60^\circ$
$x^2 - 4x + 4 = 0$, or $(x-2)^2 = 0$
$x = 2$

30. (C) Since ΔXOY is equilateral, $\hat{XOY} = 60^\circ$.

$$\text{Area of } \Delta XOY = \frac{1}{2} \times 3 \times 3 \times \sin 60^\circ$$

$$= \frac{9\sqrt{3}}{4} \text{ cm}^2$$

Area of major sector XOY

$$= \pi r^2 \left(\frac{300^\circ}{360^\circ}\right) = \frac{5\pi}{6} r^2 = \frac{5\pi}{6}(9) = \frac{15\pi}{2} \text{ cm}^2$$

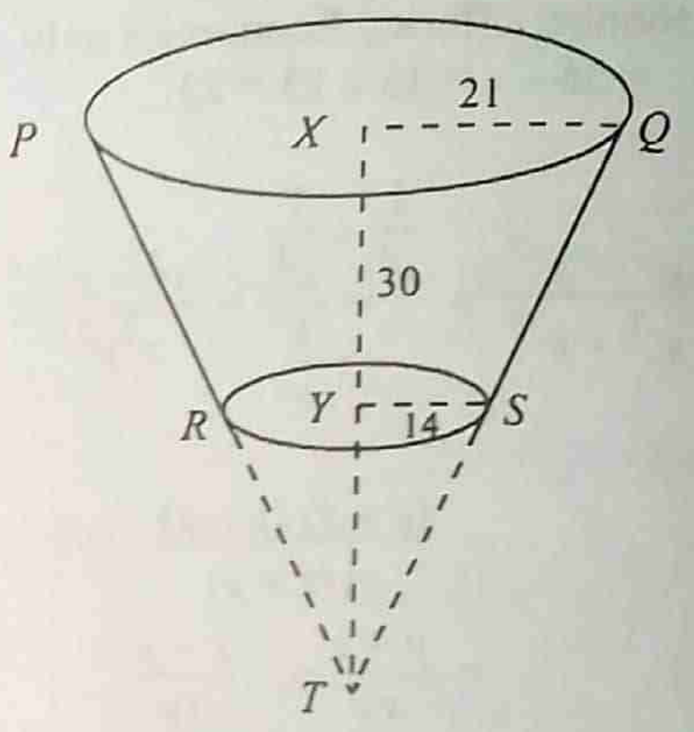
$$\text{Area of cardboard} = \frac{15\pi}{2} + \frac{9\sqrt{3}}{4}$$

$$= \frac{30\pi + 9\sqrt{3}}{4} \text{ cm}^2$$

31. (C) Volume of pyramid $= \frac{1}{3}$ x base area x height
Volume of cuboid = base area x height

$$\frac{\text{Vol. of pyramid}}{\text{Vol. of cuboid}} = \frac{1}{3}$$

32. (B) Complete the cone as shown in the figure.



Since Δs QXT and SYT are similar:

$$\frac{21}{30 + YT} = \frac{14}{YT}$$

$21(YT) = 14(30) + 14(YT)$, or $YT = 60$ cm
Vol. of large cone (PQT)

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 21^2 \times 90$$

$$= 41{,}580 \text{ cm}^3$$

Vol. of small cone (RST)

$$= \frac{1}{3} \times \frac{22}{7} \times 14^2 \times 60 = 12{,}320 \text{ cm}^3$$

Vol. of bucket $= 41{,}580 - 12{,}320$
$= 29{,}260 \text{ cm}^3$

33. (D) Each of the lines described in I, II and III bisects the side BC perpendicularly.

34. (C) $a \odot (b \odot c) = a \odot (b - c + 1) = a - b + c$
$(a \odot b) \odot c = (a - b + 1) \odot c$
$= a - b - c + 2$
[The candidate should check each of the other options for correctness.]

