BASIC ELECTRICAL ENGINEERING

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GOKARAJU RANGARAJU
INSTITUTE OF ENGINEERING AND TECHNOLOGY
(Autonomous)

Index:

GOKARAJU RANGARAJU INSTITUTE OF ENGG AND TECHNOLOGY

(AUTONOMOUS) ACADEMIC YEAR 2014-15, Semester-I/II

BASIC ELECTRICAL ENGINEERING (BEE)

Sub code:	**GR14 Regulation**	**L**	**T**	**P**	**C**
		3	**2**	**0**	**4**

PreRequities : Fundamental in Engineering Mathematics and Physics

Course Objectives: BEE (Basic Electric Engineering) is common to first year branches of UG Engineering(expect BT). At the end of the course the student is expected to

1. Know the fundamental of Electrical Engineering and practical.
2. Practical implementation of fundamental theory concepts.

Course Outcomes :

1. Students will learn strong basics of Electrical Engineering and practical implementation of Electrical fundamentals.
2. Students will learn different applications of commonly used electrical machinery

UNIT 1: Basic Laws: Ohm's law, Kirchhoff's voltage and current laws, Nodes-Branches and loops, Series elements and Voltage Division, Parallel elements and Current Division, Star-Delta transformation, Independent sources and Dependent sources, source transformation.

UNIT 2: AC Fundamentals-I: Reviews of Complex Algebra, Sinusoids, phasors, Phasor Relations of circuit elements, Impedance and admittance, Impedance Combinations, Series and Parallel combination of Inductors and capacitor, Mesh analysis and Nodal analysis .

UNIT 3: AC Fundamental-II: RMS and average values, Form factors, Steady state Analysis of series, Parallel and Series Parallel combination of R,L,C with Sinusoidal excitation, Instantaneous power, Real power, Reactive power and Apparent power, concept of Power factor, Frequency.

UNIT 4: Network Theorems and Resonance: Superposition theorem, Thevinin's theorem, Nortorn's theorem, Maximum Power Transfer Theorem, Reciprocity theorem, Resonance in Electrical circuits: Analysis of series and parallel Resonance.

UNIT 5: Fundamentals of Electrical Machines: Construction, Principle, Operation and Application of –(i) Single phase Transformer (ii) Single phase Induction motor (iii) DC Motor.

TEXTBOOK:

1. Fundamentals of Electrical Circuits by Charles k.Alexander, Mattew N.O.Saidiku, Tata McGraw Hill company.

Reference Book:

1.Circuit theory(Analysis and Synthesis) by A. Chakrabarti-Dhanpat Rai&Co.

2.Network Theory by Prof.B.N.Yoganarasimham.

3. Circuit Theory by Sudhakar and ShyamMohan.

4.Electrical Machines-I by B.I.Theraja.

UNIT 1: Basic Laws: Ohm's law, Kirchhoff's voltage and current laws, Nodes-Branches and loops, Series elements and Voltage Division, Parallel elements and Current Division, Star-Delta transformation, Independent sources and Dependent sources, source transformation.

OHM'S LAW

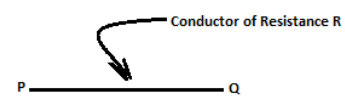
At constant temperature, the current flowing through a conductor is directly proportional to the potential difference(p.d) in volts across the two ends of the given conductor and inversely proportional to the resistance (R) in ohms (Ω) between the ends of the same conductor.

$$I \propto \frac{PD}{R} \propto volts/ohms$$

In all practical problems of electrical calculations, it is assumed that the temperature rise is within limits, so that electrical properties such as insulation and conduction properties of the *given material are not destroyed. Hence ohm's law is mathematically stated as*

$$I = V/R \quad or \quad V = IR \ or \ R = V/I$$

$$I = \frac{V_R - V_Q}{R} = \frac{V}{R} \text{ A}$$



Solved Examples:

1. An electrical iron carrying 2A at 120V. Find resistance of the device?

Soln : $R = (v/i) = 120/2 = 60\Omega$

2. The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 12Ω at 110V?

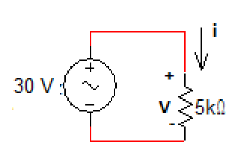
Soln: 9.167 A

3. In the given circuit, calculate current I, the conductance G and the power P?

Soln: $i = \frac{V}{R} = 6mA$

$G = \frac{1}{R} = 0.2m$ mhos

$P = VI = I^2 R = 180mW$



4. A voltage source of 20Sinπt V is connected across a 5KΩ resistor. Find the current through the resistor and power dissipated.

Soln: $I = \frac{V}{R} = 4Sin(\pi t)mA \;, P = VI = 80Sin^2(\pi t)\, mW.$

5. A resistor absorbs an instantaneous power of $20Cos^2t$ mW when connected to a voltage source V=10Cost V. Find I and R.

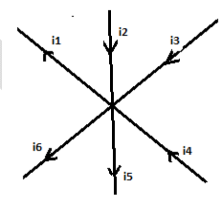
Soln: I = 2Cos(t) mA R = 5KΩ.

KIRCHOFF'S CURRENT AND VOLTAGE LAWS (KCL and KVL)

KIRCHOFF'S CURRENT LAW : KCL states that the total current entering a junction is equal to the total current leaving the junction.

(or)

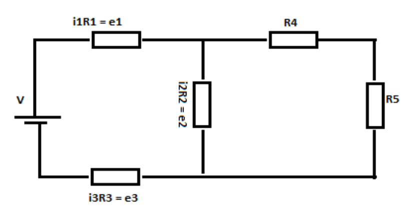
The algebraic sum of the currents at the junction (node) will be zero.



At node n, $(i2 + i3 + i4) = (i1 + i6 + i5)$

Or $i2 + i3 + i4 - i1 - i6 - i5 = 0.$

KIRCHOFF'S VOLTAGE LAW: KVL is based on the law of the law of conservation of the energy, states that the algebraic sum of voltage drops in a closed loop is zero.



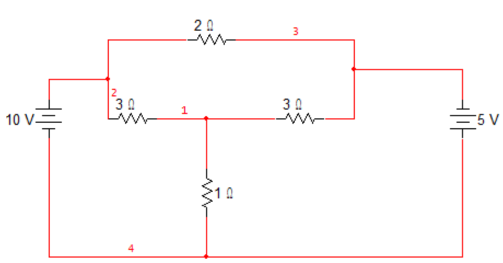
$e1 + e2 + e3 = V$

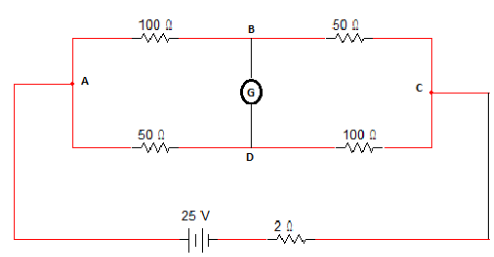
$e1 + e2 + e3 - V = 0$

{Flow of currents in loop is assumed +ve from higher potential to lower potential in **elements** and +ve from lower to higher potential in **Sources}**

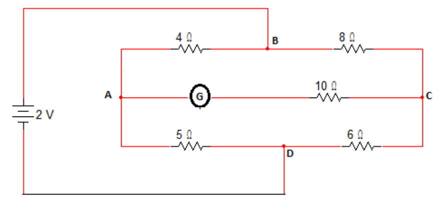
Problem:

1. Write KCL and KVL equations for the given circuits:



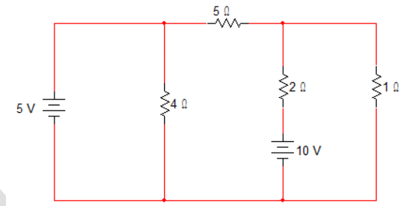


2. Find the current measured in galvanometer shown in the wheatstone's bridge.



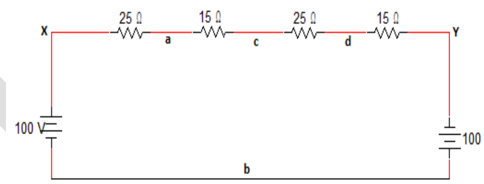
Soln: 0.015625 A (C to A)

3. Determine the voltage across 1Ω resistor in the given circuit.



Soln: 2.353 V

4.
5. From the given circuit, find the voltage across ab, cb, and db.



Soln: $V_{ab} = 37.5\,V \; V_{cb} = 0 \; V_{db} = -62.5V$

NODES, BRANCHES AND LOOPS:

Network is an interconnection of elements or devices, circuit is a network providing one or more closed paths.

A **BRANCH** represents a single element such as voltage source or a resistor.

A **NODE** is the point of connection of two or more branches.

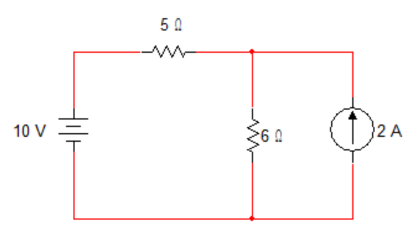
A **LOOP** is any closed path in a circuit.

A network with 'b' branches, 'l' loops and 'n' nodes should satisfy the theorem of n/w topology.

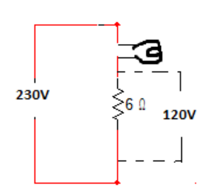
$$b = l + n - 1$$

Solved examples:

1. Determine the number of branches and nodes in the circuit shown. Identify which elements are in series and which are in parallel.



Soln: Branches 4; Nodes 3: 10V source and 5Ω resistance are connected in series. The combination is in parallel with 6Ω resistance and 2A source.



2. A bulb rated 110V , 60W has to be operated from a 230V supply. Show the arrangement.

   Soln: $I = \frac{60}{110} = 0.5454A;$

   $R = 120/I = 220.02\Omega$

3. A 50Ω and a 100Ω rheostats are each rated at 100W. What is the maximum voltage that may be applied without causing overheating of either rheostat (i) when they are connected in series and (ii) when they are in parallel.

Soln:

SERIES ELEMENTS AND VOLTAGE DIVISION

Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current.

The equivalent resistances of any number of resistors connected in series is the sum of the individual resistances.

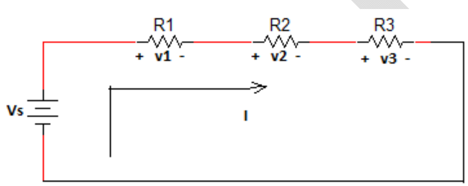


fig 1

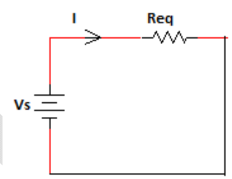


fig 2

Three resistances R1,R2,R3 are connected in series. The current passing through all the elements connected in series is same, I.

In the above figure1, according to KVL, sum of voltage drops in a closed loop is zero.

$Vs - V_1 - V_2 - V_3 = 0$ (V_1, V_2, V_3 are drop across R_1, R_2, R_3)

$Vs = IR_1 + I R_2 + IR_3 = I(R_1 + R_2 + R_3)$……………………(1)

In figure2, $V_s = IR_{eq}$ …………………………(2)

From equations (1) and (2) $R_{eq} = R_1 + R_2 + R_3$

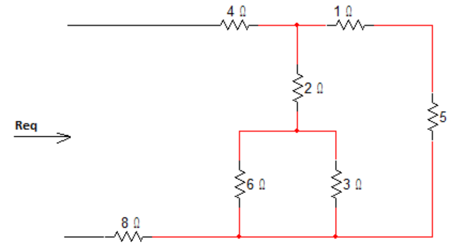
Therefore voltage applied divides across the series connected elements.

Applications of Series Circuits :

1. Decorative serial sets, number of bulbs with low voltage ratings connected in series across existing rated voltage.
2. Voltage distribution or tapping.

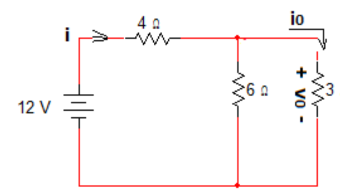
Solved Examples:

1. Find R_{eq} for the given circuit.



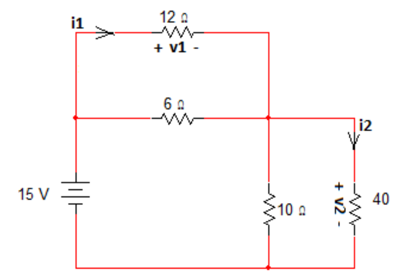
Soln : 14.4Ω

2. Find i_o and v_o in the given circuit. Calculate the power dissipated in 3Ω.



Soln : $i_o = 3.33A, \; v_o = 4V, \quad P_o = 5.33W.$

3. Find V_1 , V_2 , i_1 , i_2 and power dissipated in 12Ω and 40Ω.



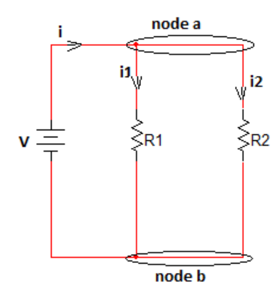
Soln : $V_1 = 5V, \; V_2 = 10V \;, i_1 = 416.7mA$,

$i_2 = 250mA$, $P_{12} = 2.083W$, $P_{40} = 2.5W.$

PARALLEL ELEMENTS AND CURRENT DIVISION

Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

The parallel connected elements have the same voltage across them.



$V = i_1R_1 = i_2R_2$………………………….…(1)

From KCL, at node A, $i = i_1 + i_2$…………….(2)

From (1) , $i = V/R_1 + V/R_2$

$i/V = 1/R_1 + 1/R_2$ or $R_{eq} = (R_1R_2)/(R_1 + R_2)$

The equivalent resistance of the parallel resistors is equal to the product of their resistances divided by their sum.

$G_{eq} = G_1 + G_2$ G : conductance

The equivalent conductance connected in parallel is the sum of their individual conductances.

From (2) , $i_1 = i - i_2 = V(\frac{1}{R_{eq}} - \frac{1}{R_2})$

$$(iR_{eq})/R_1 = (iR_1R_2)/(R_1(R_1 + R_2)$$

Therefore $i_1 = (iR_2)/(R_1 + R_2)$ and $i2 = (iR_1)/(R_1 + R_2)$

Solved Examples:

1. Two batteries of 24V and 20V with internal resistances of 0.4Ω and 0.25Ω respectively are connected in parallel across a load of 4Ω. Calculate (i) the current supplied by each battery and (ii) voltage across the load.

Soln: 8.1482A and -2.963A , 20.741V.

2. Two coils are connected in parallel and a voltage of 200V is applied to the terminals. The total current taken is 25A and the power dissipated in one of the coils is 1500W. What is the resistance of each coil?

Soln: 26.67Ω and 11.43Ω

3. Three impedances $Z_1 = (5 + j5)\Omega$, $Z_2 = -j8\Omega$ and $Z_3 = 4\Omega$ are connected in series to an unknown voltage source V. find I and V, if the voltage drop across Z_3 is $63.2\angle 18.45°V$.

Soln: $Z_{total} = Z_1 + Z_2 + Z_3 = 9.487\angle -18.434° \ \Omega$

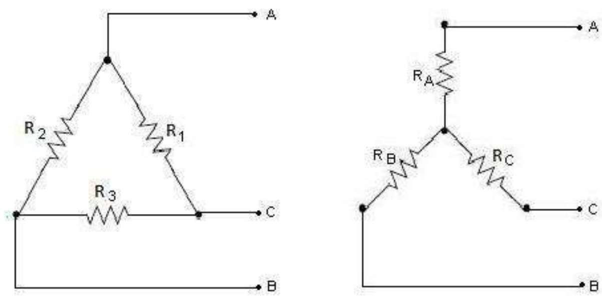
I = 15.8∟18.45°A V= 150∟0°V.

STAR – DELTA TRANSFORMATION

Star connection and delta connection are the two different methods of connecting three basic elements which cannot be further simplified into series or parallel.

The two ways of representation can have equivalent circuits in either form.

Assume some voltage source across the terminals AB.



$R_{eq} = R_a + R_b$

$R_{eq} = R_1(R_2 + R_3)/(R_1 + R_2 + R_3)$

Therefore $R_a + R_b = R_1(R_2 + R_3)/(R_1 + R_2 + R_3)$…………(1)

Similarly $R_b + R_c = R_3(R_1 + R_2)/(R_1 + R_2 + R_3)$…………(2)

$R_c + R_a = R_2(R_3 + R_1)/(R_1 + R_2 + R_3)$…………..(3)

Subtracting (2) from (1) and adding to (3) ,

$R_a = R_1R_2/(R_1 + R_2 + R_3)$……………(4)

$R_b = R_1R_3/(R_1 + R_2 + R_3)$……………(5)

$R_c = R_2R_3/(R_1 + R_2 + R_3)$……………(6)

A delta connection of R_1 R_2 R_3 can be replaced by an equvivalent star connection with the values from equations (4),(5),(6).

Multiply (4)(5) ; (5)(6) ; (4)(6) and then adding the three we get,

$$R_aR_b + R_bR_c + R_cR_a = R_1R_2R_3/(R_1 + R_2 + R_3)$$

Dividing LHS by R_a gives R_3 , by R_b gives R_2 , by R_c gives R_1.

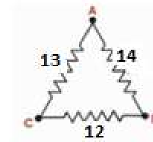
$R_1 = (R_aR_b + R_bR_c + R_cR_a) / R_c$

$R_2 \quad = \quad (R_aR_b + R_bR_c + R_cR_a) \,/\, R_b$

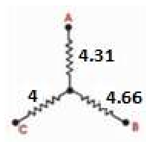
$R_3 \quad = \quad (R_aR_b + R_bR_c + R_cR_a) \,/\, R_a$

Solved Examples :

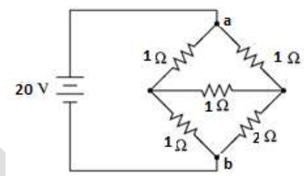
1. Obtain the star connected equivalent for the given delta circuit.



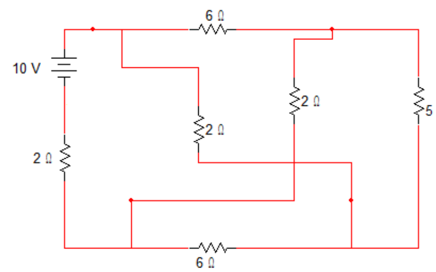
   Soln :



2. For the bridge network shown, determine the total resistance seen from the terminals AB using star-delta transformation.

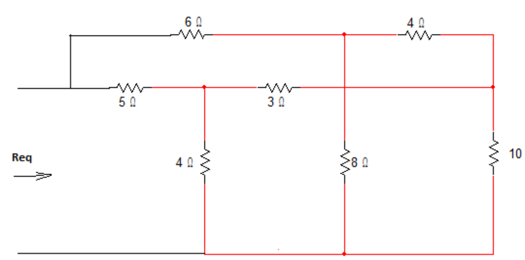


   Soln: 1.182Ω



3. Calculate the voltage across AB in the network and indicate the polarity of the voltage using star-delta conversion.

   Soln:



4. Determine the equivalent resistance Req.

   Soln: 4.93Ω

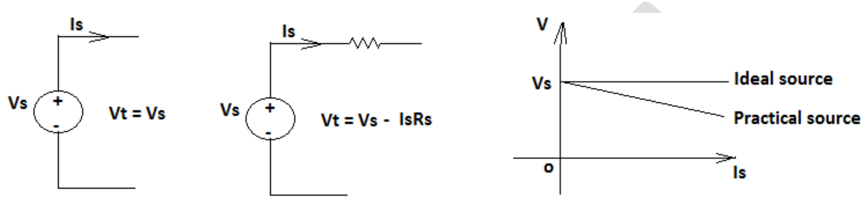
INDEPENDENT SOURCES AND DEPENDENT SOURCES

An ideal voltage source is one which maintains a constant voltage at its terminals, irrespective of the current delivered to the network.

Practically a voltage sources has internal resistances and hence, when it delivers a current, there is always an internal voltage drop which increases as the supplies more and more current.

Thus its terminals voltage progressively falls.

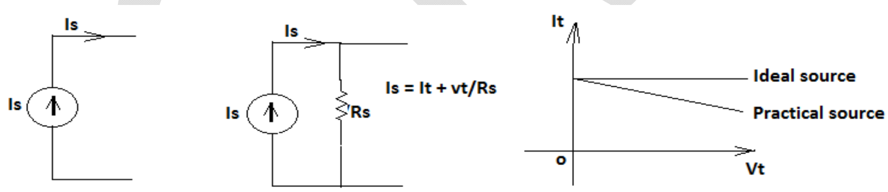
Ideal Voltage Source   Practical Voltage Source



An IDEAL CURRENT SOURCE is one which delivers a current of constant magnitude, totally independent of the external network connected.

In practical, no current source can be ideal, practical current source is always shown with resistance in parallel.

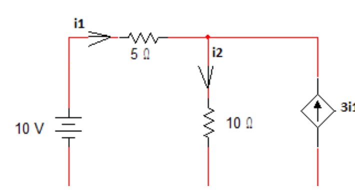
Ideal Current Source   Practical Current Source

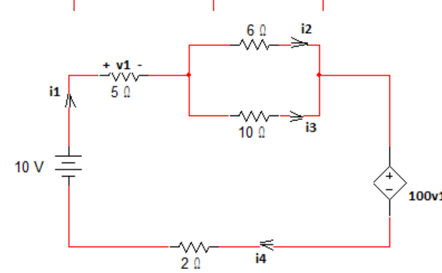


Solved Examples:

1. In the given circuit, determine all branch currents.

   Soln : $i_1 = 0.222A$ and $i_2 = 0.888A$.



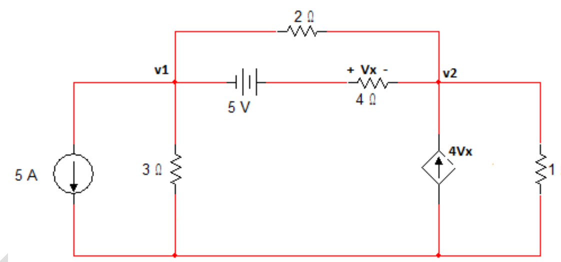


2. For the given circuit, find V_1, i_1, i_2, i_3, i_4.

Soln : $i_1 = 0.0196$A ; $i_2 = 0.01225$A ; $i_3 = 0.00735$A ;

$i_4 = 0.0196$A and $V_1 = 0.098$V.

3. In the given circuit, find voltage across 4Ω using nodal analysis.

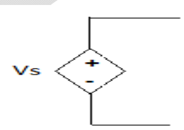


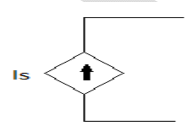
Soln: $V_x = 0.01$V.

DEPENDENT OR CONTROLLED SOURCES:

In some network, in which some of the voltage sources or current sources are controlled by changing of current or voltage elsewhere in the circuit. Such sources are termed as "Dependent or Controlled sources".

There are four types of dependent sources.





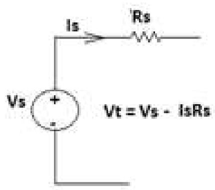
Dependent Voltage Source(DVS) Dependent Current Source(DCS)

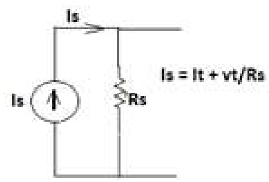
- CDVS(Current DVS)
- CDCS(Current DCS)
- VDVS(Voltage DVS)
- VDCS(Voltage DCS)

SOURCE TRANSFORMATION:

A practical voltage source can be replaced by a current source and vice versa.

This can be established if an equivalence between a voltage source and a current source.





$V_t = V_s - IR_s = V_s - I_tR_s$ since $I_t = I$

Or $I_t = (V_s - V_t)/R_s$ ....(1)

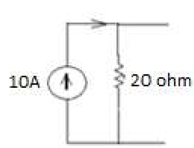
For Current source , $I_s = I_t + V_t/R_s$ or $I_t = I_s - V_t/R_s$

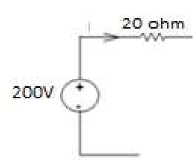
Comparing (1) and (2) , $I_s = V_s/R_s$.

A voltage source V_s with resistance R_s in series with its equivalent to a current source

$I_s = V_s/R_s$, in parallel with resistance R_s.

Example : Obtain the equivalent voltage source for the current source as shown in figure.





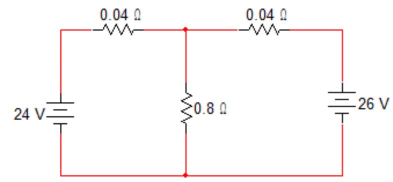
$I_s = 10A$ and $R_s = 20\Omega$, $V_s = 200V$.

EXERCISE PROBLEMS

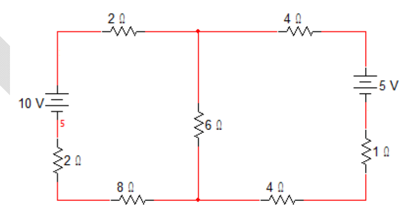
1. Two batteries of 24V and 20V with internal resistances of 0.4Ω and 0.25Ω respectively are connected in parallel across a load of 4Ω. Calculate (i) the current supplied by each battery and (ii) voltage across the load.
(Soln: 8.1482A , -2.963A , 20.741V)

2. A resistance R is connected in series with a parallel circuit comprising of resistances of 4Ω and 6Ω respectively. When the applied voltage is 15V, the power dissipated in 4Ω resistor is 36W, calculate R.
(Soln : 0.6Ω)

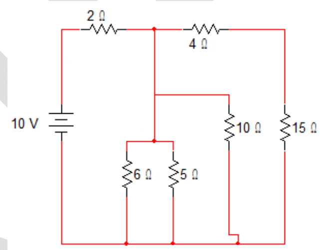
3. In the circuit shown, find current through R_L using Kirchhoff's laws.
(Soln : 30.488A)



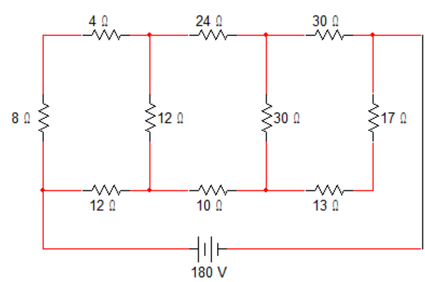
4. Find using KCL, the current through the 6Ω resistor of the circuit shown.
(Soln : 0.641A flowing from A to B)



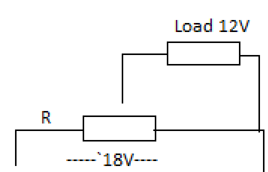
5. Find current through 10Ω resistor in the given circuit, use nodal analysis.



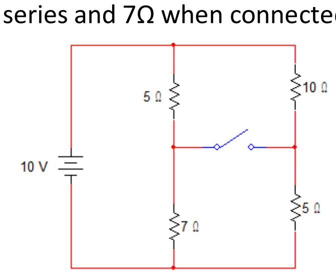
6. Determine the current in 10Ω resistor in the network shown , use star-delta conversion.