35. (B) $y = (x^2 + 1)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}(x^2 + 1)^{-1/2} \cdot 2x = \frac{x}{\sqrt{x^2 + 1}}$$

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At $x = -1$, $\frac{dy}{dx} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

36. (A) Let $y = \frac{3}{(1-2x)^2} = 3(1-2x)^{-2}$

$$\frac{dy}{dx} = -6(1-2x)^{-3}(-2) = \frac{12}{(1-2x)^3}$$

37. (D) $\int\left(x - \frac{1}{x}\right)^2 dx = \int(x^2 - 2 + x^{-2})\,dx$

$$= \frac{1}{3}x^3 - 2x - x^{-1} + c$$

$$= \frac{x^3}{3} - 2x - \frac{1}{x} + c$$

38. (C) This area is equal to

$$\int_0^{\pi/2} \sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/2}$$

$$= -\frac{1}{2}\cos\pi - \left(-\frac{1}{2}\cos 0\right)$$

$$= -\frac{1}{2}(-1) + \frac{1}{2} = 1$$

39. (B) The tangent to $y = \sin 2x$ has the equation

$$\frac{dy}{dx} = 2\cos 2x$$

The line $y = x + 2$ has a slope of 1

Hence $2\cos 2x = 1$, or $\cos 2x = \frac{1}{2}$

$2x = 60^\circ$ and $x = 30^\circ$

40. (C) Slope of $XY = \frac{1-2}{0-(-1)} = -1$

Equation of XY is $\frac{y-2}{x+1} = -1$

or $y + x = 1$

Point (3, –2) lies on this line.

41. (D) QR must be perpendicular to PQ and be of the same length.

Slope of $PQ = \frac{1-3}{5-1} = -\frac{1}{2}$

Hence slope of $QR = \left(\frac{-1}{-\frac{1}{2}}\right) = 2$

This is satisfied by (6, 3) or (7, 5)

$|PQ| = \sqrt{(5-1)^2 + (1-3)^2} = \sqrt{20}$

If $R = (6, 3)$: $QR = \sqrt{(6-5)^2 + (3-1)^2} = \sqrt{5}$

If $R = (7, 5)$: $QR = \sqrt{(7-5)^2 + (5-1)^2}$

$$= \sqrt{20}$$

Hence the point (7, 5) satisfies both conditions.

42. (D) $P^T = \begin{vmatrix} 2 & 1 \\ -3 & 2 \end{vmatrix} = (2 \times 2) - (1 \times -3) = 7$

$$P = \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} = (2 \times 2) - (1 \times -3) = 7$$

$PP^T = 7 \times 7 = 49$

43. (A) There are 5 numbers, hence

$$\frac{6 + 8 + 10 + x + 2x - 2}{5} = 14$$

$3x + 22 = 70$, $x = 16$ and

$2x - 2 = 2(16) - 2 = 30$, which is the largest number.

44. (A)

Class	Mid-value x_i	f_i	$f_i x_i$
10–19	14.5	2	29
20–29	24.5	4	98
30–39	34.5	3	103.5
40–49	44.5	1	44.5
		10	275

Mean, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{275}{10} = 27.5$

45. (B) Modal class = 20–29

True lower limit, $L = 19.5$

$c = 29.5 - 19.5 = 10$

$d_1 = 4 - 2 = 2$, $d_2 = 4 - 3 = 1$

$$\text{Mode} = L + c\left[\frac{d_1}{d_1 + d_2}\right]$$

$$= 19.5 + 10\left[\frac{2}{2+1}\right] = 26.2$$

46. (D) Median class = 20–29 (which contains the 5th value)

True lower limit, $L_m = 19.5$, $c = 10$

$f_m = 4$, $N = 10$, $F = 2$

$$\text{Median} = L_m + \frac{c}{f_m}\left[\frac{N}{2} - F\right]$$

$$= 19.5 + \frac{10}{4}\left(\frac{10}{2} - 2\right) = 27.0$$

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47. **(C)** Mean $\bar{x} = \dfrac{3+6+9+12+15}{5} = \dfrac{45}{5} = 9$

$x_i - \bar{x} = -6, -3, 0, 3, 6$

$(x_i - \bar{x})^2 = 36, 9, 0, 9, 36$

$\Sigma (x_i - \bar{x})^2 = 36 + 9 + 0 + 9 + 36 = 90$

$$\text{Variance} = \frac{\Sigma (x_i - \bar{x})^2}{n} = \frac{90}{5} = 18$$

48. **(C)** Total number of entries $N = 11$

Lower quartile is the $\dfrac{N+1}{4}$th entry, i.e. the 3rd entry = 8

Upper quartile is the $\dfrac{3(N+1)}{4}$th entry, i.e. the 9th entry = 26

49. **(B)** The possibilities are:

$1 + 2 = 3, \quad 1 + 3 = 4, \quad 1 + 4 = 5$

$2 + 3 = 5, \quad 2 + 4 = 6, \quad 3 + 4 = 7$

Out of six possibilities, four are odd

$\therefore$ Probability required $= \dfrac{4}{6} = \dfrac{2}{3}$

50. **(A)** Once the parents are seated, the number of arrangements of the four children is

$4! = 4 \times 3 \times 2 \times 1 = 24$.

But the parents can also swap positions, hence the total number of arrangements is $2 \times 24 = 48$.

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Appendix

Summary of Basic Concepts and Formulae

A. NUMBERS AND NUMERATION

Integers: any positive or negative whole number, including zero: {..., –3, –2, –1, 0, 1, 2, 3, ...}

Natural numbers: any member of the set of positive integers: {1, 2, 3, 4, ...}

Rational numbers: any number which can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, e.g. $\frac{2}{5}$, $\frac{3}{7}$, $\frac{11}{20}$, ...

Irrational numbers: any number which does not have a terminating or recurring decimal, e.g. $\sqrt{3}$, $\sqrt{5}$, π (= 3.14159 ...), ...

Real numbers: any member of the set of rational and irrational numbers, and integers.

Prime numbers: any number that is divisible only by itself and 1, e.g. 2, 3, 5, 7, 11, ...

A–1 Indices and Logarithms

1. $x^a \cdot x^b = x^{a+b}$
2. $\frac{x^a}{x^b} = x^{a-b}$
3. $(x^a)^b = x^{ab}$
4. $(xy)^a = x^a \cdot y^a$
5. $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$
6. $x^{-a} = \frac{1}{x^a}$
7. $x^{a/b} = \sqrt[b]{x^a}$
8. If $\log_b X = m$, the $X = b^m$
9. $\log_b XY = \log_b X + \log_b Y$
10. $\log_b \frac{X}{Y} = \log_b X - \log_b Y$
11. $\log_b X^a = a\log_b X$
12. $\log_b X = \frac{\log_a X}{\log_a b}$
13. $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$
14. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
15. $\frac{a}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$
16. $\sqrt{a^m} = a^{m/2}$

A–2 Sequences and Series

1. The nth term (t_n) of an arithmetic sequence is
   $t_n = a + (n-1)d$
   where a is the first term and d is the common difference.
2. The sum (S_n) of the first n terms of an arithmetic sequence is $S_n = \frac{n(a+t_n)}{2} = \frac{n}{2}[2a + (n-1)d]$
3. The nth term of a geometric sequence is $t_n = ar^{n-1}$, where a is the first term and r is the common ratio.
4. The sum of the first n terms of a geometric sequence is
   $S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}$.
5. The sum of an *infinite* geometric series is
   $S_n = \frac{a}{1-r}$.

B. ALGEBRAIC CONCEPTS AND FORMULAE

1. $a^2 - b^2 = (a-b)(a+b)$
2. $(a+b)^2 = a^2 + 2ab + b^2$
3. $(a-b)^2 = a^2 - 2ab + b^2$
4. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
5. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

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6. The roots of the equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

7. The equation $ax^2 + bx + c = 0$ has
   - two *real, distinct* roots of $b^2 > 4ac$
   - *equal* roots of $b^2 = 4ac$
   - *imaginary* roots of $b^2 < 4ac$.

8. *Factor Theorem:* If a is a factor of a polynomial $F(x)$, then $F(a) = 0$.

9. *Remainder Theorem:* If a is **not** a factor of a polynomial $F(x)$, them $F(a)$ is the remainder when $F(x)$ is divided by $x - a$.