7. For the 100Ω potentiometer shown, find the resistance R for a load current of 10A.

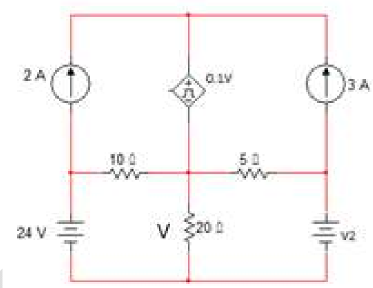


8. The resistance of two wires is 15Ω when connected in series and 7Ω when connected in parallel, find the resistances of each one.



9. Find the equivalent resistance across the battery terminals with
   (a) Switch S open
   (b) Switch S closed.

10. In the given circuit, find the value of V2 that will cause the voltage across 20Ω to be zero by using mesh analysis.



Unit II: AC Fundamentals-I:

Review of Complex Algebra, Sinusoids, Phasors, Phasor Relations of Circuit elements, Impedance and Admittance, Impedance Combination, Series and Parallel combination of Inductors and Capacitors, Mesh analysis and Nodal Analysis.

Review of Complex Number:

- In order to analyze AC circuit, it is necessary to represent multi-dimensional quantities. In order to accomplish this task, scalar numbers were abandoned and complete numbers were used to express the two dimensions of frequency and phase shift at one time
- In mathematics, 'i' is used to represent imaginary numbers. In the study of electricity and electronics, j is used to represent imaginary numbers so that there is no confusion with I, which in electronics represents current (i). It is also customary for scientist to write the complex number in the form of $a + jb$.
    1. A complex number may be written in Rectangular form as:
       $Z = X + jY$. Where $X = r\,cos(\theta)$ is real and $Y = r\,sin(\theta)$ is imaginary number.
       $\theta = tan^{-1}(Y/X)$.
    2. A second way of representing the complex number is by specifying the magnitude(r) and angle (θ) in polar form
       $Z = r\angle\theta$;
       $r = \sqrt{X^2 + Y^2}$; $\theta = tan^{-1}(Y/X)$.
    3. The third way of representing the complex number is the Exponential form.
       $Z = re^{j\theta}$,$\theta = \tan^{-1}(\frac{Y}{X})$, $r = \sqrt{X^2 + Y^2}$

Mathematical Operations of Complex Numbers:

Assume $Z_1 = X_1 + jY_1 \; or \; r_1\angle\theta_1$ and $Z_2 = X_2 + Y_2 \; or \; r_2\angle\theta_2$ are two complex numbers,

$Addition$: $Z_1 + Z_2 = (X_1 + X_2) + j(Y_1 + Y_2)$

$Subtraction$: $Z_1 - Z_2 = (X_1 - X_2) + j(Y_1 - Y_2)$

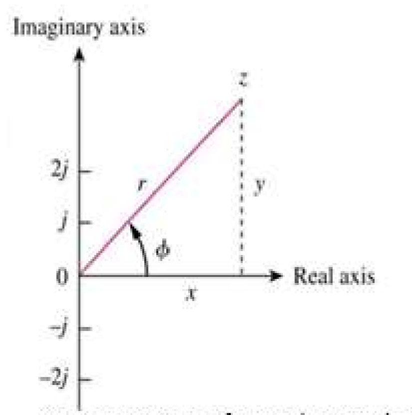
$Multiplication$: $Z_1 Z_2 = r_1 r_2\angle(\theta_1 + \theta_2)$

$Division$: $Z_1/Z_2 = (r_1/r_2)\angle(\theta_1 - \theta_2)$

$Reciprocal$: $1/Z_1 = (1/r_1)\angle - \theta_1$

Square root: $\sqrt{Z_1} = \sqrt{r_1}\angle\frac{\theta_1}{2}$

$\boldsymbol{complex\ Conjugate}$: $\boldsymbol{Z^* = X - jY = \sqrt{r}\angle\theta = re^{-j\theta}}$

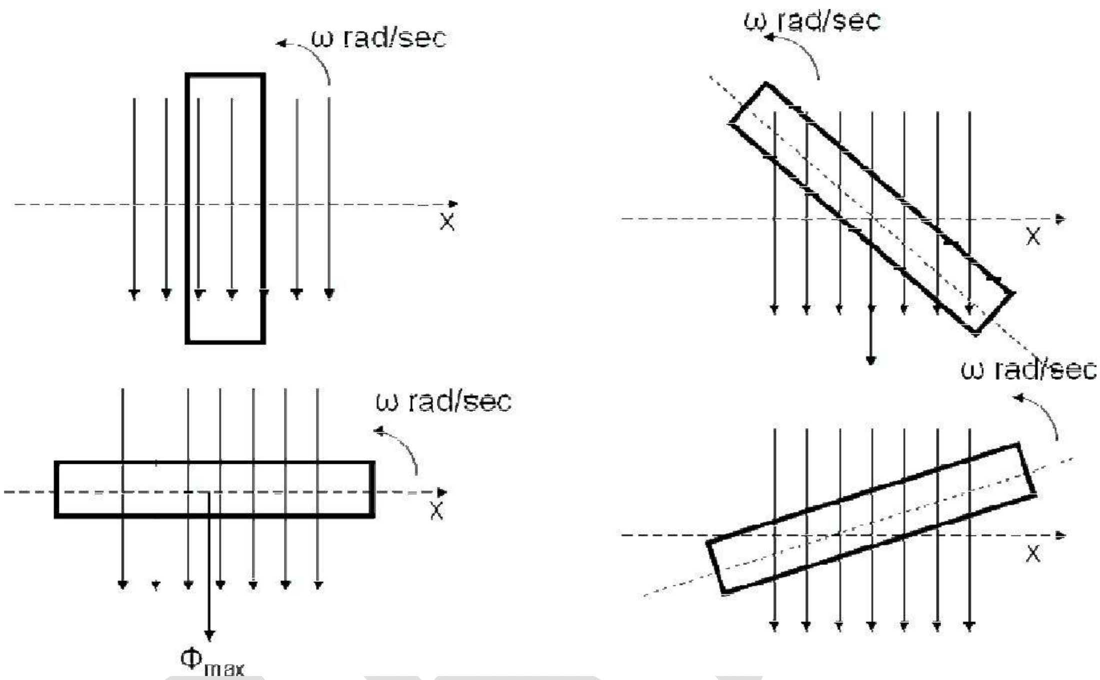


Representation of Complex number in co-ordinates plan

Explain Generation of Sinusoidal wave?

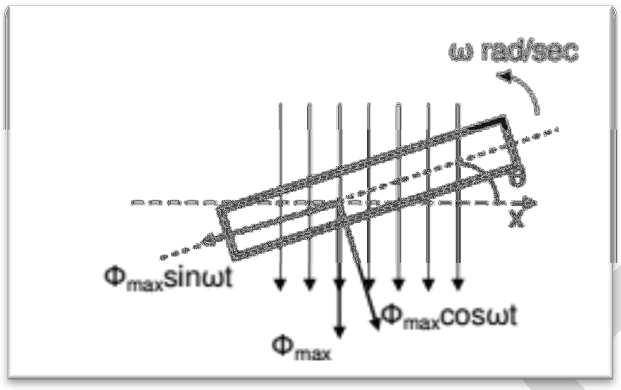
Generation of sinusoidal AC voltage

Consider a rectangular coil of N turns placed in a uniform magnetic field as shown in the figure. The coil is rotating in the anticlockwise direction at an uniform angular velocity of ω rad/sec.



When the coil is in the vertical position, the flux linking the coil is zero because the plane of the coil is parallel to the direction of the magnetic field. Hence at this position, the emf induced in the coil is zero. When the coil moves by some angle in the anticlockwise direction, there is a rate of change of flux linking the coil and hence an emf is induced in the coil according to **Faradays Law**. When the coil reaches the horizontal position, the flux linking the coil is maximum, and hence the emf induced is also maximum. When the coil further moves in the anticlockwise direction, the emf induced in the coil reduces. Next when the coil comes to the vertical position, the emf induced becomes zero. After that the same cycle repeats and the emf is induced in the opposite direction. When the coil completes one complete revolution, one cycle of AC voltage is generated.

The generation of sinusoidal AC voltage can also be explained using mathematical equations. Consider a rectangular coil of N turns placed in a uniform magnetic field in the position shown in the figure. The maximum flux linking the coil is in the downward direction as shown in the figure. This flux can be divided into two components, one component acting along the plane of the coil $\Phi_{max}\sin\omega t$ and another component acting perpendicular to the plane of the coil $\Phi_{max}\cos\omega t$.



The component of flux acting along the plane of the coil does not induce any flux in the coil. Only the component acting perpendicular to the plane of the coil ie $\Phi_{max}\cos\omega t$ induces an emf in the coil.

$$\emptyset = \emptyset_{max}\cos(\omega t)$$

$$e = -N\frac{d\emptyset}{dt}$$

$$e = -N\frac{d(\emptyset_{max}\cos(\omega t))}{dt}$$

$$e = N\emptyset_{max}\omega * \sin(\omega t)$$

$$e = E_{max}\sin(\omega t)$$

Hence the emf induced in the coil is a sinusoidal emf. This will induce a sinusoidal current in the circuit given by

$$i = i_m sin\,(\omega t)$$

Where

i *is instantaneous value,*
i_m *maximum value*
ω *is angular velocity*

Angular Frequency (ω)

Angular frequency is defined as the number of radians covered in one second(ie the angle covered by the rotating coil). The unit of angular frequency is rad/sec.

$$\omega = \frac{2\pi f}{T}$$

Problem-An alternating current i is given by $i = 141.4\, sin\, (314t)$

Find i) The maximum value

ii) Frequency

iii) Time Period

iv) The instantaneous value when $t = 3ms$ $i = 141.4\, sin(314t)$.

Solution:

$i = i_m sin\,(\omega t)$________________(1)

Compare given equation with eq-1,

$Maximum\ value\ i_m = 141.4 Volts$

$\omega = 314\ raad/sec$

$f = \omega/2\pi = 50Hz$

$T = 1/f = 0.02 sec.$

$t = 3msec$,

$i = 141.4\, sin(\,314 * 3 * 10^{-3}) = 114.35A$

Explain about phasor, and Lead and lagging ?

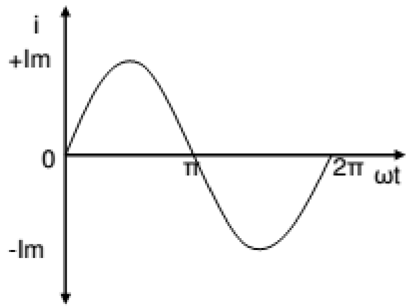
Phasor Representation:

An alternating quantity can be represented using

i) Waveform
ii) Equations
iii) Phasor

A sinusoidal alternating quantity can be represented by a rotating line called a **Phasor.** A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity

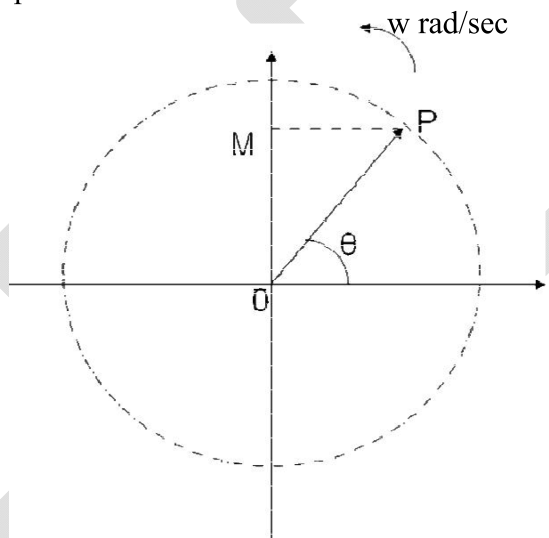
The waveform and equation representation of an alternating current is as shown. This sinusoidal quantity can also be represented using phasors.



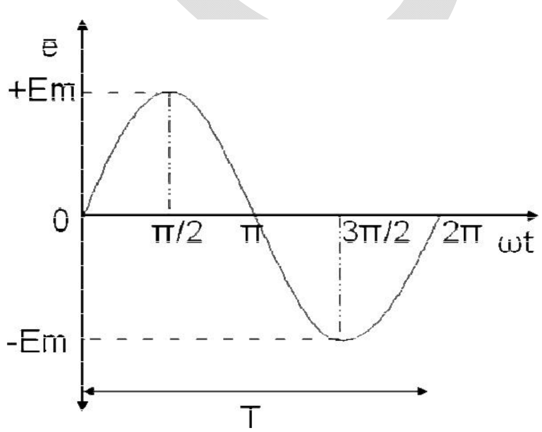
$$i = I_m \sin \omega t$$

In phasor form the above wave is written as $\bar{I} = I_m \angle 0^0$

Draw a line OP of length equal to I_m. This line OP rotates in the anticlockwise direction with a uniform angular velocity ω rad/sec and follows the circular trajectory shown in figure. At any instant, the projection of OP on the y-axis is given by OM=OPsinθ = $I_m$sinωt. Hence the line OP is the phasor representation of the sinusoidal current.



Phase

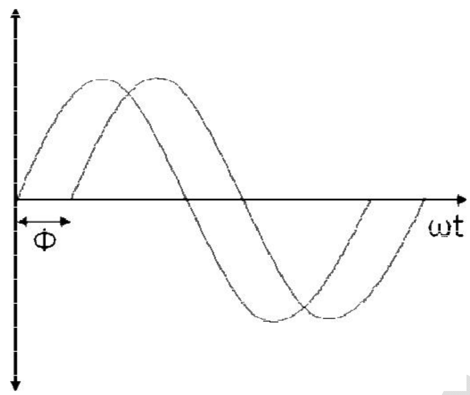


Phase is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference

Phase of $+E_m$ is $\pi/2$rad or T/4 sec

Phase of $-E_m$ is $\pi/2$rad or 3T/4 sec

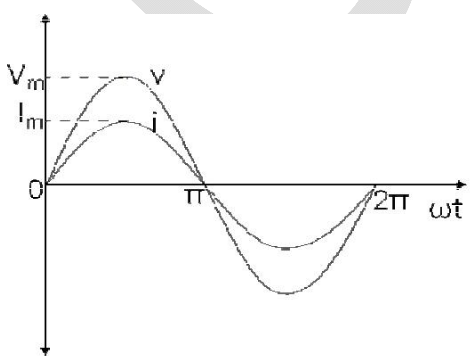
Phase Difference

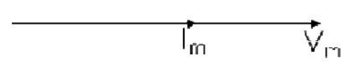


When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference.

In Phase

Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same. The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.

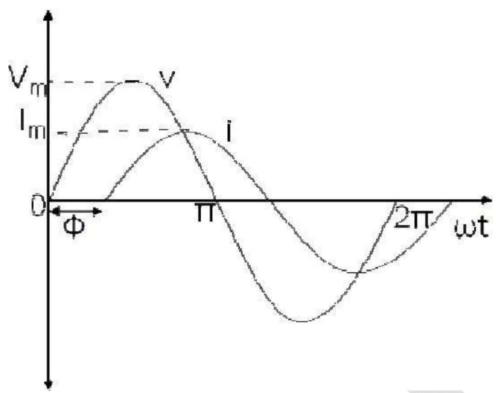


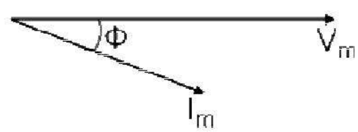


$$v = v_m \sin(\omega t)$$
$$i = i_m \sin(\omega t)$$

Lagging

In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage. The waveform, phasor and equation representation is as shown.

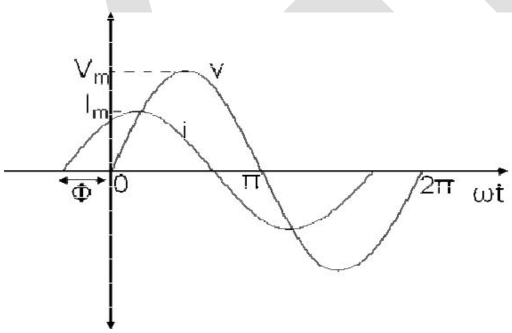


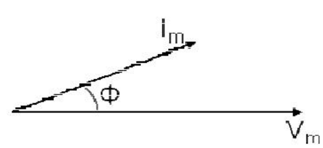


$$v = v_m \sin(\omega t) => \bar{V} = V_m \angle 0^0$$
$$i = i_m \sin(\omega t - \theta) => \bar{I} = I_m \angle -\theta$$

Leading

In the figure shown, the zero point of the current waveform is before the zero point of the voltage waveform. Hence the current is leading the voltage. The waveform, phasor and equation representation is as shown.



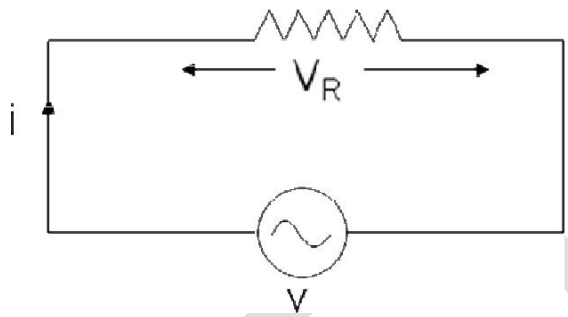


$$v = v_m \sin(\omega t) => \bar{V} = V_m \angle 0^0$$
$$i = i_m \sin(\omega t + \theta) => \bar{I} = I_m \angle \theta^0$$

Problem: if $i_1 = 4cos(\omega t + 30^0)A$ *and* $i_2 = 5sin(\omega t - 20^0)A$, find their sum and draw phasor diagram.

Q: Explain Phasor Relationship with Circuit Elements:

AC circuit with a pure resistance



Consider an AC circuit with a pure resistance R as shown in the figure. The alternating voltage v is given by

$$v = v_m \sin(\omega t)$$

The current flowing in the circuit is i. The voltage across the resistor is given as V_R which is the same as v.

Using ohms law, we can write the following relations

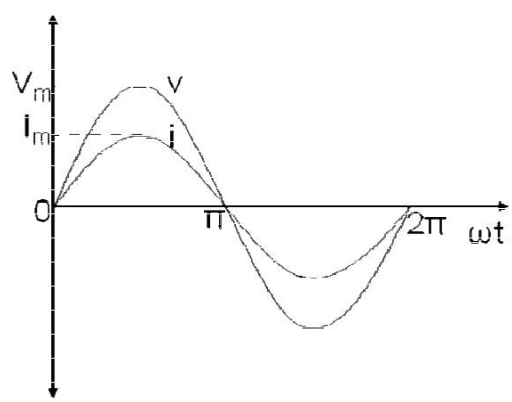
$$i = \frac{v}{R} = \frac{v_m \sin(\omega t)}{R}$$

$$i = i_m \sin(\omega t)$$

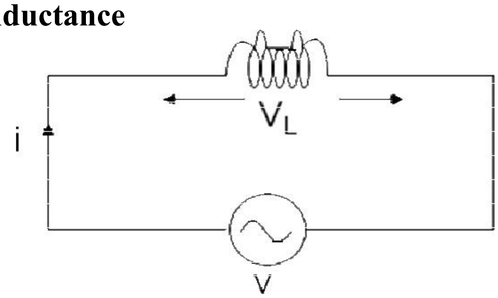
Where

$$i_m = \frac{v_m}{R}$$

From equation (1) and (2) we conclude that in a pure resistive circuit, the voltage and current are in phase. Hence the voltage and current waveforms and phasors can be drawn as below.



AC circuit with a pure inductance



Consider an AC circuit with a pure inductance L as shown in the figure. The alternating voltage v is given by

$$v = v_m \sin(\omega t)$$

The current flowing in the circuit is i. The voltage across the inductor is given as V_L which is the same as v.

We can find the current through the inductor as follows

$$i = L\frac{di}{dt}$$

$$v_m \sin(\omega t) = L\frac{di}{dt}$$

$$di = \frac{v_m}{L} * \sin(\omega t)$$

$$i = \frac{v_m}{L} * \int \sin(\omega t)\, dt$$

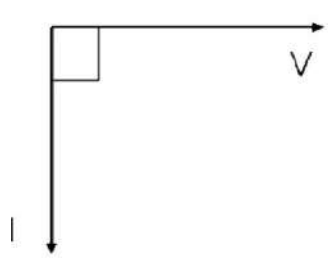
$$i = \frac{v_m}{\omega L} * -\cos(\omega t)$$

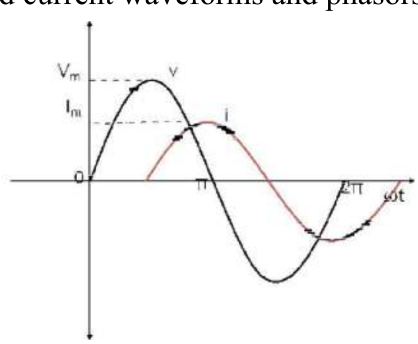
$$i = \frac{v_m}{\omega L} * sin(\omega t - \frac{\pi}{2})$$

$$i = i_m * sin(\omega t - \frac{\pi}{2})$$

Where $i_m = \frac{v_m}{\omega L}$

From equation (1) and (2) we observe that in a pure inductive circuit, the current lags behind the voltage by 90^0. Hence the voltage and current waveforms and phasors can be drawn as below.





Inductive reactance

The inductive reactance X_L is given as

$$X_L = 2\pi f L$$

$$i_m = \frac{v_m}{X_L}$$

$$in\ phasor\ form\ \bar{V} = j\omega L\bar{I}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms ( )

Problem:

The voltage $\boldsymbol{v = 12\cos(60t + 45^0)}$ is applied to a 0.1H inductor. Find the steady-state current through the inductor.

Solution: from equation $\bar{V} = j\omega L\bar{I}$

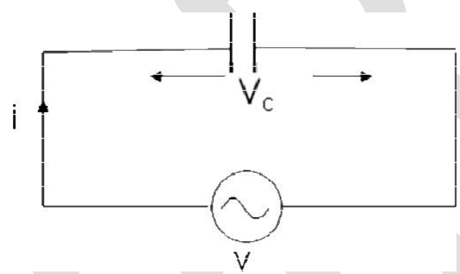
$$\bar{\bar{V}} = 12\angle 45^0\ volts$$

$$\omega = 60\frac{rad}{s}$$

$$\bar{I} = \frac{12\angle 45^0}{j * 60 * 0.1} = \frac{12\angle 45^0}{60 * 0.1\angle 90^0} = 2A$$

$$i = 2\cos(60t - 45^0)\ A$$

AC circuit with a pure capacitance



Consider an AC circuit with a pure capacitance C as shown in the figure. The alternating voltage v is given by

$$v = v_m \sin(\omega t) => \bar{V} = V_m\angle 0^0 \_\_\_\_\_\_\_\_\_(1)$$

The current flowing in the circuit is i. The voltage across the capacitor is given as V_C which is the same as v.

We can find the current through the capacitor as follows

$$q = Cv$$

$$q = Cv_m \sin(\omega t)$$

$$\frac{dq}{dt} = \omega C v_m \cos(\omega t)$$

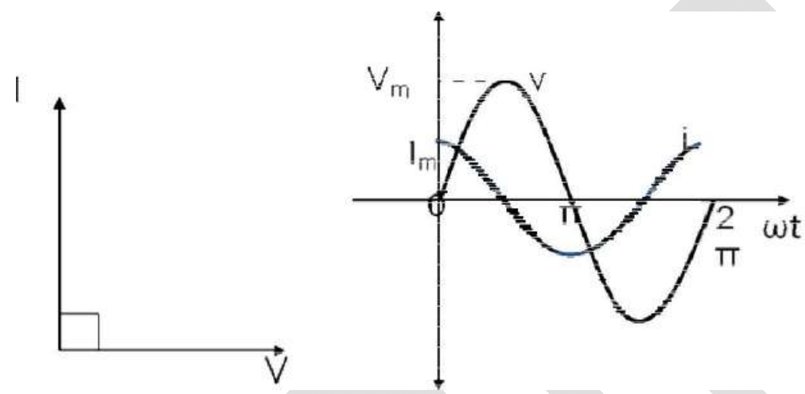
$$i = \omega C v_m \cos(\omega t)$$

$$i = \omega C v_m \sin(\omega t + \frac{\pi}{2}) \_\_\_\_\_\_\_\_ (2)$$

$$i = i_m \sin(\omega t + \frac{\pi}{2})$$

$$i_m = \omega C v_m => X_C = \frac{v_m}{i_m} = \frac{1}{\omega C}$$

From equation (1) and (2) we observe that in a pure capacitive circuit, the current leads the voltage by 90^0. Hence the voltage and current waveforms and phasors can be drawn as below.



Capacitive reactance

The capacitive reactance X_C is given as

$$X_C = \frac{1}{2\pi f C}$$
$$i_m = \frac{v_m}{X_C}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms ( )

$$\bar{V} = V_m \angle 0^0 = V + j0$$
$$\bar{I} = I_m \angle 90^0 = 0 + j\, I$$

$$\frac{\bar{V}}{\bar{I}} = \frac{V_m \angle 0^0}{I_m \angle 90^0} = \frac{\mathrm{V_m}}{\mathrm{I_m}} \angle -90^0 = \mathrm{X_C} \angle -90^0$$

Problem:

The voltage $v = 10\, cos(100t + 30^0)$ is applied to a $50\, \mu F$ capacitor, calculate the current through the capacitor, and Draw phasor diagram?

Impedance:

relationship between Current and Voltage to different circuit elements are,

1. To Resistor: $R = \frac{\overline{V}}{\overline{I}}$
2. To Capacitor: $X_C = \frac{\overline{V}}{\overline{I}}$
3. To inductor: $X_L = \frac{\overline{V}}{\overline{I}}$

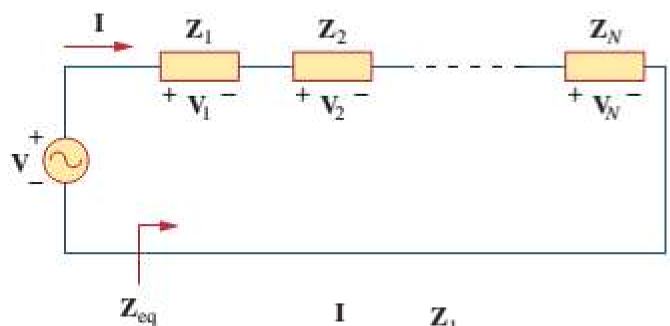
This shows that a pure resistance within an AC circuit produces a relationship between its voltage and current phasors in exactly the same way as it would relate the same resistors voltage and current relationship within a DC circuit.

Reciprocal of impedance is called as *Admittance* and units are mhos.

Impedance Combinations:

Series:

Let us assume $Z_1, Z_2, \ldots, Zn$ are N impedances are connected in series than equivalent impedance is Z_{eq} obtained by



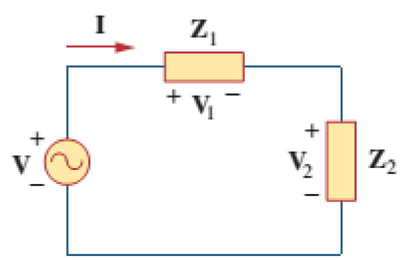
Apply KVL in loop

$V = (I\,Z_1 + IZ_2 +, \ldots \ldots . IZ_n)$

$\frac{V}{I} = Z_1 + Z_2 + \cdots Z_n$

$Z_{eq} = Z_1 + Z_2 + \cdots Z_n$

Suppose N=2:



$Z_{eq} = Z_1 + Z_2$

$V_1 = \frac{Z_1 V}{Z_1 + Z_2}$

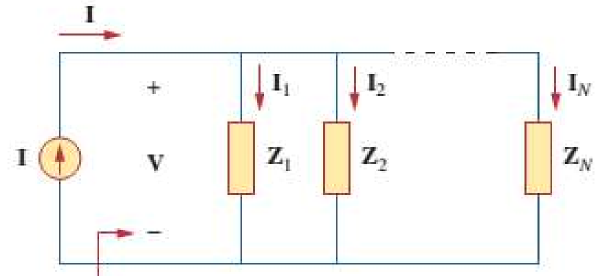
$V_2 = \frac{Z_2 V}{Z_1 + Z_2}$

Parallel:

Let us assume $Z_1, Z_2, \ldots, Zn$ are N impedances are connected in parallel ,than equivalent is given by

Apply current (I) between two nodes and assume voltage across nodes=V,

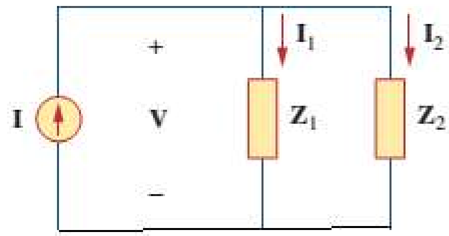
According to the KCL $I = I_1 + I_2 \ldots I_n$



$$\frac{V}{Z_{eq}} = \frac{V}{Z_1} + \frac{V}{Z_2} + \cdots + \frac{V}{Z_n}$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}$$

Suppose N=2; than



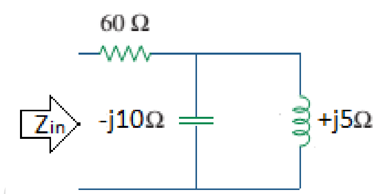
$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$I_1 = \frac{Z_2}{Z_1+Z_2} I$$

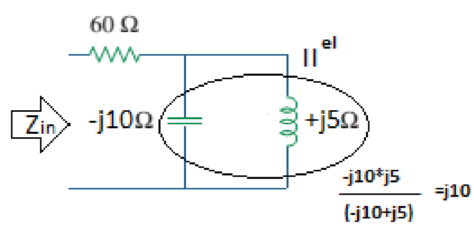
$$I_2 = \frac{Z_1}{Z_1+Z_2} I$$

Problem:

Find equivalent impedance of below circuit.



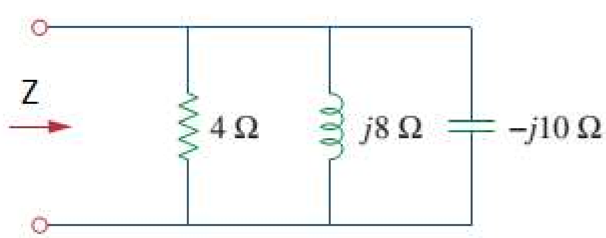
Solution: -j10 and +j5 are in $||^{el}$ after simplifying



Therefore Z_{in} = (60 series in with j10) = 60+ j10

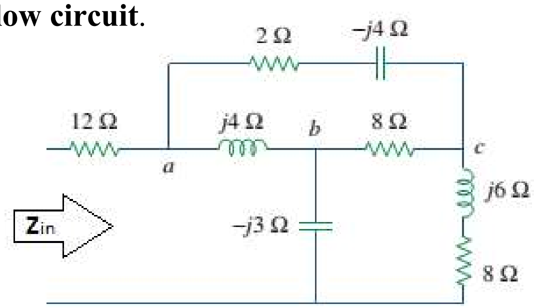
Problem:

Find Z in below figure



Problem:

Find impedance Z_{in} of below circuit.



Solution:

Solution:

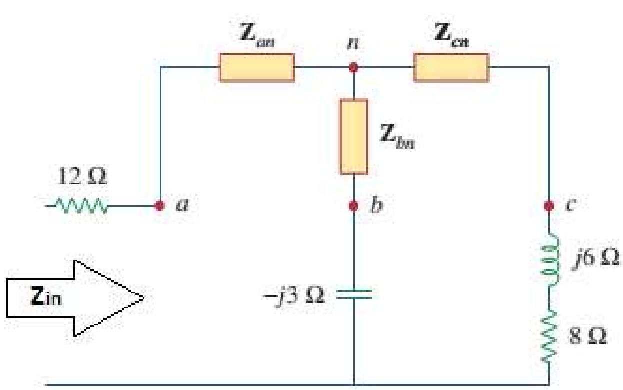
The delta network connected to nodes a, b, and c can be converted to the Y network of Fig. 9.29. We obtain the Y impedances as follows using Eq. (9.68):

$$\mathbf{Z}_{an} = \frac{j4(2 - j4)}{j4 + 2 - j4 + 8} = \frac{4(4 + j2)}{10} = (1.6 + j0.8)\ \Omega$$

$$\mathbf{Z}_{bn} = \frac{j4(8)}{10} = j3.2\ \Omega, \qquad \mathbf{Z}_{cn} = \frac{8(2 - j4)}{10} = (1.6 - j3.2)\ \Omega$$

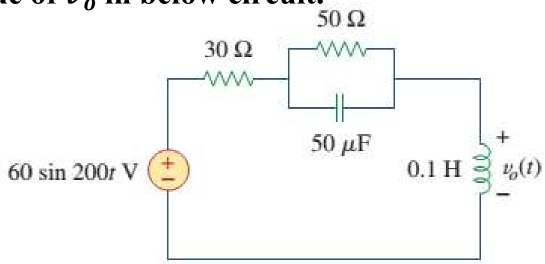
The total impedance at the source terminals is

$$\begin{aligned}\mathbf{Z} &= 12 + \mathbf{Z}_{an} + (\mathbf{Z}_{bn} - j3) \parallel (\mathbf{Z}_{cn} + j6 + 8)\\ &= 12 + 1.6 + j0.8 + (j0.2) \parallel (9.6 + j2.8)\\ &= 13.6 + j0.8 + \frac{j0.2(9.6 + j2.8)}{9.6 + j3}\\ &= 13.6 + j1 = 13.64\underline{/4.204^\circ}\ \Omega\end{aligned}$$



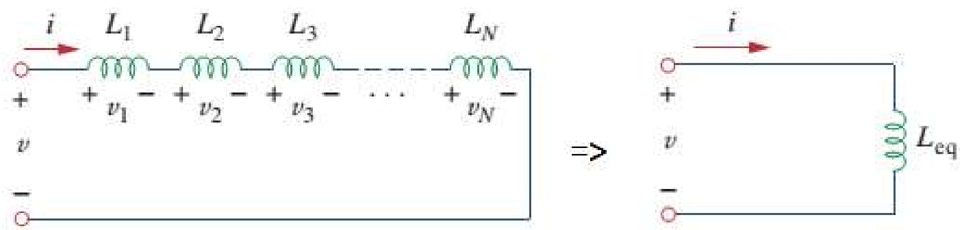
Problem:

Calculate the value of v_o in below circuit.



Series and Parallel combination of Inductors:

Series: Consider N inductor are connected in series, and voltage drop across each inductor is $v_1, v_2, \ldots\ldots v_n$.



According to the KVL

$$v = v_1 + v_2 \ldots\ldots + v_n$$

But $v = L\frac{di}{dt}$

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} \ldots\ldots + L_n\frac{di}{dt}$$

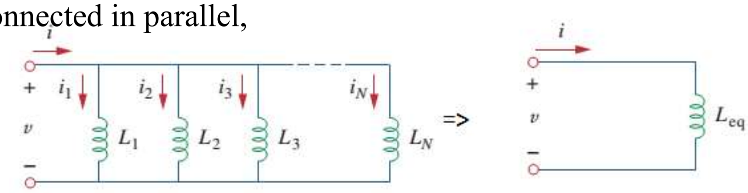
$$v = (L_1 + L_2 \ldots\ldots + L_n)\frac{di}{dt}$$

$$v = L_{eq}\frac{di}{dt}$$

So $L_{eq} = L_1 + L_2 \ldots\ldots + L_n$

The equivalent inductance of series connected inductors is the sum of the individual inductors.

Parallel: Consider N inductor are connected in parallel,



According to KCL

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

But $i_k = \frac{1}{L_k}\int_{t_0}^{t} v\,dt + i_k(t_0)$; hence,

$$i = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v\,dt + i_2(t_0) + \cdots + \frac{1}{L_N}\int_{t_0}^{t} v\,dt + i_N(t_0)$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right)\int_{t_0}^{t} v\,dt + i_1(t_0) + i_2(t_0) + \cdots + i_N(t_0)$$

$$= \left(\sum_{k=1}^{N} \frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt + i(t_0)$$

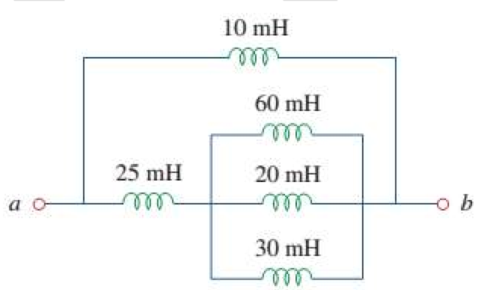
where

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

The equivalent impedance of parallel inductors is reciprocal of the sum of the reciprocals of the individual inductances.

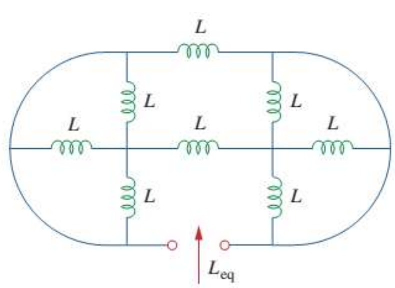
Problem:

Find L_{eq} between 'ab' terminals in below figure.



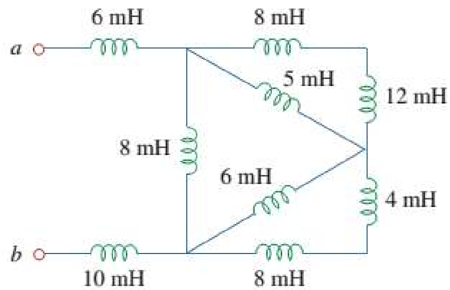
Problem:

Find L_{eq} In below figure.



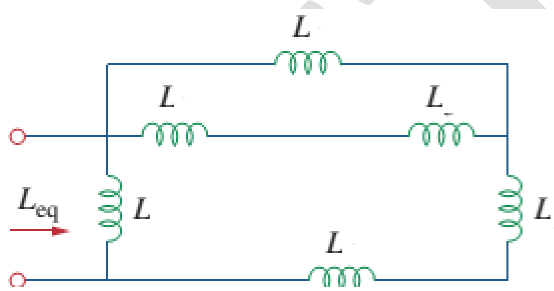
Problem:

Find L_{eq} between 'ab' terminals in below figure?



Problem:

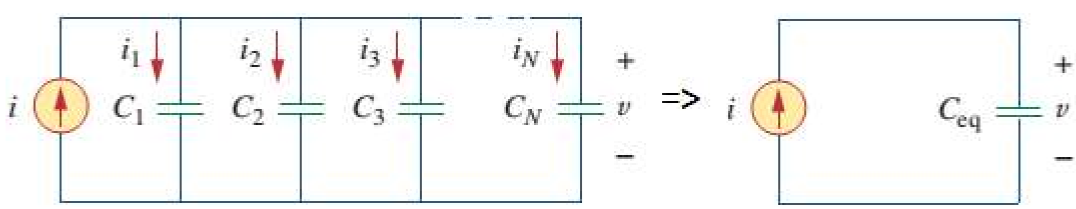
Find L_{eq} in below figure.



Series and parallel connection of capacitors:

Parallel Connection of Capacitors:

Consider N capacitors are connected in parallel,



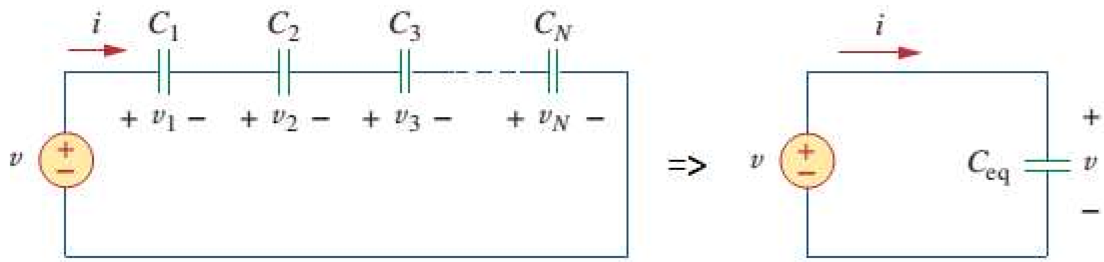
Apply KCL
$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$$
$$= \left(\sum_{k=1}^{N} C_k\right)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$$

where

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$

The equivalent capacitance of N parallel connected capacitors is the sum of all individual capacitance.

Series connection of capacitors: Consider N capacitors are connected in series.



Apply KVL above circuit (left)

$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

But $v_k = \frac{1}{C_k}\int_{t_0}^{t} i(\tau)d\tau + v_k(t_0)$. Therefore,

$$v = \frac{1}{C_1}\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i(\tau)d\tau + v_2(t_0)$$

$$+ \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(\tau)d\tau + v_N(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}\right)\int_{t_0}^{t} i(\tau)d\tau + v_1(t_0) + v_2(t_0)$$

$$+ \cdots + v_N(t_0)$$

$$= \frac{1}{C_{eq}}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

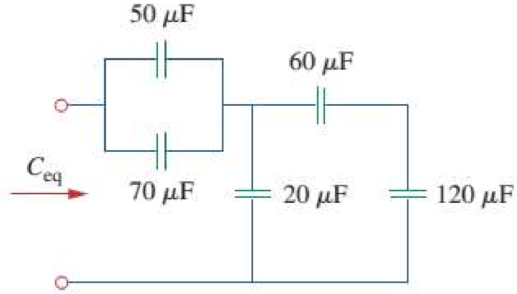
where

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

The equivalent capacitance of series connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitors.

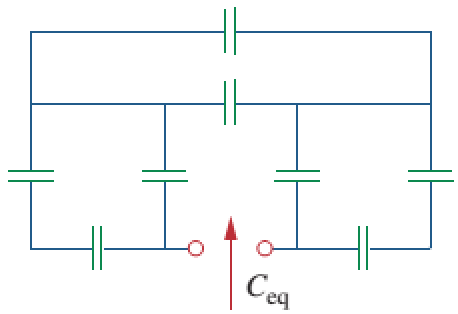
Problem:

Find C_{eq} in below figure.



Problem:

Find C_{eq} in below figure, If the value of all capacitor 4mf



Mesh Analysis:

Mesh analysis provides another general procedure for analyzing circuits, using *mesh current* as the circuit variables.

Definition: **Mesh** is a loop which does not contain any other loop within it.

Steps to Determine Mesh Currents:

Step 1: Determine the number of meshes n.

Step 2: Assign mesh current i_1, i_2, , i_n, to the n meshes.

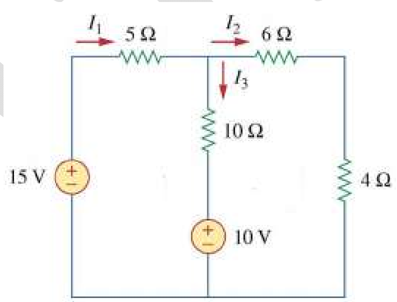
- The direction of the mesh current is arbitrary-(clockwise or counterclockwise)-and does not affect the validity of the solution.
- For convenience, we de ne currents flow in the clockwise (CW) direction.

Step 3: From the current direction in each mesh, denote the voltage drop polarities.

Step 4: Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh current.

Step 5: Solve the resulting n simultaneous equations to get the mesh current.

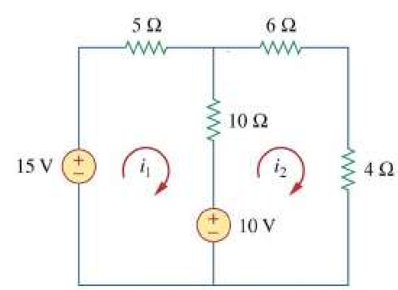
Problem: Find I_1, I_2 and I_3 in the below circuit using mesh analysis.



Solution:

Step1: In given circuit, two node are present.

Step2: Current in to meshes is assumed as i_1 and i_2.



Step-1 and Step-2

Step3:

. KVL equation to mesh 1, is

$-15+5\,i_1+10(i_1-i_2)+10=0$---------------(1)

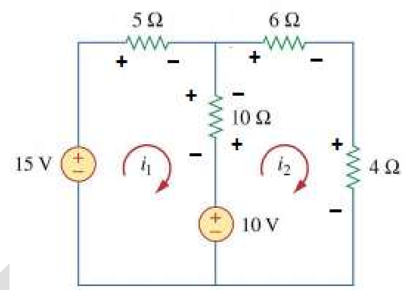
KVL equation to mesh 2, is

$+6i_2+4i_2-10+10(i_2-i_1)=0$---------------(2)

By solving (1) and (2) we will get

$i_1=2.0$ A and $i_2=1.50$A

But comparing given circuit with circuit in Step3,



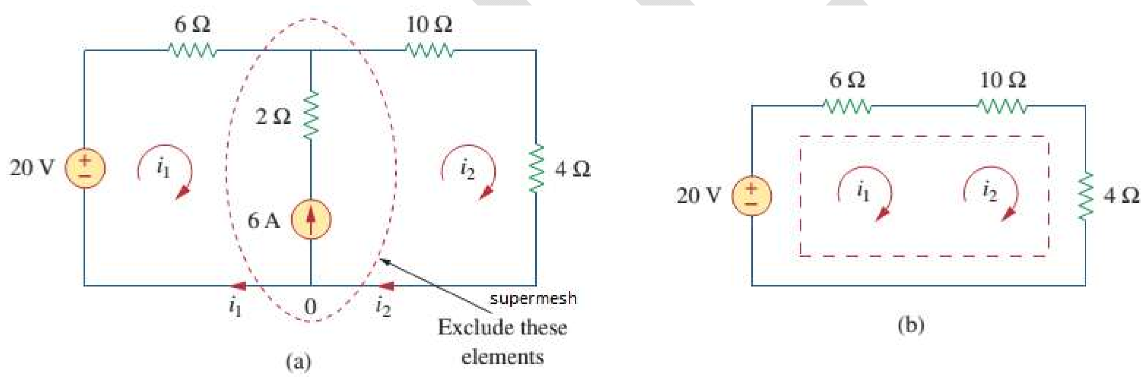
Step-3

$I_1=i_1=2.0$A

$I_2=i_2=1.5$A

$I_3=i_1-i_2=0.5$A

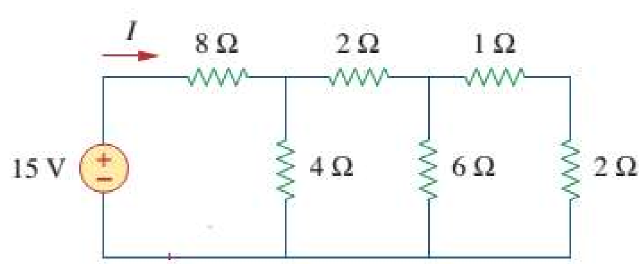
Supermesh: A *supermesh* results when two meshes have a (dependent or independent) current source in common.



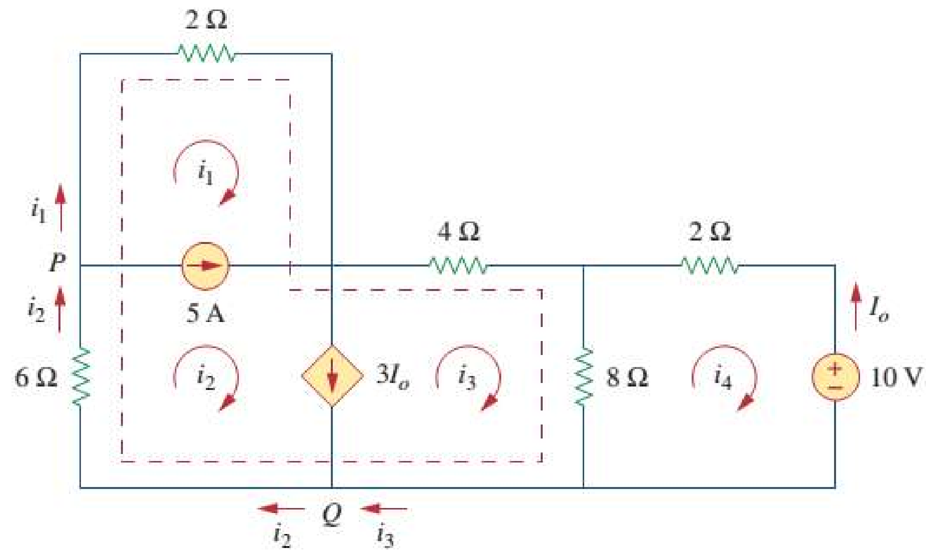
(a) Two meshes having a current source in common, (b) a supermesh, created by excluding the current source.

Problem:

Find I using mesh analysis.



Problem: Find i_1 and i_4 in the following circuit using mesh analysis.



Solution:

Note that meshes 1 and 2 form a supermesh since they have an independent current source in common. Also, meshes 2 and 3 form another supermesh because they have a dependent current source in common. The two supermeshes intersect and form a larger supermesh as shown. Applying KVL to the larger supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

or

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \quad \text{\_\_\_\_\_\_\_\_(1)}$$

For the independent current source, we apply KCL to node P:

$$i_2 = i_1 + 5 \quad \text{-----------(2)}$$

For the dependent current source, we apply KCL to node Q:

$$i_2 = i_3 + 3I_o$$

But $I_o = -i_4$, hence,

$$i_2 = i_3 - 3i_4 \quad \text{-----------(3)}$$

Applying KVL in mesh 4,

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

or

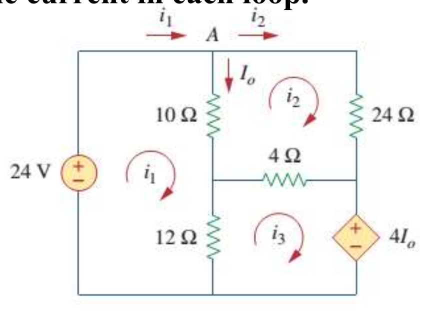
$$5i_4 - 4i_3 = -5 \quad \text{-----------(4)}$$

From Eqn. 1 to 4

$i_1 = -7.5$ A, $\quad i_2 = -2.5$ A, $\quad i_3 = 3.93$ A, $\quad i_4 = 2.143$ A

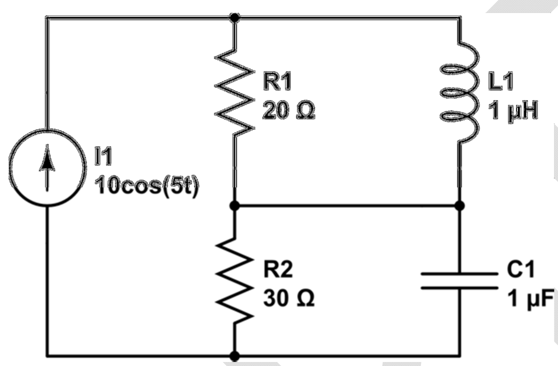
Problem:

Determine current in each loop.



Problem:

Determine current in each loop using mesh analysis



Node Analysis:

Steps to determine Node Analysis:

Step1: Determine the number of nodes n.

Step2: Select a node as reference node (ground node). Assign voltages V_1, V_2, …….V_{n-1} to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.

- Ground node is assumed to have 0(zero) potential.

Step3: Apply KCL to each of the n-1 non-reference nodes. Use Ohm's law to express the branch current in terms of node voltages.

Step4: Solve the resulting simultaneous equations to obtain the un-known node voltages.

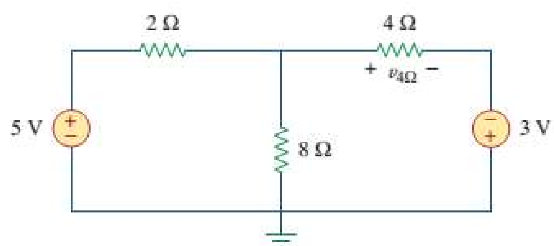
a) Current flows from a higher potential to lower potential in resistor.

b) If a voltage source is connected between the reference node and a non-reference node, we simply set the voltage at the non-reference node equal to the voltage of the source.

c) Multiple methods to solve the simultaneous equations in Step4.
   1) Method 1: Elimination technique (good for few variables)
   2) Method 2: Write in terms of matrix and vectors (write Ax=b) then use Cramer's rule.
   3) Method 3: Use computer or calculators.

Problem:

Find the value of V_4 in below circuit.



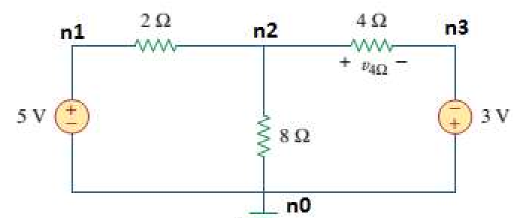
Step1: Total number of nodes are 4 they are n0, n1,n2 and n3.

Step2: node n0 is selected as reference node. $V_{n0}=0$

and voltage at other nodes are V_{n1}, V_{n2} and $V_{n3.}$

Step3: Assume voltage at node n2 is more than all other node voltages.

So current is going away from this node in all branches.