10. The quadratic function $f(x) = ax^2 + bx + c$ is a perfect square if $b^2 = 4ac$.

11. The rational expression $\frac{P(x)}{Q(x)}$ is undefined when $Q(x) = 0$.

B–1 Inequalities

For any group of real numbers a, b, c and k:

1. If $a < b$ and $b < c$, then $a < c$
2. If $a < b < c$, then $a < b$ and $b < c$
3. If $a > b > c$, then $a > b$ and $b > c$
4. If $a < b$, then $a + k < b + k$
5. If $a > b$, then $a + k > b + k$
6. If $a < b$, then $-a > -b$
7. If $a > b$, then $-a < -b$
8. If $ab > 0$, then **either** $a > 0$ and $b > 0$, **or** $a < 0$ and $b < 0$.
9. If $ab < 0$, then **either** $a < 0$ and $b > 0$, **or** $a > 0$ and $b < 0$.

B–2 Graphs

1. The equation of a straight line has the form $y = mx + c$ where m is the slope and c is the intercept on the y-axis.

2. The slope (m) of a straight line which passes through two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

3. The equation of a straight line which passes through two points (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2}$$

4. The mid-point of a straight line joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_i + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

5. A line with a positive slope ($m > 0$) rises from left to right.

6. A line with a negative slope ($m < 0$) falls from left to right.

7. A horizontal line has zero slope.

8. A vertical line has infinite slope.

9. Two lines are parallel if their slopes are equal.

10. Two lines are perpendicular if the product of their slopes equals –1.

11. The x-intercept is the point of intersection with the x-axis ($y = 0$).

12. The y-intercept is the point of intersection with the y-axis ($x = 0$).

13. A parabola of the form $y = ax^2 + bx + c$ intersects the x-axis at *two distinct points* if $b^2 > 4ac$, at *only one point* if $b^2 = 4ac$, and at *no point* if $b^2 < 4ac$.

14. The x-intercepts correspond to the solution to the equation $ax^2 + bx + c = 0$.

15. If $a > 0$, the parabola $y = ax^2 + bx + c$ faces upward and has a *minimum* point.

16. If $a < 0$, the parabola $y = ax^2 + bx + c$ faces downward and has a *maximum* point.

17. The inequality $y > ax + b$ is represented graphically by all points lying *above* the line $y = ax + b$ while the inequality $y < ax + b$ is represented by all points lying *below* the line $y = ax + b$.

C. PLANE GEOMETRY

C–1 Triangles and Polygons

1. The sum of the angles of a triangle is 180^o.
2. A right angle equals 90^o.
3. An acute angle is less than 90^o.
4. An obtuse angle is greater than 90^o.
5. An equilateral triangle has all sides equal.
6. An isosceles triangle has two of its sides equal.
7. A right-angled triangle has one of its angles as 90^o.

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8. Two triangles are **congruent** if
   (i) three sides of one triangle are each equal to the corresponding sides of the other triangle (*SSS*).
   (ii) two sides and the included angle of one triangle are each equal to the corresponding sides and angle of the other triangle (*SAS*).
   (iii) two angles and the included side of one triangle are each equal to the corresponding angles and side of the other triangle (*ASA*).
   (iv) two angles and a side not included in the angles of one triangle are equal to the corresponding angles and side of the other triangle (*AAS*).

9. Two triangles are **similar** if
   (i) the angles of one triangle are equal to the corresponding angles of the other triangle (*AAA*).
   (ii) the ratio of the sides of one triangle is equal to the ratio of the sides of the other triangle.
   (iii) the ratio of two sides of one triangle is equal to the ratio of the corresponding sides of the other triangle, and their included angles are equal.

10. The sum of the interior angles of a polygon with n sides is $2n - 4$ right angles, or $(n - 2) \times 180^\circ$.

11. The sum of the exterior angles of any polygon is 360°.

C-2 Circles

1. A radius which is perpendicular to a chord bisects the chord.

2. Two chords which are equal in length are equidistant from the centre of the circle.

3. The angle subtended by an arc of a circle at the centre is twice the angle subtended at the circumference.

4. The diameter of a circle subtends an angle of 90° at the circumference.

5. Angles in the same segment are equal.

6. Angles in opposite segments are supplementary.

7. The radius of a circle is perpendicular to the tangent at the point of contact.

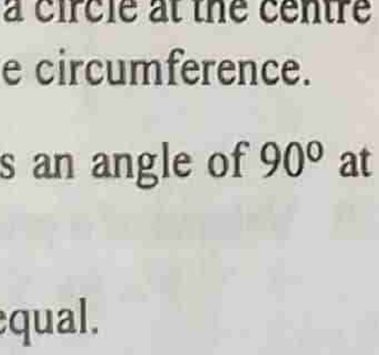
8. The angle which a chord makes with the tangent at the point of contact with the circle is equal to the angle in the alternate segment.