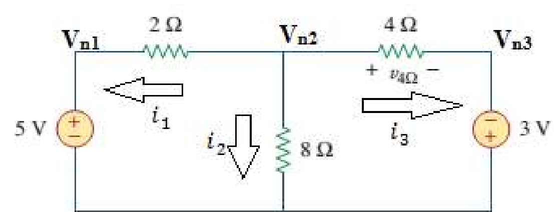


Step-1

$$i_1 = \frac{V_{n2}-V_{n1}}{2} = \frac{V_{n2}-5}{2}$$

$$i_2 = \frac{V_{n2}-V_{no}}{8} = \frac{V_{n2}-0}{8}$$

$$i_3 = \frac{V_{n2}-V_{n3}}{4} = \frac{V_{n2}-(-3)}{4}$$



Step-2 and Step-3

$$applying\ KCL\ at\ node\ n2 \quad i_1 + i_2 + i_3 = 0$$

$$\frac{V_{n2}-5}{2} + \frac{V_{n2}-0}{8} + \frac{V_{n2}-(-3)}{4} = 0$$

By solving above equation $V_{n2} = 2\ V$

$$current\ i_3 = \frac{V_{n2}-(-3)}{4} = \frac{5}{4} = 1.25A$$

$according\ to\ Ohm's\ law\ v_4 = i_3 * 4 = 1.25 * 4 = 5\ volts$

Supernode: A *supernode* is formed by enclosing a (dependent or independent voltage source connected between two non-reference nodes and any elements connected parallel with it.

Problem: For the circuit shown in Fig. 1 find the node voltages.

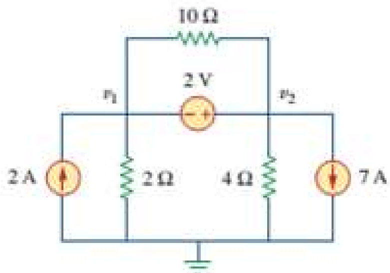


Fig-1

Solution:

The supernode contains the 2-V source, nodes 1 and 2, and the 10-Ω resistor. Applying KCL to the supernode as shown in Fig. 1 gives

$$2 = i_1 + i_2 + 7$$

Expressing i_1 and i_2 in terms of the node voltages

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \quad \Rightarrow \quad 8 = 2v_1 + v_2 + 28$$

or

$$v_2 = -20 - 2v_1 \quad \text{————(1)}$$

To get the relationship between v_1 and v_2, we apply KVL to the circuit in Fig. 3.10(b). Going around the loop, we obtain

$$-v_1 - 2 + v_2 = 0 \quad \Rightarrow \quad v_2 = v_1 + 2 \quad \text{---------(2)}$$

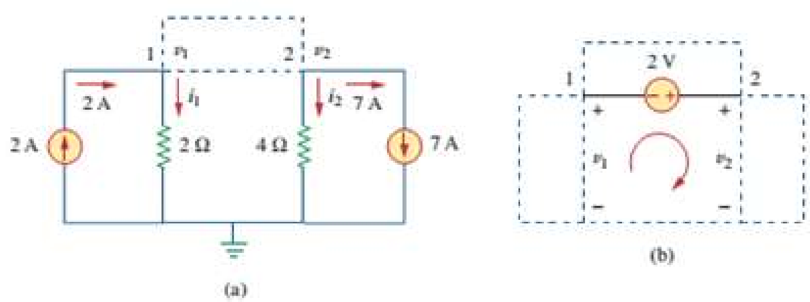
From eqn 1 and 2, we can write

$$v_2 = v_1 + 2 = -20 - 2v_1$$

or

$$3v_1 = -22 \quad \Rightarrow \quad v_1 = -7.333 \text{ V}$$

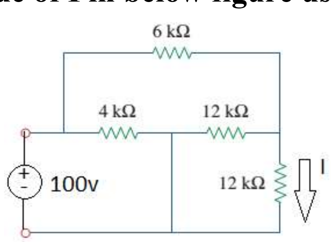
and $v_2 = v_1 + 2 = -5.333$ V. Note that the 10-Ω resistor does not make any difference because it is connected across the supernode.



Applying: (a) KCL to the supernode, (b) KVL to the loop.

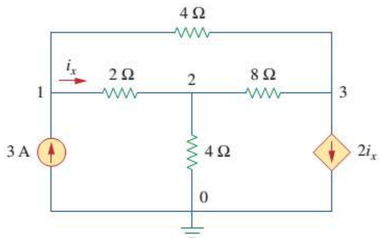
Problem:

Calculate the value of I in below figure using node analysis.



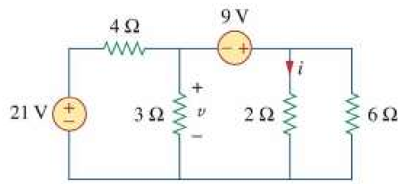
Problem:

Determine voltage at node in below figure.



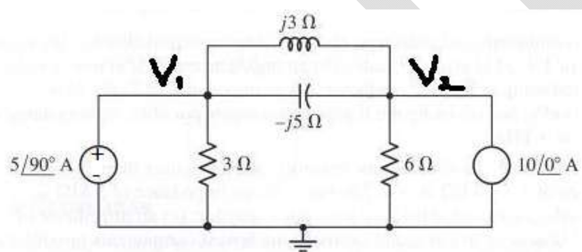
Problem:

Find v and i in below figure using node analysis.



Problem:

Find $V_1 and\ V_2$ using nodal analysis.



Two marks questions:

1. What are the different type of alternative signals and draw them.
2. Explain about phase difference.
3. What is leading, and explain with example.
4. What is lagging, and explain with example.
5. Which laws are used to solve using mesh analysis.
6. Which laws are used to solve using mesh analysis.
7. What are the different methods to solve un-know parameter in network.
8. Find the equivalent capacitance of two capacitor when
   a. Connected in parallel
   b. Connected in series.
9. Find the equivalent inductance of two inductor when
   a. Connected in series.
   b. Connected in parallel.

10. $v = 40 * \sin(\omega t + 30^0)$ and $i = 20 * cos(\omega t + 30^0)$, find phase difference between two waves.
11. Find X_L and X_C in terms of frequency.
12. How to calculate period of a given signal.
13. Draw phasor diagram for $v = 40 * \sin(\omega t + 30^0)$ and $i = 20 * cos(\omega t + 30^0)$.
14. Draw the phasor diagram for a given AC Voltage applied to
    a. Pure Resistor.
    b. Pure Inductor.
    c. Pure Capacitor.

Unit III: AC Fundamentals-II

***RMS and Average values, Form factor, Steady State Analysis of Series, Parallel and Series Parallel combinations of R, L,C with Sinusoidal excitation, Instantaneous power, Average power, Real power, Reactive power and Apparent power, concept of Power factor, Frequency*.**

Q: Define Average value, RMS value and Form Factor and Calculate for sinusoidal wave.

Average Value

The arithmetic average of all the values of an alternating quantity over one cycle is called its average value.

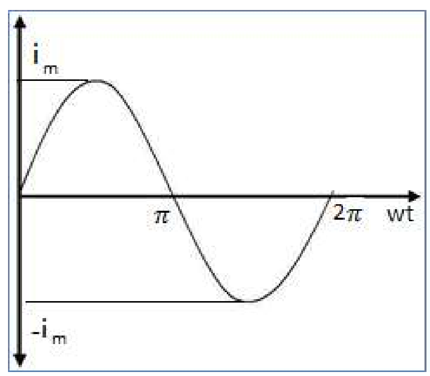
$$Average\ value = \frac{Area\ under\ one\ cycle}{Base}$$

$$v_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v\ d(\omega t)$$

For Symmetrical waveforms, the average value calculated over one cycle becomes equal to zero because the positive area cancels the negative area. Hence for symmetrical waveforms, the average value is calculated for half cycle.

$$Average\ value = \frac{Area\ under\ one\ cycle}{Base}$$

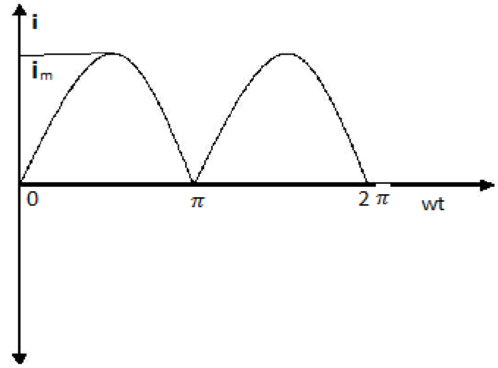
$$v_{avg} = \frac{1}{\pi}\int_0^{\pi} v\ d(\omega t)$$



Average value of a sinusoidal current:

$$i = i_m \sin(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i\ d(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i_m \sin(\omega t) d(\omega t)$$
$$i_{avg} = \frac{2i_m}{\pi} = 0.637 i_m$$

Average value of a full wave rectifier output



$$i = i_m \sin(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i\ d(\omega t)$$
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i_m \sin(\omega t) d(\omega t)$$
$$i_{avg} = \frac{2i_m}{\pi} = 0.637 i_m$$

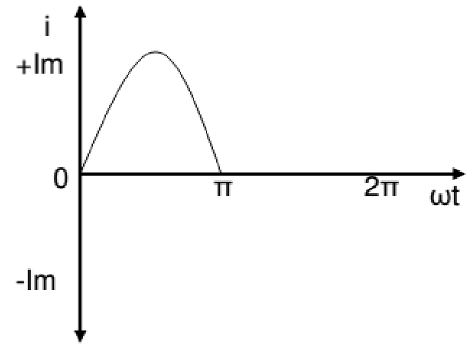
Average value of a half wave rectifier output

$$i = i_m \sin(\omega t)$$

$$i_{avg} = \frac{1}{2\pi}\int_0^{2\pi} i \, d(\omega t)$$

$$i_{avg} = \frac{1}{2\pi}\int_0^{2\pi} i_m \sin(\omega t) d(\omega t)$$

$$i_{avg} = \frac{2i_m}{2\pi} = 0.318 i_m$$



RMS or Effective Value

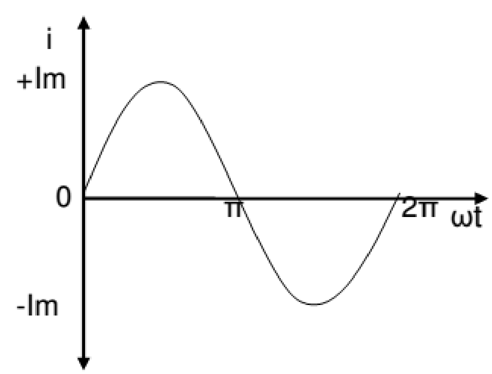
The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.

$$RMS = \sqrt{\frac{Area\ under\ squared\ curve}{Base}}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2 \, d(\omega t)}$$

RMS value of a sinusoidal current:

$$i = i_m \sin(\omega t)$$



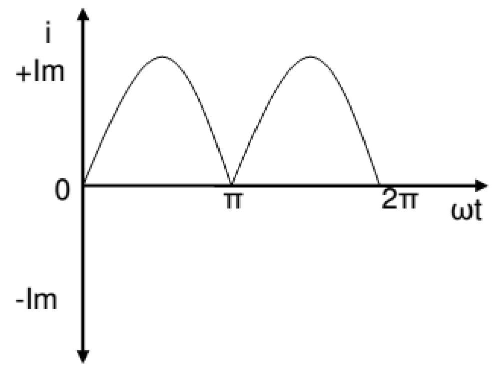
$$i_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 \, d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i_m^2 sin^2(\omega t) \, d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{i_m^2}{\pi}\int_0^{\pi} \frac{(1-\cos(2\omega t))}{2} \, d(\omega t)}$$

$$i_{rms} = \frac{i_m}{\sqrt{2}} = 0.707\ i_m$$

RMS value of a full wave rectifier output:



$$i = i_m \sin(\omega t)$$

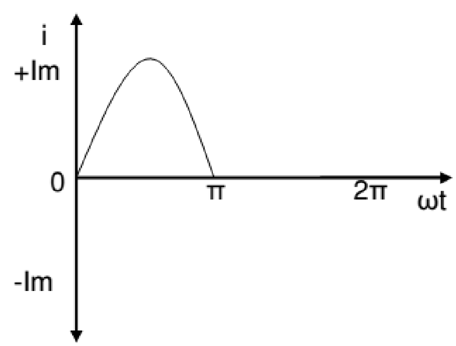
$$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2\ d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i_m^2 sin^2(\omega t)\ d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{i_m^2}{\pi}\int_0^{\pi} \frac{(1-\cos(2\omega t)}{2}\ d(\omega t)}$$

$$i_{rms} = \frac{i_m}{\sqrt{2}} = 0.707 i_m$$

RMS value of a half wave rectifier output



$$i = i_m \sin(\omega t) \qquad 0 \le \omega t \le 180^0$$

$$i = 0 \qquad 180^0 \le \omega t \le 360^0$$

$$i_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\ d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} i_m^2 sin^2(\omega t)\ d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{i_m^2}{2\pi}\int_0^{\pi} \frac{(1-\cos(2\omega t)}{2}\ d(\omega t)}$$

$$i_{rms} = \frac{i_m}{2} = 0.5 i_m$$

Form Factor:

It is the ratio of RMS value to the average value of an alternating quantity is known as Form Factor

$$FF = \frac{RMS\ value}{Avg\ Value}$$

Peak Factor or Crest Factor:
It is the ratio of maximum value to the RMS value of an alternating quantity is known as the peak factor.

$$PF = \frac{Maximum\ value}{RMS\ Value}$$

For a sinusoidal waveform and For full wave rectifier output:

$$i_{avg} = \frac{2i_m}{\pi} = 0.637 i_m$$

$$i_{rms} = \frac{i_m}{\sqrt{2}} = 0.707 i_m$$

$$FF = \frac{i_{rms}}{i_{avg}} = \frac{0.637 i_m}{0.707 i_m} = 1.11$$

$$PF = \frac{i_{rms}}{i_{avg}} = \frac{i_m}{0.707 i_m} = 1.414$$

For a Half Wave Rectifier Output:

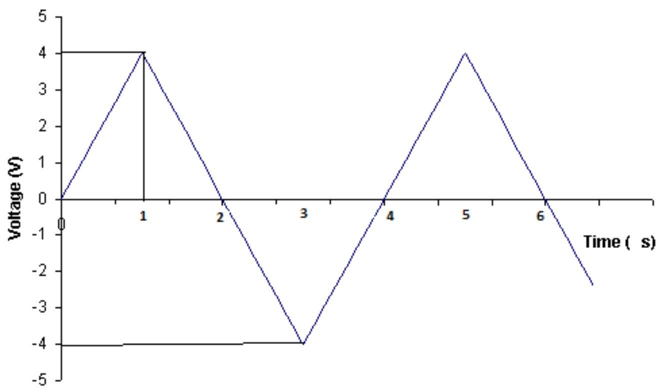
$$i_{rms} = \frac{i_m}{2} = 0.5 i_m$$

$$i_{avg} = \frac{2i_m}{2\pi} = 0.318 i_m$$

$$FF = \frac{i_{rms}}{i_{avg}} = \frac{0.318 i_m}{0.5 i_m} = 1.57$$

$$PF = \frac{i_{rms}}{i_{avg}} = \frac{i_m}{0.5 i_m} = 2$$

Problem: Find Form Factor of figure show below.



Solution:

From figure:

$$V(t) = 4 * t \qquad 0 \le t \le 1$$
$$= 4 - 4(t-1) \quad 1 \le t \le 2$$

Average value:

$$1/t \int_0^{t/2} v(t)dt = 1/4 \left(\int_0^1 4t\,dt + \int_1^2 4 - 4(t-1)\,dt \right)$$

$$¼\left(4 * \frac{t^2}{2}\Big]_0^1 + 4t]_0^1 - 4 * \frac{t^2}{2}\Big]_1^2 + 4t]_1^2\right)$$

$$\left(\frac{1}{4}\right) * (4 * [1/2] + 4 - 4[4/2 - 1/2] + 4) = 0.25$$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt\ put}\ \ T = t/2$$

$$V_{RMS} = \sqrt{\frac{1}{2}\int_0^2 v(t)^2\,dt}$$

$$V_{RMS} = \sqrt{\frac{1}{2}\left(\int_0^1 16 * (t)^2\,dt + \int_1^2 16 * (1 - 1(t-1))^2\,dt\right)}$$

$$V_{RMS} = \sqrt{\frac{16}{2}\left(\int_0^1 t^2\,dt + \int_1^2 (-(t-2))^2\,dt\right)}$$

$$V_{RMS} = \sqrt{8 * \left(\int_0^1 t^2\,dt + \int_1^2 (t^2 - 4t + 4)\,dt\right)}$$

$$V_{RMS} = \sqrt{8 * \left(\int_0^1 t^2\,dt + \int_1^2 (t-2)^2 dt\right)}$$

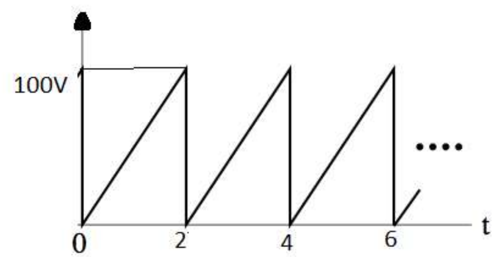
$$V_{RMS} = \sqrt{8 * \left\{ \left.\frac{t^3}{3}\right)_0^1 + (4t)_1^2 - \left.\frac{4t^2}{2}\right)_1^2 + \left.\frac{t^3}{3}\right)_1^2 \right\}}$$

$$V_{RMS} = \sqrt{8 * \left\{\frac{1}{3} - \frac{0}{3} + 4(2-1) - \left(4 * \frac{2^2}{2} - 4 * \frac{1^2}{2}\right) + \frac{2^3}{3} - \frac{1^3}{3}\right\}}$$

$$V_{RMS} = \sqrt{14 * \frac{8}{3}}$$

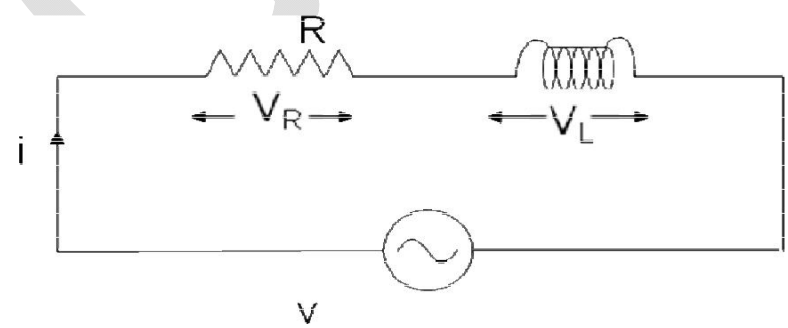
Problem:

Calculate Average value, Rms value and Form factor of the sawtooth wave show in the figure.



Steady State Analysis of Series, Parallel and Series Parallel combinations of R, L,C with Sinusoidal excitation:

R-L Series Circuit:



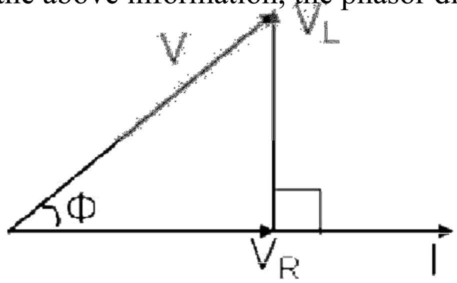
Consider an AC circuit with a resistance R and an inductance L connected in series as shown in the figure. The alternating voltage v is given by

$$v = v_m \sin(\omega t)$$

The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the inductor is V_L

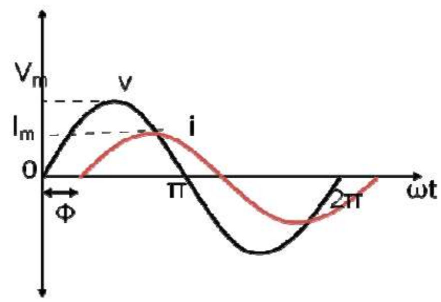
V_R=IR is in phase with I
V_L=IX_L leads current by 90 degrees

With the above information, the phasor diagram can be drawn as shown.



The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_L leads the current by 90^0. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ.
The waveform and equations for an RL series circuit can be drawn as below.

$$V = V_m \sin(\omega t)$$
$$I = I_m \sin(\omega t - \emptyset)$$



From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_L^2}$$
$$V_R = IR$$
$$V_L = IX_L$$
$$V = \sqrt{(IR)^2 + (IX_L)^2}$$
$$V = I\sqrt{R^2 + {X_L}^2}$$
$$V = IZ$$
$$where\ Z = \sqrt{R^2 + {X_L}^2}$$

The impedance in an AC circuit is similar to a resistance in a DC circuit. The unit for impedance is ohms(Ω)

Phase angle:

$$\emptyset = tan^{-1}(\frac{V_L}{V_R})$$

$\emptyset = tan^{-1}(\frac{IX_L}{IR})$
$\emptyset = tan^{-1}(\frac{X_L}{R})$
$\emptyset = tan^{-1}(\frac{\omega L}{R})$

Power Factor:

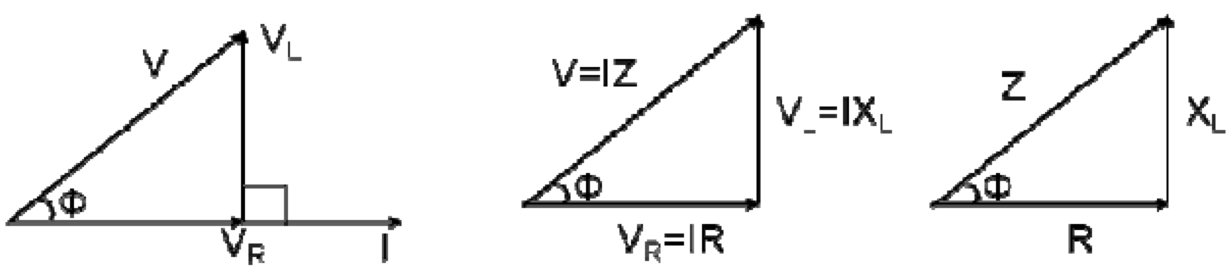
The power factor in an AC circuit is defined as the cosine of the angle between voltage and current ie., cosΦ

Problem:

A series RL circuit has a resistor 36Ω and impedance of circuit is 10 Ω, then find power factor

Impedance Triangle:

We can derive a triangle called the impedance triangle from the phasor diagram of an RL series circuit as shown



The impedance triangle is right angled triangle with R and X_L as two sides and impedance as the hypotenuse. The angle between the base and hypotenuse is Φ. The impedance triangle enables us to calculate the following things.

1. Impedance $Z = \sqrt{R^2 + X_L{}^2}$
2. Power Factor $cos\Phi = R/Z$
3. Phase angle $\emptyset = tan^{-1}(\frac{X_L}{R})$
4. Whether current is leading or lagging.

Problem:

A 200 V, 50 Hz, inductive circuit takes a current of 10A, lagging 30 degree. Find (i) the resistance (ii) reactance (iii) inductance of the coil.

Solution:

$$Z = \frac{\bar{V}}{\bar{I}} = \frac{200}{10} = 20\,\Omega$$

$$i) R = Zcos(\emptyset) = 20 * cos(30^0) = 17.32\Omega$$

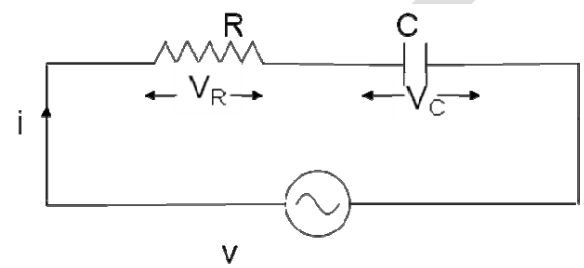
$$ii)\ X_L = Z\sin(\emptyset) = 20 * \sin(30^0) = 10\Omega$$

$$iii) L = \frac{X_L}{2\pi f} = \frac{10}{2 * 3.14 * 50} = 0.0318H$$

Problem:

A 230v, 50 Hz, is applied a series connected resistor 30 ohms and inductor 0.5mH, than find X_L, current through the circuit, voltage across each component, and also draw phasor diagram between current and voltage.

Explain the behavior of AC through RC Series Circuit:



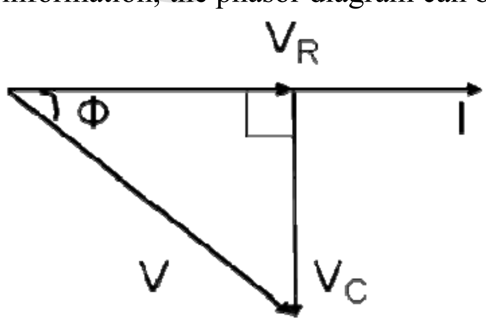
Consider an AC circuit with a resistance R and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

$$v = v_m \sin(\omega t)$$

The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the capacitor is V_C

$V_R = IR$ is in phase with I
$V_C = IX_C$ lags current by 90 degrees

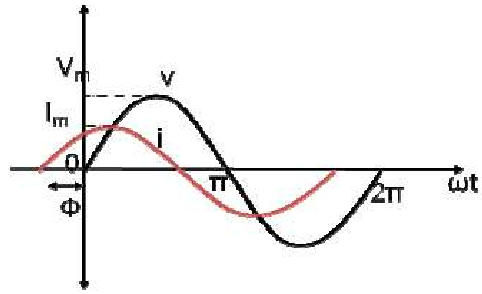
With the above information, the phasor diagram can be drawn as shown.



The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_C lags behind the current by 90^0. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

The waveform and equations for an RC series circuit can be drawn as below.

$$V = V_m \sin(\omega t)$$
$$I = I_m \sin(\omega t + \emptyset)$$



From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_C^2}$$
$$V_R = IR$$

$$V_C = IX_C$$

$$V = \sqrt{(IR)^2 + (IX_C)^2}$$
$$V = I\sqrt{R^2 + {X_C}^2}$$
$$V = IZ$$
$$where\ impedance\ Z = \sqrt{R^2 + {X_C}^2}$$

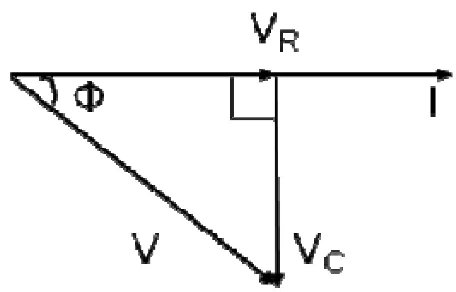
The impedance in an AC circuit is similar to a resistance in a DC circuit. The unit for impedance is ohms(Ω).

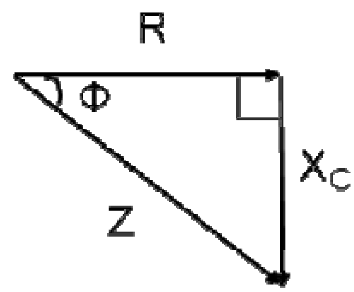
Phase angle:

$$\emptyset = tan^{-1}(\frac{V_C}{V_R})$$

$$\emptyset = tan^{-1}(\frac{IX_C}{IR})$$
$$\emptyset = tan^{-1}(\frac{X_C}{R})$$
$$\emptyset = tan^{-1}\left(\frac{\frac{1}{\omega C}}{R}\right) = tan^{-1}(\frac{1}{R\omega C})$$

Impedance triangle:





Phasor algebra for RC series circuit.

$$\bar{V} = V + j0 = V\angle 0^o$$

$$\bar{Z} = R - j\,X_C = Z\angle - \emptyset$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z}\angle\emptyset$$

Problem:

A Capacitor of capacitance 79.5µF is connected in series with a non inductive resistance of 30 across a 100V, 50Hz supply. Find (i) impedance (ii) current (iii) phase angle

Solution:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2*3.141*50*79.5*10^{-6}} = 40\Omega$$

$$i)\ Z = \sqrt{R^2 + {X_L}^2} = \sqrt{30^2 + 40^2} = 50\Omega$$

$$ii)\ I = V/Z = 100/50 = 2A$$

$$iii)\ Phase\ angle = Tan^{-1}\left(\frac{X_C}{R}\right) = Tan^{-1}\left(\frac{40}{30}\right) = 53^o$$

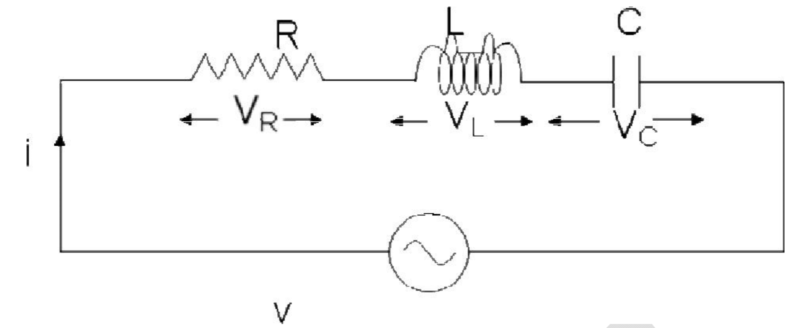
Problem:

A non-inductive resistor of 10 Ω is in series with a capacitor of 100µF across a 250V, 50Hz ac supply. Determine the current taken by the capacitor and power factor of the circuit

Problem:

A circuit consists of R and C reactance is 30Ω connected in series. Determine the value of R for which power factor of the circuit is 0.8. Draw the phasor Diagram.

Behavior AC with R-L-C Series circuit:



Consider an AC circuit with a resistance R, an inductance L and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

$$V = V_m \sin(\omega t)$$

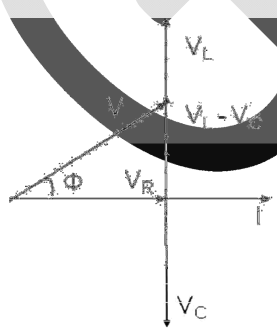
The current flowing in the circuit is i. The voltage across the resistor is V_R, the voltage across the inductor is V_L and that across the capacitor is V_C.

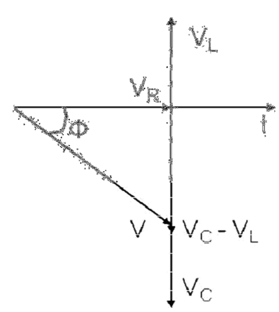
$V_R = IR$ is in phase with I

$V_L = IX_L$ leads the current by 90 degrees

$V_C = IX_C$ lags behind the current by 90 degrees

With the above information, the phasor diagram can be drawn as shown. The current I is taken as the reference phasor. The voltage V_R is in phase with I, the voltage V_L leads the current by 90^o and the voltage V_C lags behind the current by 90^o. There are two cases that can occur $V_L > V_C$ and $V_L < V_C$ depending on the values of X_L and X_C. And hence there are two possible phasor diagrams. The phasor $V_L - V_C$ or $V_C - V_L$ is drawn and then the resultant voltage V is drawn.





From the phasor diagram we observe that when $V_L > V_C$, the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ. When $V_L < V_C$, the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived

as follows.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$
$$V = I\sqrt{(R)^2 + (X_L - X_C)^2}$$
$$V = IZ$$
$$Where\ impedance\ is\ Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Phase\ angle\ \emptyset = Tan^{-1}(\frac{V_L - V_C}{V_R}) = Tan^{-1}\left(\frac{IX_L - IX_C}{IR}\right) = Tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

From the expression for phase angle, we can derive the following three cases

Case (i): When $X_L > X_C$

The phase angle Φ is positive and the circuit is inductive. The circuit behaves like a series RL circuit.

Case (ii): When $X_L < X_C$

The phase angle Φ is negative and the circuit is capacitive. The circuit behaves like a series RC circuit.

Case (iii): When $X_L = X_C$

The phase angle Φ = 0 and the circuit is purely resistive. The circuit behaves like a pure resistive circuit.

The voltage and the current can be represented by the following equations. The angle Φ is positive or negative depending on the circuit elements.

$$V = V_m \sin(\omega t)$$
$$I = I_m \sin(\omega t \pm \emptyset)$$

Phasor algebra for RLC series circuit.

$$V = V + j0 = V\angle 0^o$$

$$\bar{Z} = R + j\,(X_L - X_C) = Z\angle\emptyset$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z}\angle -\emptyset$$

Problem:

A 230 V, 50 Hz ac supply is applied to a coil of 0.06 H inductance and 2.5 resistance connected in series with a 6.8 μF capacitor. Calculate (i) Impedance (ii) Current (iii) Phase angle between current and voltage (iv) power factor

Solution:

$$X_L = 2\pi f L = 1 * 3.14 * 50 * 0.06 = 18.84\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2*3.141*50*6.8*10^{-6}} = 468\ \Omega$$

$$i)\quad Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2.5^2 + (18.84 - 468)^2} = 449.2\ \Omega$$

$$I = \frac{V}{Z} = \frac{230}{4492} = 0.512A$$

$$\emptyset = Tan^{-1}\left(\frac{X_L - X_C}{R}\right) = Tan^{-1}\left(\frac{18.84 - 468}{30}\right) = -89.7^o$$

$$Power\ factor = \cos(\emptyset) = \cos(-89.7) = 0.0056\ Lead$$

Problem:

A resistance R, an inductance L=0.01 H and a capacitance C are connected in series. When an alternating voltage v=400sin(3000t-20º)is applied to the series combination, the current flowing is 10 √2 sin(3000t-65º). Find the values of R and C.

Solution:

$$\emptyset = 65^0 - 20^0 = 45^0$$

$$X_L = 2\pi f L = \omega L = 3000 * 0.01 = 30$$

$$Tan(\emptyset) = Tan(45) = 1$$

$$Tan(\emptyset) = \frac{X_L - X_C}{R} = 1$$

$$X_L - X_C = R$$

$$Z = \frac{V_m}{I_m} = \frac{400}{10\sqrt{2}} = \sqrt{R^2 + R^2}$$

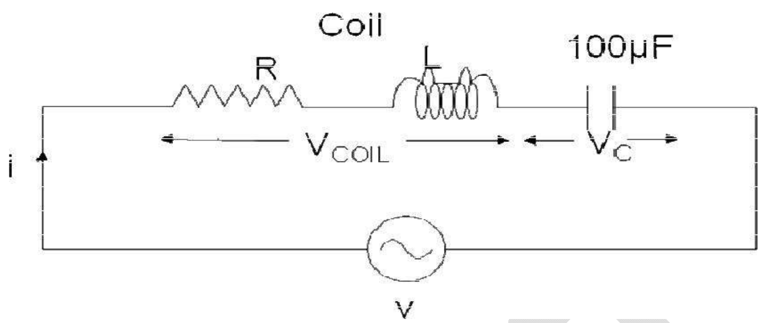
$$R = 20\Omega$$

$$X_C = X_L - R = 30 - 20 = 10\Omega$$

$$X_C = \frac{1}{2\pi f C} => C = \frac{1}{\omega X_C} = \frac{1}{3000*10} = 33.3\mu F$$

Problem:

A coil of pf 0.6 is in series with a 100µF capacitor. When connected to a 50Hz supply, the potential difference across the coil is equal to the potential difference across the capacitor. Find the resistance and inductance of the coil.

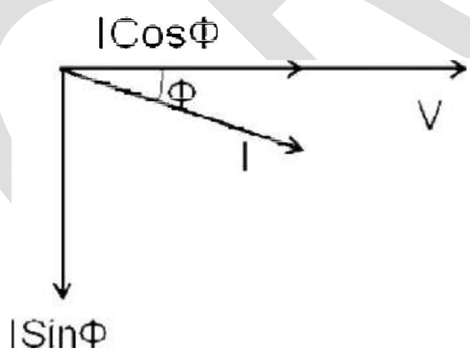


Power:

In an AC circuit, the various powers can be classified as

1. Real or Active power or Average power.
2. Reactive power
3. Apparent power

Real or active power in an AC circuit is the power that does useful work in the circuit. Reactive power flows in an AC circuit but does not do any useful work. Apparent power is the total power in an AC circuit.



Instantaneous Power:

The instantaneous power is product of instantaneous values of current and voltages and it can be derived as follows

$$P = vi$$

$$p = V_m \sin(\omega t + \theta_v) * I_m \sin(\omega t + \theta_i)$$

From trigonometric expression:

$$\cos(A - B) - \cos(A + B) = 2\sin(A)\sin(B)$$

$$p = \frac{V_m I_m}{2}(\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i))$$

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) - \frac{V_m I_m}{2}\cos(2\omega t)$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

Average Power:

From instantaneous power we can find average power over one cycle as following.

$$P = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{V_m I_m}{2}\cos(\theta_v - \theta_i) - \frac{V_m I_m}{2}\cos(2\omega t + \theta_v + \theta_i)\right)d(\omega t)$$

$$P = \frac{1}{2\pi}\left(\frac{V_m I_m}{2}\cos(\theta_v - \theta_i) * (2\pi - 0)\right) - \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2}\cos(2\omega t)\, d(\omega t)$$

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} = V_{RMS} * I_{RMS}\cos(\theta_v - \theta_i)$$

As seen above the average power is the product of the RMS voltage and the RMS current.

Problem:

Calculate the instantaneous power and average power absorbed by the passive linear network. If $v = 330\cos(10t + 20^0)\ volts$ and $i = 33\sin(10t + 60^0)\ A$.

Problem:

Calculate the average power absorbed by an impedance Z=30-j70 ohms when applied a voltage $\overline{V} = 120\angle 0^0$ is applied across it.

Problem:

$v(t) = 120\cos(377t + 45^\circ)$ V and $i(t) = 10\cos(377t - 10^\circ)$ A

find the instantaneous power and the average power absorbed by the passive linear network

Solution:

The instantaneous power is given by

$$p = vi = 1200\cos(377t + 45^\circ)\cos(377t - 10^\circ)$$

Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

gives

$$p = 600[\cos(754t + 35^\circ) + \cos 55^\circ]$$

or

$$p(t) = 344.2 + 600\cos(754t + 35^\circ) \text{ W}$$

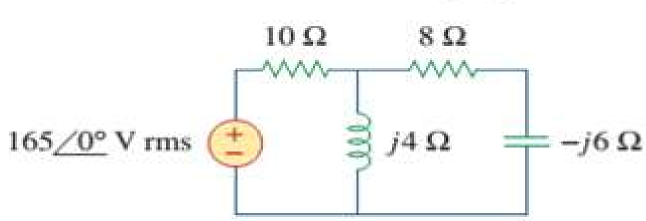
The average power is

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}120(10)\cos[45^\circ - (-10^\circ)]$$
$$= 600\cos 55^\circ = 344.2 \text{ W}$$

which is the constant part of $p(t)$ above.

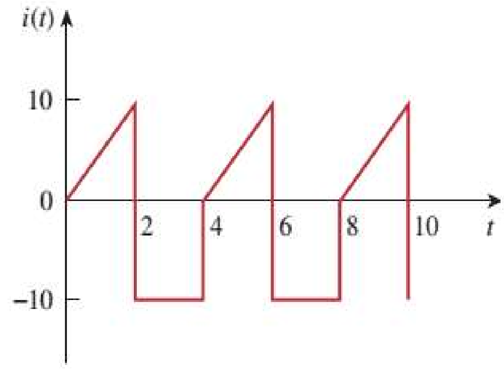
Problem:

Calculate the power factor of the entire circuit of Fig. as seen by the source. What is the average power supplied by the source?



Problem:

Determine the rms value of the current waveform in Fig. 11.14. If the current is passed through a 2-Ω resistor, find the average power absorbed by the resistor.



Solution:

The period of the waveform is $T = 4$. Over a period, we can write the current waveform as

$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

The rms value is

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2\, dt + \int_2^4 (-10)^2\, dt\right]}$$

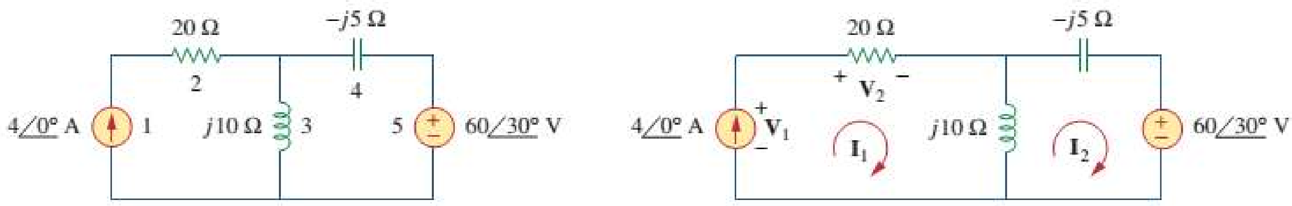
$$= \sqrt{\frac{1}{4}\left[25\frac{t^3}{3}\Big|_0^2 + 100t\Big|_2^4\right]} = \sqrt{\frac{1}{4}\left(\frac{200}{3} + 200\right)} = 8.165 \text{ A}$$

The power absorbed by a 2-Ω resistor is

$$P = I_{rms}^2 R = (8.165)^2(2) = 133.3 \text{ W}$$

Problem:

Determine the average power generated by each source and the average power absorbed by each passive element in the circuit



(a) (b)

We apply mesh analysis as shown in figure (b) For mesh 1,

$$\mathbf{I}_1 = 4 \text{ A}$$

For mesh 2,

$$(j10 - j5)\mathbf{I}_2 - j10\mathbf{I}_1 + 60\underline{/30^\circ} = 0, \qquad \mathbf{I}_1 = 4 \text{ A}$$

or

$$j5\mathbf{I}_2 = -60\underline{/30^\circ} + j40 \quad \Rightarrow \quad \mathbf{I}_2 = -12\underline{/-60^\circ} + 8$$
$$= 10.58\underline{/79.1^\circ} \text{ A}$$

For the voltage source, the current flowing from it is $\mathbf{I}_2 = 10.58\underline{/79.1^\circ}$ A and the voltage across it is $60\underline{/30^\circ}$ V, so that the average power is

$$P_5 = \frac{1}{2}(60)(10.58)\cos(30^\circ - 79.1^\circ) = 207.8 \text{ W}$$

Following the passive sign convention figure(b) this average power is absorbed by the source, in view of the direction of $\mathbf{I}_2$ and the polarity of the voltage source. That is, the circuit is delivering average power to the voltage source.

For the current source, the current through it is $\mathbf{I}_1 = 4\underline{/0^\circ}$ and the voltage across it is

$$\mathbf{V}_1 = 20\mathbf{I}_1 + j10(\mathbf{I}_1 - \mathbf{I}_2) = 80 + j10(4 - 2 - j10.39)$$
$$= 183.9 + j20 = 184.984\underline{/6.21^\circ} \text{ V}$$

The average power supplied by the current source is

$$P_1 = -\frac{1}{2}(184.984)(4)\cos(6.21^\circ - 0) = -367.8 \text{ W}$$

It is negative according to the passive sign convention, meaning that the current source is supplying power to the circuit.

For the resistor, the current through it is $\mathbf{I}_1 = 4\underline{/0^\circ}$ and the voltage across it is $20\mathbf{I}_1 = 80\underline{/0^\circ}$, so that the power absorbed by the resistor is

$$P_2 = \frac{1}{2}(80)(4) = 160 \text{ W}$$

Real Power:

The power due to the active component of current is called as the active power or real power. It is denoted by P.

$$P = V * Icos(\emptyset) = I^2Rcos(\emptyset)$$

Real power is the power that does useful power. It is the power that is consumed by the resistance. The unit for real power in Watt(W).

Reactive Power:

The power due to the reactive component of current is called as the reactive power. It is denoted by Q.

$$Q = V * Isin(\emptyset) = I^2 X_L \sin(\emptyset)$$

Reactive power does not do any useful work. It is the circulating power in the L and C components. The unit for reactive power is Volt Amperes Reactive (VAR).

Apparent Power:

The apparent power is the total power in the circuit. It is denoted by S.

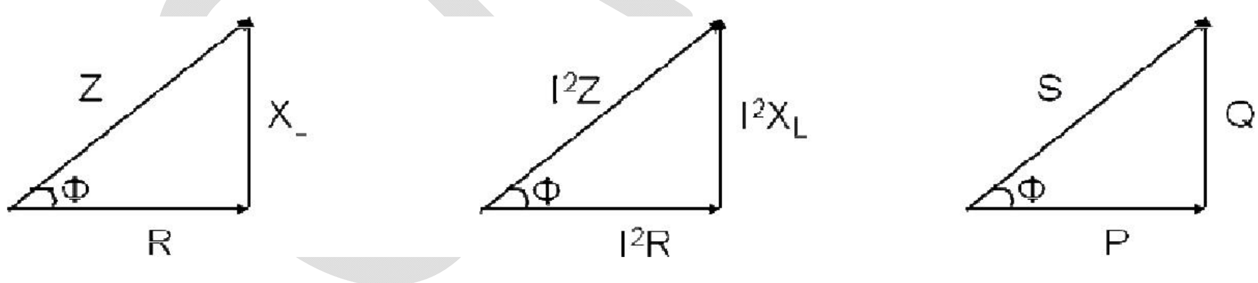
$$S = VI = I^2Z$$

$$S = \sqrt{P^2 + Q^2}$$

The unit for apparent power is Volt Amperes (VA).

Power Triangle:

From the impedance triangle, another triangle called the power triangle can be derived as shown.



The power triangle is right angled triangle with P and Q as two sides and S as the hypotenuse. The angle between the base and hypotenuse is Φ. The power triangle enables us to calculate the following things.

Apparent Power $S = \sqrt{P^2 + Q^2}$

$$Power\ factor = \cos(\emptyset) = \frac{P}{Q} = \frac{Real\ Power}{Reactive\ Power}$$

The power Factor in an AC circuit can be calculated by any one of the following

Methods

$= Cosine\ of\ angle\ between\ V\ and\ I$

$= \frac{Resistance}{Impedance} = \frac{R}{Z}$

$= \frac{Real\ power}{Apparent\ power}$

Problem:

A coil having a resistance of 7 and an inductance of 31.8mH is connected to 230V, 50Hz supply. Calculate (i) the circuit current (ii) phase angle (iii) power factor (iv) power consumed v) Reactive power vi) Apparent power.

Solution:

$X_L = 2\pi f L = 2 * 3.14 * 50 * 31.8 * 10^{-3} = 10\Omega$

$Z = \sqrt{R^2 + {X_L}^2} = \sqrt{7^2 + 10^2} = 12.2\ \Omega$

$i)\ I = \frac{V}{Z} = \frac{230}{12.2} = 18.85A$

$ii)\ \emptyset = Tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}(\frac{10}{7}) = -53^0\ lag$

$iii) PF = \cos(\emptyset) = \cos(-53) = 0.537\ lag$

$iv) Power\ Consumed\ P = VI\ cos(\emptyset) = 230 * 18.85 * 0.537 = 2.484kW$

$v) Reactive\ Power\ Q = VI \sin(\emptyset) = 230 * 18.85 * 0.795 = 3.462k\ VAR$

$vi) Apperant\ Power = \sqrt{P^2 + Q^2} = \sqrt{2.48^2 + 3.46^2} = 4.25kVA$

Problem:

A current of (120-j50)A flows through a circuit when the applied voltage is (8+j12)V. Determine (i) impedance (ii) power factor (iii) power consumed and reactive power.

Solution:

$\bar{V} = 8 + j\ 12$

$\bar{I} = 120 - j50$

$i)\ \bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{8+j\ 12}{120-j50} = 0.02 + j0.11 = 0.11\angle 79.7^0$

$Z = 0.11\Omega$

$\emptyset = 79.7^0$

$ii)\ pf = \cos(\emptyset) = \cos(79.7^0) = 0.179\ lag$

$iii) S = VI^* = (8 + j12)(120 + j50) = 360 + j1840$

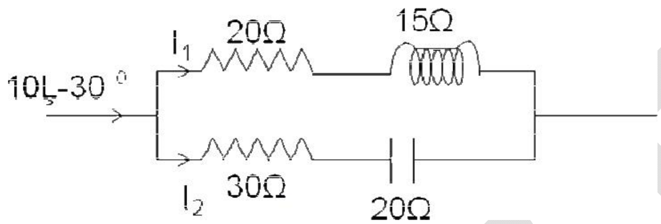
$But\ S = P + jQ$

$P = 360W$
$S = 1860VAR$

Problem:

A parallel circuit comprises of a resistor of 20Ω in series with an inductive reactance 15Ω in one branch and a resistor of 30Ω in series with a capacitive reactance of 20Ω in the other branch. Determine the current and power dissipated in each branch if the total current drawn by the parallel circuit is $\mathbf{10\angle -30\,^{0}A}$

Solution:



$Z_1 = 20 + j15$
$Z_2 = 30 - j20$
$I = 10\angle -30^0$
According to KCL
$I_1 = \frac{IZ_2}{Z_1+Z_2} = (8.66 - j5) * \frac{(30-j20)}{(20+j15)+(30-j20)} = 3.8 - j6.08 = 7.17\angle -60^0$
$I_2 = I - I_1(8.66 - j5) - (3.8 - j6.08) = 4.86 + j1.08 = 4.98\angle -12.5^0$
$P_1 = I_1^2 R_1 = 7.17^2 * 20 = 1028.2W$
$P_2 = I_2^2 R_2 = 4.98^2 * 30 = 744W$

Problem:

A circuit having a resistance of 20 and inductance of 0.07H is connected in parallel with a series combination of 50 resistance and 60µF capacitance. Calculate the total current, when the parallel combination is connected across 230V, 50Hz supply.

Problem:

An impedance coil in parallel with a 100µF capacitor is connected across a 200V, 50Hz supply. The coil takes a current of 4A and the power loss in the coil is 600W. Calculate (i) the resistance of the coil (ii) the inductance of the coil (iii) the power factor of the entire circuit.

Solution:

$Z_{coil} = \frac{V}{I} = \frac{200}{4} = 50\Omega$
$P = I^2 R = 600W$

$$R = \frac{600}{I^2} = \frac{600}{4^2} = 37.5\Omega$$

$$X_L = \sqrt{Z_{coil}^2 - R^2} = \sqrt{50^2 - 37.5^2} = 33.07\Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{33.07}{2*3.14*50} = 0.105H$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2*3.14*50*100*10^{-6}} = 31.83\ \Omega$$

$$Z_1 = R + jX_L = 37.5 + j33.07$$

$$Z_2 = -jX_C = -j31.83$$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(37.5+j33.07)(-j31.83)}{(37.5+j33.07)+(-j31.83)} = 27 - j32.72 = 42.42\angle -50.5^0$$

$$\emptyset = -50.5^0$$

$$pf = \cos(\emptyset) = \cos(-50.5^0) = 0.6365$$

Problem:

A series RLC circuit is connected across a 50Hz supply. R=100Ω, L=159.16mH and C=63.7µF. If the voltage across C is $150\angle -90^0$V. Find the supply voltage

Solution:

$$X_L = 2\pi f L = 2*3.14*50*159*16*10^{-3} = 50\Omega$$

$$X_c = \frac{1}{2\pi f C} = \frac{1}{2*3.14*50*63.7*10^{-6}} = 50\Omega$$

$$V_C = I(-X_C) = 150\angle -90^0 = -j150$$

$$I = -\frac{j150}{-jX_c} = -\frac{j150}{-j50} = 3\angle 0^0 A$$

$$Z = R + j(X_L - X_C) = 100 + j(50 - 50) = 100\ \Omega$$

$$V = IZ = 3*100 = 300$$

Problem:

An alternative voltage of 160+j120 is applied to a circuit and the current in the circuit given by 6+j8 A, Find,

i) Value of elements in circuit. ii) the power factor of circuit iii) power consumed.

Problem:

A series circuit consists of non-inductive resistor of 10 Ω, an inductor having a reactance of 50Ω, and a capacitor having a reactance of 30Ω. It is connected to a 230v ac supply. Calculate 1) the current 2) the voltage across each component. 3) Draw to scale a phasor diagram showing the supply voltage and current and voltage across each component.

Problem:

The voltage applied to a circuit is $v = 100sin(\omega t + 30^0)$, and current flowing the circuit is $i = 15sin(\omega t + 60^0)$. Determine the impedance, resistance, reactance, power and power factor of the circuit.

Problem:

A sinusoidal source supplies 1000KVR reactive power to load $Z = 250\angle -73^0$. Determine a) the power factor b) the apparent power delivered to the load, and c) the rms voltage.

Two marks Questions

1. Define RMS, Average and Form Factor.
2. Find the average value of sine wave.
3. Find RMS value of sinusoidal wave.
4. Define Real, Reactive and Apperant power.
5. Define power factor and find the power factor when $Z=30\angle 45^{0.}$
6. Write the formulae to calculate Real, Reactive and Apparent power.
7. Calculate the Reactive power when 30V DC is applied to Z=30+j60.
8. Write the relation between Real, Reactive and Apparent power.
9. Find the impedance of series connected circuit contains
    a. R=30 ohms and C=0.003F
    b. R=45 ohms and L=0.04H

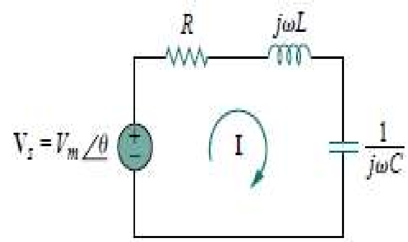
    When supplied voltage is 32V DC.
10. Draw impedance triangle and explain.
11. Draw power triangle and explain.
12. From the given Impedance triangle and current, How to find the Real, Reactive and Apparent power.

Unit-IV

Resonance is a condition in an *RLC* circuit in which the capacitive and inductive reactance's are equal in magnitude, thereby resulting in a purely resistive impedance.

Q1)Explain about Series Resonance and derive an expression for its bandwidth.