D. TRIGONOMETRY

1. For a right-angled triangle of sides a, b, c (see figure):

$\sin\theta = \frac{a}{c}$, $\tan\theta = \frac{a}{b}$

$\cos\theta = \frac{b}{c}$, $\operatorname{cosec}\theta = \frac{c}{a}$



$\cot\theta = \frac{b}{a}$, $\sec\theta = \frac{c}{b}$

2. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

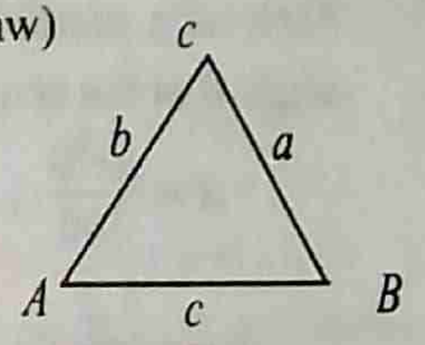
3. $\sin\theta = \cos(90^\circ - \theta)$

4. $\sin^2\theta + \cos^2\theta = 1$

5. $\tan^2\theta + 1 = \sec^2\theta$

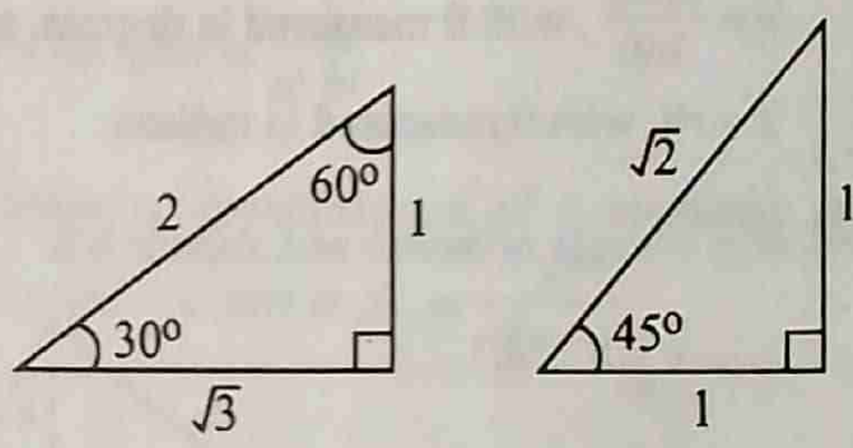
6. For any triangle ABC with sides a, b, c (see figure)

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (Sine Law)

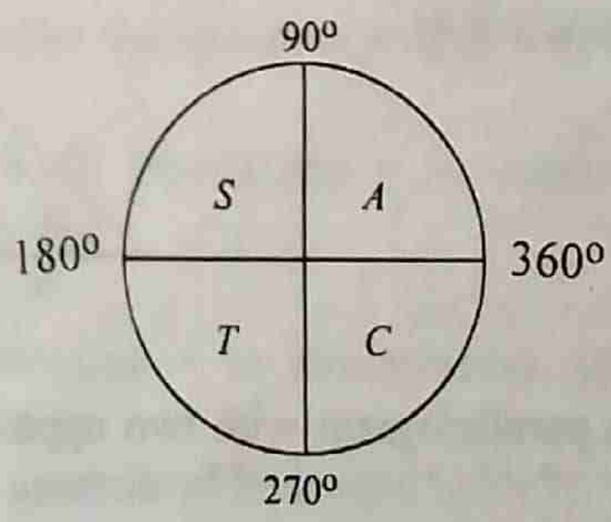


7. $a^2 = b^2 + c^2 - 2bc\cos A$
(Cosine Law)

8. The trigonometric ratios of standard angles 30°, 45° and 60° are easily found from the following standard triangles:



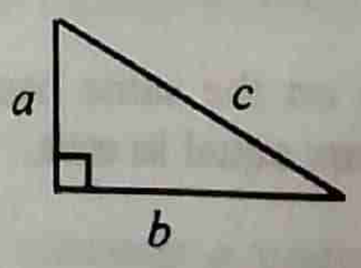
9. The signs of the trigonometric ratios (sine, cosine, tangent) are found from the following figure:



where the symbol in each quadrant indicates which ratio is positive in that quadrant.