The most prominent feature of the frequency response of a circuit may be the sharp peak (or *resonant peak*) exhibited in its amplitude characteristic. The concept of resonance applies in several areas of science and engineering. Resonance occurs in any system that has a complex conjugatepair of poles; it is the cause of oscillations of stored energy from one formto another. It is the phenomenon that allows frequency discrimination incommunications networks. Resonance occurs in any circuit that has atleast one inductor and one capacitor.
Resonant circuits (series or parallel) are useful for constructing filters, as their transfer functions can be highly frequency selective. They are usedin many applications such as selecting the desired stations in radio andTV receivers.
Consider the series *RLC* circuit shown in the frequencydomain. The input impedance is



$Z = H(\omega) = \frac{Vs}{I} = R + j\omega L + \frac{1}{j\omega C}$….(1)

$Z = R + j(\omega L - \frac{1}{j\omega C})$..(2)

Resonance results when the imaginary part of the transfer function iszero, or

$Im(Z) = \omega L - \frac{1}{\omega C} = 0$ ..(3)

The value of ω that satisfies this condition is called the ***resonant frequency***ω_0. Thus, the resonance condition is

$\omega_0 L = \frac{1}{\omega_0 C}$..(4)

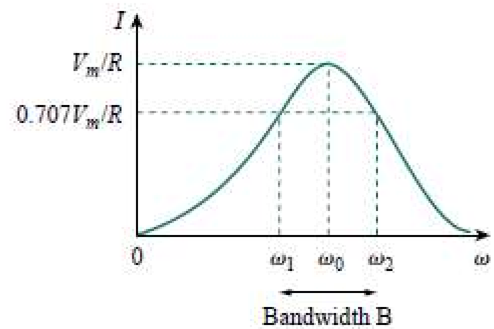
..(5) $\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{rad/s}}$

Or Since $\omega_0 = 2\pi f_0$, $f_0 = \frac{1}{2\pi\sqrt{LC}}$Hz ..(6)

Note that at resonance: The impedance is purely resistive, thus, **Z** = *R*. In otherwords, the *LC* series combination acts like a short circuit, andthe entire voltage is across *R*. The voltage **V***s* and the current **I** are in phase, so that the power factor is unity. The magnitude of the transfer function **H***(ω)* = **Z***(ω)* isminimum. The inductor voltage and capacitor voltage can be much morethan the source voltage.The frequency response of the circuit's current magnitude

$I = |I| = \dfrac{Vm}{\sqrt{R^2 + \left((\omega L - \frac{1}{\omega C})^2\right)}}$ (7)

is shown in Fig. the plot only shows the symmetry illustrated inthis graph when the frequency axis is a logarithm.



The average power
dissipated by the *RLC* circuit is$P(\omega) = \frac{1}{2} I^2 R$The highest power dissipated occurs at resonance, when $I = Vm/R$, so That $P(\omega_0) = \frac{1}{2}\frac{v_m{}^2}{R}$ ..(8)

At certain frequencies $\omega = \omega 1, \omega 2$, the dissipated power is half themaximum value; that is,

$P(\omega_1) = P(\omega_2) = \dfrac{(Vm/\sqrt{2})^2}{2R} = \dfrac{V_m{}^2}{4R}$ ..(9)

Hence, $\omega 1$ and $\omega 2$ are called the **half-power frequencies**. The half-power frequencies are obtained by setting$Z$ equal to$\sqrt{2}$R and writing$\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

Solving for ω, we obtain

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \quad ..(10)$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

We can relate the half-power frequencies with the resonant frequency. From Eqs. (5) and (10),
$\omega_0 = \sqrt{\omega 1 \omega 2}$
It is seen that the resonant frequency is the geometric mean of the half power frequencies. Notice that $\omega 1$ and $\omega 2$ are in general not symmetricalaround the resonant frequency $\omega 0$, because the frequency response is notgenerally symmetrical. However, as will be explained shortly, symmetryof the half-power frequencies around the resonant frequency is often areasonable approximation.Although the height of the curve in Fig.,it is determined by *R*, the width of the curve depends on other factors. The width of theresponse curve depends on the *bandwidth B*, which is defined as thedifference between the two half-power frequencies,

$B = \omega_2 - \omega_1$(11)
This definition of bandwidth is just one of several that are commonly used.Strictly speaking, *B* in Eq. (14.35) is a half-power bandwidth, because itis the width of the frequency band between the half-power frequencies. The "sharpness" of the resonance in a resonant circuit is measured quantitatively by the **quality factor Q**. At resonance, the reactive energyin the circuit oscillates between the inductor and the capacitor. The qualityfactor relates the maximum or peak energy stored to the energy dissipatedin the circuit per cycle of oscillation:

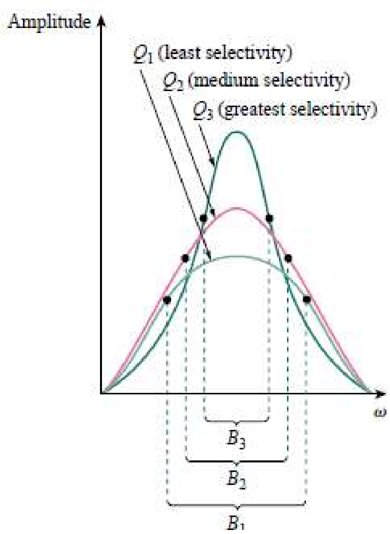
$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}} \text{..(12)}$$

It is also regarded as a measure of the energy storage property of a circuitin relation to its energy dissipation property. In the series *RLC* circuit,the peak energy stored is $\frac{1}{2}LI^2$, while the energy dissipated in one periodis $\frac{1}{2}(I^2R)(1/f)$. Hence,

$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R(\frac{1}{f})} = \frac{2\pi fL}{R} \text{..(13)}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} \text{..(14)}$$

Notice that the quality factor is dimensionless. The relationship betweenthe bandwidth *B* and the quality factor *Q* is obtained by substituting Eq.10 into Eq. (11) and utilizing Eq. (14).



$$B = \frac{R}{L} = \frac{\omega_0}{Q}$$

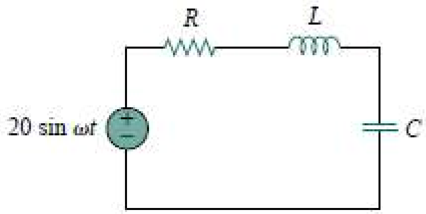
Or $B = \omega_0^2 CR$ Thus, The qualityfactor of a resonant circuit is the ratio of its resonant frequency to its bandwidth. As illustrated in Fig, the higher the value of *Q*, the moreselective the circuit is but the smaller the bandwidth. The ***selectivity*** of an *RLC* circuit is the ability of the circuit to respond to a certain frequencyand discriminate against all other frequencies. If the band of frequencies to be selected or rejected is narrow, the quality factor of the resonantcircuit must be high. If the band of frequencies is wide, the quality factormust be low.A resonant circuit is designed to operate at or near its resonantfrequency. It is said to be a *high-Q circuit* when its quality factor is equal to or greater than 10. For high-Q circuits ($Q \geq 10$), the half power frequencies are, for all practical purposes, symmetrical around theresonant frequency and can be approximated as

$$\omega_1 \cong \omega_0 - \frac{B}{2}; \quad \omega_1 \cong \omega_0 + \frac{B}{2}$$

High-Q circuits are used often in communications networks.We see that a resonant circuit is characterized by five related parameters:the two half-power frequencies $\omega 1$ and $\omega 2$, the resonant frequency$\omega 0$, the bandwidth B, and the quality factor Q.

Problem:

In the circuit in Fig. 14.24, R = 2 &, L = 1 mH, and C = 0.4 μF.(a) Find the resonant frequency and the half-power frequencies. (b) Calculatethe quality factor and bandwidth. (c) Determine the amplitude of the current at $\omega 0$, $\omega 1$, and $\omega 2$.



Solution:

(a) The resonant frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3}*0.4*10^{-6}}} = 50 \text{ krad/s}$$

The lower half-power frequency is

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$-1 + \sqrt{1 + 2500}$ krad/s = 49 krad/s

Similarly, the upper half-power frequency is

$\omega_2 = 1 + \sqrt{1 + 2500}$ krad/s = 51 krad/s

(b) The bandwidth is

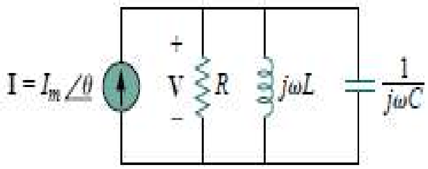
$B = \omega 2 - \omega 1 = 2$ krad/s or $B = \frac{R}{L} = \frac{2}{10^{-3}} = 2$krad/s

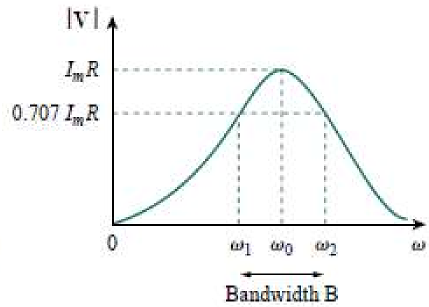
The quality factor is

$$Q = \frac{\omega_0}{B} = \frac{50}{2} = 25$$

Q2.)Explain about Parallel Resonance and derive an expression for bandwidth.

The parallel *RLC* circuit in Fig. 14.25 is the dual of the series *RLC*circuit. So we will avoid needless repetition. The admittance is





$Y = H(\omega) = \frac{I}{V} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$..(16)

$Y = \frac{1}{R} + j(\ \omega C - \frac{1}{\omega L})$ ..(17)

Resonance occurs when the imaginary part of **Y** is zero,i.e $\omega C - \frac{1}{\omega L} = 0$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{rad/s}$$

which is the same as Eq. (5) for the series resonant circuit. Thevoltage |**V**| is sketched in Fig. as a function of frequency. Noticethat at resonance, the parallel *LC* combination acts like an open circuit, sothat the entire currents flows through *R*. Also, the inductor and capacitorWe can see this from the fact thatcurrent can be much more than the source current at resonance.We exploit the duality between Figsby comparingEq. (17) with Eq. (2). By replacing *R*, *L*, and *C* in the expressionsfor the series circuit with 1/*R,* 1/*C*, and 1/*L* respectively, we obtain for the parallel circuit

$\omega_1 = -\frac{1}{2RC} + \sqrt{(\frac{1}{2RC})^2 + \frac{1}{LC}}$; $\omega_2 = \frac{1}{2RC} + \sqrt{(\frac{1}{2RC})^2 + \frac{1}{LC}}$.. (18)

$$B = \omega_2 - \omega_1 = \frac{1}{RC} \text{..(19)}$$

$$Q = \frac{\omega 0}{B} = \omega_0 RC = \frac{1}{RC} \text{..(20)}$$

Using Eqs. (18) and (19), we can express the half-power frequenciesin terms of the quality factor. The result is

$\omega_1 = \omega_0 \sqrt{1 + (\frac{1}{2Q})^2} - \frac{\omega 0}{2Q}$; $\omega_2 = \omega_0 \sqrt{1 + (\frac{1}{2Q})^2} + \frac{\omega 0}{2Q}$ ..(21)

Again, for high-*Q* circuits ($Q \geq 10$)

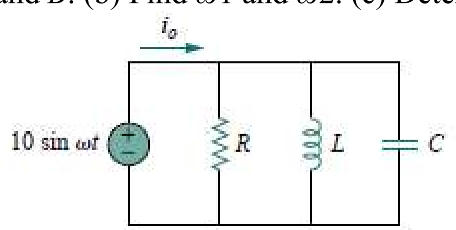
$$\omega_1 \cong \omega_0 - \frac{B}{2};\quad \omega_2 \cong \omega_0 + \frac{B}{2}$$

Table below presents a summary of the characteristics of the series andparallel resonant circuits. Besides the series and parallel*RLC* consideredhere, other resonant circuits exist..

Summary of Characteristics of resonant RLC circuits		
Characteristic	Series Circuit	Parallel Circuit
Resonant frequency , ω_0	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
Quality Factor,Q	$\frac{\omega_0 L}{R}$ or $\frac{1}{\omega_0 CR}$	$\omega_0 RC$ or $\frac{R}{\omega_0 L}$
Bandwidth B	$\frac{\omega 0}{Q}$	$\frac{\omega 0}{Q}$
Half Power frequencies ω_1, ω_2	$\omega_0 \sqrt{1+(\frac{1}{2Q})^2} \pm \frac{\omega 0}{2Q}$	$\omega_0 \sqrt{1+(\frac{1}{2Q})^2} \pm \frac{\omega 0}{2Q}$
For Q ≥ 10, ω_1, ω_2	$\omega_0 \pm \frac{B}{2}$	$\omega_0 \pm \frac{B}{2}$

Problem:

In the parallel *RLC* circuit in Fig. 14.27, let $R =$ 8 k&, $L =$ 0.2 mH, and$C =$ 8 μF. (a) Calculate $\omega 0$, Q, and B. (b) Find $\omega 1$ and $\omega 2$. (c) Determinethe power dissipated at $\omega 0$, $\omega 1$, and $\omega 2$.



$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2 * 10^{-3} * 8 * 10^{-6}}} = \frac{10^5}{4} = 25 \text{ krad/s}$$

$$Q = \frac{R}{\omega 0 L} = \frac{8 * 10^3}{\sqrt{25 * 10^3 * 0.2 * 10^{-3}}} = 1600$$

$$B = \frac{\omega 0}{Q} = 15.625 \text{ rad/s}$$

Due to the high value of *Q*, we can regard this as a high-*Q* circuit.Hence,

$$\omega_1 = \omega_0 - \frac{B}{2} = 25{,}000 - 7.812 = 24{,}992 \text{ rad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 25{,}000 + 7.812 = 25{,}008 \text{ rad/s}$$

(c) At $\omega = \omega_0$, Y = 1/R or Z= R = 8KΩ Then

$$I_0 = \frac{V}{Z} = \frac{10\angle -90}{8000} = 1.25 \angle -90 \text{ mA}$$

Since the entire current flows through *R* at resonance, the average powerdissipated at $\omega = \omega 0$ is

$$P = \frac{1}{2} |Io|^2 R = \frac{1}{2}(1.25 \text{ x } 10^{-3})^2 (8 \text{ x } 10^3) = 6.25\text{mW}$$

$P = \frac{Vm^2}{2R} = \frac{100}{2*8*10^3} = 6.25$ mW

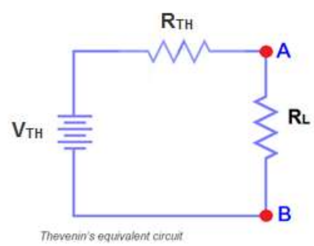
At $\omega = \omega 1, \omega 2,$

$P = \frac{Vm^2}{4R} = 3.125$ mW

Q3) Explain Thevenin's Theorem with a suitable example.

Network Theorems are useful in determining the unknown values of current, Resistance, Voltage etc in Electric Networks.

Thevenin's theorem states that any two output terminals (A & B) of an active linear network containing independent sources (it includes voltage and current sources) can be replaced by a simple voltage source of magnitude V_{th} in series with a single resistor R_{th} where R_{th} is the equivalent resistance of the network when looking from the output terminals A & B with all sources (voltage and current) removed and replaced by their internal resistances and the magnitude of V_{th} is equal to the open circuit voltage across the A & B terminals.



Simple Steps to Analyze Electric Circuit through Thevenin's Theorem

1. Open the load resistor.
2. Calculate / measure the Open Circuit Voltage. This is the Thevenin Voltage (V_{TH}).
3. Open Current Sources and Short Voltage Sources.
4. Calculate /measure the Open Circuit Resistance. This is the Thevenin Resistance (R_{TH}).
5. Now, Redraw the circuit with measured open circuit Voltage (V_{TH}) in Step (2) as voltage Source and measured open circuit resistance (R_{TH}) in step (4) as a series resistance and connect the load resistor which we had removed in Step (1). This is the Equivalent Thevenin Circuit of that Linear Electric Network or Complex circuit which had to be simplified and analyzed by Thevenin's Theorem. You have done.
6. Now find the Total current flowing through Load resistor by using the Ohm's Law $I_T = V_{TH} / (R_{TH} + R_L)$.

Example:

Find V_{TH}, R_{TH} and the load current flowing through and load voltage across the load resistor in fig (1) by using Thevenin's Theorem.

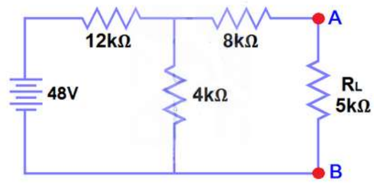


Fig 1

Step 1.
Open the 5kΩ load resistor

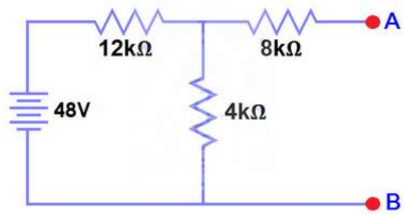


Fig 2

Step 2.
Calculate / measure the Open Circuit Voltage. This is the Thevenin Voltage (V_{TH}). We have already removed the load resistor from figure 1, so the circuit became an open circuit as shown in fig 2. Now we have to calculate the Thevenin's Voltage. Since 3mA Current flows in both 12kΩ and 4kΩ resistors as this is a series circuit because current will not flow in the 8kΩ resistor as it is open. So 12V (3mA x 4kΩ) will appear across the 4kΩ resistor. We also know that current is not flowing through the 8kΩ resistor as it is open circuit, but the 8kΩ resistor is in parallel with 4k resistor. So the same voltage (i.e. 12V) will appear across the 8kΩ resistor as 4kΩ resistor. Therefore 12V will appear across the AB terminals. So,

$\mathbf{V_{TH} = 12V}$

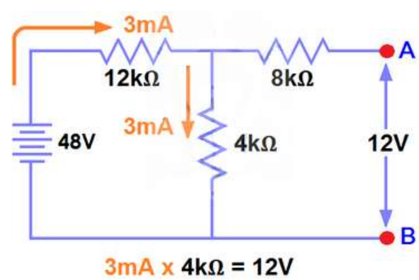


Fig 3

Step 3.
Open Current Sources and Short Voltage Sources. Fig (4)

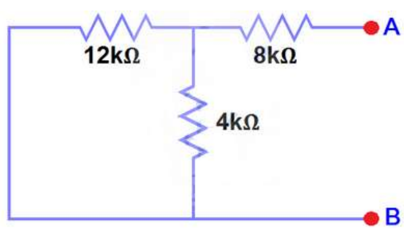


Fig 4

Step 4.
Calculate /measure the Open Circuit Resistance. This is the Thevenin Resistance (R_{TH})We have Reduced the 48V DC source to zero is equivalent to replace it with a short in step (3),as shownin

figure (3) We can see that 8kΩ resistor is in series with a parallel connection of 4kΩ resistor and 12k Ω resistor. i.e.:

8kΩ + (4k Ω || 12kΩ) ….. (|| = in parallel with)

R_{TH} = 8kΩ + [(4kΩ x 12kΩ) / (4kΩ + 12kΩ)]

R_{TH} = 8kΩ + 3kΩ

R_{TH} = 11kΩ

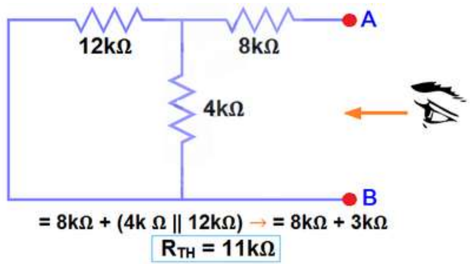
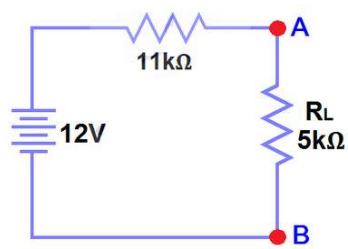


Fig 5

Step 5.

Connect the R_{TH}in series with Voltage Source V_{TH} and re-connect the load resistor. This is shown in fig (6) i.e. Thevenin circuit with load resistor. This the Thevenin's equivalent circuit



Thevenin's equivalent circuit

Step 6.

Now apply the last step i.e Ohm's law . calculate the total load current & load voltage as shown in fig 6.

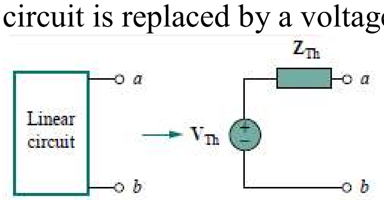
$I_L = V_{TH}/ (R_{TH} + R_L)$= 12V / (11kΩ + 5kΩ) → = 12/16kΩ

I_L= 0.75mA

And $V_L = I_L x R_L$= 0.75mA x 5kΩ

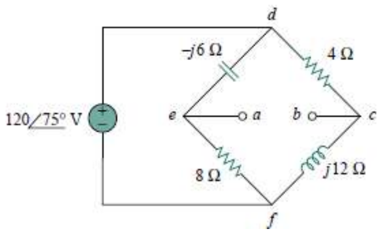
V_L= 3.75V

Thevenin's theorem is applied to ac circuits in the same way as they are to dc circuits.Thevenin's theorem, as applied to ac circuits, provides a method for reducing any circuit to an equivalent form that consists of an equivalent ac voltage source in series with an equivalent impedance. The only additional effort arises from the need to manipulate complex numbers. The frequency-domain version of a Thevenin equivalent circuit is depicted in Fig, where a linear circuit is replaced by a voltage source in series with an impedance.



Problem:

Obtain the Thevenin equivalent at terminals *a-b* of the circuit in



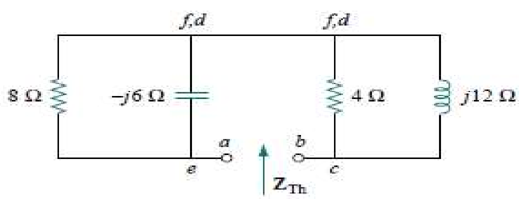
Solution:

We find $\mathbf{Z}_{Th}$ by setting the voltage source to zero. As shown in Fig. (a) below, the 8-*Ω*resistance is now in parallel with the −*j*6 reactance, sothat their combination gives

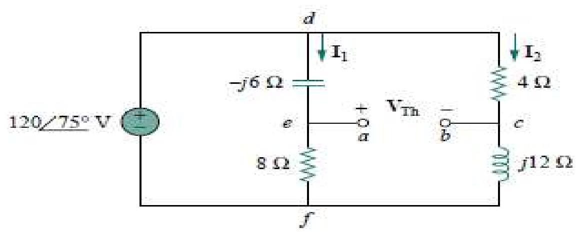
Z_1= -j6 // 8 = $\frac{-j6*8}{8-j6}$ = 2.88 – j3.84 Ω

Similarly, the 4-*Ω*resistance is in parallel with the *j*12 reactance, andtheir combination gives

Z_2= 4 // j12 = $\frac{j12*4}{4+j12}$ = 3.6 + j1.2 Ω



(a)



(b)

The Thevenin impedance is the series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$; that is,

Z_{Th}= Z_1+ Z_2 = 6.48 – j2.64 Ω

To find $\mathbf{V}_{Th}$, consider the circuit in Fig. (b). Currents $\mathbf{I}_1$ and$\mathbf{I}_2$ are obtained as

I_1= $\frac{120\angle 75}{8-j6}$A, I_2 = $\frac{120\angle 75}{4+j12}$A

Applying KVL around loop *bcdeab* in Fig.(b) gives

V_{Th}– 4I_2 + (-j6) I_1 =0

Or V_{Th}= 4I_2 + j6I_1= $\frac{480\angle 75}{4+j12}$ + $\frac{720\angle(75+90)}{8-j6}$

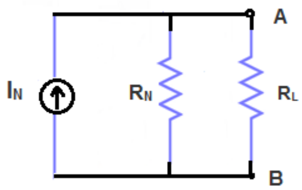
= 37.95∟3.43^0 + 72∟201.87^0

= -28.936 –j24.55 = 37.95 ∟220.31^0 V

Q4) Explain Norton's Theorem with suitable example.

This is another useful theorem to analyze electric circuits like Thevenin's Theorem, which reduces linear, active circuits and complex networks into a simple equivalent circuit. The main difference between Thevenin's theorem and Norton's theorem is that, Thevenin's theorem provides an equivalent voltage source and an equivalent series resistance, while Norton's theorem provides an equivalent Current source and an equivalent parallel resistance.

Norton's Theorem may be stated as Any Linear Electric Network or complex circuit with Current and Voltage sources can be replaced by an equivalent circuit containing of a single independent Current Source I_N and a Parallel Resistance R_N.



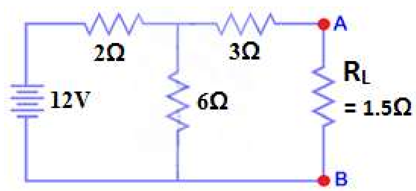
Norton's Equivalent Circuit

Simple Steps to Analyze Electric Circuit through Norton's Theorem

1. Short the load resistor
2. Calculate / measure the Short Circuit Current. This is the Norton Current (I_N)
3. Open Current Sources, Short Voltage Sources and Open Load Resistor.
4. Calculate /measure the Open Circuit Resistance. This is the Norton Resistance (R_N)
5. Now, Redraw the circuit with measured short circuit Current (I_N) in Step (2) as current Source and measured open circuit resistance (R_N) in step (4) as a parallel resistance and connect the load resistor which we had removed in Step (3). This is the Equivalent Norton Circuit of that Linear Electric Network or Complex circuit which had to be simplified and analyzed. You have done.
6. Now find the Load current flowing through and Load Voltage across Load Resistor by using the Current divider rule. $I_L = I_N / (R_N / (R_N + R_L))$

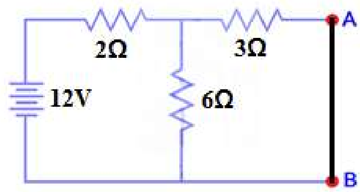
Example:

Find R_N, I_N, the current flowing through and Load Voltage across the load resistor in fig (1) by using Norton's Theorem.



Step 1.

Short the 1.5Ω load resistor as shown in (Fig 2).



Step 2.

Calculate / measure the Short Circuit Current. This is the Norton Current (I_N).

We have shorted the AB terminals to determine the Norton current, I_N. The 6Ω and 3Ω are then in parallel and this parallel combination of 6Ω and 3Ω are then in series with 2Ω.

So the Total Resistance of the circuit to the Source is:-

2Ω + (6Ω || 3Ω) ….. (|| = in parallel with).

R_T = 2Ω + [(3Ω x 6Ω) / (3Ω + 6Ω)] → I_T= 2Ω + 2Ω = 4Ω.

R_T = 4Ω

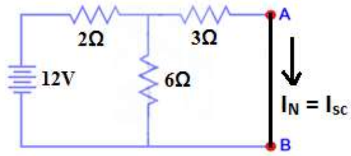
I_T = V / R_T

I_T = 12V / 4Ω= 3A..

Now we have to find I_{SC} = I_N… Apply CDR… (Current Divider Rule)…

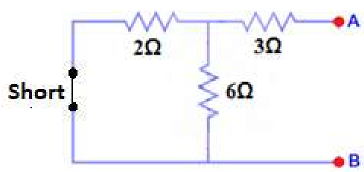
I_{SC} = I_N = 3A x [(6Ω / (3Ω + 6Ω)] = 2A.

I_{SC}= I_N = 2A.



Step 3.

Open Current Sources, Short Voltage Sources and Open Load Resistor.



Step 4.

Calculate /measure the Open Circuit Resistance. This is the Norton Resistance (R_N)

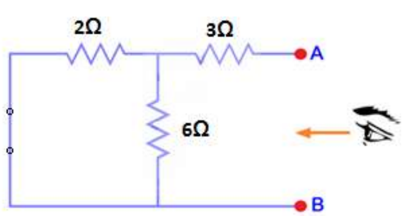
We have Reduced the 12V DC source to zero is equivalent to replace it with a short in step (3), as shown in figure (4) We can see that 3Ω resistor is in series with a parallel combination of 6Ω resistor and 2Ω resistor. i.e.:

3Ω + (6Ω || 2Ω) ….. (|| = in parallel with)

R_N = 3Ω + [(6Ω x 2Ω) / (6Ω + 2Ω)]

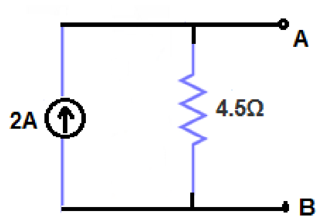
R_N = 3Ω + 1.5Ω

R_N = 4.5Ω



Step 5.

Connect the R_N in Parallel with Current Source $I_N$and re-connect the load resistor. This is shown in fig (6) i.e. Norton Equivalent circuit with load resistor.



Step 6.

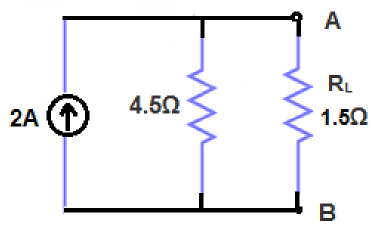
Now apply the last step i.e. calculate the load current through and Load voltage across load resistor by Ohm's Law as shown in fig 7.

Load Current through Load Resistor…

$I_L = I_N$ x $[R_N / (R_N + R_L)]$= 2A x (4.5Ω /4.5Ω +1.5kΩ) → = 1.5A

$\mathbf{I_L}$ **= 1. 5A**AndLoad Voltage across Load Resistor…

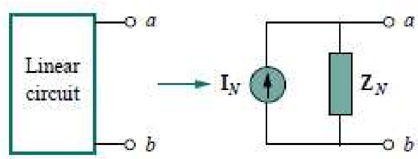
$V_L = I_L$ x R_L= 1.5A x 1.5Ω = **2.25V**



Norton's theorem is also applied to ac circuits in the same way as they are to dc circuits. The only additional effort arises from the need to manipulate complex numbers. The Norton equivalent circuit is illustrated in Fig.below, where a linear circuit s replaced by a current source in parallel with an impedance. Keep in mind that the two equivalent circuits are related as

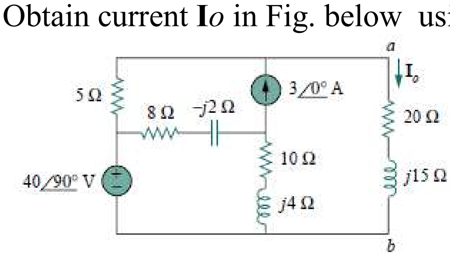
$\mathbf{V}_{Th} = \mathbf{Z}_N \mathbf{I}_N$, $\mathbf{Z}_{Th} = \mathbf{Z}_N$

just as in source transformation. $\mathbf{V}_{Th}$ is the open-circuit voltage while $\mathbf{I}_N$is the short-circuit current.



Norton equivalent.

Obtain current **I***o* in Fig. below using Norton's theorem.



Solution:

Our first objective is to find the Norton equivalent at terminals *a-b*. $\mathbf{Z}_N$is found in the same way as $\mathbf{Z}_{Th}$. We set the sources to zero as shownin Fig. (a). As evident from the figure, the *(*8 − *j*2*)* and *(*10 + *j*4*)*impedances are short-circuited, so that

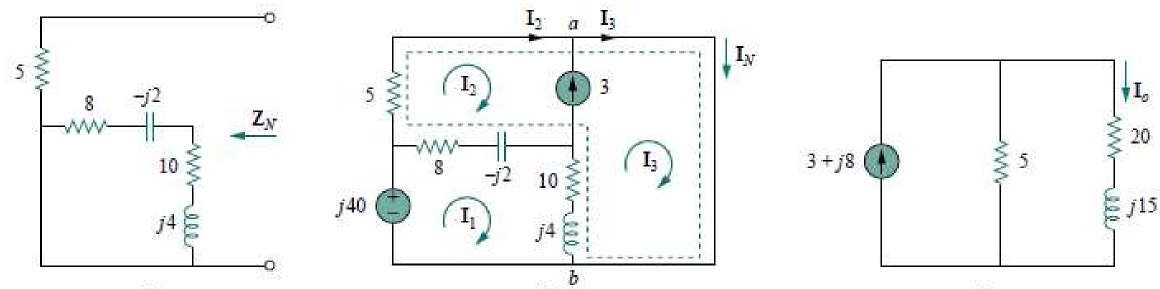
$\mathbf{Z}_N$= 5Ω

To get $\mathbf{I}_N$, we short-circuit terminals *a*-*b* as in Fig. (b) andapply mesh analysis. Notice that meshes 2 and 3 form a supermesh because of the current source linking them. For mesh 1,

$-j40 + (18 + j2)\, I_1 - (8 - j2)\, I_2 - (10 + j4)\, I_3 = 0$ …(1)

For the supermesh,

$(13 - j2) I_2 + (10 + j4)\, I_3 - (18 + j2)\, I_1 = 0$ …(2)



(a) (b) (c)

At node *a*, due to the current source between meshes 2 and 3,

$\mathbf{I}_3 = \mathbf{I}_2 + 3$ …(3)

Adding Eqs. (1) and (2) gives

$-j40 + 5\mathbf{I}_2 = 0 \Rightarrow \mathbf{I}_2 = j8$

From Eq. (3),

$\mathbf{I}_3 = \mathbf{I}_2 + 3 = 3 + j8$

The Norton current is

$\mathbf{I}_N = \mathbf{I}_3 = (3 + j8)$ A

Figure (c) shows the Norton equivalent circuit along with the impedanceat terminals *a*-*b*. By current division

$$I_0 = \frac{5}{5+20+j15} I_N = \frac{3+j8}{5+j3} = 1.465\angle 38.48^0 \text{A}$$

Q5) Explain Superposition Theorem with a suitable example.

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currentsthrough) that element due to each independent source acting alone.

The principle of superposition helps us to analyze a linear circuit withmore than one independent source by calculating the contribution of eachindependent source separately. However, to apply the superposition principle,we must keep two things in mind:

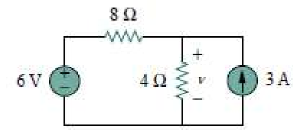
1. We consider one independent source at a time while all otherindependent sources are turned off. This implies that wereplace every voltage source by 0 V (or a short circuit), andevery current source by 0 A (or an open circuit). This way weobtain a simpler and more manageable circuit.Other terms such as killed, made inactive, deadened,or set equal to zero are often used to conveythe same idea.
2. Dependent sources are left intact because they are controlledby circuit variables.With these in mind, we apply the superposition principle in three steps:

Steps to Apply Super position Principle:

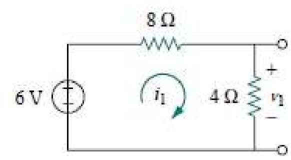
1. Turn off all independent sources except one source. Find theoutput (voltage or current) due to that active source using nodal ormesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all thecontributions due to the independent sources.Use the superposition theorem to find v in the circuit

Solution:

Since there are two sources, let
$v = v1 + v2$



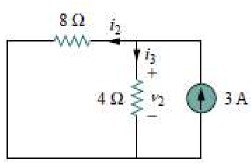
where$v1$ and $v2$ are the contributions due to the 6-V voltage source and the 3-A current source, respectively. To obtain $v1$, we set the currentsource to zero, as shown in Fig. Applying KVL to the loop in Fig.gives
$12i1 - 6 = 0 \Rightarrow i1 = 0.5$ A
Thus,$v1 = 4i1 = 2$ V



We may also use voltage division to get $v1$ by writing

$v_1 = \frac{4}{4+8}(6) = 2$V.To get $v2$, we set the voltage source to zero, as in FigUsingcurrent division,i_3

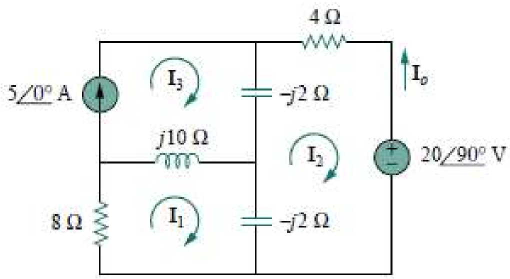
$i_3 = \frac{8}{4+8}(3) = 2$A



Hence,$v2 = 4i3 = 8$ V
And we find
$v = v1 + v2 = 2 + 8 = 10$ V

Since ac circuits are linear, the superposition theorem applies to ac circuitsthe same way it applies to dc circuits.

Use the superposition theorem to find $\mathbf{I}_o$ in the circuit



Let $I_0 = I_0' + I_0''$

Where I_0' and I_0'' are due to the voltage and current sources respectively. To find I_0', consider the circuit given below. If we let Z be the parallel combination of –j2 and 8 + j10, then

$$Z = \frac{-j2(8+j10)}{-2j+8+j10} = 0.25 - j2.25$$

And current I_0' is

$$I_0' = \frac{j20}{4-j2+Z} = \frac{j20}{4.25-j4.25}$$

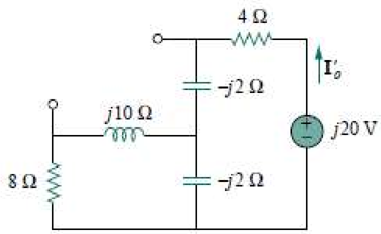


Fig (a)

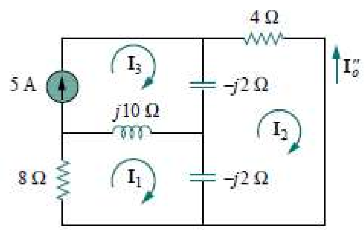


Fig (b)

$I_0' = -2.353 + j2.353$ **...(2)**

To get I_0'', consider the circuit in Fig (b) . For mesh 1,

$(8 + j8)\, I_1 - j10\, I_3 + j2I_2 = 0$ **…(3)**

For mesh 2, $(4-j4)\, I_2 + j2\, I_3 = 0$ **…(4)**

For mesh3, $I_3 = 5$ **...(5)**

From eqs (4) & (5),

$(4 - j4)I_2 + j2\, I_1 + j10 = 0$

Expressing I_1 in terms of I_2 gives

$I_1 = (2 + j2)\, I_2 - 5$ **…(6)**

Substituting Eqs. (5) and (6) into Eq. (3), we get

$(8 + j8)[(2 + j2)\mathbf{I}_2 - 5] - j50 + j2\mathbf{I}_2 = 0$

or

$I_2 = \frac{90-j40}{34} = 2.647 - j\, 1.176$

Current I_0'' is obtained as

$I_0'' = -I_2 = -2.647 + j\, 1.176$ **…(7)**

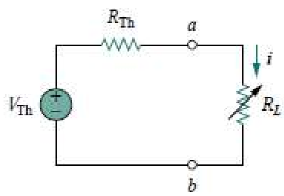
From eqs (2) & (7),

$I_0 = I_0' + I_0'' = -5 + j\, 3.529 = 6.12\angle 144.78^0$ A

Q6) Explain Maximum Power Transfer Theorem and derive an expression for Pmax.

In many practical situations, a circuit is designed to provide power to aload. While for electric utilities, minimizing power losses in the processof transmission and distribution is critical for efficiency and economicreasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load. We nowaddress the problem of delivering the maximum power to a load whengiven a system with known internal losses. It should be noted that thiswill result in significant internal losses greater than or equal to the power delivered to the load. The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the loadresistance *RL*. If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in Fig.the power delivered to the loadis

$P = i^2 R_L = \left(\frac{Vth}{Rth+Rl}\right)^2 R_L$ **..(1)**



For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance*RL*, the power delivered to the load varies as sketched in Fig. 4.49. Wenotice from Fig. 4.49 that the power is small for small or large values of*RL* but maximum for some value of *RL* between 0 and∞. We now wantto show that this maximum power occurs when *RL* is equal to *R*Th. Thisis known as the *maximum power theorem.*

Maximum power transfer theoremstates that maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

To prove the maximum power transfer theorem, we differentiate*p* in Eq. (4.21) with respect to *RL* and set the result equal to zero. Weobtain

$$\frac{dp}{d\,Rl} = Vth^2 \left[\frac{(Rth+Rl)^2 - 2Rl(Rth+Rl)}{(Rth+Rl)^4}\right]$$

$$= Vth^2 \left[\frac{(Rth+Rl-2Rl)}{(Rth+Rl)^3}\right]$$

This implies that0= (R_{Th}+ R_L − 2RL) = (R_{Th}− R_L) ..(2)
which yields

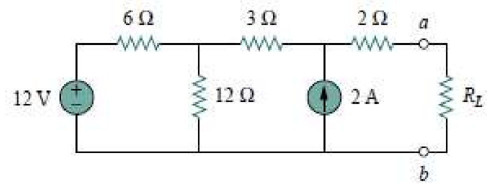
$R_L = R_{Th}$..(3)

showing that the maximum power transfer takes place when the load resistance R_L equals the Thevenin resistance R_{Th}. We can readily confirm that he maximum power is given by showing that $\frac{d_p^2}{d_{Rl}^2}$<0.The source and load are said to be *matched* when $R_L = R_{Th}$. The maximum power transferred is obtained by substituting Eq.(3) into Eq. (1), for

$$P_{max} = \frac{Vth^2}{4\ Rth}..(4)$$

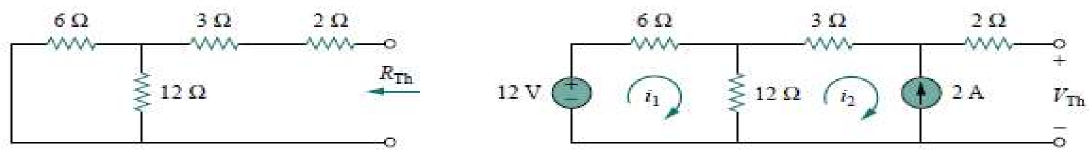
Equation (4) applies only when $R_L = R_{Th}$. When $R_L \neq R_{Th}$, wecompute the power delivered to the load using Eq. (1).

PROBLEM: Find the value of *RL* for maximum power transfer in the circuit of Fig. Find the maximum power.



Solution:
We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals *a*-*b*. To get R_{Th}, we use the circuit in Fig. (a)and obtain

$$R_{Th} = 2 + 3 + 6 \,//\, 12 = 5 + \frac{6*12}{18} = 12\Omega$$



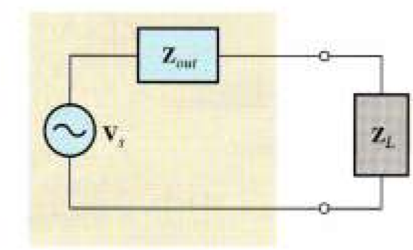
(a) (b)

To get V_{Th}, we consider the circuit in Fig. (b). Applying mesh analysis,
$-12 + 18i_1 - 12i_2 = 0,\ i_2 = -2$ A
Solving for $i1$, we get $i_1 = -2/3$. Applying KVL around the outer loopto get V_{Th} across terminals *a*-*b*, we obtain
$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0$ _$\Rightarrow V_{Th} = 22$ V
For maximum power transfer, $R_L = R_{Th} = 9\Omega$
and the maximum power is

$$P_{max} = \frac{Vth^2}{4\ Rth} = \frac{22^2}{4*9} = 13.44\text{W}$$

Maximum Power Transfer Theorem for AC Circuits:

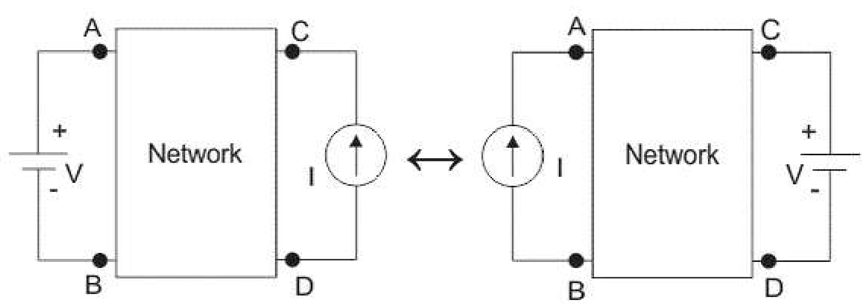
When a load is connected to a circuit, maximum power is transferred to the load when the load impedance is the complex conjugate of the circuit's output impedance. The complex conjugate of R - jX_C is R + jX_L, where the resistances and the reactances are equal in magnitude. The output impedance is effectively Thevenin's equivalent impedance viewed from the output terminals. When Z_L is the complex conjugate of Z_{out}, maximum power is transfered from the circuit to the load. An equivalent circuit with its output impedance and load is shown below



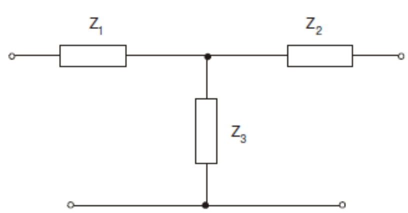
Q7)Prove Reciprocity Theorem.

> The ratio of excitation to response remains invariant in a reciprocal network with respect to an interchange between the points of application of excitation and measurement of the response.

Under Basic Electrical Engineering In many electrical networks it is found that if the positions of voltage source and ammeter are interchanged, the reading of ammeter remains the same. Suppose a voltage source is connected to a passive network and an ammeter is connected to other part of the network to indicate the response. Now any one interchanges the positions of ammeter and voltage source that means he or she connects the voltage source at the part of the network where the ammeter was connected and connects ammeter to that part of the network where the voltage source was connected. The response of the ammeter means current through the ammeter would be the same in both the cases. This is where the property of reciprocity comes in the circuit. The particular circuit that has this reciprocal property, is called reciprocal circuit. This type of circuit perfectly obeys **reciprocity theorem**.



Verify reciprocity theorem for the *T*-circuit.

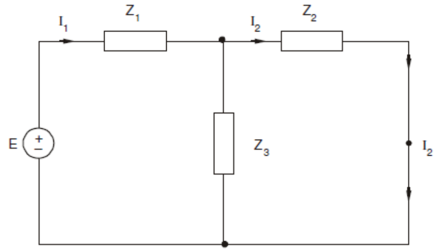


Let us find response *I*2 of voltage source *E* in the position shown in Figure

$Z_{eff} = (Z_2 // Z_3) + Z_1 = \left(\frac{Z2*Z3}{Z2+Z3}\right) + Z_1 = \frac{Z2Z3 + Z2Z1 + Z3Z1}{Z2+Z3}$

$I_1 = \frac{E}{Zeff}$

$I_2 = I_1 \cdot \frac{Z3}{Z2+Z3} = \frac{E.Z3}{Z1Z2+Z2Z3+Z3Z1}$



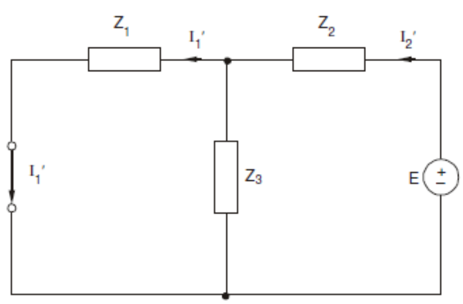
Case 2

When positions of source, i.e. *E*, and response, i.e. *I* are interchanged asshown in next Figure

$Z_{eff} = (Z_1 // Z_3) + Z_2 = \frac{Z1Z2 + Z2Z3 + Z3Z1}{Z1+Z3}$

$I_2' = \frac{E}{Zeff}$

$I_1' = I_2' \cdot \frac{Z2}{Z1+Z3} = \frac{E.Z3}{Z1Z2+Z2Z3+Z3Z1}$

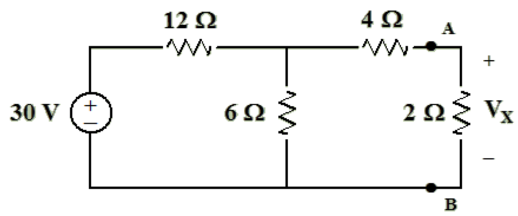


From the above, it can be seen that $I_2 = I_2'$Hence theorem is verified. The ratio$\frac{E}{I2}$ or $\frac{E}{I_1'}$ is called the transfer impedance and is given by

$$Z_T = \frac{Z1Z2+Z2Z3+Z3Z1}{Z3} = Z_1 + Z_2 + \left[\frac{Z1Z2}{Z3}\right]$$

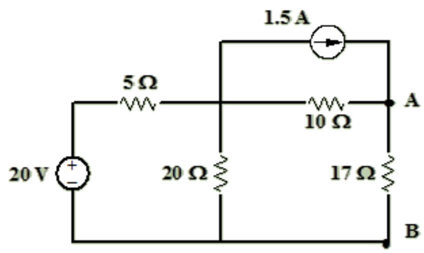
Problems:

1) Find Vx by first finding Vth and Rth.



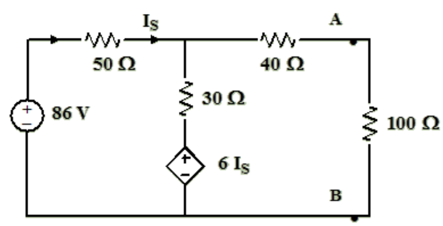
(Ans: Vx = 2V)

2) For the circuit given below find V_{AB} by first finding Thevenins equivalent circuit.



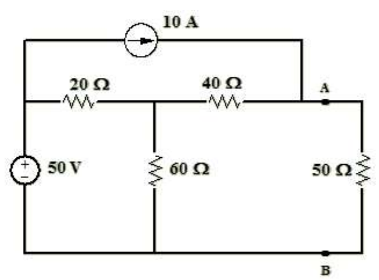
(Ans: Vab=17V)

3) Find the voltage across 100Ω resistor by finding Thevenins equivalent circuit.



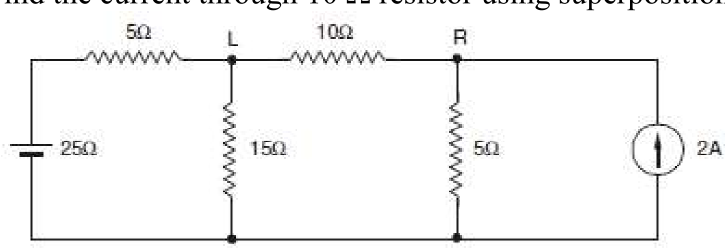
(Ans: V_{100}= 22.9 V)

4) Find the current in the 50Ω resistor by drawing Nortons equivalent circuit.



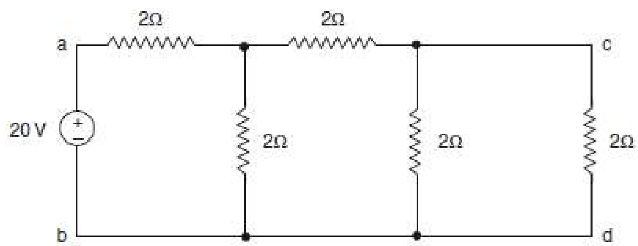
(Ans:I_N= 10.7 A)

5) Find the current through 10 Ω resistor using superposition theorem

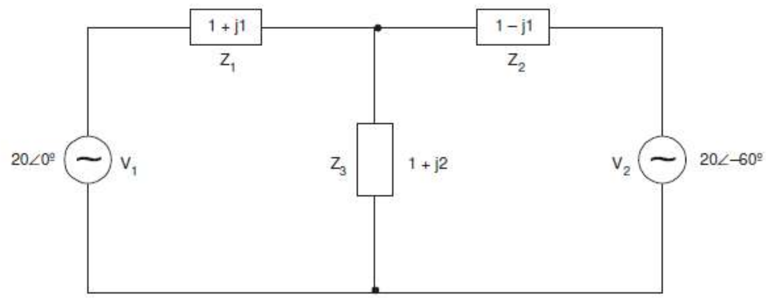


(Ans: I = 7/15 A)

6) Verify Reciprocity theorem for the network shown below, with source and response positions being ab and cd.

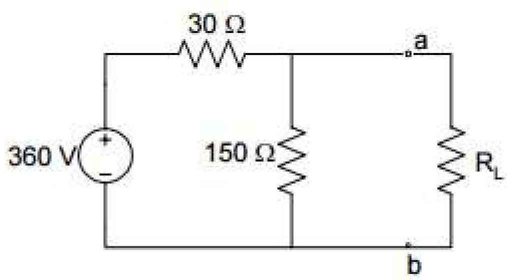


7) Calculate current in impedance Z_3using Superposition Theorem.



(Ans: I_3= 9.65∟-75^{0}A)

8) Find current in *Z*3 using Thevenin's theorem in above network.
9) Find current in *Z*3 using Norton's theorem in above network.
10) Find the value of RL that results in maximum power being transferred to RL.



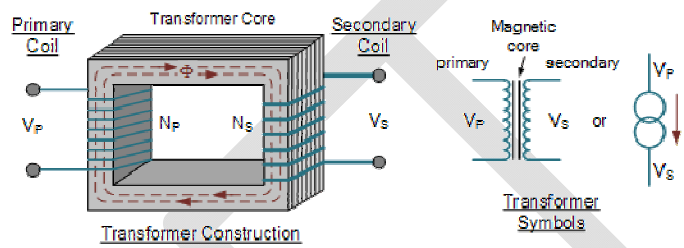
Unit V : Fundamentals of Electrical Machines :

Construction, Principle, Operation and Applications of (i) Single Phase Transformer (ii) Single Phase Induction motor (iii) DC Motor.

Single Phase Transformer :

To overcome losses, the electricity from a generator is passed through a step up transformer, which increases the voltage. Throughout the distribution system, the voltages are changed using step-down transformers to voltages suitable to the applications at industry and homes.

Construction:



Where:

- V_P - is the Primary Voltage
- V_S - is the Secondary Voltage
- N_P - is the Number of Primary Windings
- N_S - is the Number of Secondary Windings
- Φ (phi) - is the Flux Linkage

Elements of Transformer:

Two coils having mutual inductance:

Laminated steel core.the two coils are insulated from each other and the steel core. Some suitable container(Tank) for the assembled core and windings. Medium(Transformer oil) for insulating the core and its windings from its container.

Suitable bushings(porcelain, oil filled or capacitor type), for insulating and bring out the terminals of windings from the tank.