10. Pythagoras' theorem for a right-angled triangle:

$c^2 = a^2 + b^2$



11. Triangles with the ratio of sides $1 : 1 : \sqrt{2}$, $3 : 4 : 5$ and $5 : 12 : 13$ are right-angled triangles.

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E. MENSURATION

1. Area (A) of a circle of radius r is
$A = \pi r^2$

2. Circumference (C) of a circle of radius r is
$C = 2\pi r = \pi d$
where d = diameter of the circle = $2r$.

3. 1 radian = $\frac{180}{\pi}$ degrees.

Area of a sector of circle, radius r, which subtends angle θ at the centre of the circle is

$A = \frac{\pi r^2 \theta}{360}$, if θ is measured in degrees, and

$A = \frac{1}{2} r^2 \theta$, if θ is measured in radians.

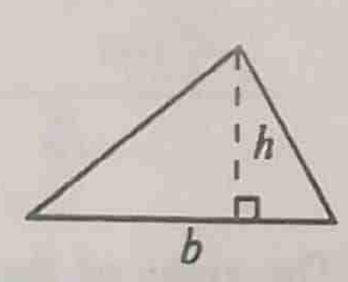
Length of arc of a circle, radius r, which subtends angle θ at the centre of the circle is

$s = \frac{2\pi r\theta}{360}$, with θ measured in degrees, and

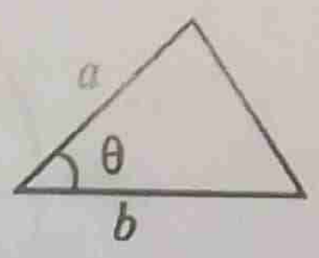
$s = r\theta$, with θ measured in radians.

Area of a triangle of base b and altitude h is

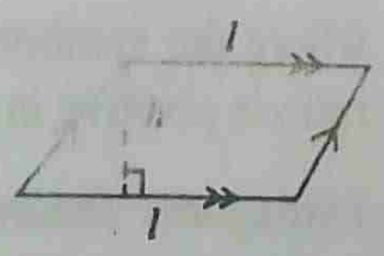
$A = \frac{1}{2} bh$



7. Area of a triangle with two given sides a, b and included angle θ is

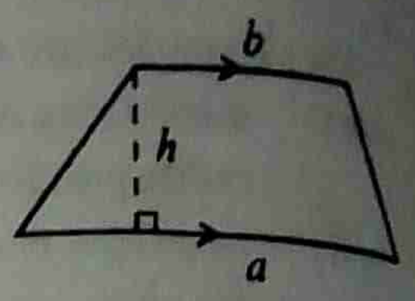


8. Area of a parallelogram with two opposite, parallel sides each of side l separated by distance h is



9. Triangles on the same base and between the same parallels are equal in area.

$A = \frac{1}{2} h(a + b)$



11. Area of a rhombus with diagonals of lengths a, b is
$A = \frac{1}{2} ab$

12. Surface area (S) of a cube of side l is
$S = 6l^2$

13. Volume (V) of a cube of side l is
$V = l^3$

14. Surface area of a sphere of radius r is
$S = 4\pi r^2$

15. Volume of a sphere of radius r is
$V = \frac{4}{3}\pi r^3$

16. Curved surface area (A_c) of a cylinder of radius r and height h is
$A_c = 2\pi rh$

17. Volume of a circular cylinder of base radius r and height h is
$V = \pi r^2 h$

18. Volume of a cone or pyramid of base area B and height h is
$V = \frac{1}{3} Bh$

19. Volume of a prism of base area B and height h is
$V = Bh$

F. STATISTICS AND PROBABILITY

1. The mean, $\bar{x}$, of a discrete set of data $x_1, x_2, x_3, \cdots x_n$ is

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

where n is the total number of observations.