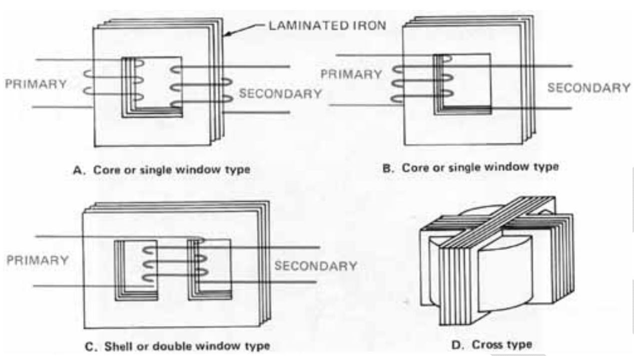
Transformer core is constructed of transformer sheet steel laminations assembled to provide a continuous magnetic path with a minimum airgap included. The steel used is of high silicon content, sometimes heat treated to produce a high permeability and low hysteresis loss at the usual operating flux densities. The eddy current loss is minimized by laminating the core, the laminations being insulated from each other by a light coat of core-plate varnish or by an oxide layer on the surface. The thickness of laminations vary from 0.35mm for a frequency 50hz to 0.5mm for frequency 25Hz. The joints if the core laminations in the alternate layers are staggered in order to avoid the presence of narrow gaps right through the cross section of the core. Such staggered joints are said to be imbricated.

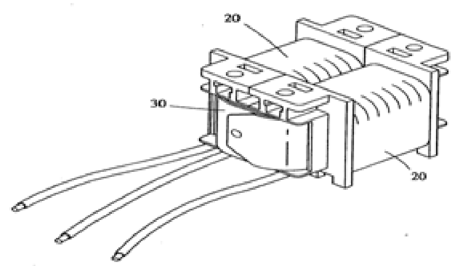
The transformers can be classified into two types depending on the manner in which the coils are wound on the core.

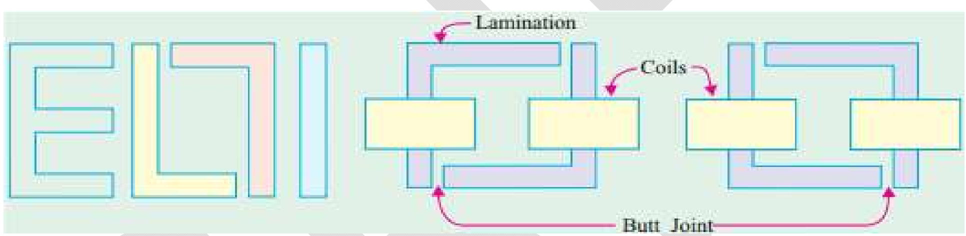
i) **Core type**: Winding surrounded by a considerable part of the core.

ii) **Shell type**: Core surrounded by a considerable portion of windings.

iii) **Spirakore type**: In both core and shell type transformers, the individual laminations are cut in the form of long strips of **Es,Ls and Is**







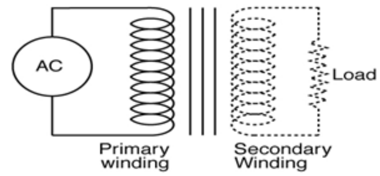
Another way of classifying transformer is depending on the method of cooling employed.

a) Oil filled self cooled:
The assembled winding and cores of such transformers are mounted in the welded, oil tight steel tank provided with steel cover. After putting the core at its proper place, the tank is filled with purified high quality insulating oil. The oil serves to convey the heat from the core and the winding to the case from where it is radiated out to the surroundings. For small size, the tanks are usually smooth surfaced, but for large sizes, the cases are frequently corrugated or fluted to get greater heat radiation area without increasing the cubical capacity of the tank still large size are provided with radiators or pipes.

b) Oil filled water cooled: the winding and the core are immersed in the oil, but there is mounted near the surface of the oil a cooling coil through which cold water is kept circulating. The heat is carried away by this water. The largest transformers such as those used with high voltage transmission lines are constructed in the manner.

c) Air-blast type: for voltage below 25KV, transformers can be built for cooling by means of an air blast. The transformer is not immersed in oil, but is housed in a thin sheet-metal box open at both ends through which air is blown from the bottom to the top by means of a fan or blower.

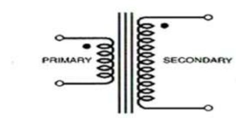
Conventional Representation of Transformer in Electrical Circuits:



Principle of operation:

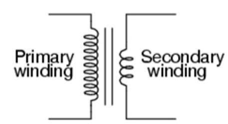
Transformer is a static piece of electric device.

- If transformer electric power I one circuit to electrical power of same frequency to another circuit.
- The transformation is done by the process of mutual induction, where the two electric circuits are in mutual inductive influence of each other without any physical contact.
- The first coil in which electric energy is fed from AC supply is called primary winding and the other from which energy is drawn out is called secondary coil.
  It secondary wind is a closed circuit a current flows in it.
  The working of the transformer can be explained as two methods:
  1) **Step up transformer**:



If the primary coil has 3 loops and secondary coil has 30, the voltage

is stepped up 10 times.



2) **Step down transformer**:

If primary coil has 30 loops and secondary coil has 3 loops, the voltage is stepped down 10 times.

Emf Equation of A Transformer:

N_1= no. of primary turns,
N_2=No. of secondary turns.
$\emptyset_m$= B_m*A : maximum flux in core in webers.
f=frequency of AC i/p In Hz.
Average rate of change of flux=$\frac{\emptyset_m}{\frac{1}{4f}} = 4f\emptyset_m \;\; \frac{wb}{s}$ or volts.

Rate of change of flux per turn (Induced emf in volts)=$\frac{Avg\ Emf}{turn} = 4f\emptyset_m\ volts.$

$$Form\ Factor = \frac{RMS}{Avg} = 1.11$$

$$RMS\ value\ of \frac{emf}{turn} = 1.11 * 4f\emptyset_m = 4.44fN_1\emptyset_m\ volts$$

$$RMS\ value\ of\ induced\ EMF\ in\ whole\ primary\ winding = 4.44f\emptyset_m N_1$$

$$E_1 = 4.44f\emptyset_m N_{1=}4.44fN_1B_mA$$
$$RMS\ value\ of\ EMF\ induced\ in\ secondary\ E_2 = 4.44fN_2\emptyset_m\ volts$$

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44f\emptyset_m = constant$$
$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = k(\ votlage\ Transformation\ ratio)$$
$$N_2 > N_1 => \ \ k > 1 => Step\ Up$$
$$N_2 < N_1 => \ \ k < 1 => Step\ Down$$

Transformer - Losses And Efficiency

Losses In Transformer

In any electrical machine, 'loss' can be defined as the difference between input power and output power. An electrical transformer is an static device, hence mechanical losses (like windage or friction losses) are absent in it. A transformer only consists of electrical losses (iron losses and copper losses). Transformer losses are similar to losses in a DC machine, except that transformers do not have mechanical losses.
Losses in transformer are explained below -

(I) Core Losses Or Iron Losses

Eddy current loss and hysteresis loss depend upon the magnetic properties of the material used for the construction of core. Hence these losses are also known as **core losses** or **iron losses**.

- **Hysteresis loss in transformer**: Hysteresis loss is due to reversal of magnetization in the transformer core. This loss depends upon the volume and grade of the iron, frequency of magnetic reversals and value of flux density. It can be given by, Steinmetz formula:
  $W_h = \eta B_{max}^{1.6} fV$ (watts)
  where, η = Steinmetz hysteresis constant
  V = volume of the core in m^3
- **Eddy current loss in transformer**: In transformer, AC current is supplied to the primary winding which sets up alternating magnetizing flux. When this flux links with secondary winding, it produces induced emf in it. But some part of this flux also gets linked with other conducting parts like steel core or iron body or the transformer, which will result in induced emf in those parts, causing small circulating current in them. This current is called as eddy current. Due to these eddy currents, some energy will be dissipated in the form of heat.

(Ii) Copper Loss In Transformer

Copper loss is due to ohmic resistance of the transformer windings. Copper loss for the primary winding is $I_1^2R_1$ and for secondary winding is $I_2^2R_2$. Where, I_1 and I_2 are current in primary and secondary winding respectively, R_1 and R_2 are the resistances of primary and secondary winding respectively. It is clear that Cu loss is proportional to square of the current, and current depends on the load. Hence copper loss in transformer varies with the load.

Efficiency Of Transformer

Just like any other electrical machine, **efficiency of a transformer** can be defined as the output power divided by the input power. That is efficiency = output / input .
Transformers are the most highly efficient electrical devices. Most of the transformers have full load efficiency between 95% to 98.5% . As a transformer being highly efficient, output and input are having nearly same value, and hence it is impractical to measure the efficiency of transformer by using output / input. A better method to find efficiency of a transformer is using, efficiency = (input - losses) / input = 1 - (losses / input).

Condition For Maximum Efficiency

Let,
Copper loss = I12R1
Iron loss = Wi

$$\text{efficiency} = 1 - \frac{\text{losses}}{\text{input}} = 1 - \frac{I_1^2R_1 + W_i}{V_1I_1\cos\Phi_1}$$

$$\eta = 1 - \frac{I_1R_1}{V_1\cos\Phi_1} - \frac{W_i}{V_1I_1\cos\Phi_1}$$

differentiating above equation with respect to I_1

$$\frac{d\eta}{dI_1} = 0 - \frac{R_1}{V_1\cos\Phi_1} + \frac{W_i}{V_1I_1^2\cos\Phi_1}$$

η will be maximum at $\frac{d\eta}{dI_1} = 0$

Hence efficiency η will be maximum at

$$\frac{R_1}{V_1\cos\Phi_1} = \frac{W_i}{V_1I_1^2\cos\Phi_1}$$

$$\frac{I_1^2R_1}{V_1I_1^2\cos\Phi_1} = \frac{W_i}{V_1I_1^2\cos\Phi_1}$$

$$I_1^2R_1 = W_i$$

Hence, **efficiency of a transformer** will be maximum when copper loss and iron losses are equal.
That is Copper loss = Iron loss.

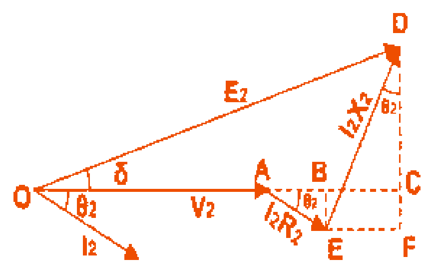
VOLTAGE REGULATION OF A SINGLE PHASE TRANSFORMER:
The voltage regulation is the percentage of voltage difference between no load and full load voltages of a transformer with respect to its full load voltage.

Explanation:
Say an electrical power transformer is open circuited, means load is not connected with secondary terminals. In this situation, the secondary terminal voltage of the transformer will be its secondary induced emf E_2. Whenever full load is connected to the secondary terminals of the transformer, rated current I_2 flows through the secondary circuit and voltage drop comes into picture. At this situation, primary winding will also draw equivalent full load current from source. The voltage drop in the secondary is I_2Z_2 where Z_2 is the secondary impedance of transformer. Now if at this loading condition, any one measures the voltage between secondary terminals, he or she will get voltage V_2 across load terminals which is obviously less than no load secondary voltage E_2 and this is because of I_2Z_2 voltage drop in the transformer.

Expression of Voltage Regulation of Transformer
Expression of **Voltage Regulation of Transformer**, represented in percentage, is

$$Voltage\ regulation(\%) = \frac{E_2 - V_2}{V_2} \times 100\%$$

Voltage Regulation of Transformer for Lagging Power Factor
Now we will derive the expression of voltage regulation in detail. Say lagging power factor of the load is $\cos\theta_2$, that means angle between secondary current and voltage is θ_2



Voltage Regulation at Lagging Power Factor

Here, from the above diagram,

$$OC = OA + AB + BC$$

$$Here,\ OA = V_2$$

$$Here,\ AB = AE\cos\theta_2 = I_2R_2\cos\theta_2$$

$$and,\ BC = DE\sin\theta_2 = I_2X_2\sin\theta_2$$

Angle between OC & OD may be very small, so it can be neglected and OD is considered nearly equal to OC i.e.

$$E_2 = OC = OA + AB + BC$$

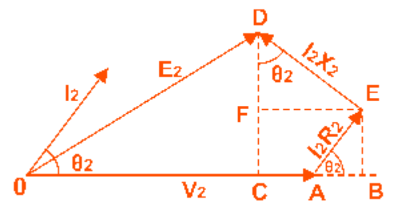
$$E_2 = OC = V_2 + I_2R_2\cos\theta_2 + I_2X_2\sin\theta_2$$

Voltage regulation of transformer at lagging power factor,

$$Voltage\ regulation\ (\%) = \frac{E_2 - V_2}{V_2} \times 100(\%)$$

$$= \frac{I_2R_2\cos\theta_2 + I_2X_2\sin\theta_2}{V_2} \times 100(\%)$$

Voltage Regulation of Transformer for Leading Power Factor



Voltage Regulation at Leading Power Factor

Let's derive the expression of voltage regulation with leading current, say leading power factor of the load is $\cos\theta_2$, that means angle between secondary current and voltage is θ_2.

Here, from the above diagram,

$$OC = OA + AB - BC$$

$$Here,\ OA = V_2$$

$$Here,\ AB = AE\cos\theta_2 = I_2R_2\cos\theta_2$$

$$and,\ BC = DE\sin\theta_2 = I_2X_2\sin\theta_2$$

Angle between OC & OD may be very small, so it can be neglected and OD is considered nearly equal to OC i.e.

$$E_2 = OC = OA + AB - BC$$

$$E_2 = OC = V_2 + I_2R_2\cos\theta_2 - I_2X_2\sin\theta_2$$

Voltage regulation of transformer at leading power factor,

$$Voltage\ regulation\ (\%) = \frac{E_2 - V_2}{V_2} \times 100(\%)$$
$$= \frac{I_2R_2\cos\theta_2 - I_2X_2\sin\theta_2}{V_2} \times 100(\%)$$

Why is Transformer Rated in KVA, not in KW ?

Copper losses (I^2R) depends on Current which passing through transformer winding while Iron Losses or Core Losses or Insulation Losses depends on Voltage. So the Cu Losses depend on the rated current of the load so the load type will determine the power factor, that is why the rating of Transformer in kVA, and not in kW.

Solved Example:

A 40KVA single phase transformer has 400 turns on primary and 100 turns on secondary the primary is connected to 200V, 50Hz supply, Determine

1) The secondary voltage on open circuit.
2) The current flowing through the two windings on full load.
3) The maximum value of flux.

Solution: Tr, rating=40KVA, N_1=400,N_2=100

Primary induced voltage V_1=200V,

1) Secondary voltage on open circuit: V_2

$$\frac{V_2}{V_1} = \frac{N_1}{N_2}$$

$$V_2 = \frac{N_1}{N_2} * V_1 = 2000 * \frac{100}{400} = 500volts$$

2)Primary Current (I_1): at full load,

$$I_1 = \frac{KVA*100}{V_1} = 40 * \frac{1000}{200}=20A$$

$$secondary\ current\ at\ full\ load\ I_2 = \frac{KVA*100}{V_2} = 40 * \frac{1000}{500}=80A$$

3)Maximum value of flux

$$EMF\ equation \quad E = 4.44 f N_1 \emptyset_m$$

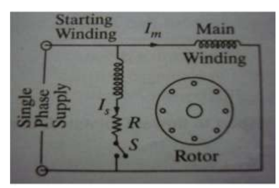
$$\emptyset_m = \frac{V_1}{4.44 * f * N_1} = \frac{200}{4.44 * 50 * 200} = 0.022\ wb$$

Single Phase Induction Motor:

Single phase induction motor perform a great varity of useful services in the home , the office , factory, business establishments on the farm and many other places where electricity is available.

There are different type of Single Phase Induction Motor:

1) Split Phsae Motor:
    a. Resistance start motors.
    b. Capacitor start motors.
    c. Permanent –split(single value) capacitor motor.
    d. Two value capacitor motor.
2) Shaded pole induction motor.
3) Reluctance start induction motor.
4) Repulsion start induction motor.



Construction and Working Principle:

An induction motor is simply an electric transformer whose primary winding is stationary and secondary winding is free to rotate.

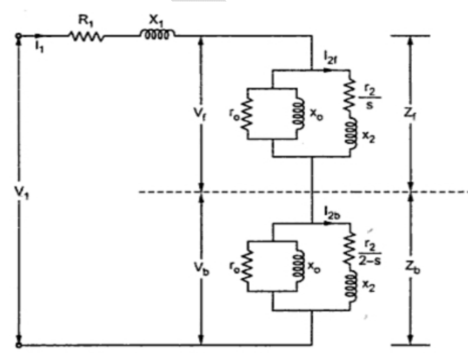
This motor has

1) Its stator is provided with a single phase winding and
2) A centrifugal switch is used in same types of motors in order to cutout a winding , used only for starting purpose. It has distributed stator winding and a squirrel cage rotor. When fed from a single phase supply , its stator winding , produces a flux(field) which is only alternating i.e., one

which alternates along one space axis only. Now an alternating or pulsating flux acting on a stationary squirrel cage rotor cannot produce rotation(only a revolving flux can). That is why a single phase motor is not self starting.
However, if the motor of such machine is given an initial start by a small external force, in either direction , then immediately a torque arises and the motor accelerates to its final speed. This behaviors is explained in two ways:

1) Double-field revolving theory.
2) Cross-field theory:
   This theory makes use of the idea that an alternating uni - axis quantity can be represented by two oppositely rotating vectors of half magnitude. Accordingly, an alternating sinusoidal flux can be represented by two revolving fluxes, each equal to half the value of the alternating flux and each rotating synchronously ($Ns = 120 / f$) in opposite direction. However if the rotor is started somehow say, in the clockwise direction, the clockwise torque starts increasing and the same time, the anticlockwise torque starts decreasing. Hence there is a certain amount of net torque in the clockwise direction which accelerated the motor to full speed. Depending on different starting methods, the different type of induction motor are mentioned above.

Equivalent Circuit of Single Phase Induction Motor:



The single phase induction motor has been imagined to be made up of

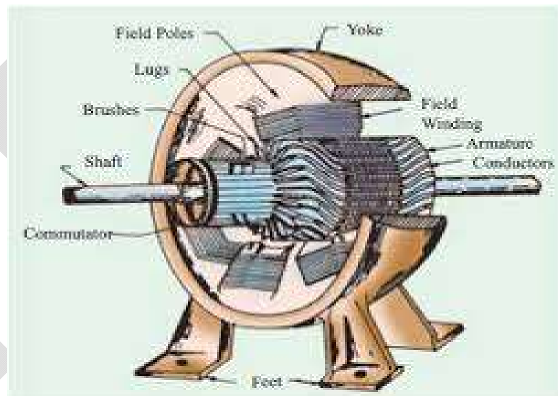
1) One stator winding and
2) Two imaginary rotor. The stator impedance is $Z_1=R_1+j\,X_1$. the impedance of each rotor is (r_2+jx_2) where r_2 and x_2 represent half the actual rotor values in stator terms.(x_2 stands for half the standstill reactance of the rotor, as referred to stator) Iron loss has been neglected, the exciting branch is shown consisting of exciting reactance only. Each rotor has been assigned half the magnetizing reactance(x_m : half the actual reactance).
   Impedance of the forward running motor $Z_f = \frac{jx_m(\frac{r_2}{s}+jx_2)}{\frac{r_2}{s}+j(x_m+x_2)}$ and it runs with a slip 's'
   Impedance of backward running motor$Z_b = \frac{jx_m(\frac{r_2}{2-s}+jx_2)}{\frac{r_2}{2-s}+j(x_m+x_2)}$
   And it runs with a slip of (2-s).
   Under standstill condition, V_f is 90 to 95% of applied voltage.
   Forward torque is synchronous watts $T_f = \frac{I_3^2 r_2}{s}$
   Backward Torque is $T_b = \frac{I_5^2 r_2}{2-s}$
   Total Torque $T = T_f - T_b$

DC Motor:

An electrical motor is a machine which converts electrical energy in to mechanical energy. Its action is based on the principle that when a current carrying conductor is placed in a magnetic field, it experiences a mechanical force whose direction is given by FLEMINGS LEFT hand rule and whose magnitude is given by F=BIL Newton.

The same DC machine can be operated both like a generator as well as motor. So the construction of the machine will be same for both DC motor and DC generator.

Construction of Dc Machine:



The different parts of the machine and their working are explained as follows.

Yoke: The outer frame of the machine is called yoke and it serves two purposes.

1) Mechanical support for the poles and a protecting cover for the whole machine.
2) It carries magnetic flux produced by pole.
   This is made of cast iron/cast steel or rolled steel.

Pole Cores and Pole Shoes: These are built of the thin laminations of annealed steel which are riveted together under hydraulic pressure. The thickness of laminations vary from 1mm to 0.25mm. The laminated poles are secured to the yoke by means of screws bolted through the yoke into the pole body.

The pole shoe serve two purposes.

1) The spread out the flux in the air gap and also being of large cross section, reduce the reluctance of the magnetic path.
2) They support exciting (or) field coils.

Pole Coils: The field or pole coils are former wound and placed on the pole core. The current passing through these coils electromagnetics the poles which produces the necessary flux that is cut by the revolving armature.

Armature Core: The houses the armature coils and causes them to rotate and hence cut the magnetic flux.

It is a cylindrical or drum shaped and built up of circular sheet steel discs or laminations approximately 0.5mm thickness.

Usually the laminations are performed for air ducts which permits axials flow of air through the armature for cooling purpose. A complete circular lamination is made up of four or six or even eight segmental laminations. The two keyways are notched in each segment and are dovetailed or wedge shaped to make the laminations self locking in position. The purpose of laminations is to reduce the eddy current losses.

Armature winding: These are usually former wound. These are first wound in the form of flat rectangular coils and are them pulled into their proper shape in a coil puller. Various conductors of the coils are insulated from each other. The conductors are placed in the armature slots which are lined with tough insulating material, the slot insulation is folded over above the armature conductors placed in the slot and is secured in place by special hard wooden or fiber wedge.

Commutator: The function of commutator is to facilitate collection of current(or supply) from armature conductors. It rectifies AC current in armature conductors into DC current in external load circuit if generator or rectifies DC current from supply to ac current in armature if motor.

Commutator segments are insulated from each other by mica. Number of segments are equal to number of armature coils.

Brushes and Bearings: Their function is to collect current from commutator. These are made of carbon or graphite.

Working Principle:

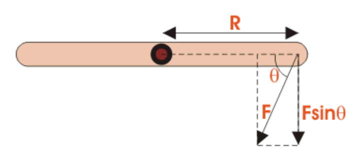
When the field magnets are excited and its armature conductors are supplied with current from the supply mains, they experience a force tending to rotate the armature. Armature conductors under N-pole are assumed to carry current downwards and those under S-pole carry current upwards. By applying FLEMINGS LEFT hand rule, the direction of the force on each conductor can be found. It will be seen that each conductor experiences a force F which tends to rotate the armature in anti-clockwise direction. These forces collectively produce a driving torque which sets the armature rotating.

TORQUE EQUATION OF DC MOTOR

Under Electrical Motor The term torque as best explained by Dr. Huge d Young is the quantitative measure of the tendency of a force to cause a rotational motion, or to bring about a change in rotational motion. It is in fact the moment of a force that produces or changes a rotational motion.

The equation of torque is given by, $\tau = FR\sin\theta$..................(1)

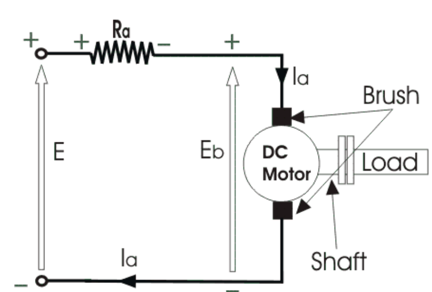
Where F is force in linear direction.



R is radius of the object being rotated,

and θ is the angle, the force F is making with R vector

The dc motor as we all know is a rotational machine, and **torque of dc motor** is a very important parameter in this concern, and it's of utmost importance to understand the **torque equation of dc motor** for establishing its running characteristics.



To establish the torque equation, let us first consider the basic circuit diagram of a dc motor, and its voltage equation.

Referring to the diagram beside, we can see, that if E is the supply voltage, E_b is the back emf produced and I_a, R_a are the armature current and armature resistance respectively then the voltage equation is given by,

$$E = E_b + I_a R_a \;\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

But keeping in mind that our purpose is to derive the **torque equation of dc motor** we multiply both sides of equation (2) by I_a.

$$Therefore,\; EI_c = E_b I_a + I_a^2 R_a \;\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Now $I_a^2.R_a$ is the power loss due to heating of the armature coil, and the true effective mechanical power that is required to produce the desired torque of dc machine is given by,

$$P_m = E_b I_a \;\ldots\ldots\ldots\ldots\ldots (4)$$

The mechanical power P_m is related to the electromagnetic torque T_g as,

$$P_m = T_g \omega \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Where ω is speed in rad/sec.

Now equating equation (4) & (5) we get, $E_b I_a = T_g \omega$

Now for simplifying the torque equation of dc motor we substitute.

$$E_b = \frac{P\varphi Z N}{60A} \;\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Where, P is no of poles,

ϕ is flux per pole,

Z is no. of conductors,

A is no. of parallel paths,

and N is the speed of the D.C. motor.

$$Hence\ \omega = \frac{2\pi N}{60} - - - - - - - - - - - - - (7)$$

Substituting equation (6) and (7) in equation (4), we get: $T_g = \frac{PZ\varphi I_a}{2\pi A}$

The torque we so obtain, is known as the electromagnetic torque of dc motor, and subtracting the mechanical and rotational losses from it we get the mechanical torque.

Therefore, $T_m = T_g$ mechanical losses.

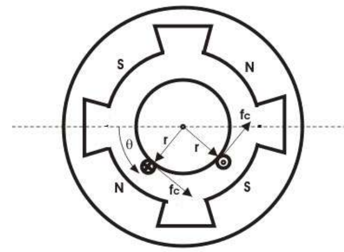
This is the torque equation of dc motor. It can be further simplified as:

$$T_g = K_a \emptyset I_A$$

$$where\ k_a = \frac{PZ}{2\pi A}$$

Which is constant for a particular machine and therefore the torque of dc motor varies with only flux ϕ and armature current I_a.

The Torque equation of a dc motor can also be explained considering the figure below.



Here we can see Area per pole $A_r = \frac{2\pi rL}{P}$

$$B = \frac{\varphi}{A_r}$$

$$B = \frac{P\varphi}{2\pi rL}$$

Current / conductor $I_c = I_a / A$

Therefore, force per conductor = $f_c = BLI_a/A$

Now torque $T_c = f_c.r = BLI_a.r/A$

$$T_c = \frac{P\varphi I_a}{2\pi A}$$

Hence the total torque developed of a dc machine is,

$$T_c = \frac{P\varphi ZI_a}{2\pi A}$$

This torque equation of dc motor can be further simplified as:

$$T_g = K_a \emptyset I_a$$

where $k_a = \frac{PZ}{2\pi A}$

Which is constant for a particular machine and therefore the torque of dc motor varies with only flux ϕ and armature current I_a.