2. The mean of a grouped set of data is

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f}$$

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3. The mode is that value which occurs most frequently in a given set of data.

4. For a grouped set of data,

$$\text{Mode} = L + c\left[\frac{d_1}{d_1 + d_2}\right]$$

where L = true lower limit of the modal class
c = class width
$d_1 = f_m - f_{m-1}$
$d_2 = f_m - f_{m+1}$

5. The median is that value which lies in the middle of a set of data when the values are arranged in order.

6. For a grouped set of data,

$$\text{Median} = L_m + \frac{c}{f_m}\left[\frac{N}{2} - F\right]$$

where L_m = true lower limit of the median class
N = sum of all the frequencies = Σf_i
F = cumulative frequency up to the group immediately preceding the median group.

7. The range of a set of data is the difference between the largest value and the smallest value.

8. The mean deviation of a discrete set of data is

$$\text{Mean Deviation} = \frac{1}{n}\sum_{i=1}^{n} |x_i - \bar{x}|$$

9. For a grouped set of data,

$$\text{Mean Deviation} = \frac{\Sigma f_i |x_i - \bar{x}|}{\Sigma f_i}$$

where x_i = mid-point of the ith class.

10. The variance of a discrete set of data is given by

$$\text{Variance} = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

11. For a grouped set of data,

$$\text{Variance} = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}$$

12. Standard deviation is the square root of variance.

$P(E) = m/n$, where $0 \le P(E) \le \cdots$

14. For two mutually exclusive events A and B,
$P(A \cup B) = P(A) + P(B)$

15. If A and B are mutually exclusive and exhaustive then
$P(A) + P(B) = 1$

16. If A and B are not mutually exclusive, then
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

17. If A and B are independent,
$P(A \cap B) = P(A) \times P(B)$

18. Number of permutations of a set containing n different elements is $n! = n(n-1)(n-2)\cdots(2)(1)$.

19. Number of permutations of a set of n elements, of which a elements are alike while b elements of another kind are alike is $\dfrac{n!}{a!\,b!}$

20. Number of combinations of n elements taking r elements at a time is ${}_nC_r = \dfrac{n!}{r!(n-r)!}$

G. MISCELLANEOUS TOPICS

1. Given a set of elements $S = (a, b, c, d, \ldots)$, the set S is closed under the operation $*$ if $(a * b) \in S$.

2. The binary operation $*$ is commutative if $a * b = b * a$.

3. The operation $*$ is associative if $(a * b) * c = a * (b * c)$.

4. The operation $*$ is distributive if $a * (b + c) = (a * b) + (a * c)$.

5. The operation $*$ has an identity element e such that $a * e = e * a = a$

6. The inverse i of an element a under the binary operation $*$ is such that $a * i = i * a = e$.

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7. Derivatives of common functions:

Function	Derivative
constant k	0
x^n , n = constant	nx^{n-1}
$(ax + b)^n$	$na(ax + b)^{n-1}$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$
$\tan ax$	$a\sec^2 ax$
e^{ax}	ae^{ax}
$\ln x$	$1/x$
$1/x$	$-1/x^2$

8. Integrals of common functions:

Function	Integral
constant k	$kx + c$
x^n	$\frac{1}{n+1}x^{n+1} + c$
$(ax + b)^n$	$\frac{(ax + b)^{n+1}}{a(n+1)} + c$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{!}{a}\sin ax + c$
e^{ax}	$\frac{1}{a}e^{ax} + c$
$\frac{1}{x}$	$\ln x + c$

9. Distance d between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

10. Perpendicular distance D of a point (h, k) from the line $ax + by + c = 0$ is

$$D = \left| \frac{ah + bk + c}{\sqrt{a^2 + b^2}} \right|$$

11. Angle θ between two straight lines $y_1 = m_1x + c_1$ and $y = m_2x + c_2$ is

$$\theta = \tan^{-1}\left[\frac{m_1 - m_2}{1 + m_1 m_2}\right]$$

12. A straight line with intercepts of a and b on the x and y axes respectively has an equation of the form

$$\frac{x}{a} + \frac{y}{b} = 1$$

where $a \neq \theta$ and $b \neq 0$.

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